STAT 6021: Project Two

Medical Insurance Costs

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1 Executive Summary:

Insurance companies calculate premiums by using models based on certain demographics. We wanted to ascertain what variables influenced claims and how influential these predictors were. Through Kaggle, we were able to obtain a data set that contained the claims of people based on age, body mass index (BMI), sex, region, smoking, and children.

At first, we individually compared each variable against charges and saw that our data appeared to be right skewed. This implied that a transformation of the data would be necessary in order to normalize our data. Before we did that, we wanted to discern which variables were significant to keep in our model.

The linear regression output for our initial model, which contained all the variables, showed the northwest region was linearly related some other variable, therefore suggesting it should be dropped from the model. However, no matter how we manipulated the predictors, the data seemed to have non-constant variance which would give no value to any statistical analysis we did on these models. Due to the complexity of data manipulation, we used a logarithmic binomial model.

In this approach, we split the groups into charges that were above 20,000 USD dollars and charges that were below 20,000. The equation we found to be the best fit for our data was:

$$\ln \frac{\pi_i}{1 - \pi_i} = -7.86104 + 0.04087age + 0.10134bmi + 4.75564smokeryes$$

This shows that the odds of having charges over \$20,000 gets multiplied by a factor of 116 if one is a smoker (while holding age and BMI constant), gets multiplied by a factor 1.13 for each BMI increase (while holding age and smoking constant), and gets multiplied by a factor of 1.04 for every addition year (while holding smoking and BMI constant).

2 Exploratory Data Analysis:

The data was contained in the file datasets_13720_18513_insurance.csv included with this project.

- The variables were as follows:
 - Predictors
 - * **x1**: **age**: age of primary beneficiary.
 - * x2: sex: insurance contractor gender, female, male.
 - * **x3**: **bmi**: Body mass index, providing an understanding of body, weights that are relatively high or low relative to height, objective index of body weight (kg / m 2 2) using the ratio of

height to weight, ideally 18.5 to 24.9.

- * x4: children: Number of children covered by health insurance / Number of dependents.
- * x5: smoker: Smoking
- * **x6**: **region**: the beneficiary's residential area in the US, northeast, southeast, southwest, northwest.

- Response Variable

* Y: charges: Individual medical costs billed by health insurance.

• The main objectives for this project were:

- 1. Explore relationship between response variable **charges** & the six other predictor variables (x1-x6).
- 2. Analyze the correlation and directionality of the dataset.
- 3. Create a model that is the best fit model to predict the insurance **charges** based the demographic predictor variables and evaluate the validity and usefulness of this model.

Additionally, we planned to utilize model selection tools to give us a deeper understanding of how different potential models compare. We want to recommend a best fit model and end our section by exploring the pros and cons of our models under consideration.

Exploratory data analysis started with investigating the dataset.

```
bmi children smoker
                                        region
                                                  charges
  age
         sex
  19 female 27.900
                            0
                                 yes southwest 16884.924
   18
        male 33.770
                            1
                                  no southeast
                                               1725.552
        male 33.000
3
   28
                            3
                                  no southeast
                                                 4449.462
        male 22.705
   33
                            0
                                  no northwest 21984.471
5
   32
        male 28.880
                            0
                                  no northwest
                                                 3866.855
   31 female 25.740
                            0
                                  no southeast
                                                 3756.622
```

There were six predictors and a response variable **charges**. The dataset had 1338 rows, and the data appeared to need little cleaning and did not contain missing values.

```
'data.frame': 1338 obs. of 7 variables:

$ age : int 19 18 28 33 32 31 46 37 37 60 ...
```

: Factor w/ 2 levels "female", "male": 1 2 2 2 2 1 1 1 2 1 ... \$ sex

\$ bmi : num 27.9 33.8 33 22.7 28.9 ...

\$ children: int 0 1 3 0 0 0 1 3 2 0 ...

\$ smoker : Factor w/ 2 levels "no", "yes": 2 1 1 1 1 1 1 1 1 1 ...

\$ region : Factor w/ 4 levels "northeast", "northwest",..: 4 3 3 2 2 3 3 2 1 2 ...

\$ charges : num 16885 1726 4449 21984 3867 ...

Inspecting the data types of variables, we observed that the predictor variables sex, smoker, and region were categorical variables and were automatically converted as a factor by R when loading the dataset because because of the option stringsAsFactors = TRUE used while reading the csv file

age	sex	bmi	-	chil	ldren	smoker
Min. :18.00	female:662	Min. :	15.96	Min.	:0.000	no :1064
1st Qu.:27.00	male :676	1st Qu.:	26.30	1st Qu	.:0.000	yes: 274
Median:39.00		Median :	30.40	Median	:1.000	
Mean :39.21		Mean :	30.66	Mean	:1.095	
3rd Qu.:51.00		3rd Qu.:	34.69	3rd Qu	.:2.000	
Max. :64.00		Max. :	53.13	Max.	:5.000	
region	charges					
northeast:324	Min. : 112	2				
northwest:325	1st Qu.: 474	0				
southeast:364	Median : 938	2				
southwest:325	Mean :1327	0				
	3rd Qu.:1664	0				
	Max. :6377	0				
Min. 1st Qu.	Median Me	an 3rd Qu	ı. Ma	х.		

From the summary we made the following observations:

9382

1122

4740

• The observations seemed to be evenly distributed across region.

13270

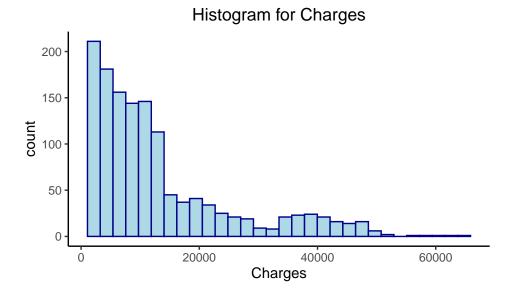
- The age varied between low of 18 and a max of 64.
- The observations were almost evenly distributed by sex.
- The dataset had almost 4:1 non-smoker to smoker ratio or only 20.5% people smoke.

16640

• The bmi varied between a min of 15.96 and max of 53.13.

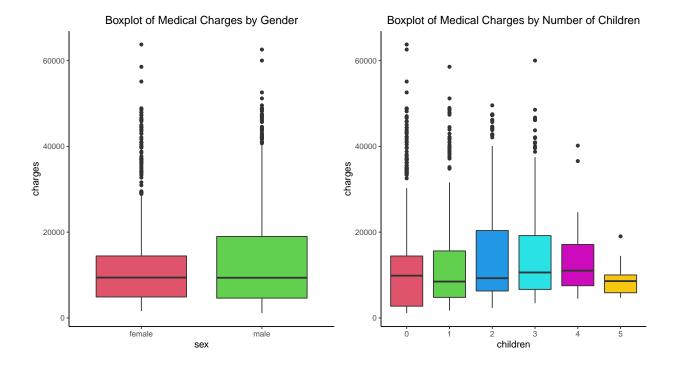
63770

The response variable mean was greater than median, this was an indication that data is right-skewed. This could be confirmed by the histogram of **charges** shown below.



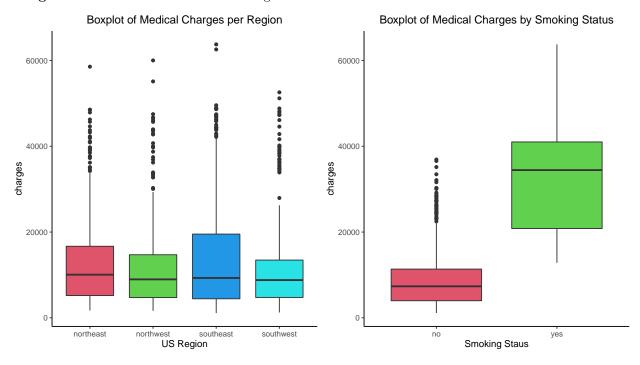
In the box plot shown below for medical **charges** by **sex** the median value of the medical **charges** for both male and female appeared to be almost the same. The third quartile for male seemed to be greater than female, so the data may be skewed towards the men.

The box plot of medical **charges** by number of **children**, we made an interesting observation that the medical **charges** for people with 5 children were lower than people with one to four children and people with no children had the lowest medical charges.



In the box plot of medical **charges** per **region** the median value of the medical **charges** across all four US regions was almost the same. The people in the southeast seemed to have higher medical expenses then the people in the other areas.

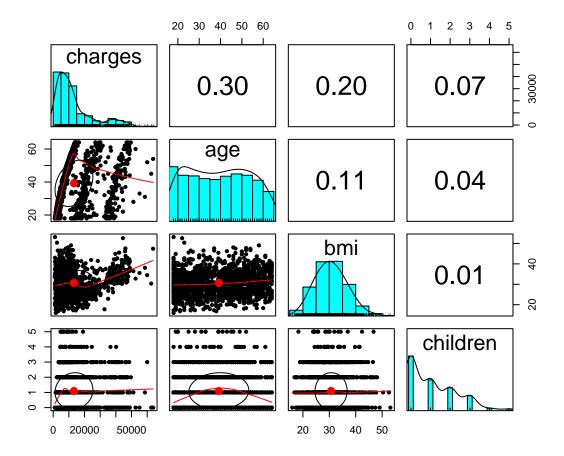
However, exploring the box plot of medical **charges** by **smoking** status, we could see that the medical **charges** for those who smoke were much higher than those who do not smoke.



The Correlation matrix:

	charges	age	bmi	children
charges	1.00000000	0.2990082	0.1983410	0.06799823
age	0.29900819	1.0000000	0.1092719	0.04246900
bmi	0.19834097	0.1092719	1.0000000	0.01275890
children	0.06799823	0.0424690	0.0127589	1.00000000

We observed that **age** and **charges** were moderately correlated, therefore as age increased, the medical charges also increased moderately. There was also a moderate correlation between **age** and **bmi**, and **children** and **charges**



Computational Exploration A model with all predictors was considered as an initial starting point. Additional candidate models were calculated by applying model automatic predictor search procedures. The R^2_{adj} value and the BIC metrics were used to identify likely models since these both approaches

penalized for adding more terms.

The following models were the two automatic search procedure recommended models:

- + The model with lowest BIC (-1817.233) was: $\$ charges = $\$ + $\$ deta_1 age + $\$ + $\$ beta_2 bmi + $\$ + $\$ beta_3
 - The model with highest adjusted R^2 was:

```
charges = \beta_0 + \beta_1 age + \beta_2 bmi + \beta_3 children + \beta_4 smokeyes + \beta_5 regions out heast + \beta_6 region 5 south west
```

We also considered the models with the highest R^2 , lowest Cp, and lowest MSE values. The best Cp and best MSE were both on the the same model as the best adjusted R^2

The model with the best R² value has all predictors as adjusted R² in addition to regionnorthwest

Summary of Exploratory Data Analysis: The following observations were made from the exploratory data analysis phase:

- 1. The smokers had more medical expenses than non-smokers
- 2. None of the correlations from the correlation matrix appeared to be strong
- 3. The quantitative predictors age, bmi, and children were moderately correlated with response variable
- 4. From computational analysis, we observed that categorical variable sex and region may be considred as significant predictors.
- 5. There may be a possibility of the data set being skewed, particularly charges

3. Initial Model Considered:

Based on the results from the model search procedures, the intial model considered was:

$$charges = \beta_0 + \beta_1 age + \beta_2 bmi + \beta_3 children + \beta_4 smokeyes + \beta_5 region + \beta_6 sex$$

```
initalmodel <- lm(charges ~ age + bmi + children + smoker + region +sex, data=data)
summary(initalmodel)</pre>
```

```
Call:
```

```
lm(formula = charges ~ age + bmi + children + smoker + region +
sex, data = data)
```

Residuals:

Min 1Q Median 3Q Max -11304.9 -2848.1 -982.1 1393.9 29992.8

Coefficients:

	Estimate Std.	Error t	value	Pr(> t)	
(Intercept)	-11938.5	987.8 -	-12.086	< 2e-16	***
age	256.9	11.9	21.587	< 2e-16	***
bmi	339.2	28.6	11.860	< 2e-16	***
children	475.5	137.8	3.451	0.000577	***
smokeryes	23848.5	413.1	57.723	< 2e-16	***
regionnorthwest	-353.0	476.3	-0.741	0.458769	
regionsoutheast	-1035.0	478.7	-2.162	0.030782	*
regionsouthwest	-960.0	477.9	-2.009	0.044765	*
sexmale	-131.3	332.9	-0.394	0.693348	

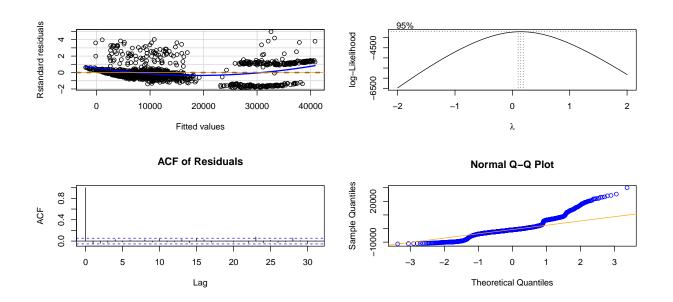
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6062 on 1329 degrees of freedom

Multiple R-squared: 0.7509, Adjusted R-squared: 0.7494

F-statistic: 500.8 on 8 and 1329 DF, $\,$ p-value: < 2.2e-16

Next the linear regression assumptions were validated:



In the plots displayed above, we observed that the variance was not constant as seen in the box-cox plot and that the residual plot did not have constant variance.

In our hypothesis, we stated that, age, people who smoke and people with high bmi (bmi>30) may be at high risk and so their medical costs may be higher. Based on that hypothesis and considering that our initial model suffered from non-linearity and non-constant variance issues. We will transformed both response variable and predictors.

The following transformations were applied: 1. Transformed **charges** (y) to fix non-constant variance 2. Transformed age by adding a non-linear term 3. Created a indicator variable for bmi (obesity indicator) 4. Specified an interaction between smokers and bmi indicator predictor

[1] TRUE

Call:

Residuals:

Coefficients:

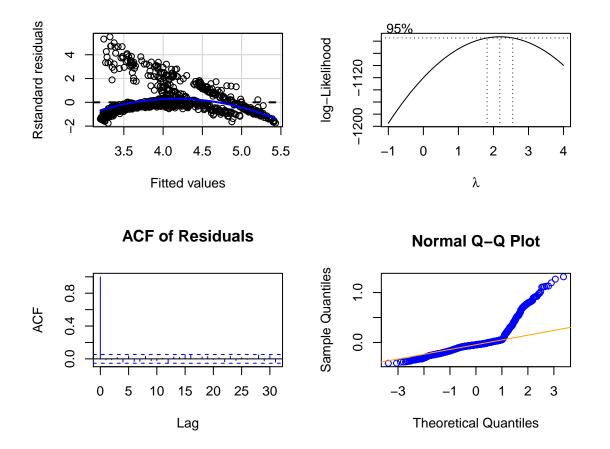
```
Estimate Std. Error t value Pr(>|t|)
(Intercept)
                 2.816e+00 7.348e-02 38.323 < 2e-16 ***
                                     7.527 9.53e-14 ***
                 2.428e-02 3.226e-03
age
                -6.299e-05 4.024e-05 -1.565 0.117753
age2
                5.109e-02 5.710e-03 8.948 < 2e-16 ***
children
bmi
                 4.007e-03 1.848e-03
                                     2.169 0.030288 *
                -4.518e-02 1.318e-02 -3.429 0.000625 ***
sexmale
                -2.074e-02 2.280e-02 -0.910 0.363243
bmi301
smokeryes
                7.202e-01 2.372e-02 30.360 < 2e-16 ***
regionnorthwest -3.253e-02 1.883e-02 -1.727 0.084396 .
regionsoutheast -8.001e-02 1.896e-02 -4.220 2.61e-05 ***
regionsouthwest -7.718e-02 1.890e-02 -4.083 4.71e-05 ***
bmi301:smokeryes 4.531e-01 3.261e-02 13.896 < 2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.2397 on 1326 degrees of freedom Multiple R-squared: 0.8063, Adjusted R-squared: 0.8047 F-statistic: 501.9 on 11 and 1326 DF, p-value: < 2.2e-16

Multiple R^2 and Adjusted R^2 measured how well our model explained the response variable. The transformed model had higher Multiple $R^2=0.8063$ and Adjusted $R^2=0.8047$ compared to initial model Multiple $R^2=0.7509$ and Adjusted $R^2=0.7494$

We also observed from the model summary, age2 the second order variable is insignificant based on t value and high p-value greater than 0.05. The interaction term bmi301:somkeryes is significant.

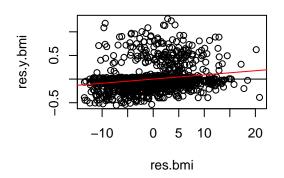
The next step was to verify the linear regression model assumptions:

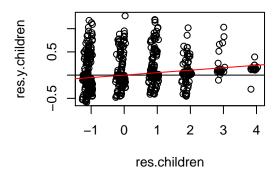


While box cox plot now showed that non-constant variance issue was addressed, the residual plot still appeared to have non-constant variance. It was not clear that the transform solved the non-constant and non-linearity issue, we further explored which predictors could be removed by creating partial regression plots.

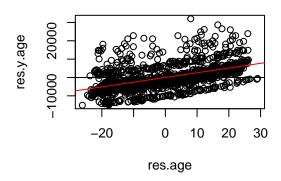
parital regression plot of bmi

parital regression plot of children





parital regression plot of age



From the partial regression plot, we observed a leaner pattern for all three quantiative variables, this means the linear terms for the predictors **bmi**, **age** and **children** seemed appropriate

'data.frame': 1338 obs. of 9 variables:

\$ age : int 19 18 28 33 32 31 46 37 37 60 ...

sex: Factor w/ 2 levels "female", "male": 1 2 2 2 2 1 1 1 2 1 ...

\$ bmi : num 27.9 33.8 33 22.7 28.9 ...

\$ children: int 0 1 3 0 0 0 1 3 2 0 ...

\$ smoker : Factor w/ 2 levels "no","yes": 2 1 1 1 1 1 1 1 1 1 ...

 $\$ region $\ :$ Factor w/ 4 levels "northeast", "northwest", ...: 4 3 3 2 2 3 3 2 1 2

\$ charges : num 16885 1726 4449 21984 3867 ...

\$ age2 : num 361 324 784 1089 1024 ...

\$ bmi30 : Factor w/ 2 levels "0","1": 1 2 2 1 1 1 2 1 1 1 ...

Call:

```
lm(formula = charges^0.15 ~ log(age) + children + log(bmi) +
    smoker + region, data = data)
```

Residuals:

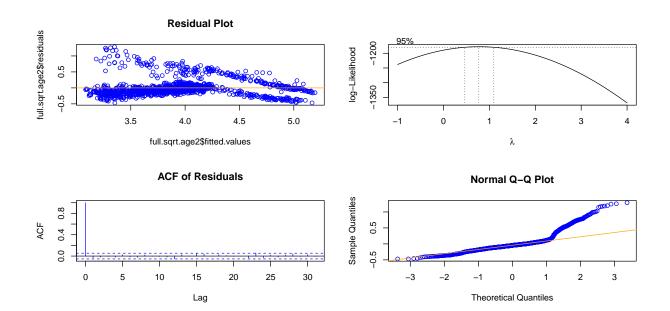
Min 1Q Median 3Q Max -0.47069 -0.13211 -0.04842 0.04722 1.28840

Coefficients:

Estimate Std. Error t value Pr(>|t|) (Intercept) 0.318640 0.133283 2.391 0.016955 * log(age) 0.685523 0.018340 37.379 < 2e-16 *** children 0.041804 0.005906 7.078 2.36e-12 *** log(bmi) 0.286829 0.036637 7.829 9.98e-15 *** 0.017604 54.249 < 2e-16 *** smokeryes 0.955017 regionnorthwest -0.035919 0.020353 -1.765 0.077820 . regionsouthwest -0.073450 0.020434 -3.595 0.000337 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.259 on 1330 degrees of freedom
Multiple R-squared: 0.7731, Adjusted R-squared: 0.7719
F-statistic: 647.3 on 7 and 1330 DF, p-value: < 2.2e-16



[1] 3.529468

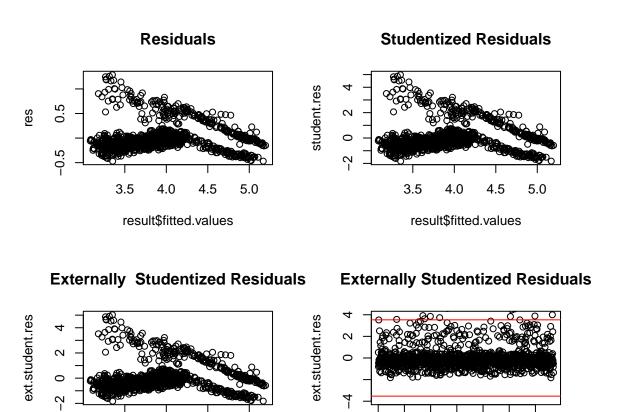
3.5

4.0

result\$fitted.values

4.5

5.0



200

0

1000

600

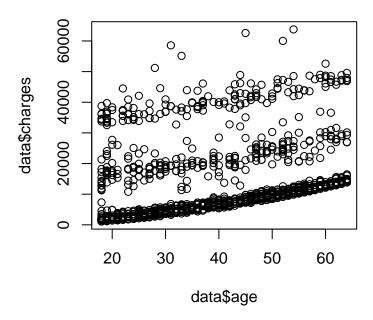
Index

469	431	398	355	341	220	141	103
3.728462	4.865511	3.850610	3.631488	3.915308	4.907768	3.570630	4.619736
1329	1196	1040	1028	1020	1009	527	517
3.997432	3.880408	4.535482	4.672036	4.450827	3.835164	4.585884	5.031708

Even after applying transformations, the model fit is still did not seem to satisfy linear regression assumptions. We still observed the presence of non-linearity and non-constant variance potentially due to outliers in the data. This model may be adequate to explore the relationship between the predictors and the response variable. However, the predicted values may be unrealistic.

We observed that the scatter plot of charges against age had three distinct relationships, where the medical charges increased with age at a very slight increasing rate in three segments. Since this relationship was odd, we wished to explore if age was the reason for skew in the data.

plot(data\$age, data\$charges)



Age was removed from the model and the and the response variable was transformed. When validating the assumptions through a box-cox plot, non-constant variation was still present thus not making it a good model for consideration

```
without.age <- lm(charges^.35 ~ + children + smoker + sex + region + bmi, data=data)
summary(without.age)</pre>
```

Call:

```
lm(formula = charges^0.35 ~ +children + smoker + sex + region +
bmi, data = data)
```

Residuals:

Min 1Q Median 3Q Max -13.8671 -4.2115 0.0291 3.5759 17.5451

Coefficients:

	Estimate S	td. Error	t value	Pr(> t)	
(Intercept)	15.5451	0.7931	19.600	< 2e-16	***
children	0.7732	0.1192	6.485	1.25e-10	***
smokeryes	14.8532	0.3577	41.526	< 2e-16	***
sexmale	-0.5982	0.2882	-2.076	0.038119	*
regionnorthwest	-0.4948	0.4124	-1.200	0.230442	
regionsoutheast	-1.4895	0.4142	-3.596	0.000335	***
regionsouthwest	-1.0330	0.4138	-2.496	0.012675	*
bmi	0.2304	0.0246	9.367	< 2e-16	***
Signif. codes:	0 '***' 0.	001 '**' 0	0.01 '*'	0.05 '.'	0.1 ' ' 1

```
Residual standard error: 5.249 on 1330 degrees of freedom

Multiple R-squared: 0.5835, Adjusted R-squared: 0.5813

F-statistic: 266.2 on 7 and 1330 DF, p-value: < 2.2e-16
```

However, since age independently had the highest correlation with charges, the adjusted R-squared value fell to 0.5813 in the model without a period. Therefore, we do not believe it made sense to use this model as a predictor, especially given the significant trade-off in predictability.

```
par(mfrow=c(2,2))
# without.age = without age and with y transformed to achieve lambda = 1 maximized

plot(without.age$fitted.values,without.age$residuals, main="Residual Plot", col='blue')

abline(h=0, col="orange")

boxcox.lambda <- boxcox(without.age, main = "Box-Cox", col='blue',lambda=seq(-1,4, by=0.1))

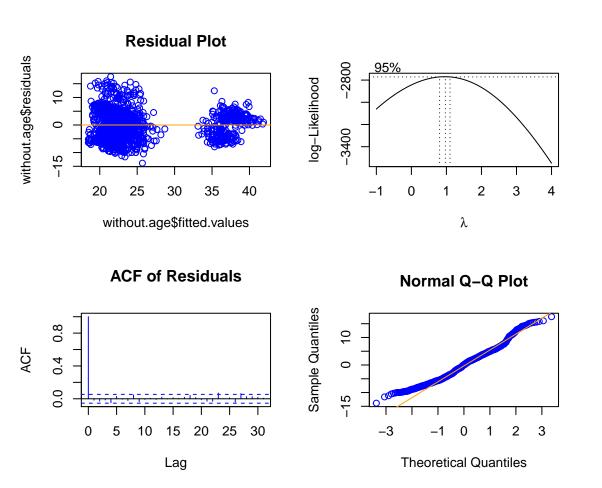
##ACF plot of residuals

acf(without.age$residuals, main="ACF of Residuals", col='blue')

##Normal probability or QQ plot of residuals

qqnorm(without.age$residuals, col='blue')

qqline(without.age$residuals, col="orange")</pre>
```



Initial Model Summary: We increased the adjusted R^2 for our model by transforming the initial model we considered, however the non-constant variance and non-linearity issues in the data set were not fully addressed. We acknowledged that our initial model could be used to explore the relationship between

response and predictor variables. However, predictions may not be accurate.

4 Alternate Models Considered:

Based on the analysis of the linear regression model, the goal was modified to find a model that predicted the charges to be above or below a certian threshold value. For example above or below \$20,000. A logistic regression was then considered instead of linear regression.

The response variable was convered to a categorical variable and the data was split the data into a training and a testing dataset.

Call:

Deviance Residuals:

```
Min 1Q Median 3Q Max
-1.8184 -0.3834 -0.2428 -0.1373 3.1248
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-7.91973	1.07038	-7.399	1.37e-13 ***	¢
age	0.03953	0.01153	3.428	0.000607 ***	¢
bmi	0.11077	0.02582	4.289	1.79e-05 ***	¢
smokeryes	4.85604	0.37410	12.981	< 2e-16 ***	¢
regionnorthwest	0.22102	0.44125	0.501	0.616451	
regionsoutheast	-0.47459	0.41783	-1.136	0.256019	
regionsouthwest	-0.81043	0.45047	-1.799	0.072008 .	
sexmale	0.05906	0.30390	0.194	0.845916	
children	0.03180	0.12272	0.259	0.795552	

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 664.52 on 668 degrees of freedom Residual deviance: 321.06 on 660 degrees of freedom

AIC: 339.06

Number of Fisher Scoring iterations: 6

The higher the difference between null deviance and residual deviance, the better the model's predictability would be. Our data supported the claim that our logistic regression model was useful in estimating the log odds of whether medical **charges** are greater or less than \$20000

The model summary showed, based Z-value (Wald test) age, bmi, and smoker are significant predictors with p-value of less than 0.05. Furthermore, the region sex and children predictors seem insignificant, hence removing the model.

Hypothesis-testing: H_0 : coefficients for all predictors is =0 and

H₁: at least one coefficient is not zero

```
1-pchisq(lrmodel1$null.deviance - lrmodel1$deviance,8)
```

[1] 0

Based on the small p-value we rejected the null hypothesis that at least one of these coefficients was not zero.

```
lrmodel2 <-glm(lrcharges ~ age + bmi + smoker, family="binomial" , data = lrdata_train)
summary(lrmodel2)</pre>
```

```
Call:
```

```
glm(formula = lrcharges ~ age + bmi + smoker, family = "binomial",
    data = lrdata_train)
```

Deviance Residuals:

Min 1Q Median 3Q Max

```
-1.7030 -0.3739 -0.2481 -0.1506 3.1720
```

Coefficients:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept) -7.86104
                       1.03377 -7.604 2.87e-14 ***
            0.04087
                       0.01150
                                 3.554 0.000379 ***
age
bmi
            0.10134
                       0.02465
                                 4.111 3.94e-05 ***
smokeryes
            4.75564
                       0.35844 13.268 < 2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 664.52 on 668 degrees of freedom
Residual deviance: 327.44 on 665 degrees of freedom
AIC: 335.44
```

Number of Fisher Scoring iterations: 6

An obersvation from the Wald test was that the region, sex, and children predictors were insignificant, so we will conducted the delta G^2 test to see if these predictors can be removed from the model.

```
#test if additional predictors have coefficients equal to 0
1-pchisq(lrmodel2$deviance - lrmodel1$deviance,5)
```

[1] 0.2701389

The p-value was 0.0941 greater than 0.05, so we could not reject the null. We then chose a simpler model with just the three predictors **age**, **bmi** and **smoker**.

Logistic Regression model validation We then tested how well this logistic regression model performed in predicting an outcome that medical **charges** were greater than or less than \$20000 given the values of other predictors, using the probability of the observations in the test data of being in each class, we will choose a threshold of 0.5 for the confusion matrix.

False Positive Rate: When it was actually no, how often would it predict yes = 0.0625

False Negative Rate: When it was actually yes, how often would it predict yes = 0.2198582Sensitivity out of all the positive classes, how much was predicted correctly = 0.7801418Specificity determined the proportion of actual negatives that were correctly identified = 0.9375

The AUC value for our model was 0.8999704. The AUC value was higher than 0.5, which meant the model did better than random guessing the classifying observations.

5. Conclusion:

- 1. Even after applying transformations, the model fit is still did not satisfy linear regression assumptions.
- 2. There were still observed non-linearity, and non-constant variance issues that not addressed in the linear model.
- 3. The regression assumption issues could be due to skewed data or outliers in the dataset.
- 4. The conclusion was that our initial transformed model is useful for exploring the relationship between predictor and response variables. However, the predicted values would be unreliable.
- 5. The data is skewed when it comes to age & smokers, producing more balanced dataset may improve the predictability of our initial MLR model.
- 6. The alternate logistic regression model would be a better predictor of the likelihood of charges above 20,000 USD when other variables were held constant.

The recommendation would be to use with the logistic regression model for better predictability.

$$\pi = ln(P(charges > 20000) == 1)$$

$$\ln \frac{\pi_i}{1 - \pi_i} = -7.86104 + 0.04087age + 0.10134bmi + 4.75564smokeryes$$