可靠度分析與應用(Reliability Analysis and Applications)
Final Exam

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1. Find by derivation the mean and variance of the following Poisson distribution both have the value λ. (10%)

$$P(n;\lambda) = \frac{e^{-\lambda}\lambda^n}{n!},\ n = 0, 1, 2, \cdots$$

$$\mu = \sum_{x=0}^{\infty} np(n;\lambda) = \lambda e^{\lambda} \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda$$

$$\begin{aligned}\sigma^2 &= \sum_{n=0}^{\infty} (n-\lambda)^2 p(n;\lambda) \\ &= \sum_{n=0}^{\infty} [n(n-1) - (2\lambda - 1)n + \lambda^2] p(n;\lambda) \\ &= \sum_{n=0}^{\infty} n(n-1)p(n;\lambda) - (2\lambda-1)\sum_{n=0}^{\infty} np(n;\lambda) + \lambda^2 \sum_{n=0}^{\infty} p(n;\lambda) \\ &= \lambda^2 - (2\lambda - 1)\lambda + \lambda^2 = \lambda\end{aligned}$$

2. A packaging machine (cartoner) in a food processing facility will jam with a constant probability of 0.003 per application (per carton). Twelve cans of coffee are combined into a single case for shipment to buyers. The production rate is 20 cans of coffee every minute. What is the probability (reliability) of no jams during 1–hr production run? (10%)

$$\Delta t = \frac{12}{20} = 0.6\ \text{(cartons/min)},\ \lambda = \frac{P}{\Delta t} = \frac{0.003}{0.6} = 0.005\ \text{(failures/min)},\ R(60) = e^{-0.005\times 6} \approx 0.7405$$

3. A cutting tool wears out with a time to failure that is log-normally distributed with a mean of 10 working days and a standard deviation of 2.5 days.
   (a) Determine its design life for a reliability of 0.99. (5%)
   (b) Find the reliability (at the end of a day) if the tool is replaced every day. (5%)
   (c) Determine the probability that the cutting tool will last one more day given it has been in use successfully for 5 days. (5%)

$$t_{med} e^{\frac{s^2}{2}} = 10,\ 2.5^2 = {t_{med}}^2 e^{s^2}(e^{s^2} - 1) \implies s = 0.246\ \text{and}\ t_{med} = 9.7$$

(a) $R(t) = 1 - \Phi(\frac{1}{0.246}\ln\frac{t}{9.7}) = 0.99 \implies t = 5.475$ (days)

(b) $R(1) = 1 - \Phi(\frac{1}{0.246}\ln\frac{1}{9.7}) = 1 - \Phi(-9.23628) \simeq 1$

(c) $\frac{R(5+1)}{R(5)} = \frac{1-\Phi(\frac{1}{0.246}\ln\frac{6}{9.7})}{1-\Phi(\frac{1}{0.246}\ln\frac{5}{9.7})} = \frac{1-\Phi(-1.953)}{1-\Phi(-2.694)} \simeq \frac{0.9745}{0.9965} \simeq 0.9779$

4. A system consisting of four components in series is designed to operate for 100 days. If each component has constant failure rate of 0.0001/day.
   (a) Compute the system reliability. (5%)
   (b) If two units of components 1 and 2 are available, determine the high–level redundancy reliability. (5%)
   (c) If two units of components 1 and 2 are available, determine the low–level redundancy reliability. (5%)

(a) $R_4(100) = e^{-100\times 0.0001} \approx 0.99,\ R_s = {R_4}^4(100) \simeq 0.99^4 \simeq 0.960596$

(b) $R_s \simeq [1 - [1 - (0.99\times 0.99)]^2] \times 0.99 \times 0.99 \simeq 0.9797$

(c) $R_s \simeq [1 - (1-0.99)^2][1 - (1-0.99)^2] \times 0.99 \times 0.99 \simeq 0.9799$

5. A system requires three out of its four components to operate in order to achieve its mission.
   (a) If each component has a reliability of 0.995, determine the system reliability. (5%)
   (b) If each component has a constant failure rate of 0.0001/day, determine the MTTF of the system. (5%)

(a) $R_s = \sum_{x=3}^{4} C_3^4 \times (0.995)^x \times (0.005)^{4-x} = 4 \times (0.995)^3 \times 0.995 + (0.995)^4 \simeq 0.9999$

(b) $\text{MTTF}_s = 10000 \times \left(\frac{1}{3} + \frac{1}{4}\right) \simeq 5833.33$ (mins) $\simeq 97.222167$ (hrs) $\simeq 4.050924$ (days)

6. A contractor must decide between two different sump pump systems to be installed in a new housing development. The option is to install a single $1,000$ gallon per minute (gpm) system or two 500–gpm pumps. If the two–pump system is used, one pump can carry most of the load in the event the other pump fails. Both of the 500–gpm pumps have an MTTF of 800 hr when working together. Their individual MTTF is 200 hr. The $1,000$–gpm system has a rated MTTF of 700 hr. Which system is preferred on the basis of system MTTF of the wind turbine? Which system has the best design life for a reliability of 0.9? (10%)

$$\text{Single } 1,000 \text{ gpm pump: } \mathrm{MTTF} = 700 \text{ (hrs) and } t_d = -\mathrm{MTTF}\ln R = -700\ln 0.9 = 73.7524 \text{ (hrs)}$$

$$\text{Load Sharing System: } \mathrm{MTTF} = 600 \text{ (hrs)} \implies \mathrm{MTTF} = 700 \text{ is the preferred } \mathrm{MTTF}$$

$$R(t) = e^{-2\lambda t} + \frac{2\lambda}{2\lambda - \lambda^+}\left[e^{-2\lambda^+ t} - {}^{-2\lambda t}\right] = e^{-\frac{t}{400}} - \left[e^{-\frac{t}{200}} - e^{-\frac{t}{400}}\right] = 0.9$$

$$\implies t_R = 152.052 \text{ (hrs) is preferred designed life}$$

7. In consideration of strong typhoon, a wind turbine is subject to Poisson distributed random wind gusts at an average rate of 2 per year. The wind turbine is designed to withstand winds up to 250 km/hr (deterministic). Wind speed during gusts, however, is random with a Weibull distribution having a shape parameter of 2 and a scale parameter 200 km/hr. Determine the reliability function and MTTF of the wind turbine. (10%)

$$P = (X \le Y) = P(X \le 100) = 1 - e^{-(\frac{250}{200})^2} \simeq 0.79038$$

$$R(t) = e^{(1-0.79038)2t} = e^{-0.419240t} = e^{-\lambda t}$$

$$\lambda = 0.419240 \text{ and } \mathrm{MTTF} = \frac{1}{\lambda} = 2.385 \text{ (days)}$$

8. Two alternative components are being considered for use in a certain communication system. One component has a unit cost of $\$820$ and a Weibull failure distribution with shape parameter 2 and scale parameter $10,000$ hr. The other has a unit cost of $\$880$ and an exponential failure distribution with failure rate 1×10^{-4} per hr. It is estimated that the selected component will experience $2,000$ operation hours a year, and the system is being designed for a 20–yr life. Any component failures will require a replacement at the current unit cost. Assuming a 2.0 percent interest rate, which is the preferred component? (10%)

For Component 1: $\mathrm{MTTF} = 10000\Gamma(1 + \frac{1}{2}) = 8862.3$

$$\begin{aligned} &\mathrm{Cost}_1 &&= 820[1 + (\mathrm{P/A}, 0.02, 20)2000/8862.3] &&= \$3845.81 \\ &\mathrm{Cost}_2 &&= 820[1 + (\mathrm{P/A}, 0.02, 20)2000/10000] &&= \$3458.60 \end{aligned}$$

The component 2 is the preferred component.

9. A system is comprised of three components in series each having a Weibull life distribution with scale parameters $8,000$, $10,000$ and $9,000$ respectively, and shape parameters 1.8, 1.5 and 1.9 respectively. The cost of improving the current reliability is assumed to be linear and equal for each of the components. The system reliability goal is 0.99 at $1,200$ hours of operation. Determine the percentage improvement required for each component in order to achieve the system goal. (10%)

$$R_A = 0.99$$

Component i	**Scale Parameters** α	**Shape Parameters** β	$R^i(1200) = e^{(-\frac{1200}{\alpha})^\beta}$	$x_i = \sqrt[3]{R_A} - R^i$	**Percentage Improvement** $\frac{x_i}{R^i(1200)}$ $(\%)$
1	$8,000$	1.8	0.979496	0.0290035	2.99%
2	$10,000$	1.5	0.959283	0.0373720	3.89%
3	$9,000$	1.9	0.967652	0.0181690	1.86%

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