Generative models using Probabilistic Principal Component Analysis

Jay Joshi (1401005), Akash Soni (1401047), Mihir Gajjar (1401076),
Nishi Shah (1401099), Shruti Agrawal (1644007)

***Abstract*—Computer vision tasks include methods for acquiring, processing, analyzing and understanding digital images, and extraction of high-dimensional data from the real world in order to produce numerical or symbolic information. Our project aims at implementing different generative models for computer vision and make a comparative study of different models.**

***Index Terms*—Probabilistic PCA, Latent Variables, Covariance, Expectation, Maximization.**

I. Introduction

The PPCA and the EM algorithms on the Iris Data set are implemented.

A. Applications of PPCA in Computer Vision:-

In Robotics for Navigation, Obstacle avoidance. In Medical field for Vision guided robotic Surgery and detection of tumor. In Transportation for Autonomous Vehicle, Object Sorting, Motion estimation. In Security for Image restoration, face recognition, surveillance, fingerprint.

II. Generative Models

A. Principle Component Analysis and its limitations:-

- Used to analyze data and project data into lower dimensional space.

Limitations:-

- No probabilistic model for missing data is observed.
- Difficult to compute for high dimensional datasets.
- Linearity model does not always give best solution.
- non-parametric model for given data.

B. Probablilistic Principle Component Analysis:-

Probabilistic PCA is an example of the linear-Gaussian framework, in which all of the marginal and conditional distributions are Gaussian. The Gaussian prior distribution p(z) over the latent variable and together with a Gaussian conditional distribution $p(x|z)$ for the observed variable x conditioned on the value of the latent variable is represented as follows:

$$p(Z) = N(z|0, I) \tag{1}$$

$$p(X|Z) = N(x|Wz + \mu, \sigma^2 I) \tag{2}$$

in which the mean of x is a general linear function of z governed by the D x M matrix W and the D-dimensional vector Îij. More general view of the observed variable x is defined by a linear, transformation of the M-dimensional latent z plus additive Gaussian 'noise', so that

$$x = Wz + \mu + \epsilon \tag{3}$$

where z is an M-dimensional Gaussian latent variable, and ϵ is a D-dimensional zero-mean Gaussian- distributed noise variable with covariance σ^2I

C. Expectation - Maximization algorithm :-

Given a joint distribution $p(X, Z|\theta)$ over observed variables X and latent variables Z, governed by parameters θ, the goal is to maximize the likelihood function $p(X|\theta)$ with respect to θ.

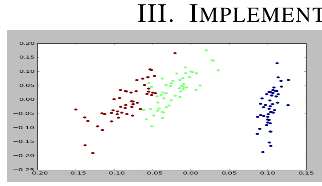
1) Choose an initial setting for the parameters θ_{old}.
2) E step : Evaluate $p(Z|X, \theta_{old})$.
   Using the current parameters, we compute log-likelihood for all data, observed and unobserved, and, then, marginalize it with respect to the unobserved data.
3) M step : Evaluate θ_{new} given by

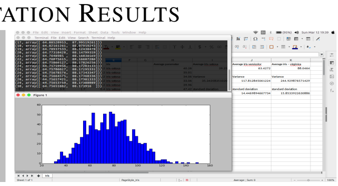
$$\theta_{new} = \underset{\theta}{argmax} Q(\theta, \theta^{old}) \tag{4}$$

$$Q(\theta, \theta_{old}) = \sum_z p(Z|X, \theta_{old}) ln p(X, Z|\theta) \tag{5}$$

   We choose those values for the model parameters that maximize the expectation expression.
4) Check for convergence of either the log likelihood or the parameter values.If the convergence criterion is not satisfied, then let0 $\theta_{old} \leftarrow \theta_{new}$ and return to step 2.

III. Implementation Results





. Figure: 1 Figure:2

IV. Interpretation and Conclusion

- Figure 1 and Figure 2 shows the implementation of PPCA and EM algorithm on the Iris data set respectively.
- In PPCA to deal with missing values some methods explicitly compute the sample covariance matrix and have complexities $O(nd^2)$ where as E-M algorithm does not require computation of sample covariance matrix, so complexity reduces to O(dnq).

References

[1] Tipping, M. E., & Bishop, C. M. (1999). Probabilistic principal component analysis. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 61(3), 611-622.

[2] Christopher Bishop, "Pattern Recognition And Machine Learning",1^{th} edition,New York,Springer,2006,ch 12.