

Mixing of Fluids

Derek Hammar

Jason Ngo



THE MATHEMATICS OF TAFFY PULLERS

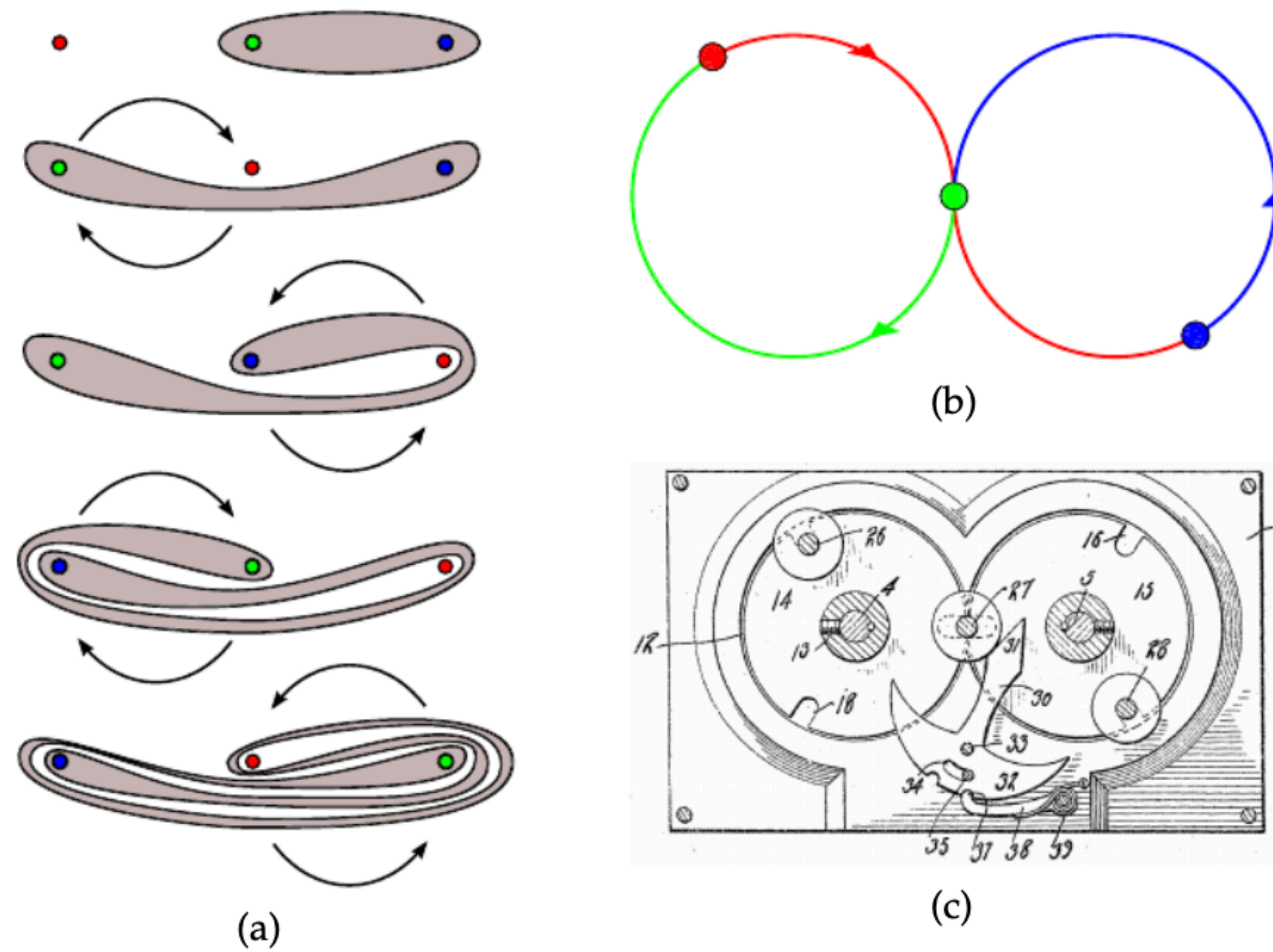
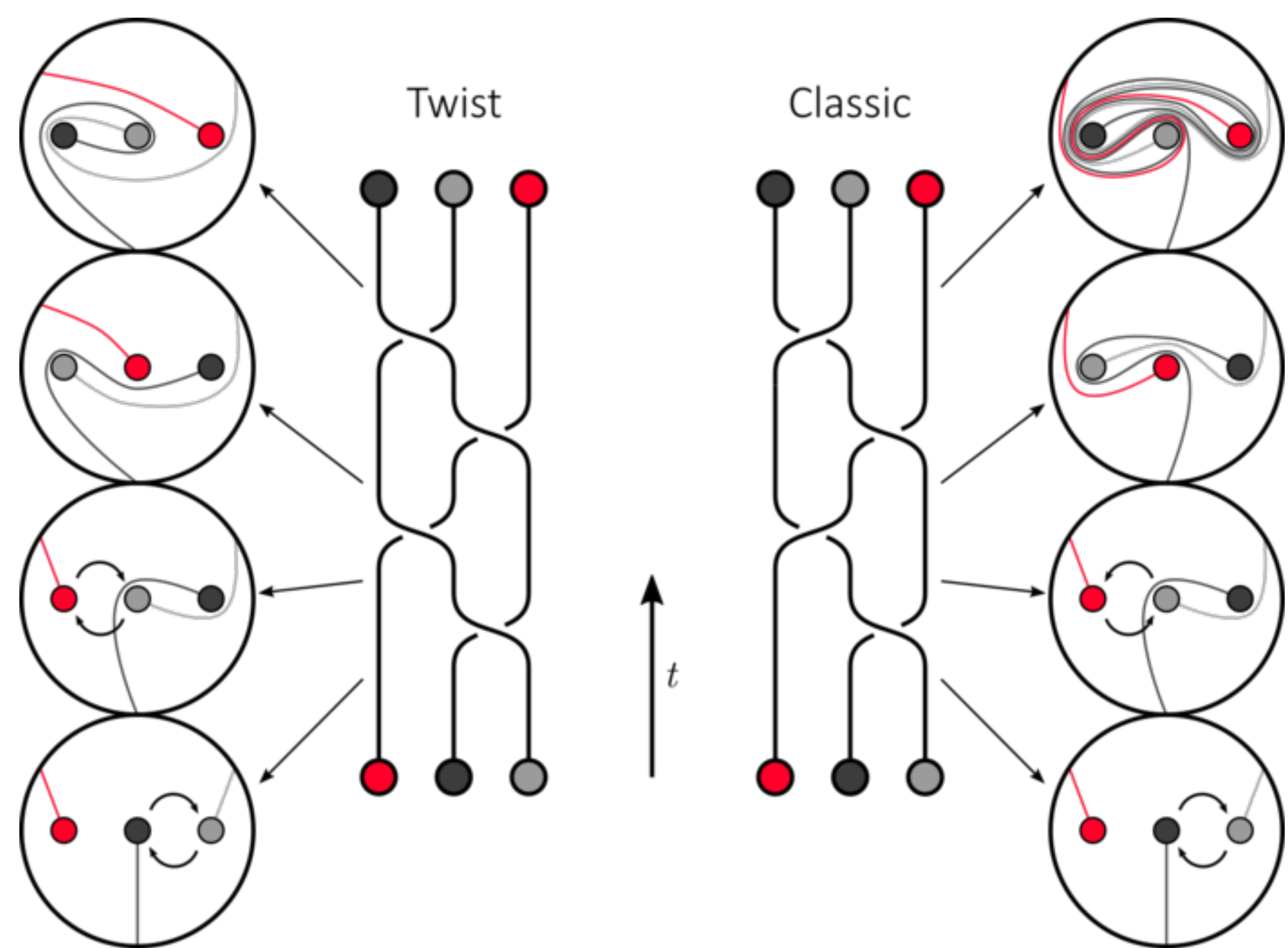


FIGURE 3. (a) The action of a 3-rod taffy puller. The first and second rods are interchanged clockwise, then the second and third rods are interchanged counterclockwise. (b) Each of the three rods moves in a Figure-8. (c) Taffy puller from the patent of Nitz (1918), where rods alternate between the two wheels.

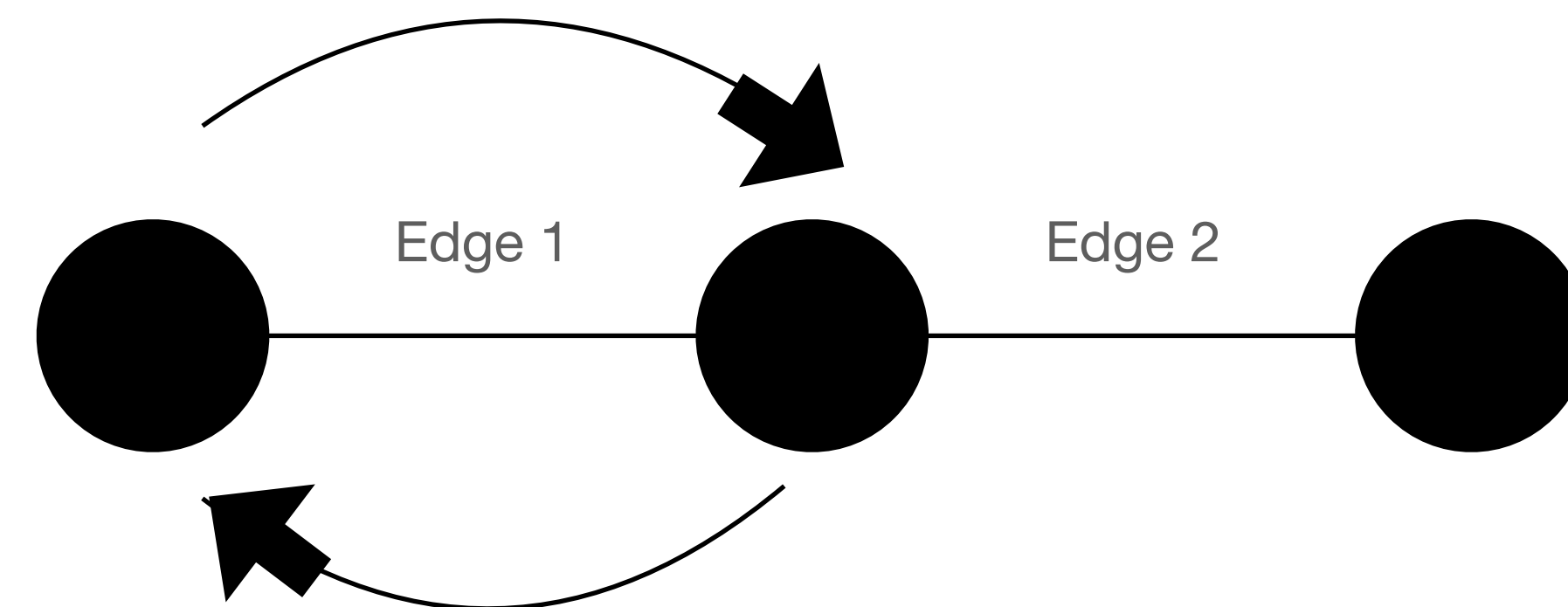
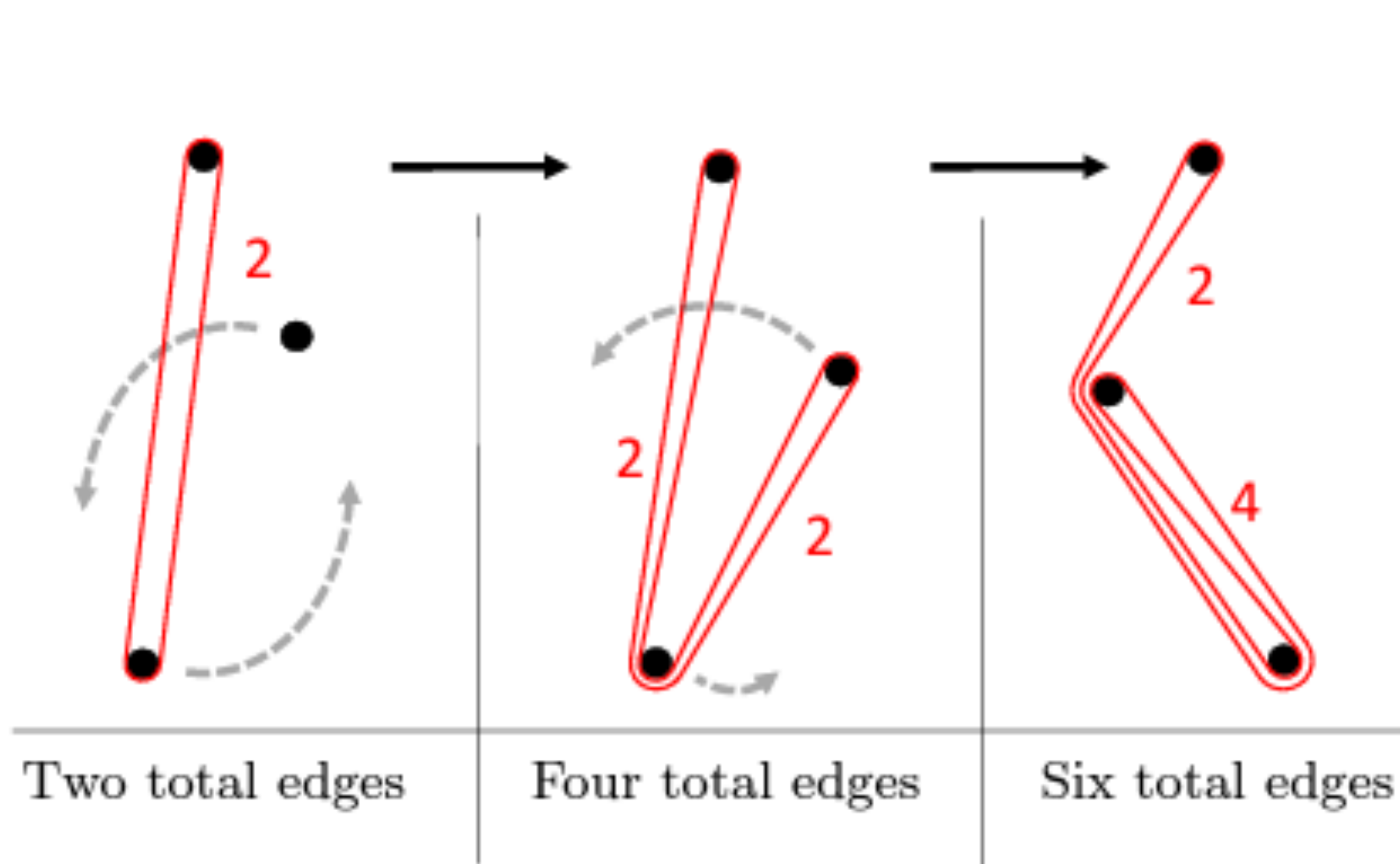


Bad Mixing

Good Mixing

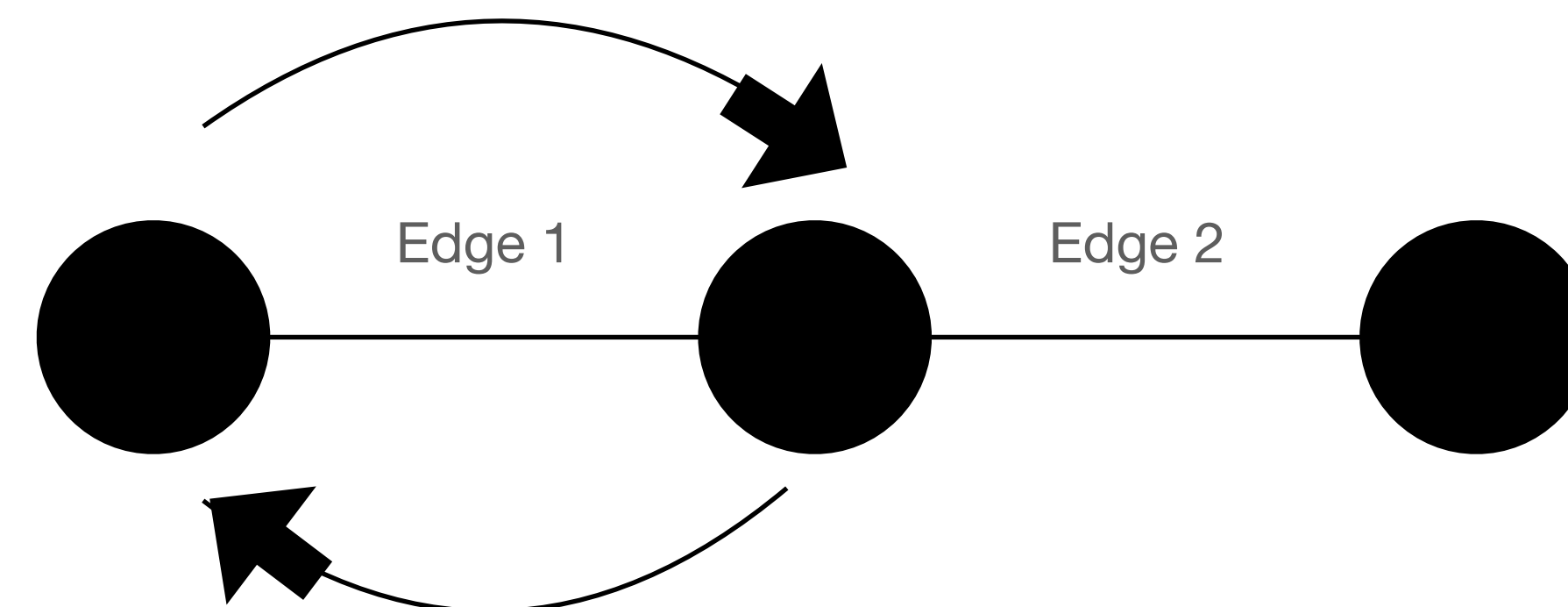
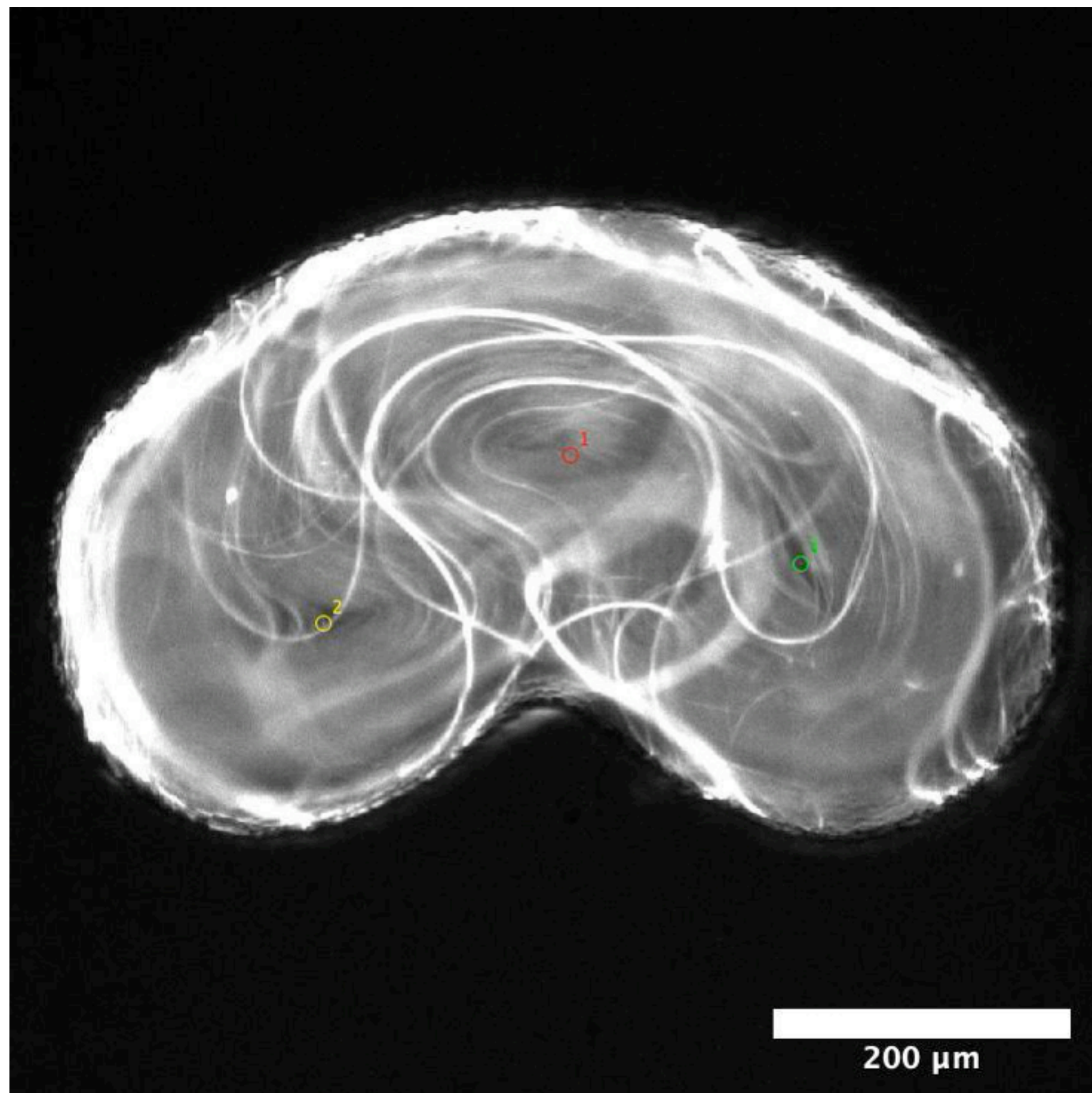


How good is the mixing?



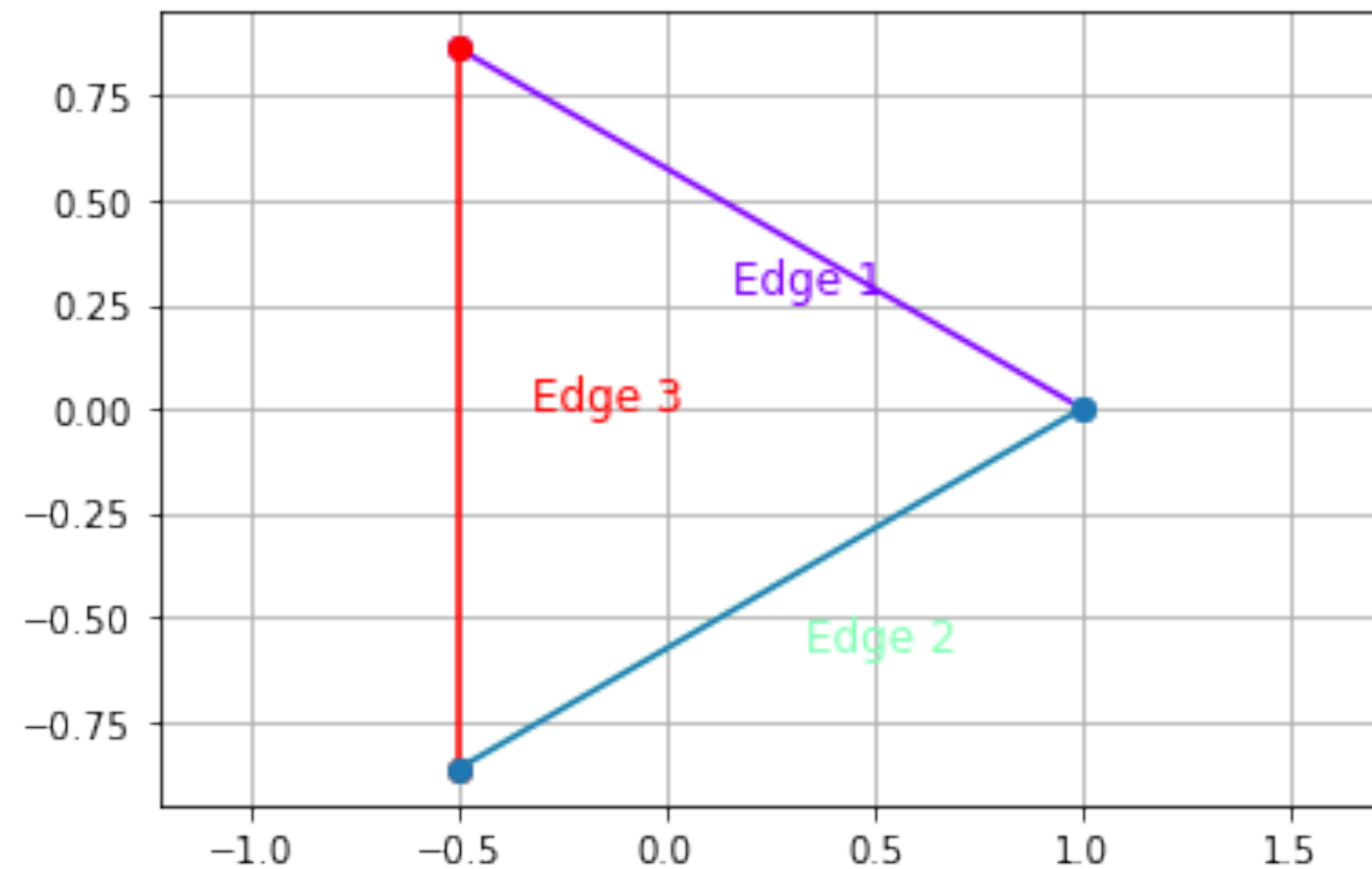
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- Set 2: Edge 1 - ccw
- Set 3: Edge 2 - cw
- Set 4: Edge 2 - ccw

1D and 2D Mixing

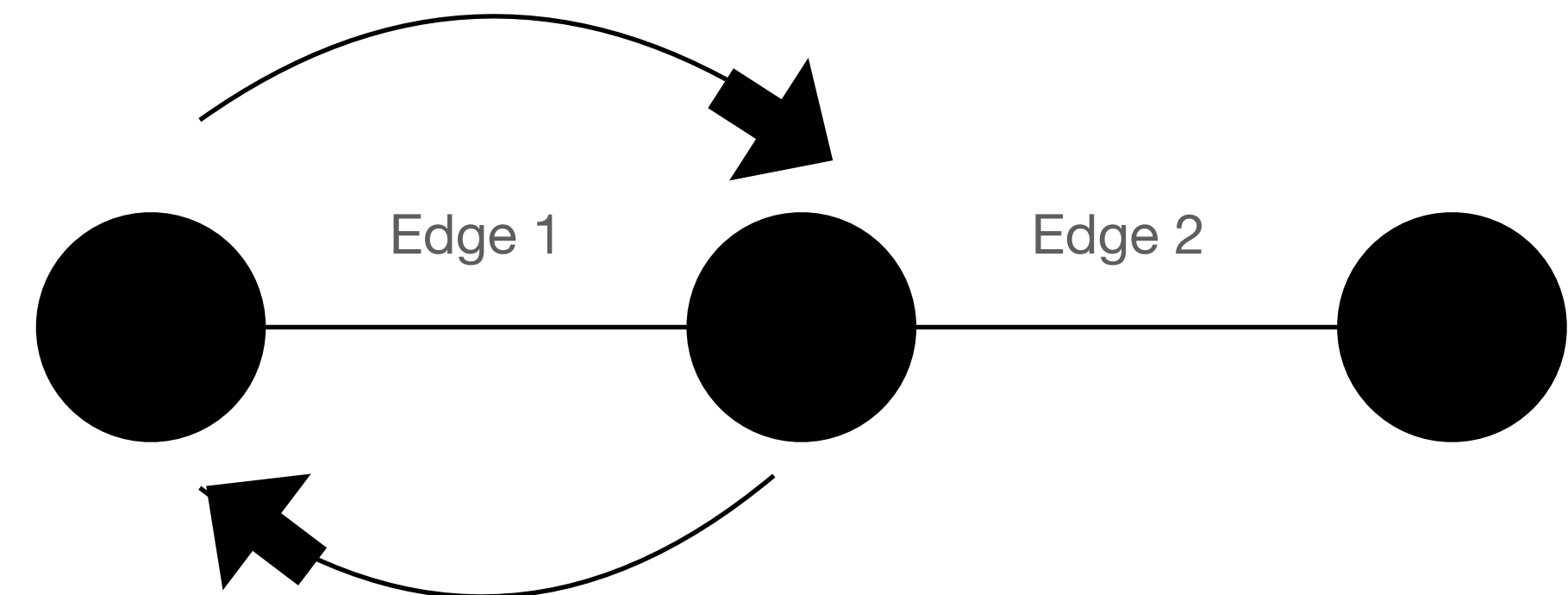


Set 1: Edge 1 - CW
Set 2: Edge 1 - CCW
Set 3: Edge 2 - CW
Set 4: Edge 2 - CCW

1D and 2D Mixing



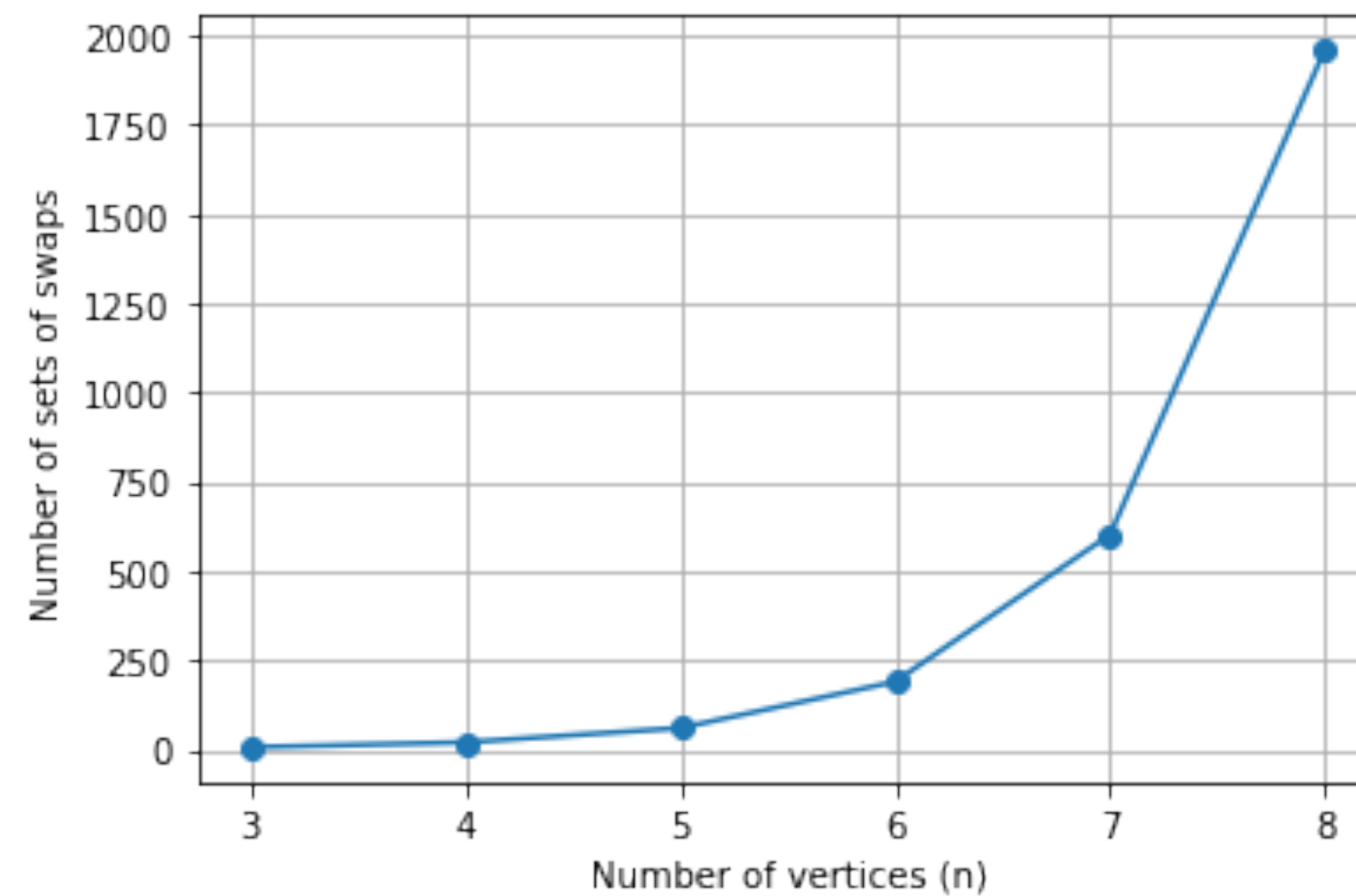
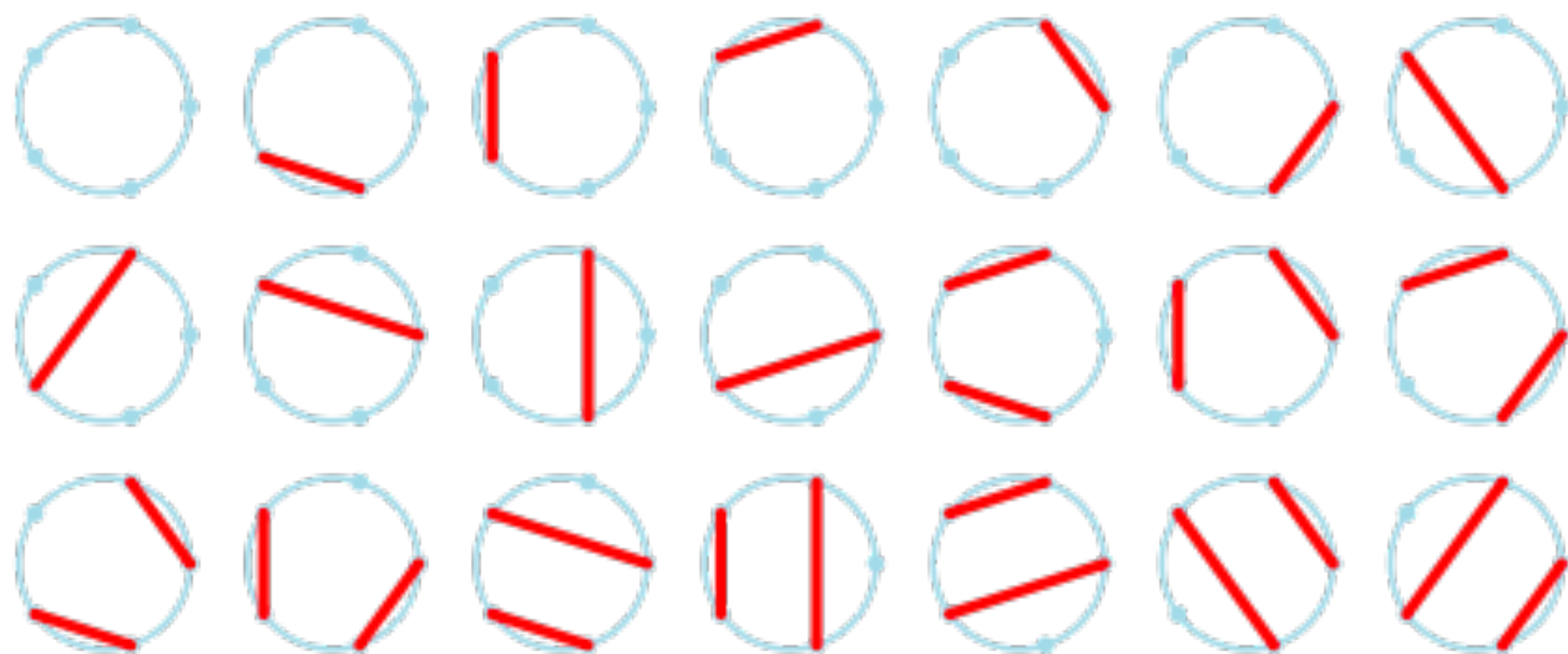
Set 1: Edge 1 - cw
Set 2: Edge 1 - ccw
Set 3: Edge 2 - cw
Set 4: Edge 2 - ccw
Set 5: Edge 3 - cw
Set 6: Edge 3 - ccw



Set 1: Edge 1 - cw
Set 2: Edge 1 - ccw
Set 3: Edge 2 - cw
Set 4: Edge 2 - ccw

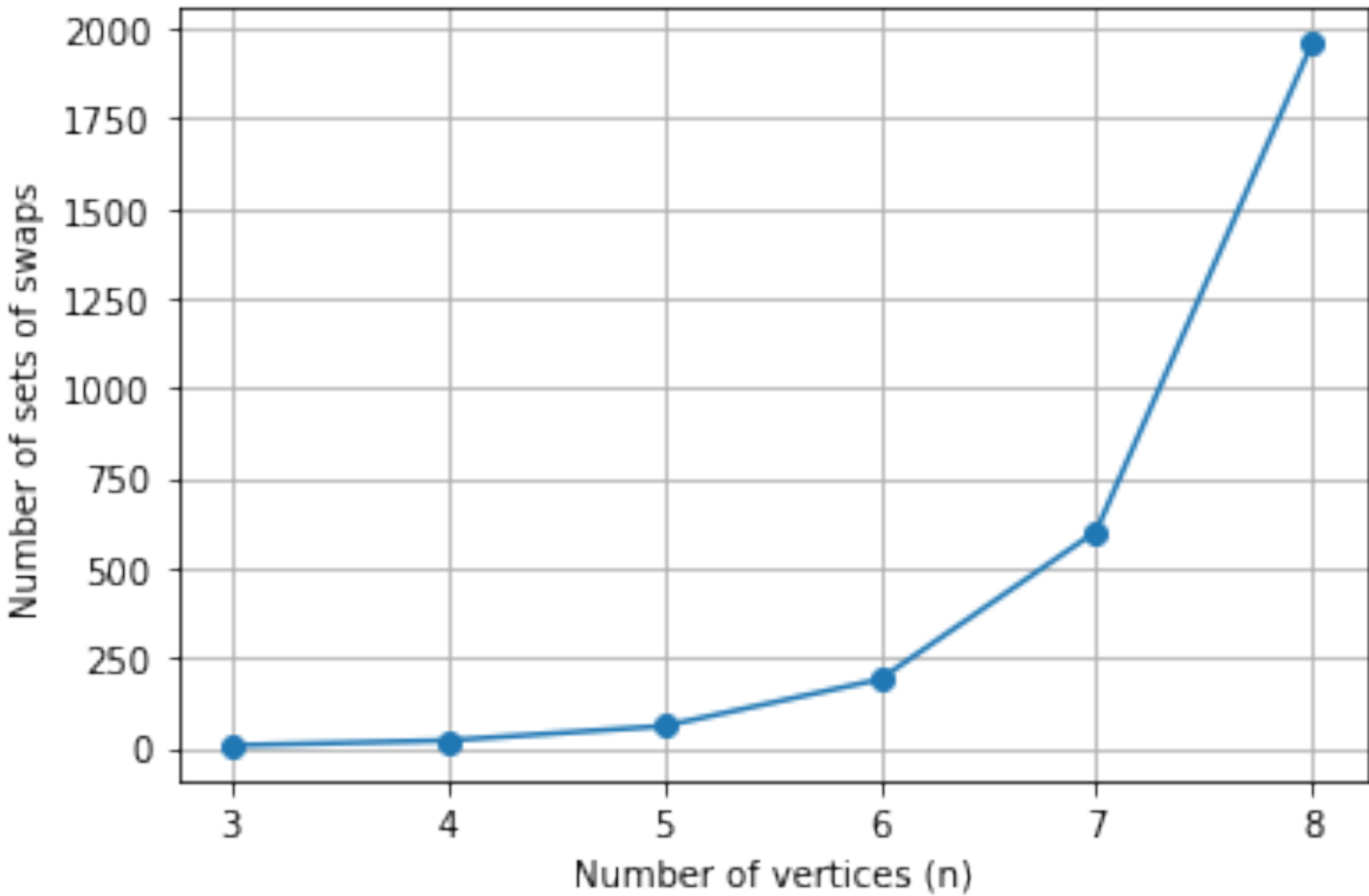
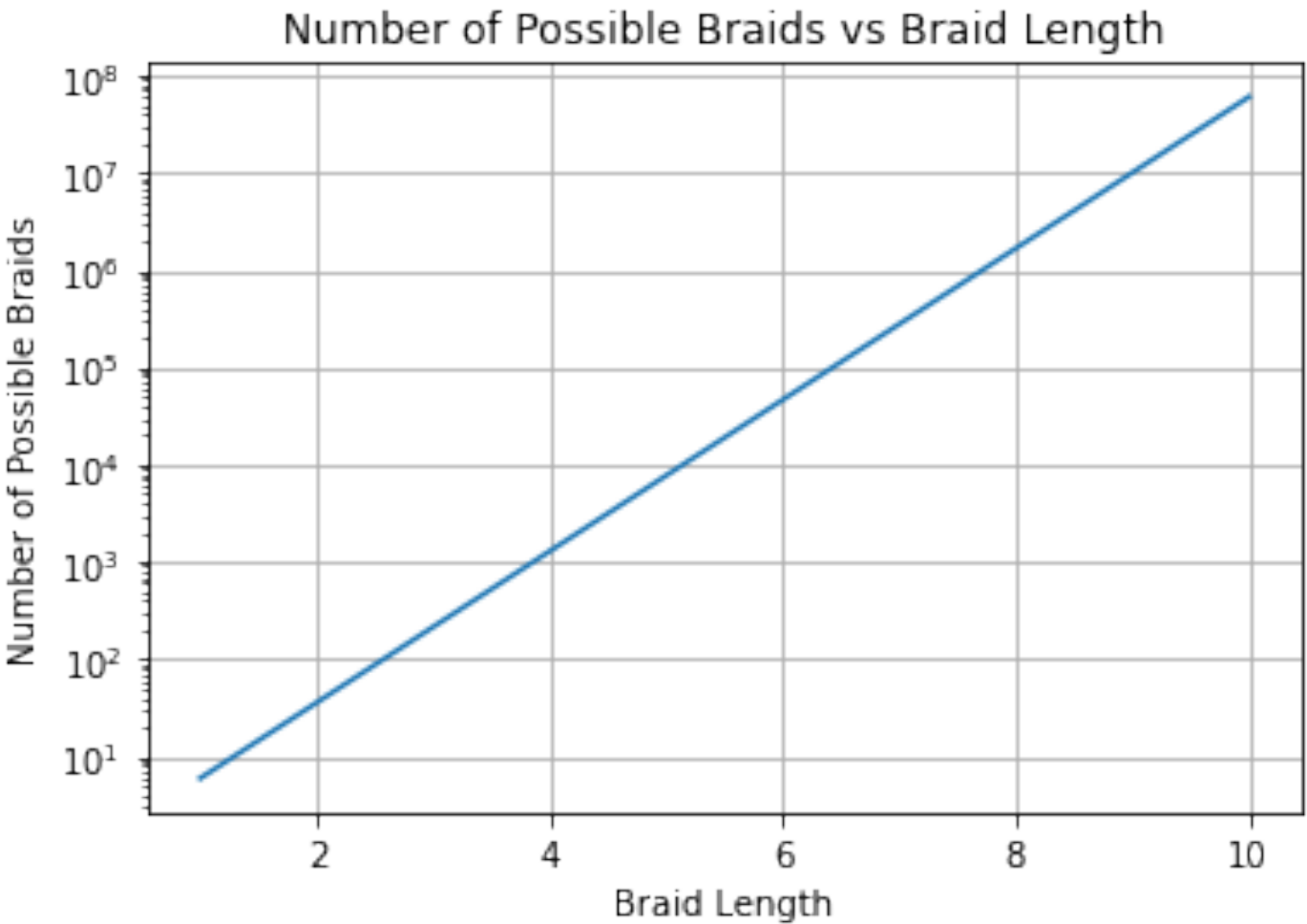
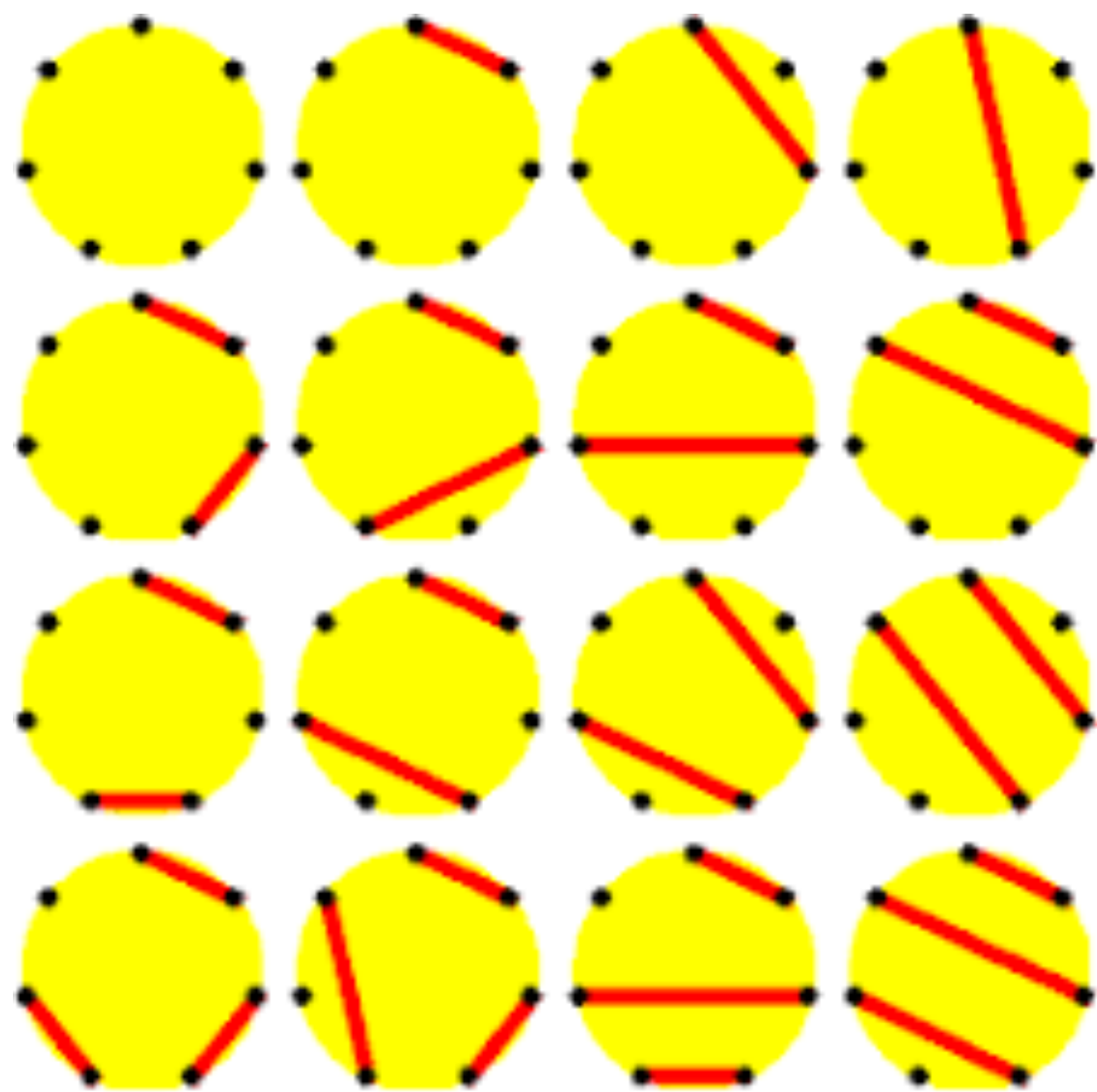
How many ways to mix?

$n=5$

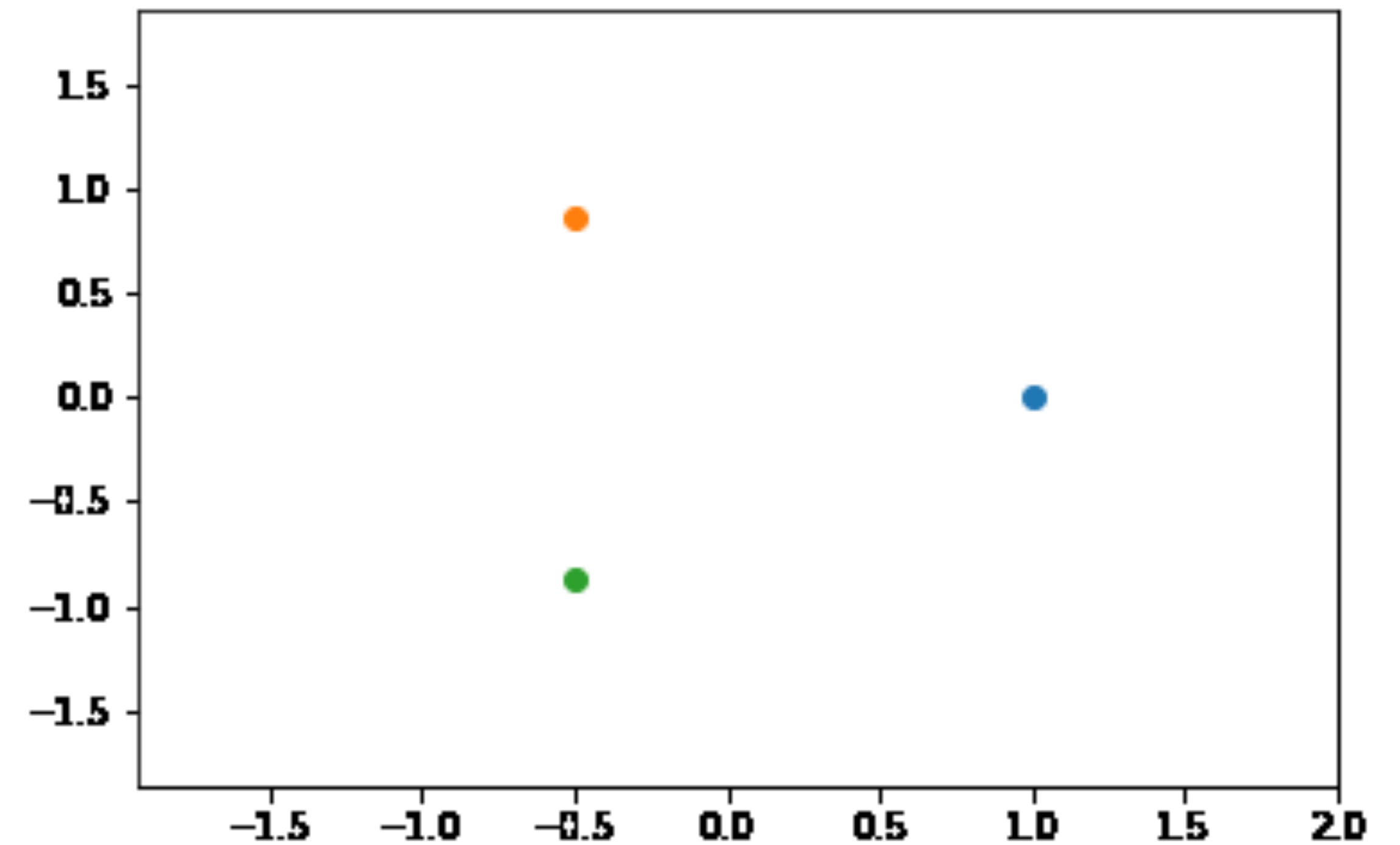
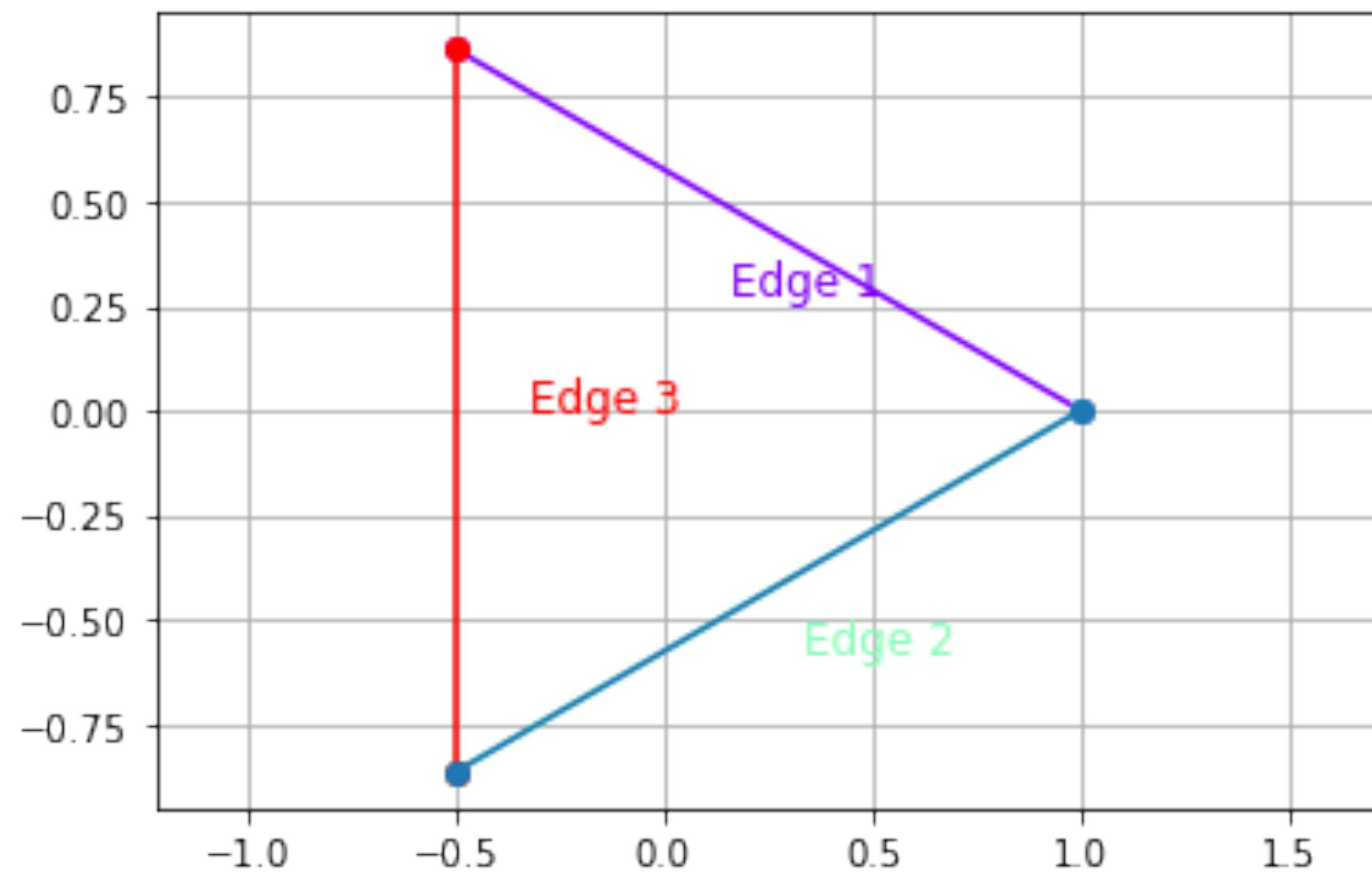


How many ways to mix?

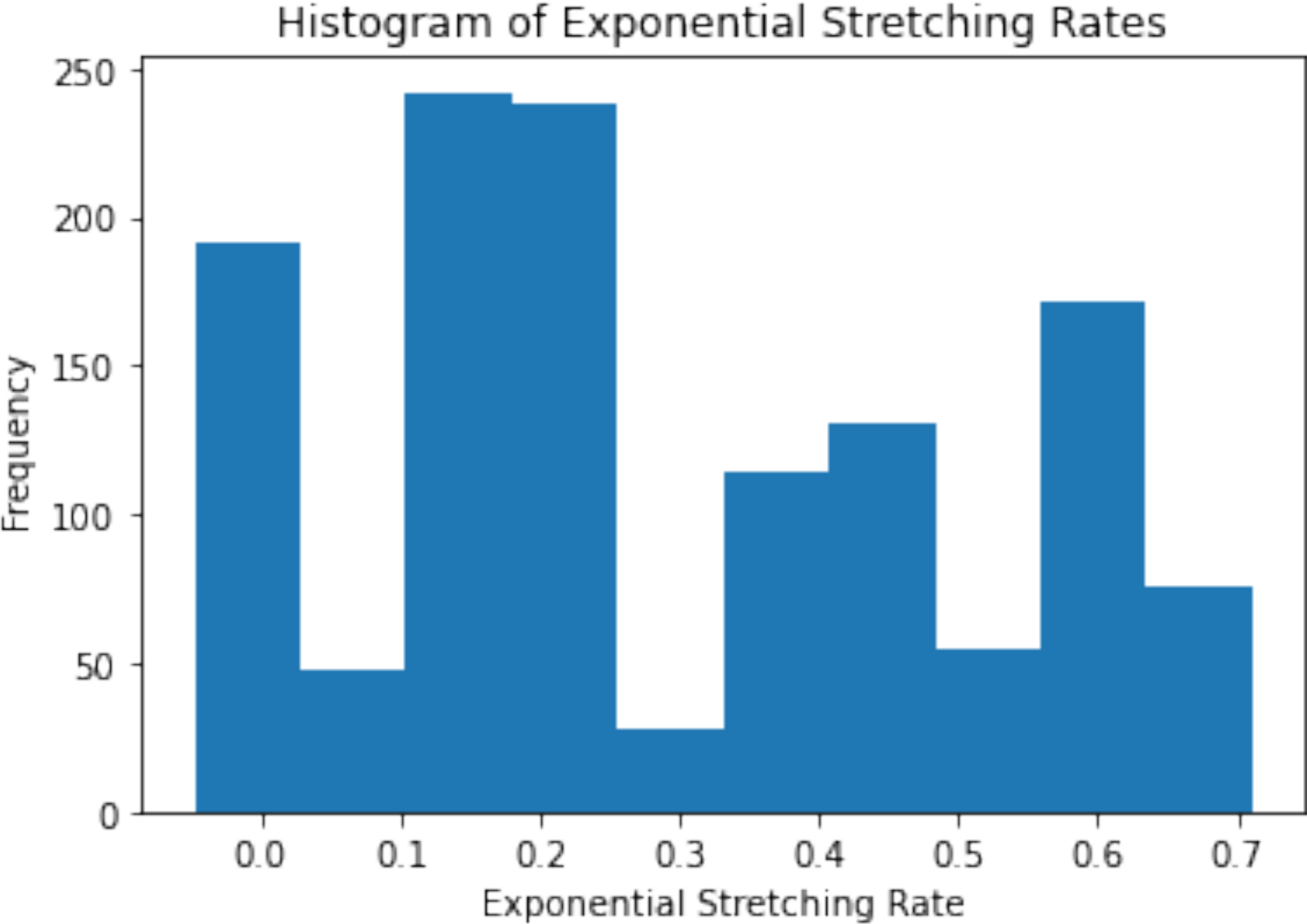
n=6



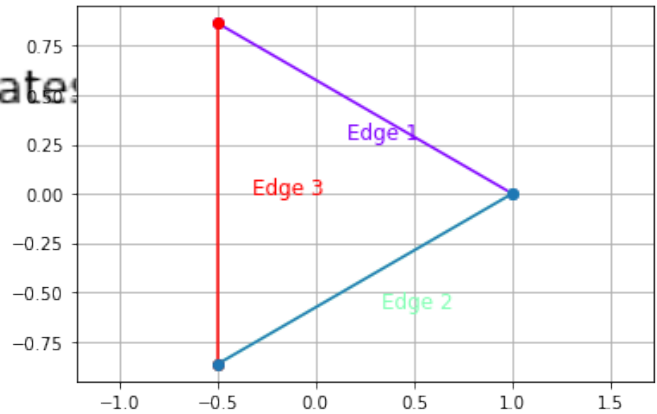
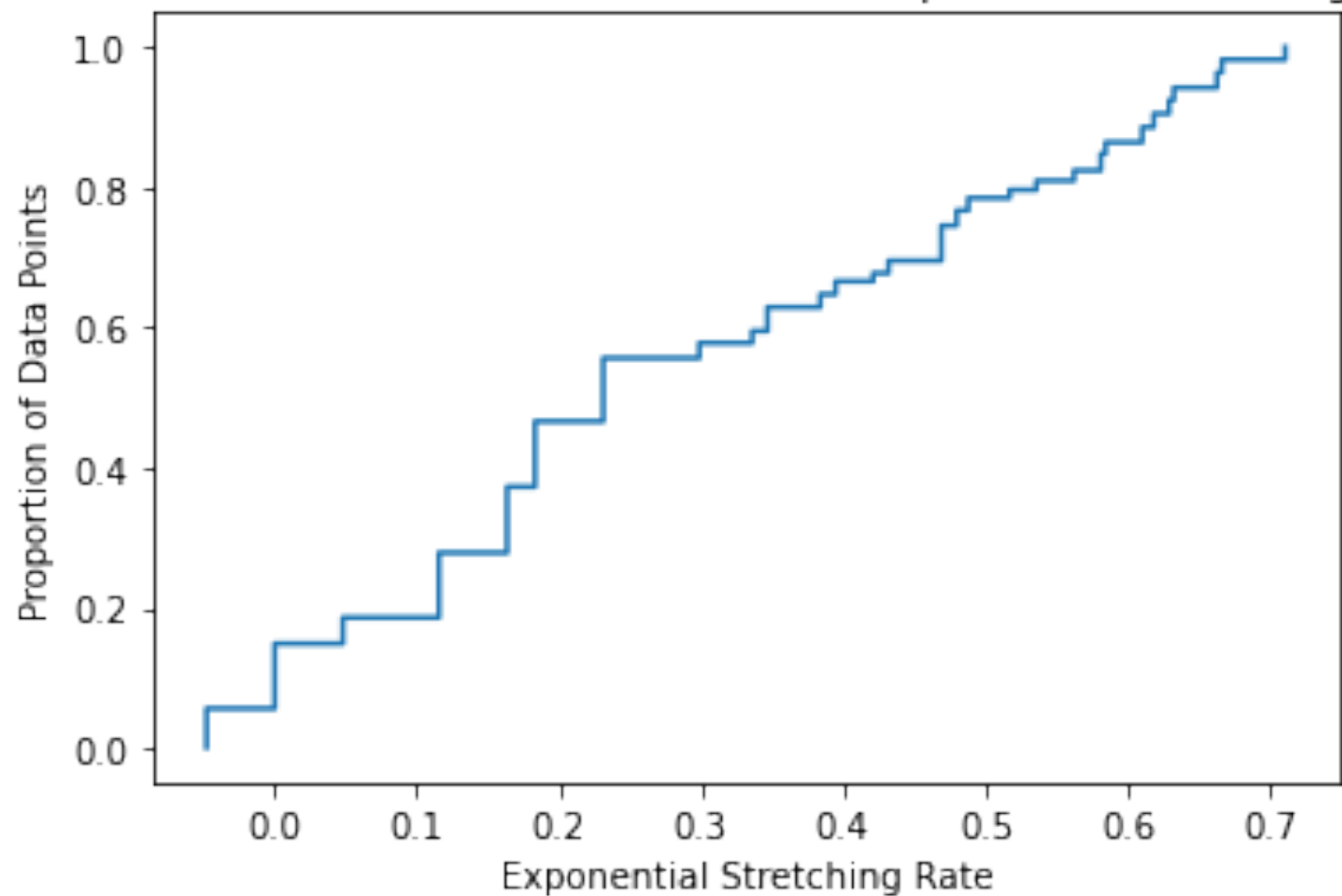
3-Rod Mixing



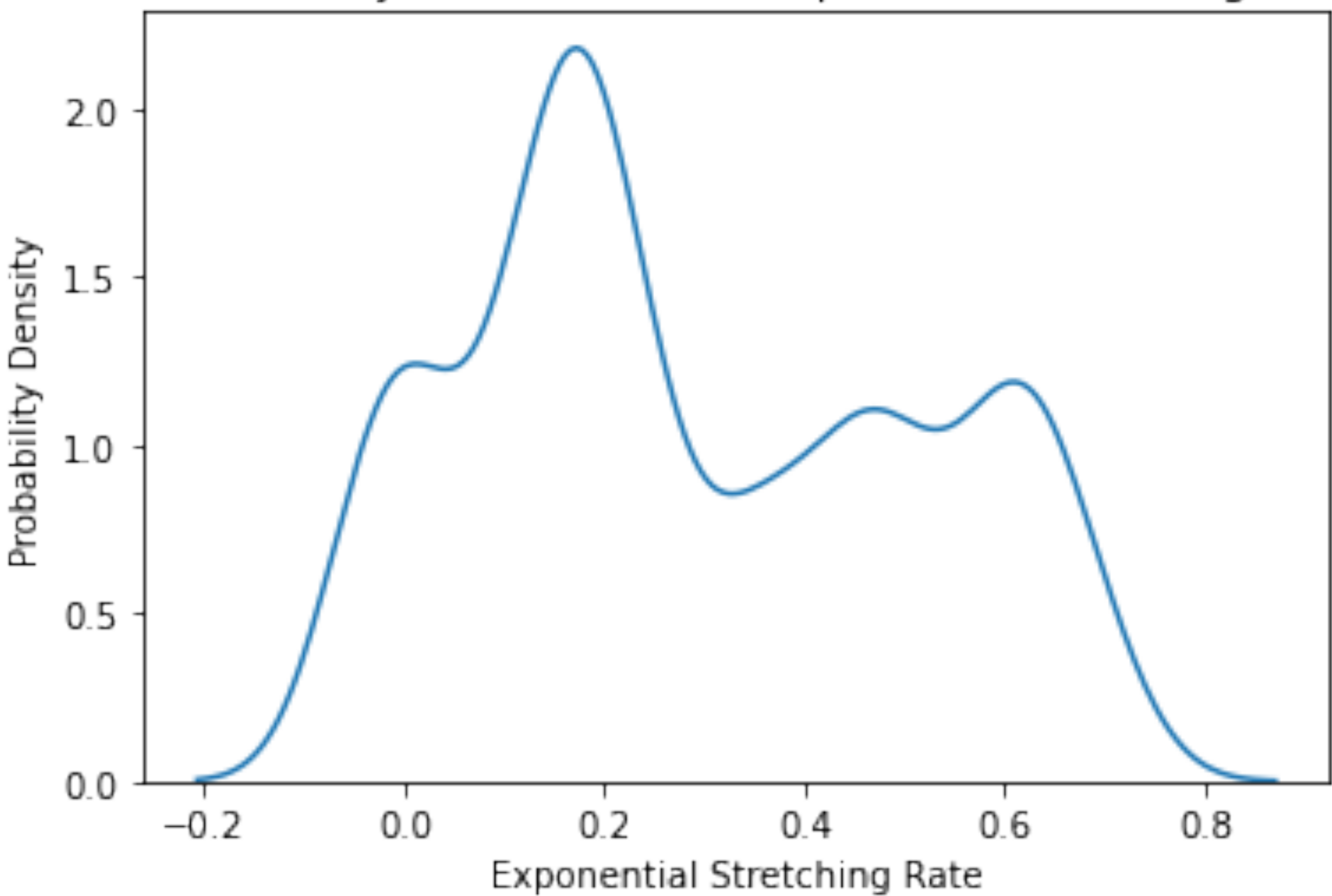
Mixing of N=3 L=4



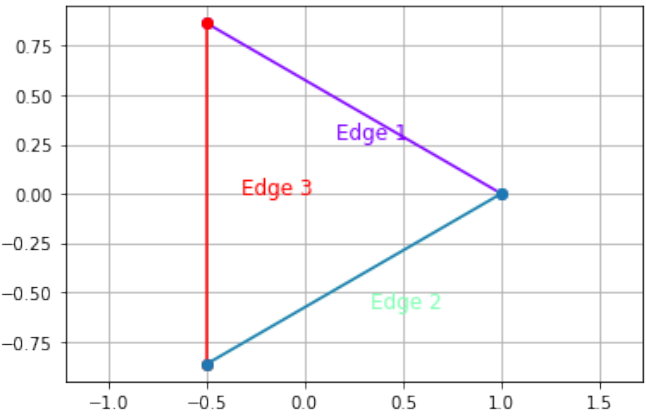
Cumulative Distribution Function Plot of Exponential Stretching Rates



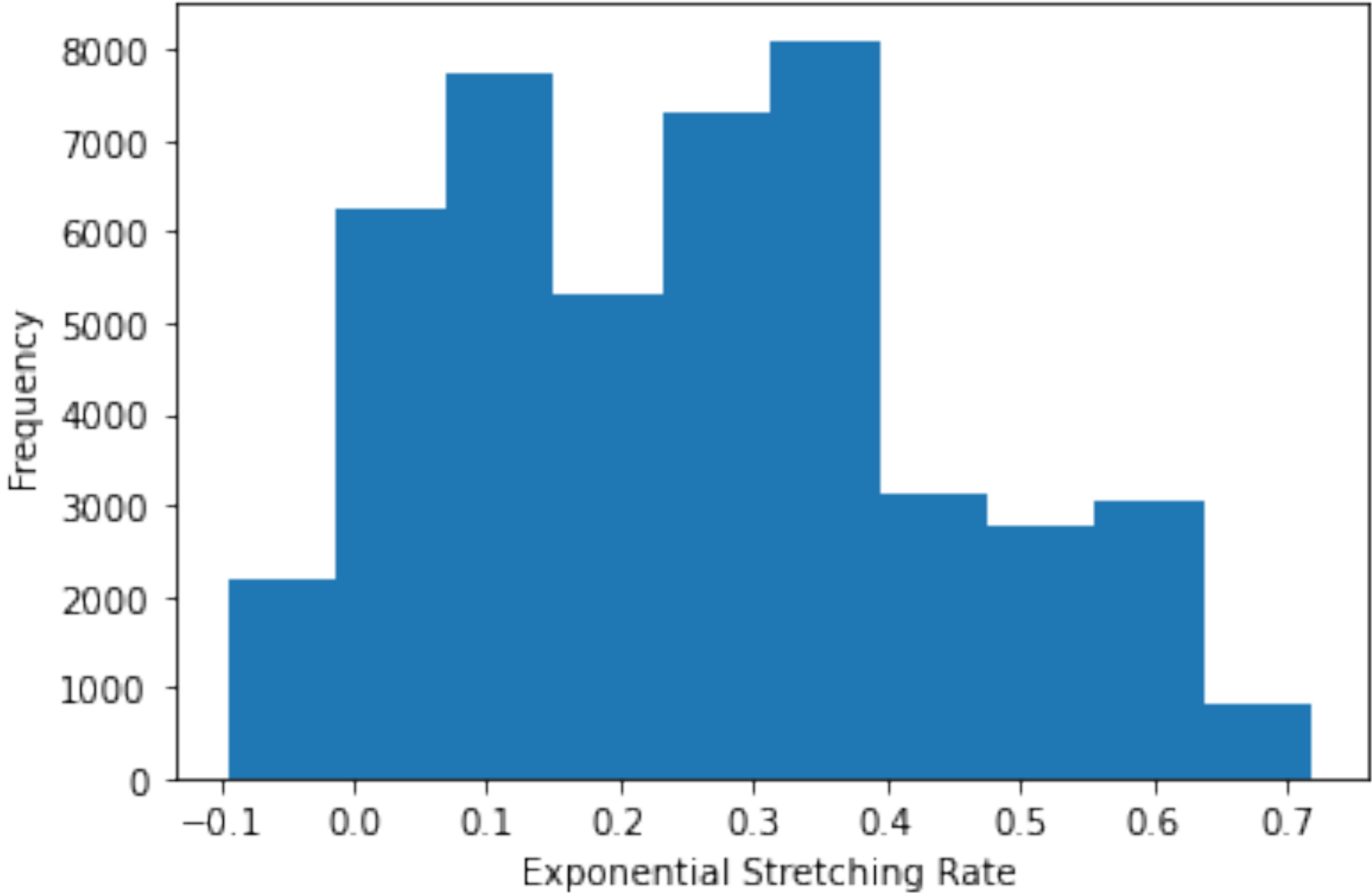
Kernel Density Estimate Plot of Exponential Stretching Rates



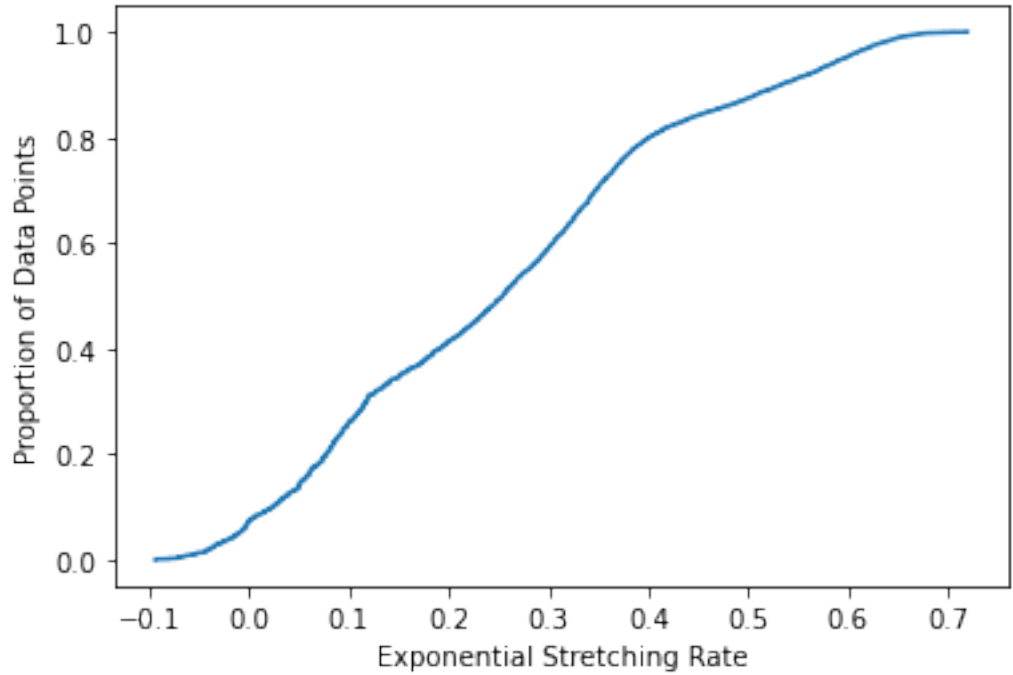
Mixing of N=3 L=6



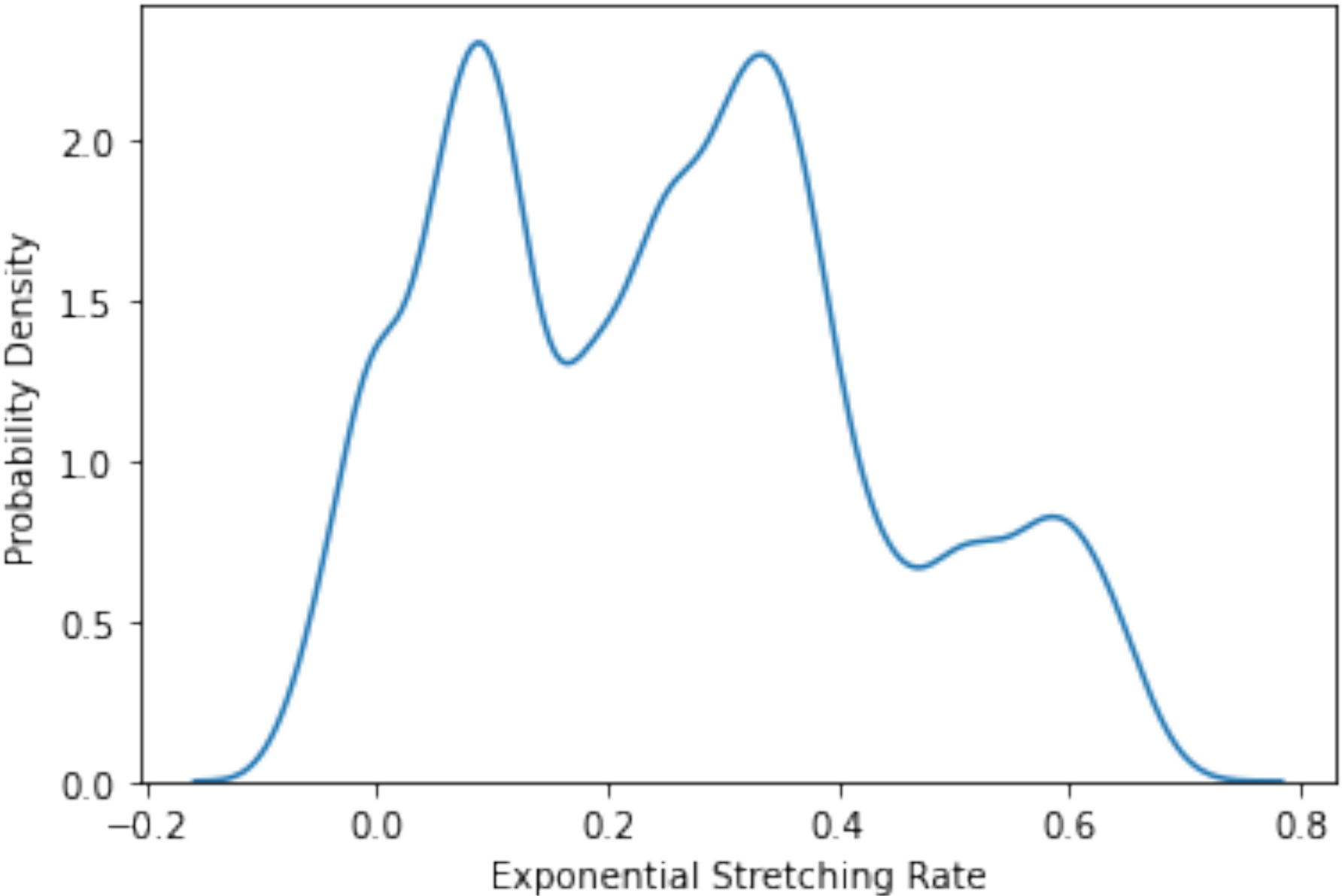
Histogram of Exponential Stretching Rates



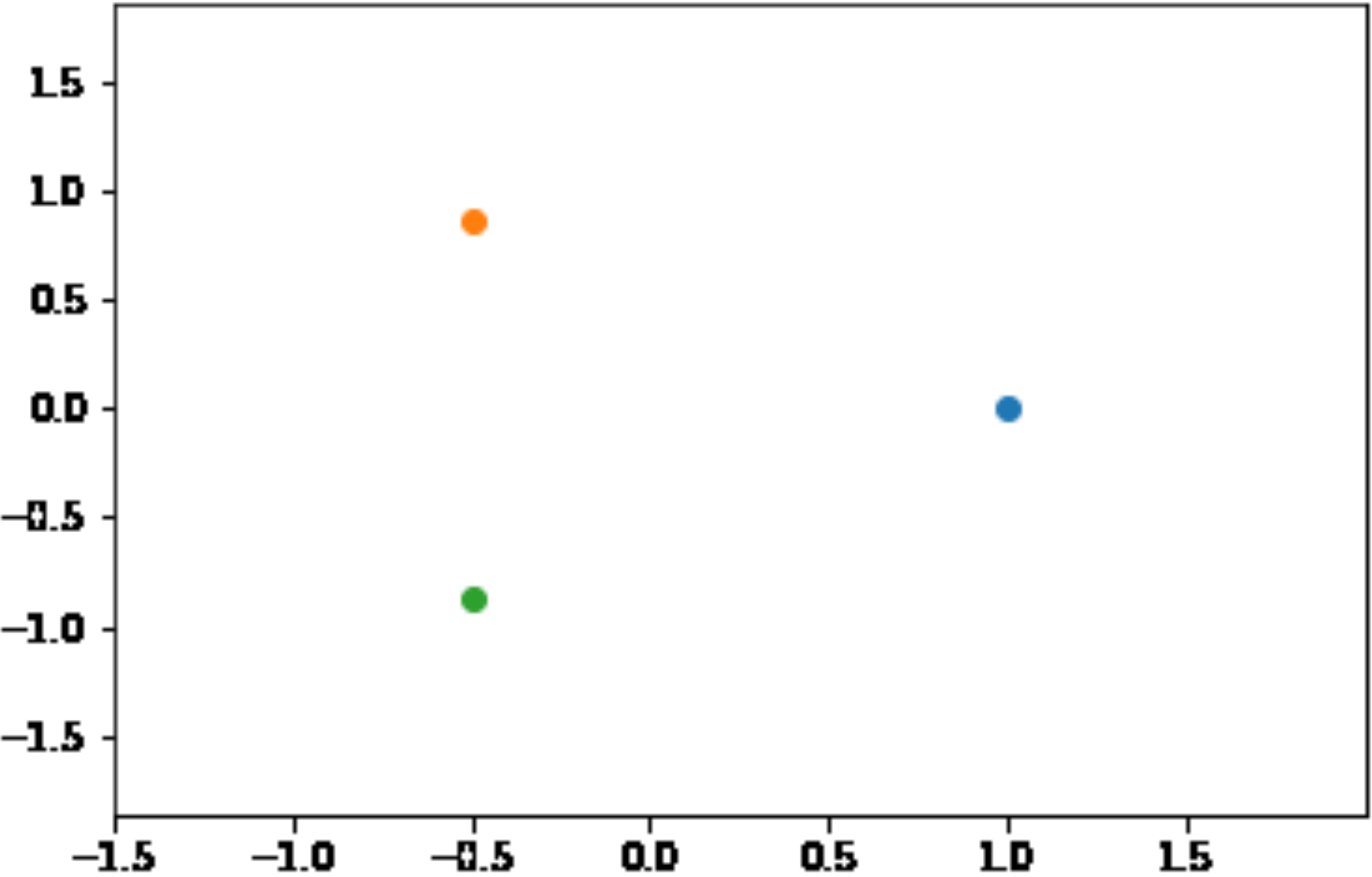
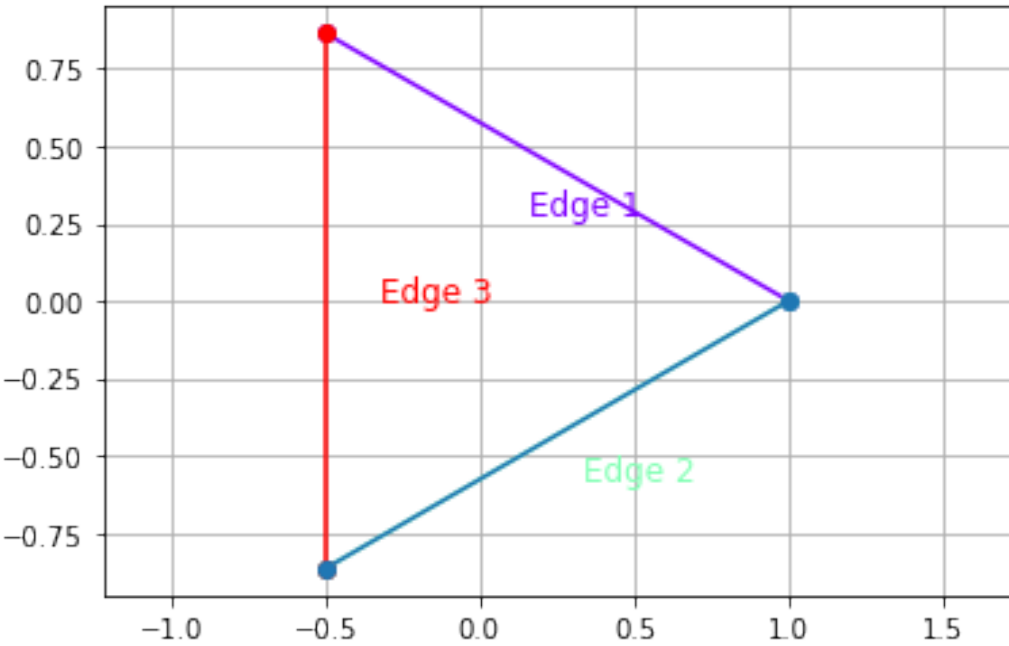
Cumulative Distribution Function Plot of Exponential Stretching Rates



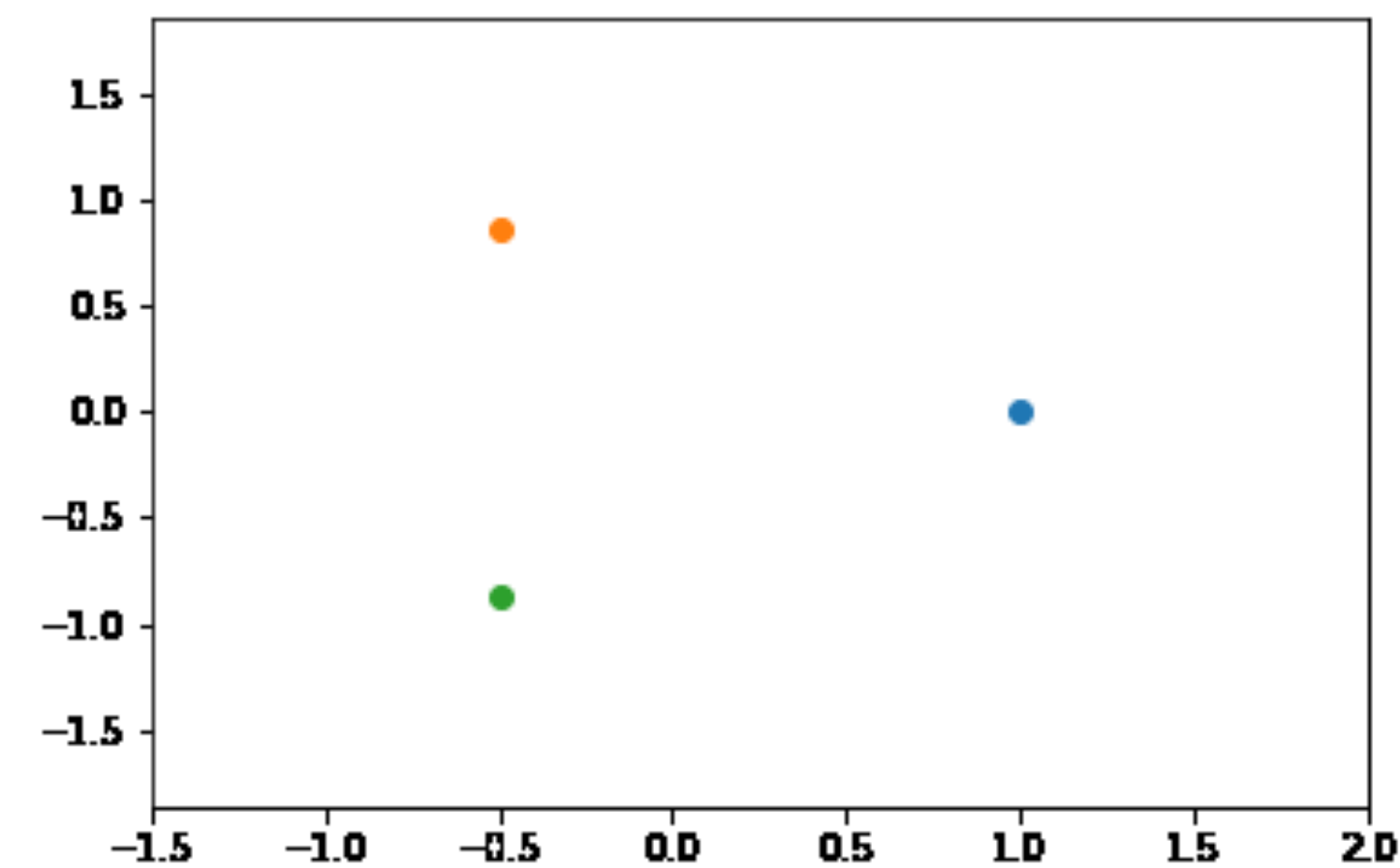
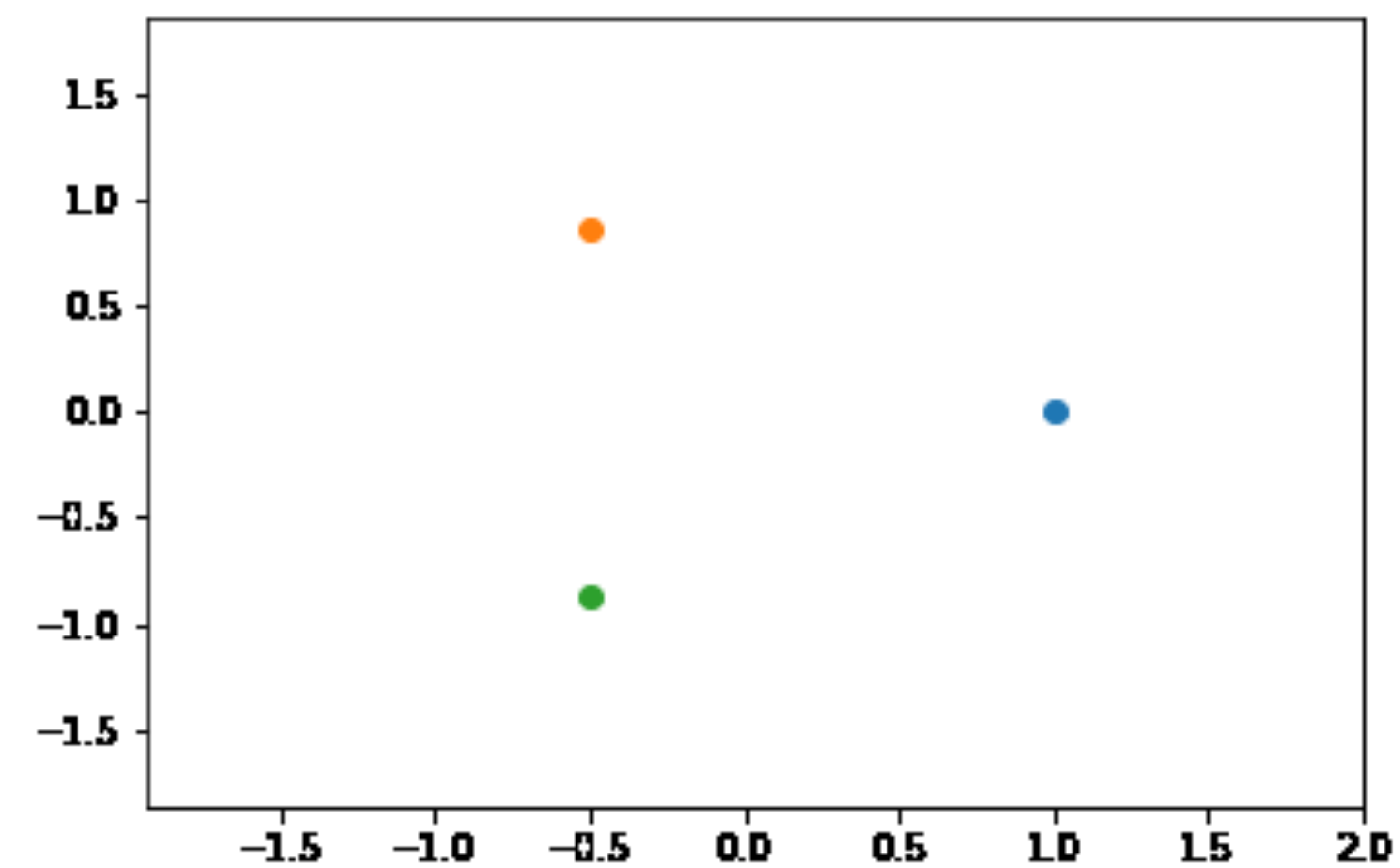
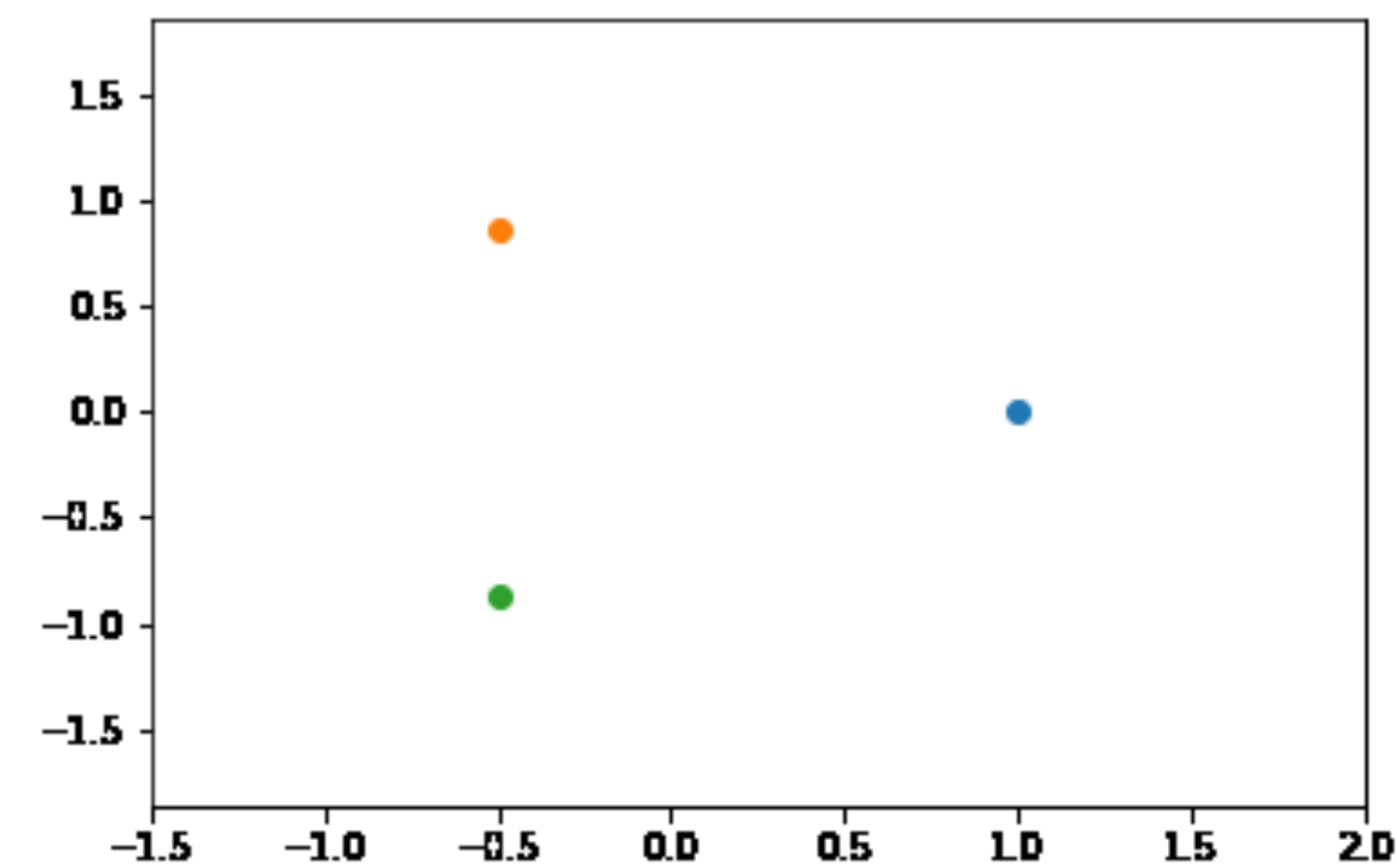
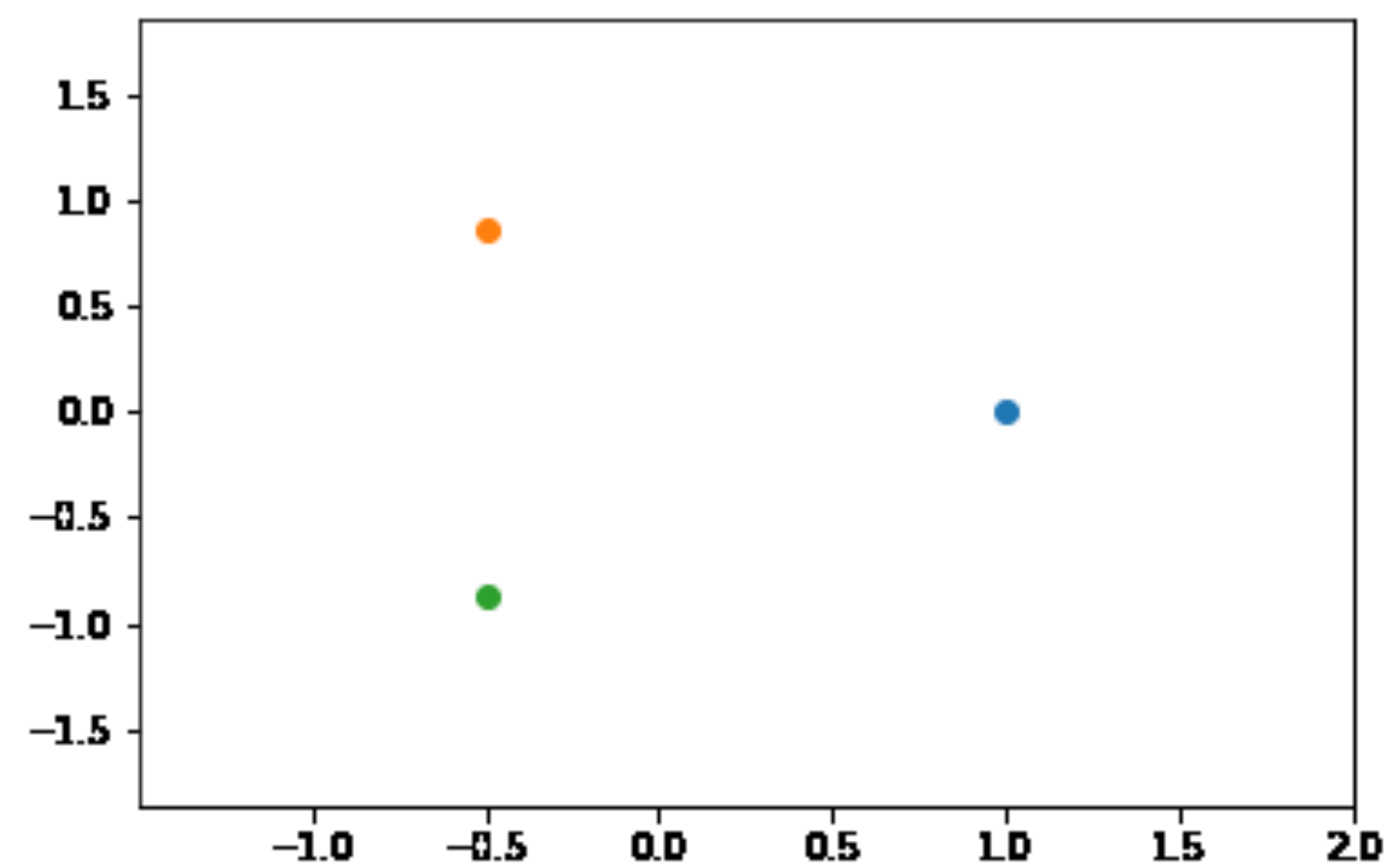
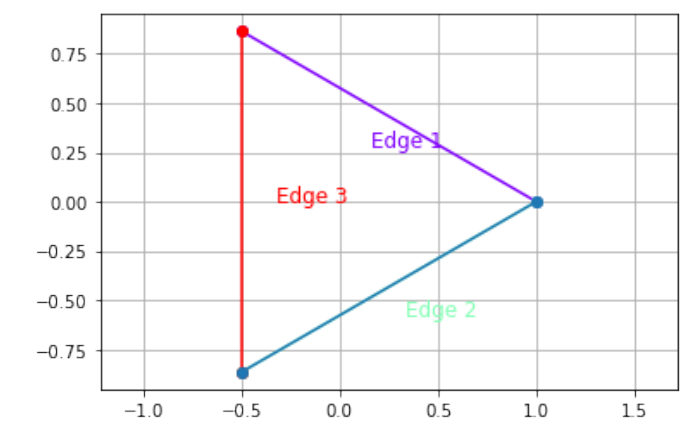
Kernel Density Estimate Plot of Exponential Stretching Rates



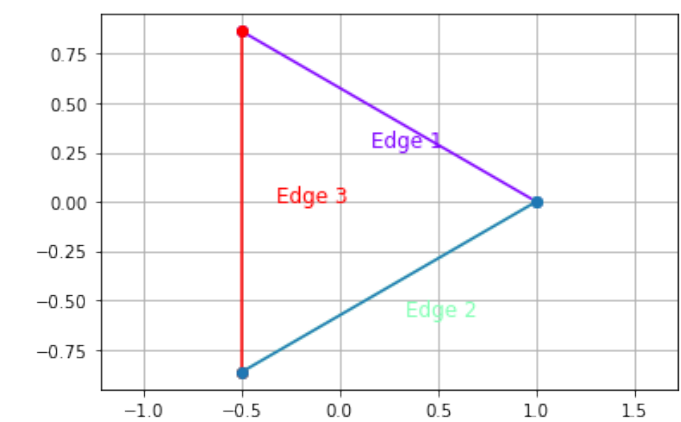
Lowest entropy mixing N=3 L=4



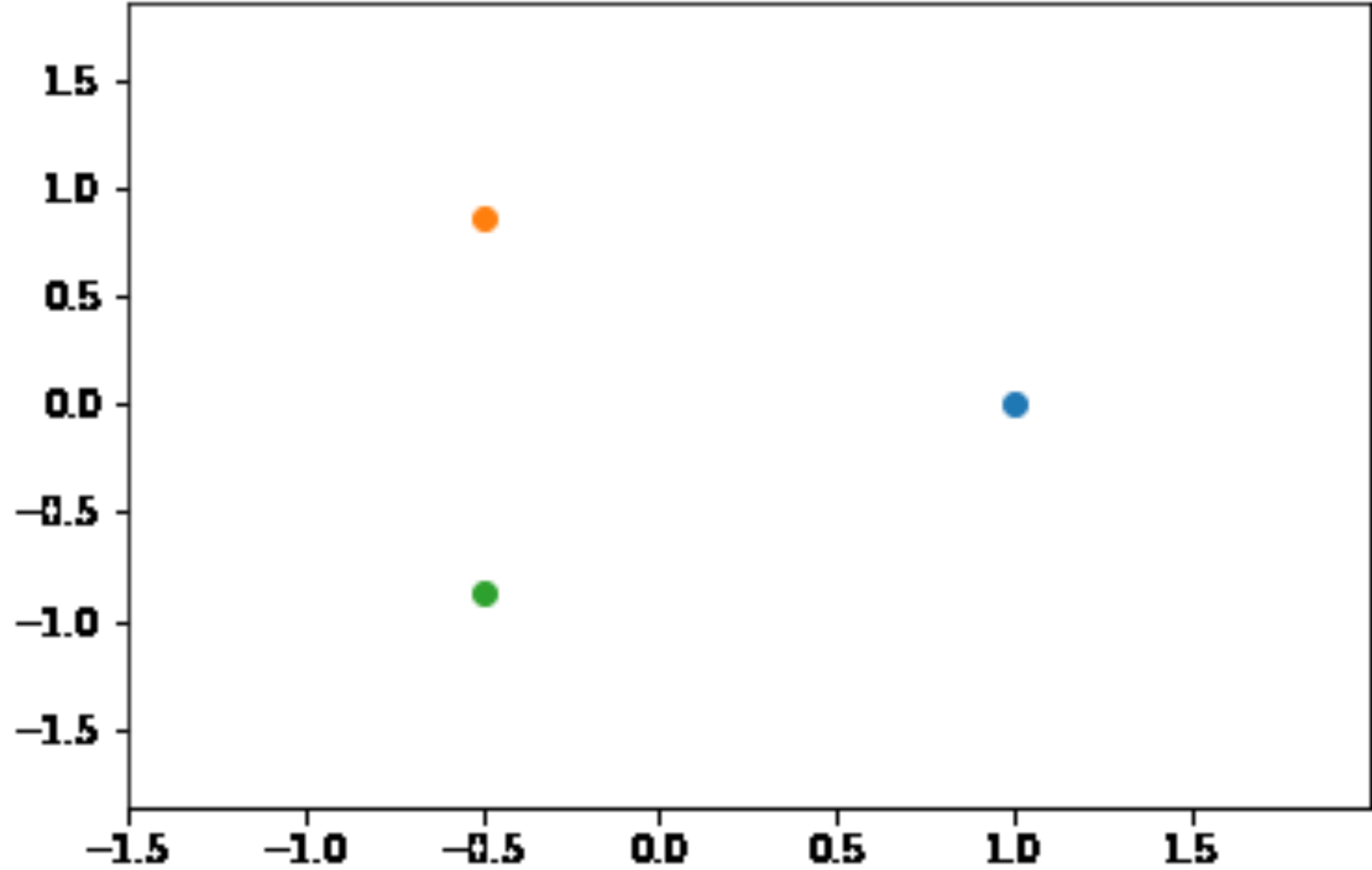
Highest entropy mixing N=3 L=4



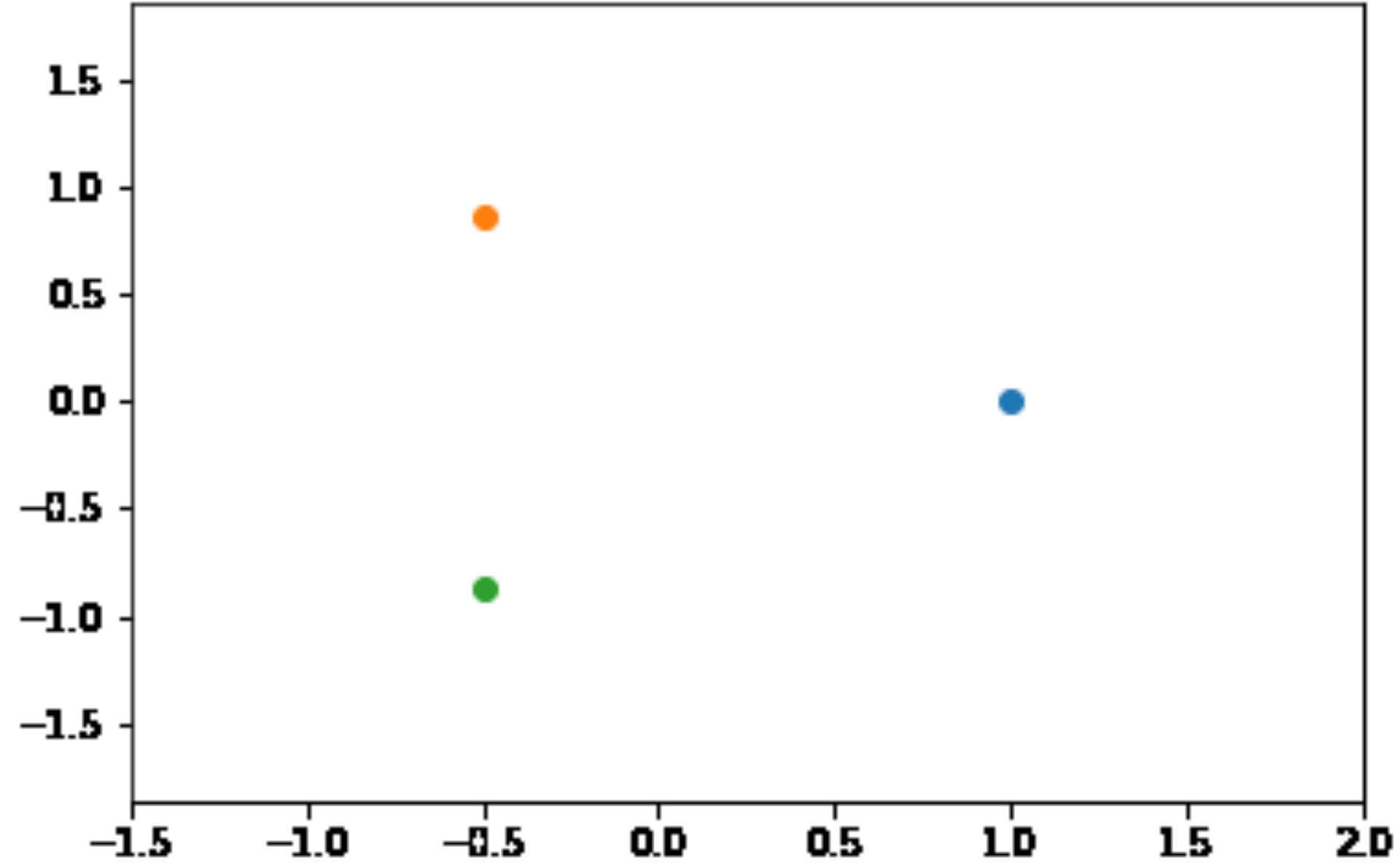
Comparison N=3 L=4



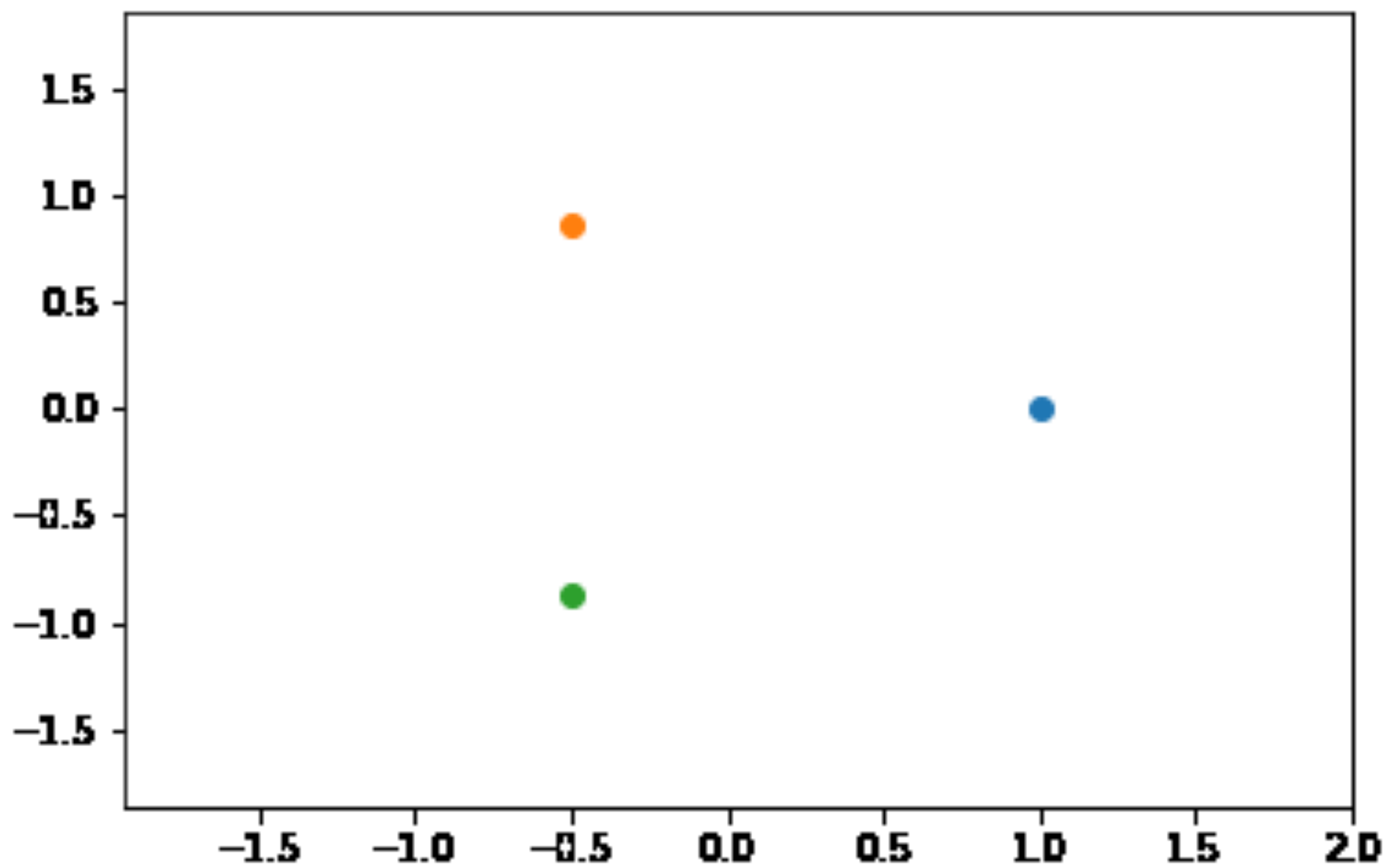
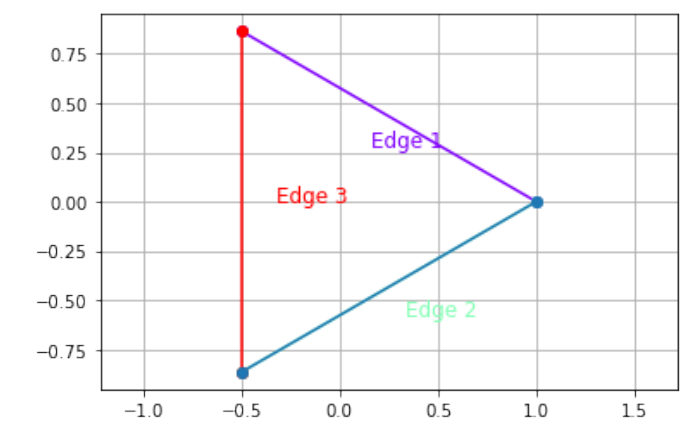
Low



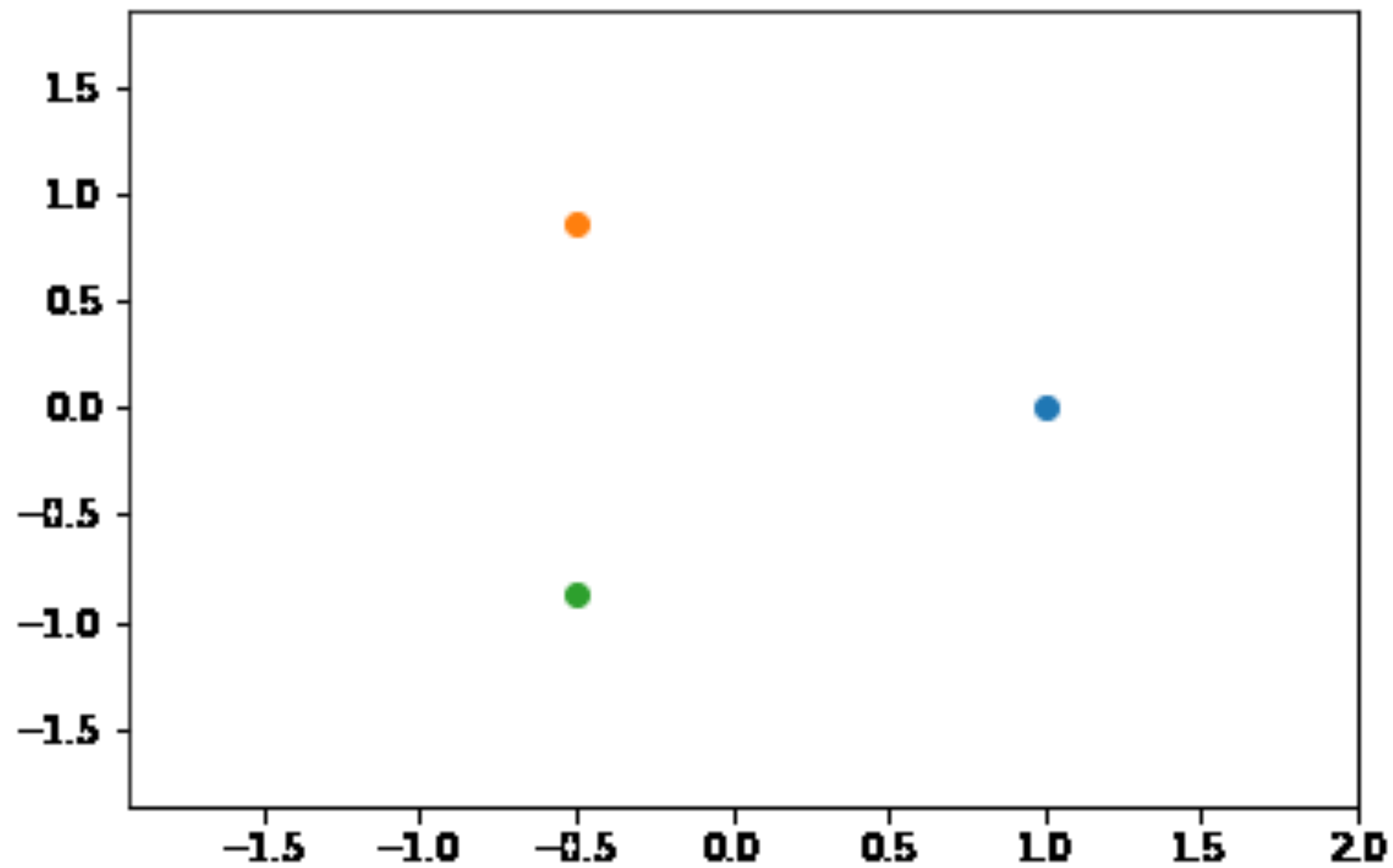
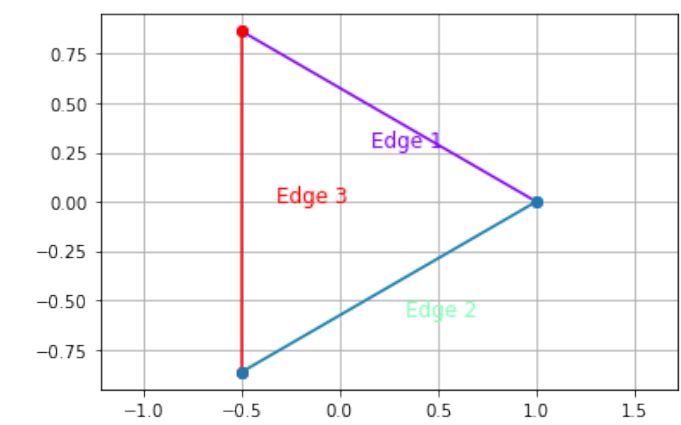
High



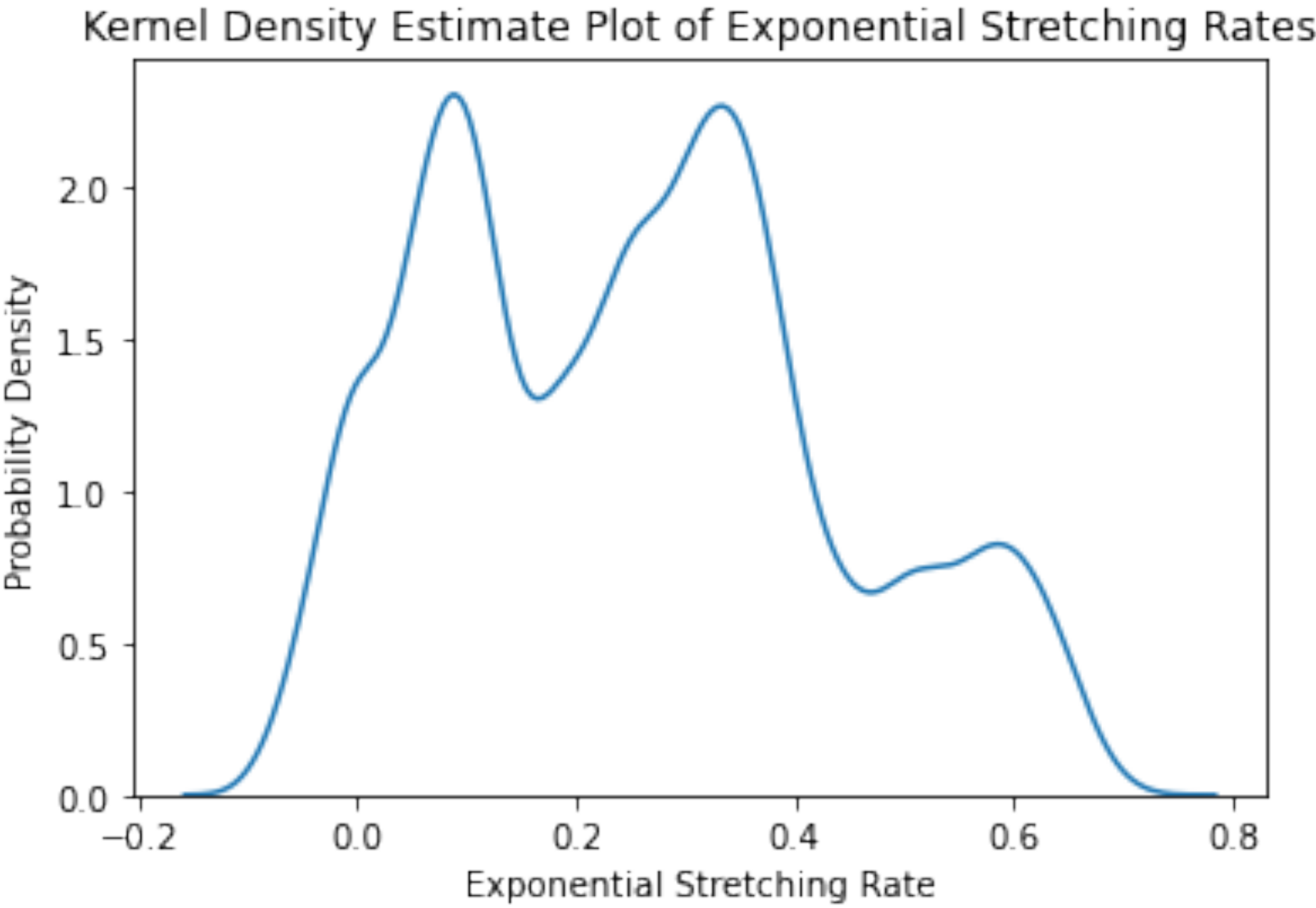
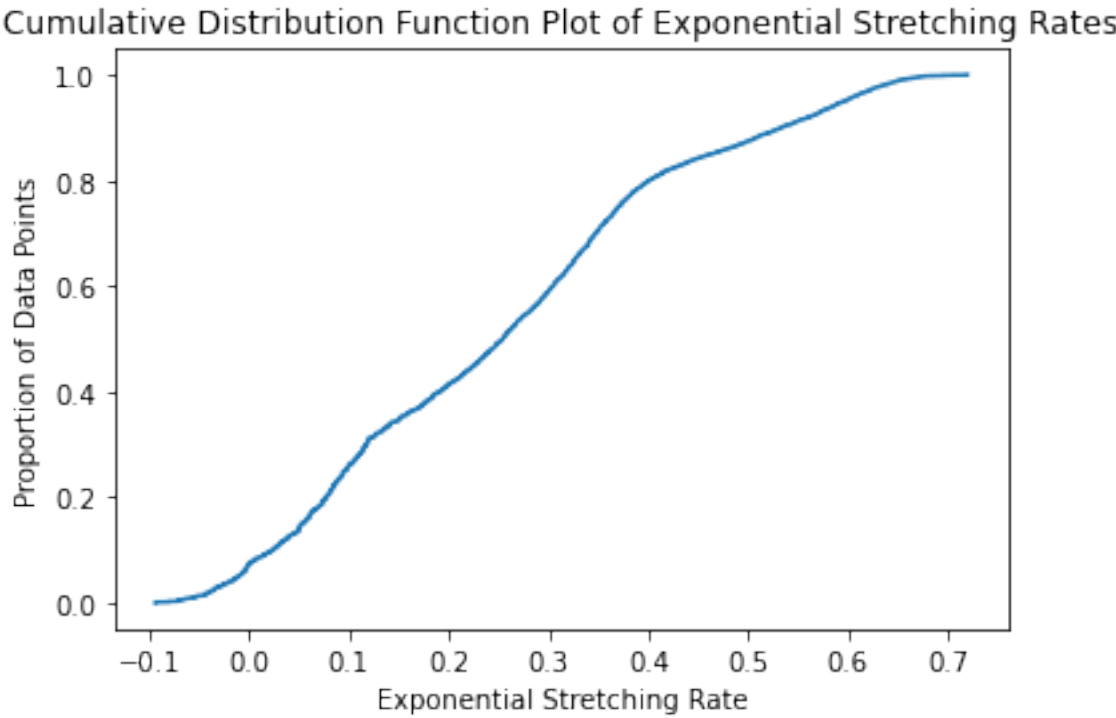
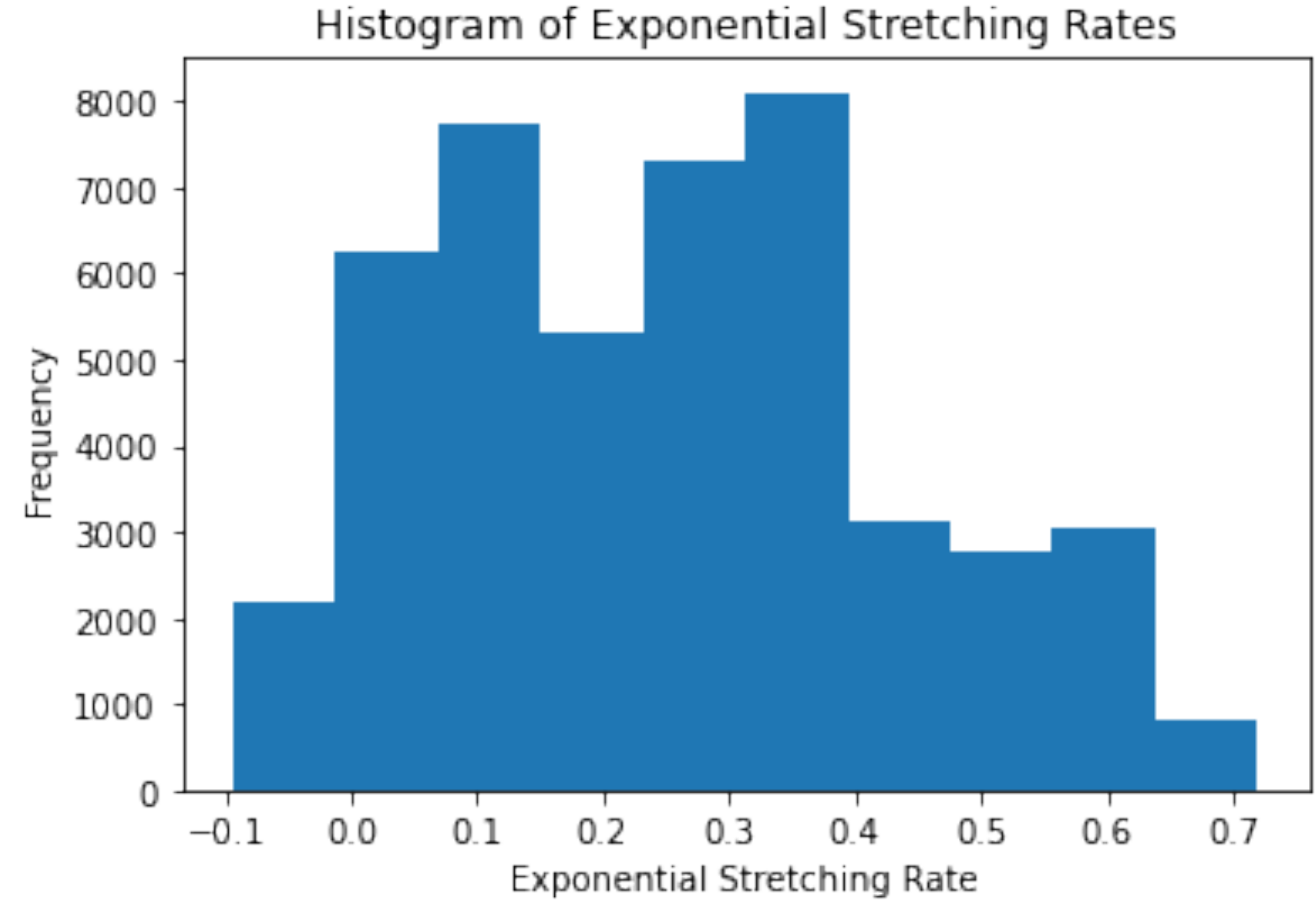
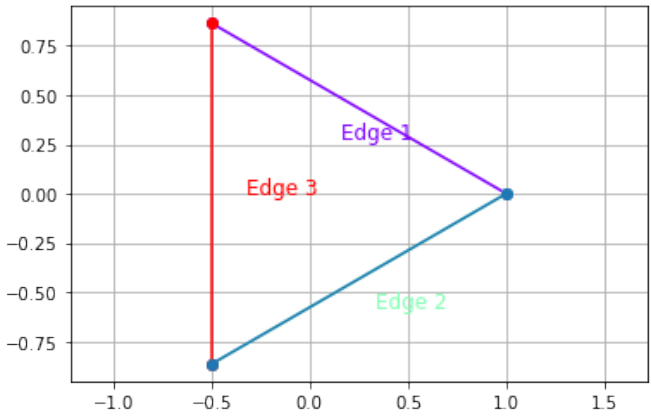
Lowest entropy mixing N=3 L=6



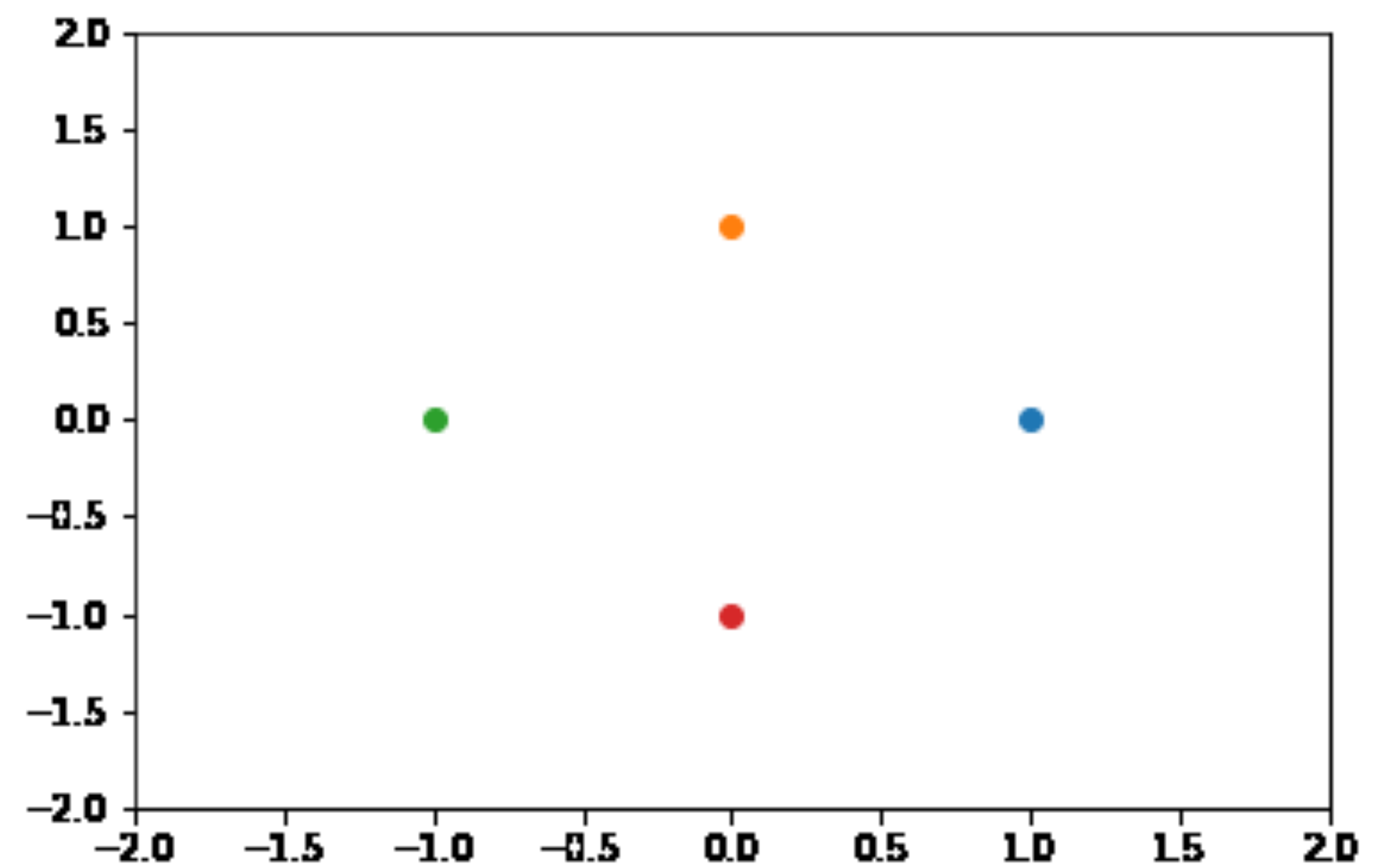
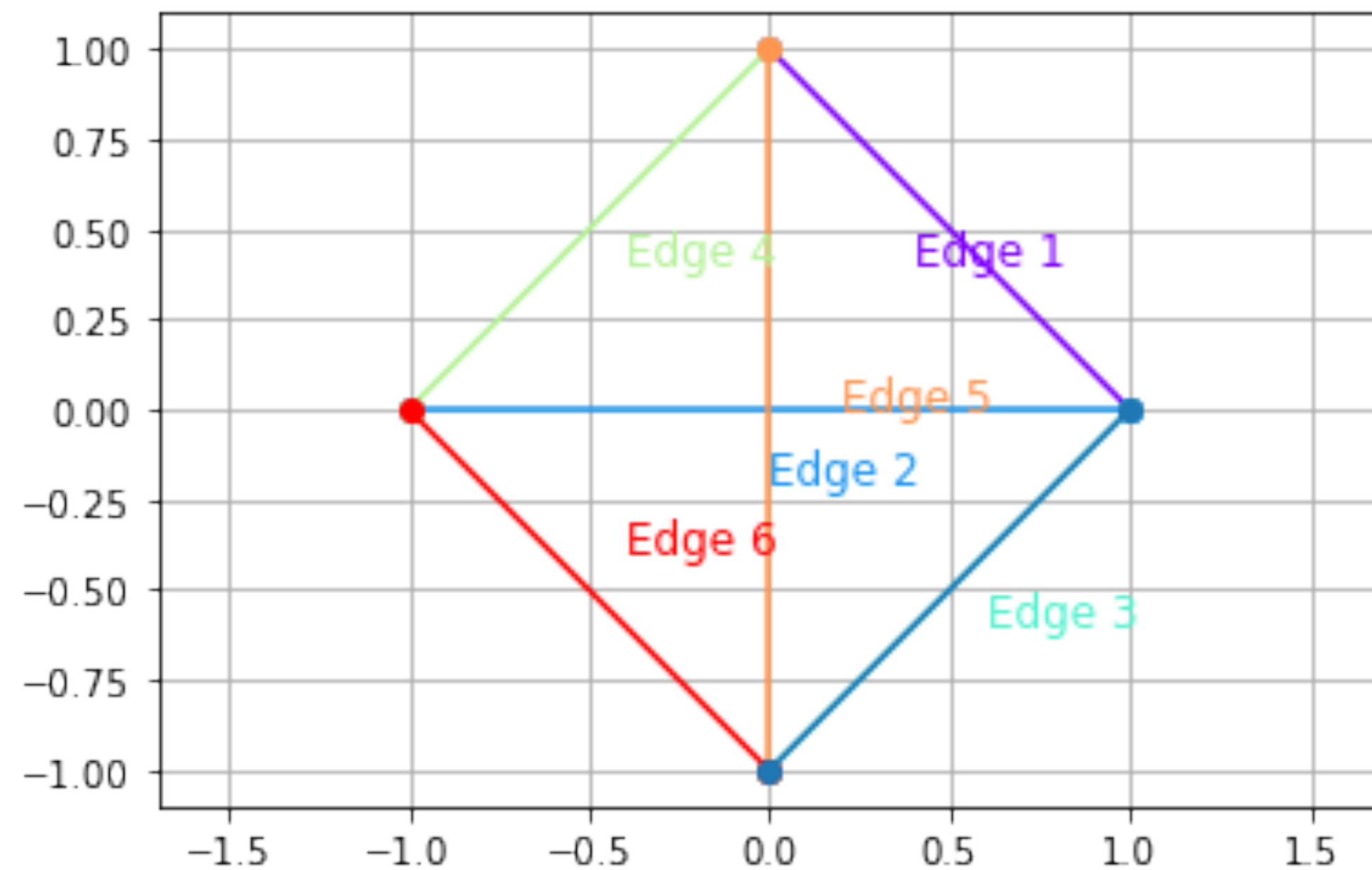
Highest entropy mixing N=3 L=6



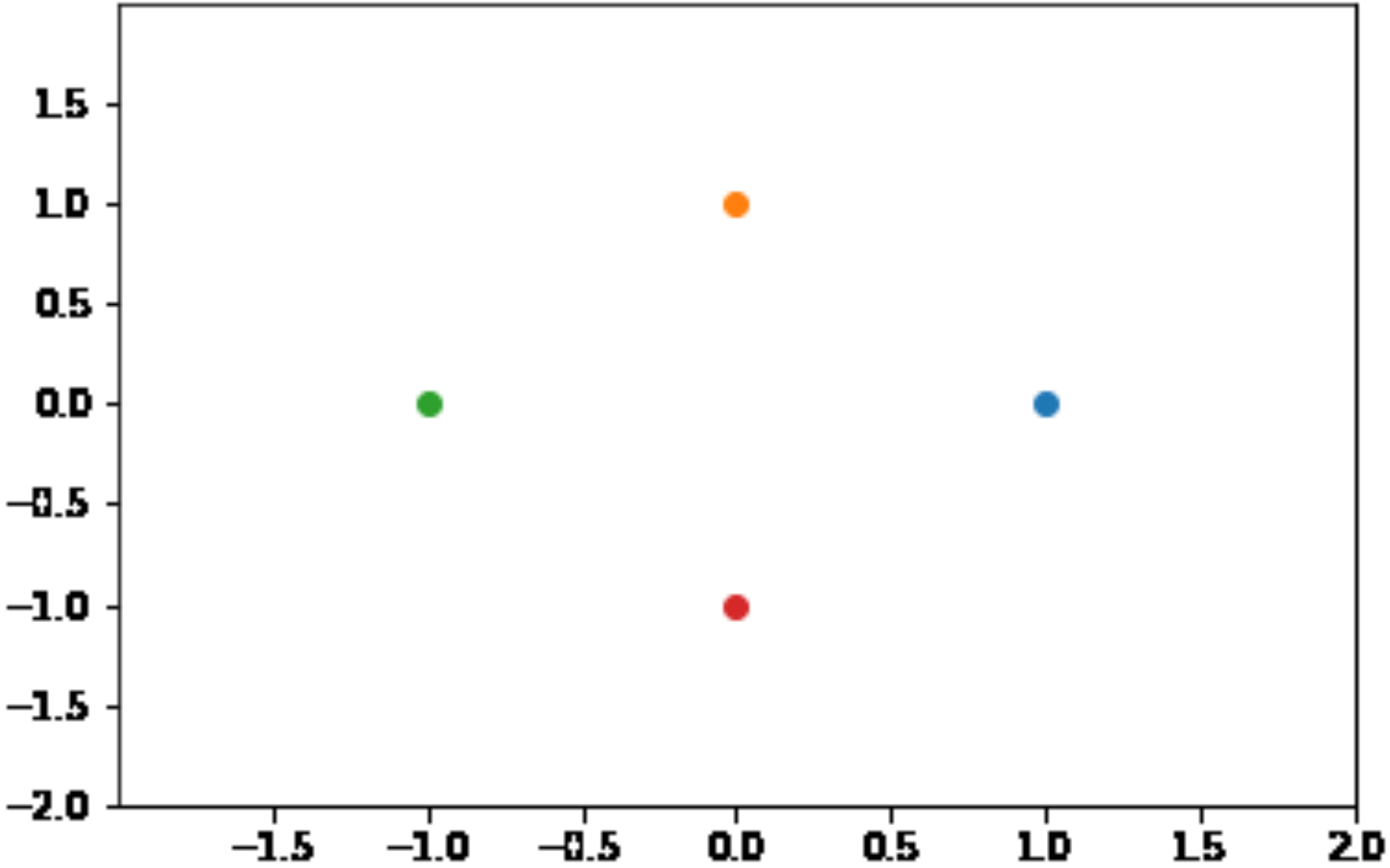
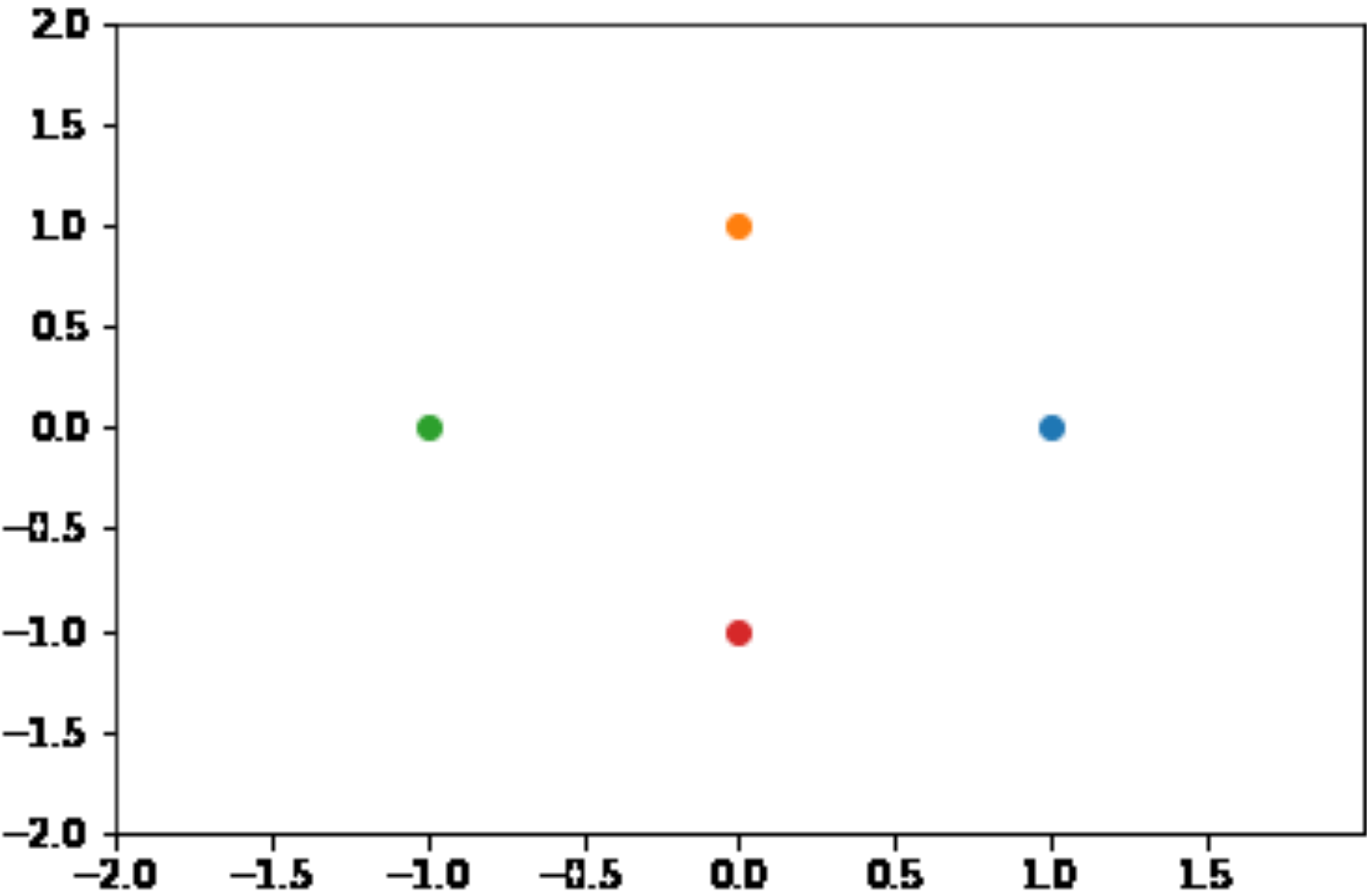
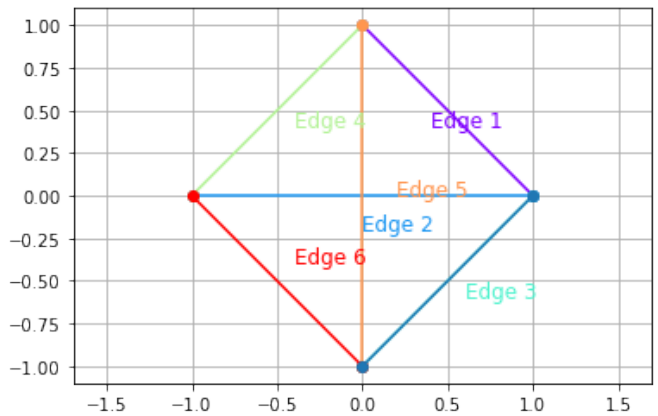
Mixing of N=3 L=6



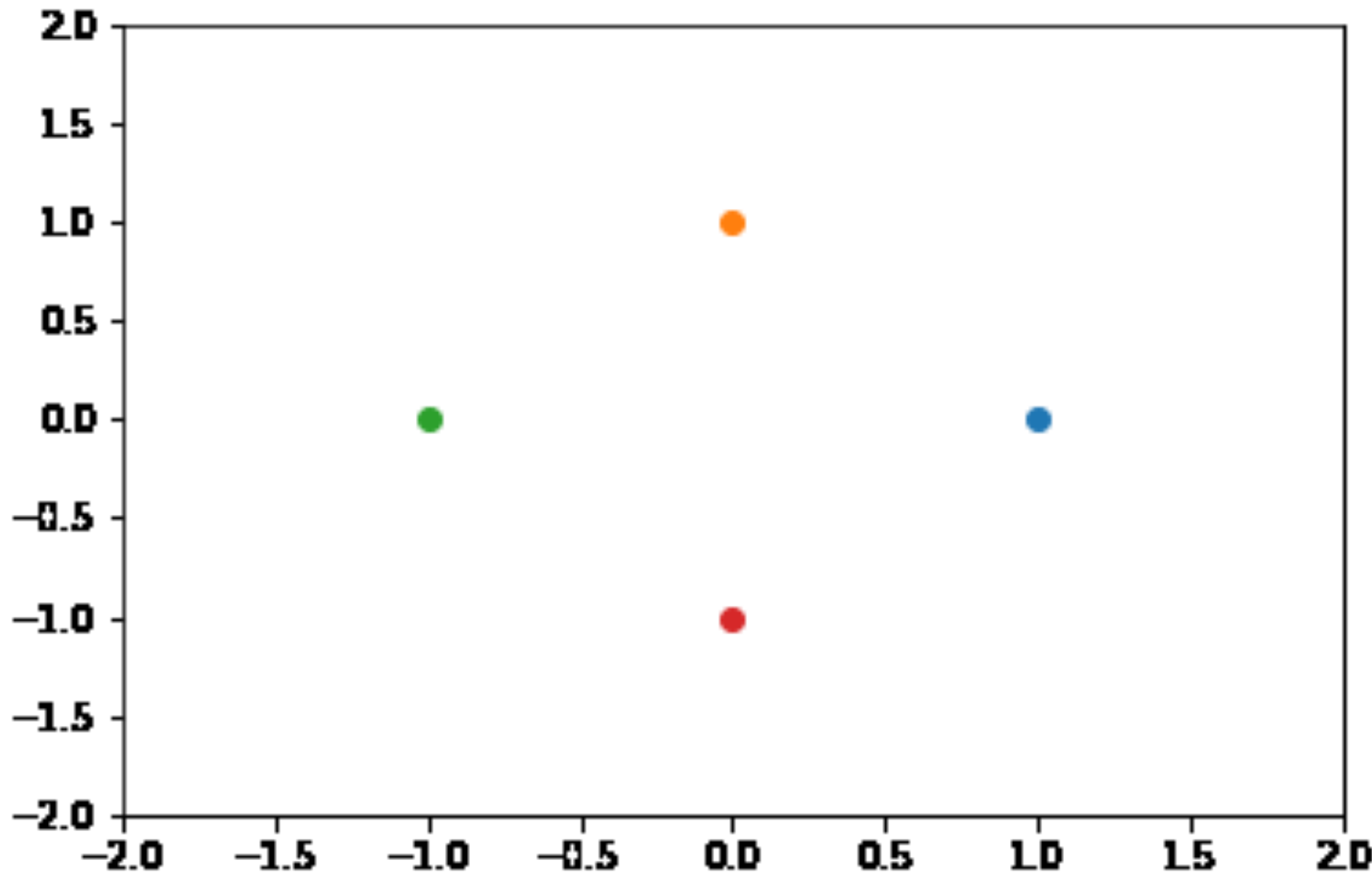
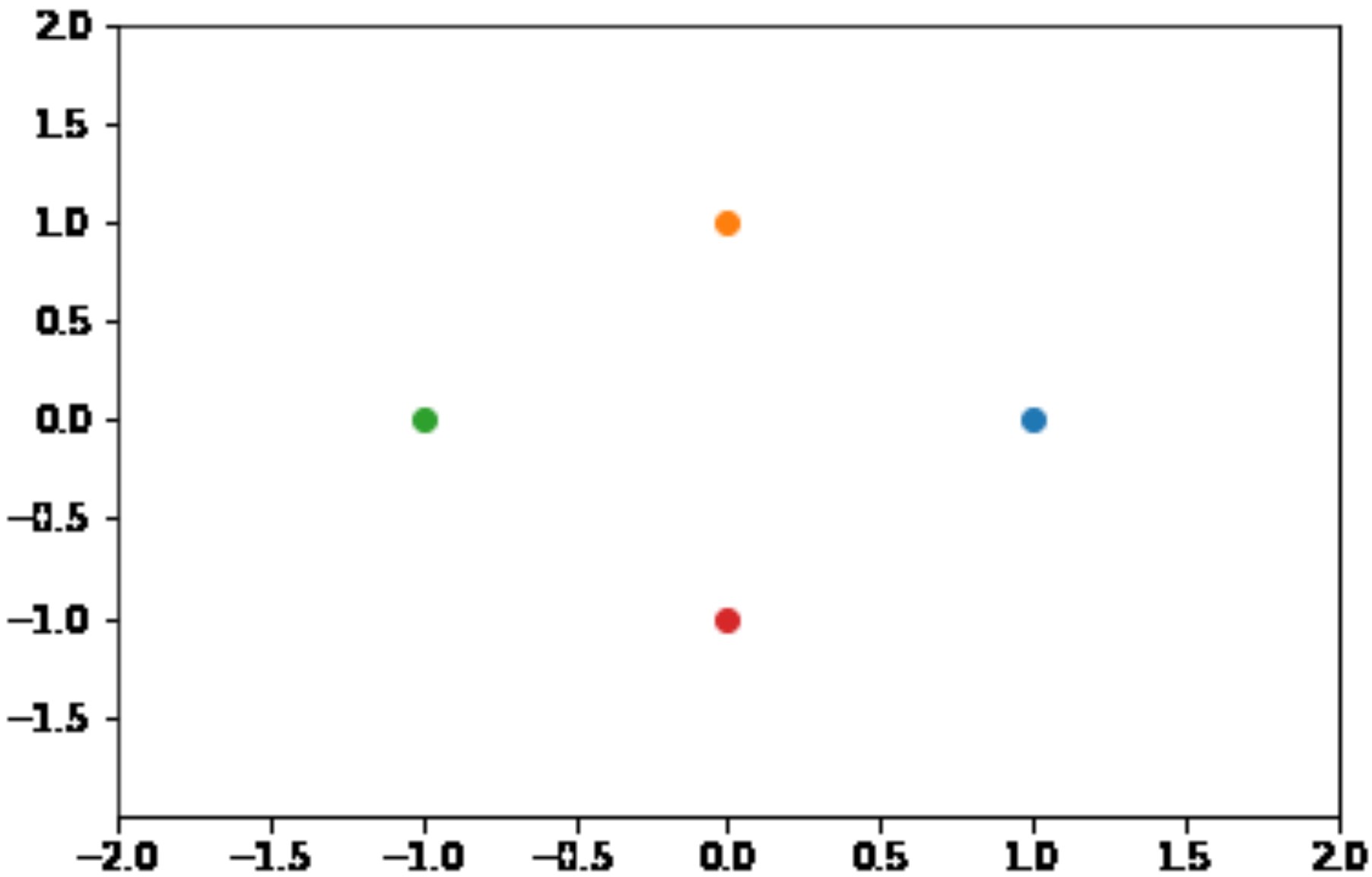
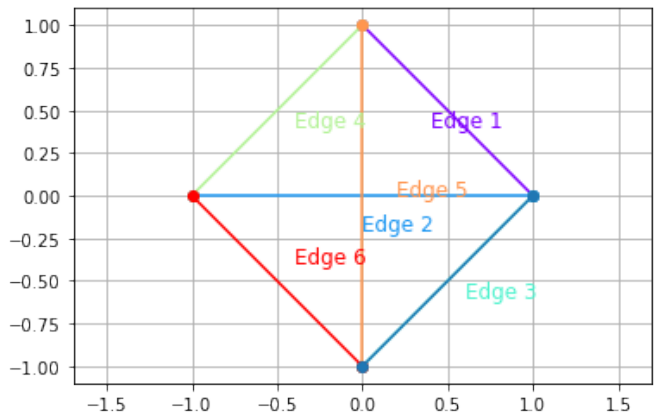
4-Rod Mixing



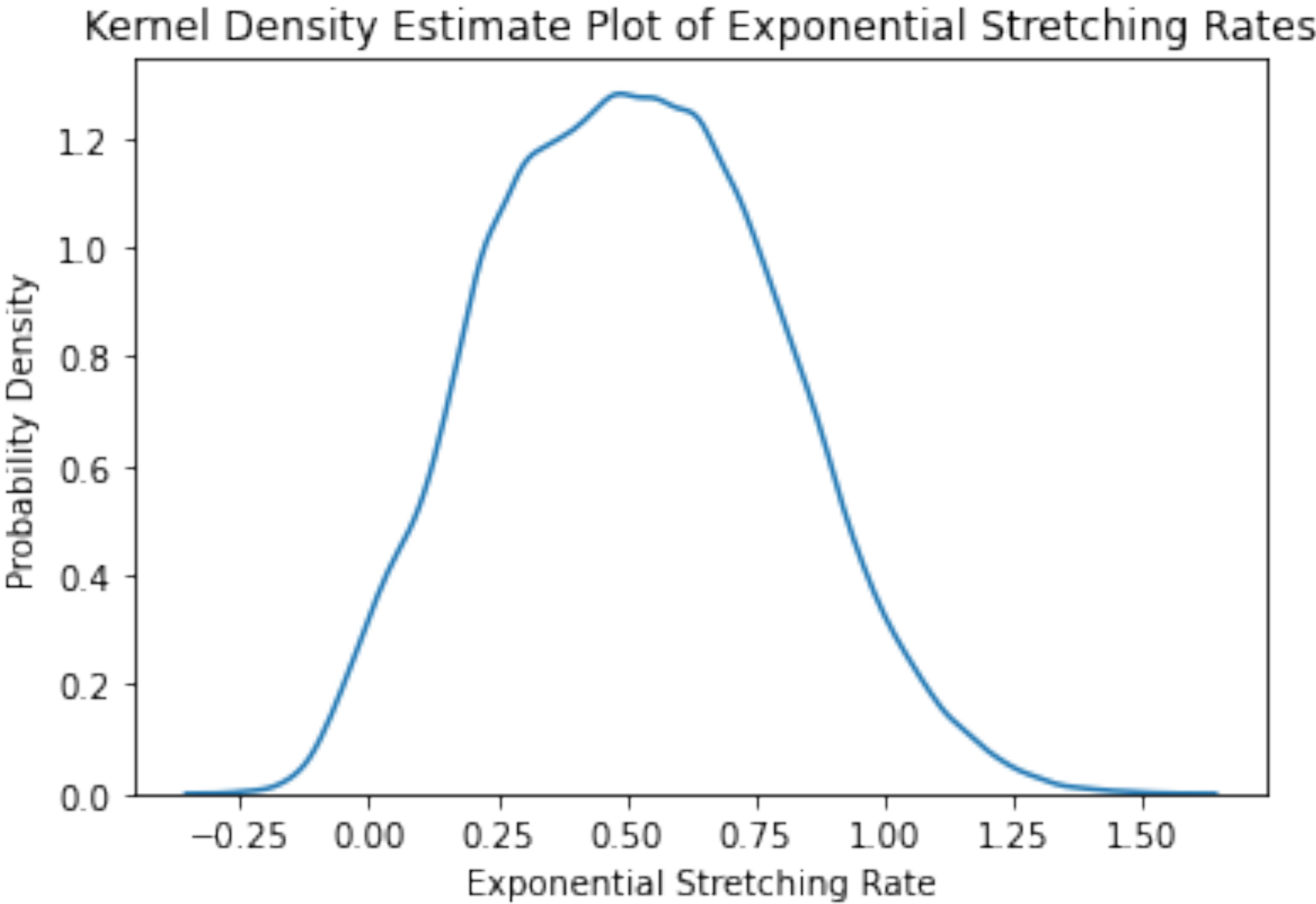
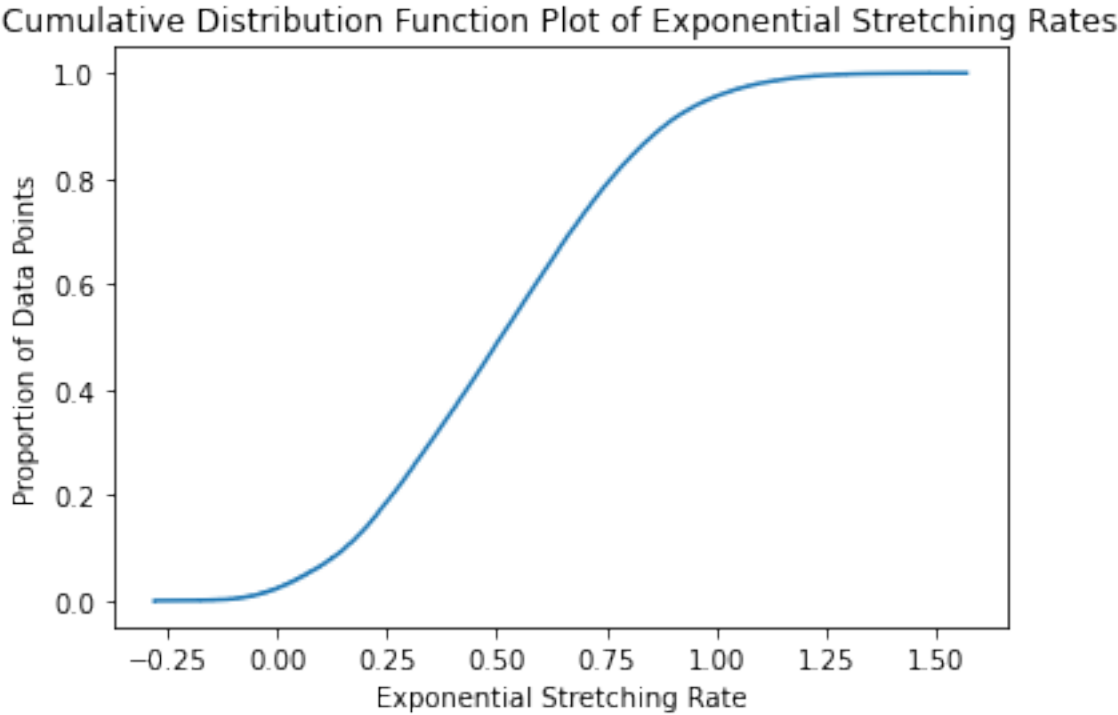
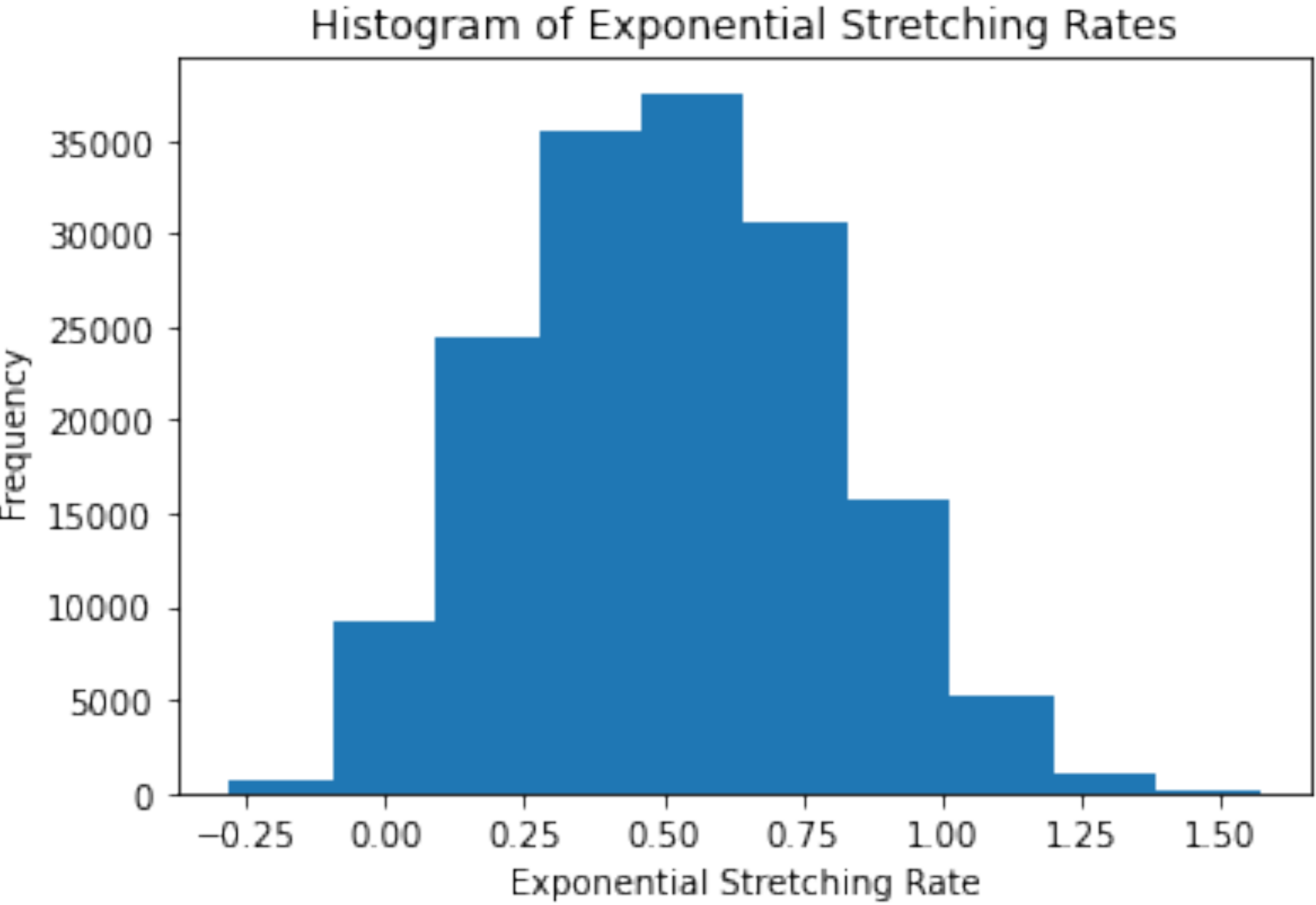
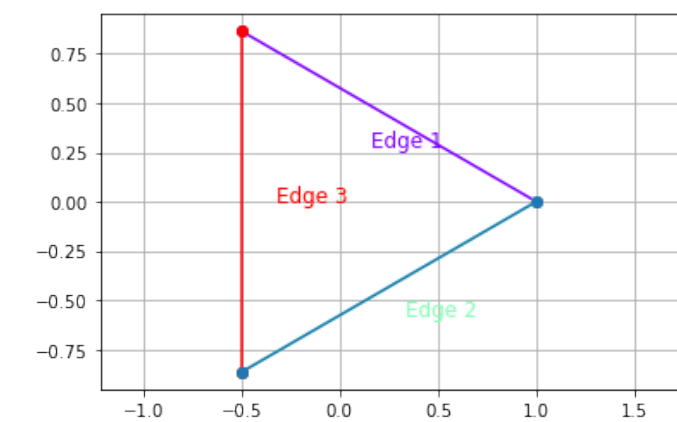
Lowest entropy mixing N=4 L=4



Highest entropy mixing N=4 L=4



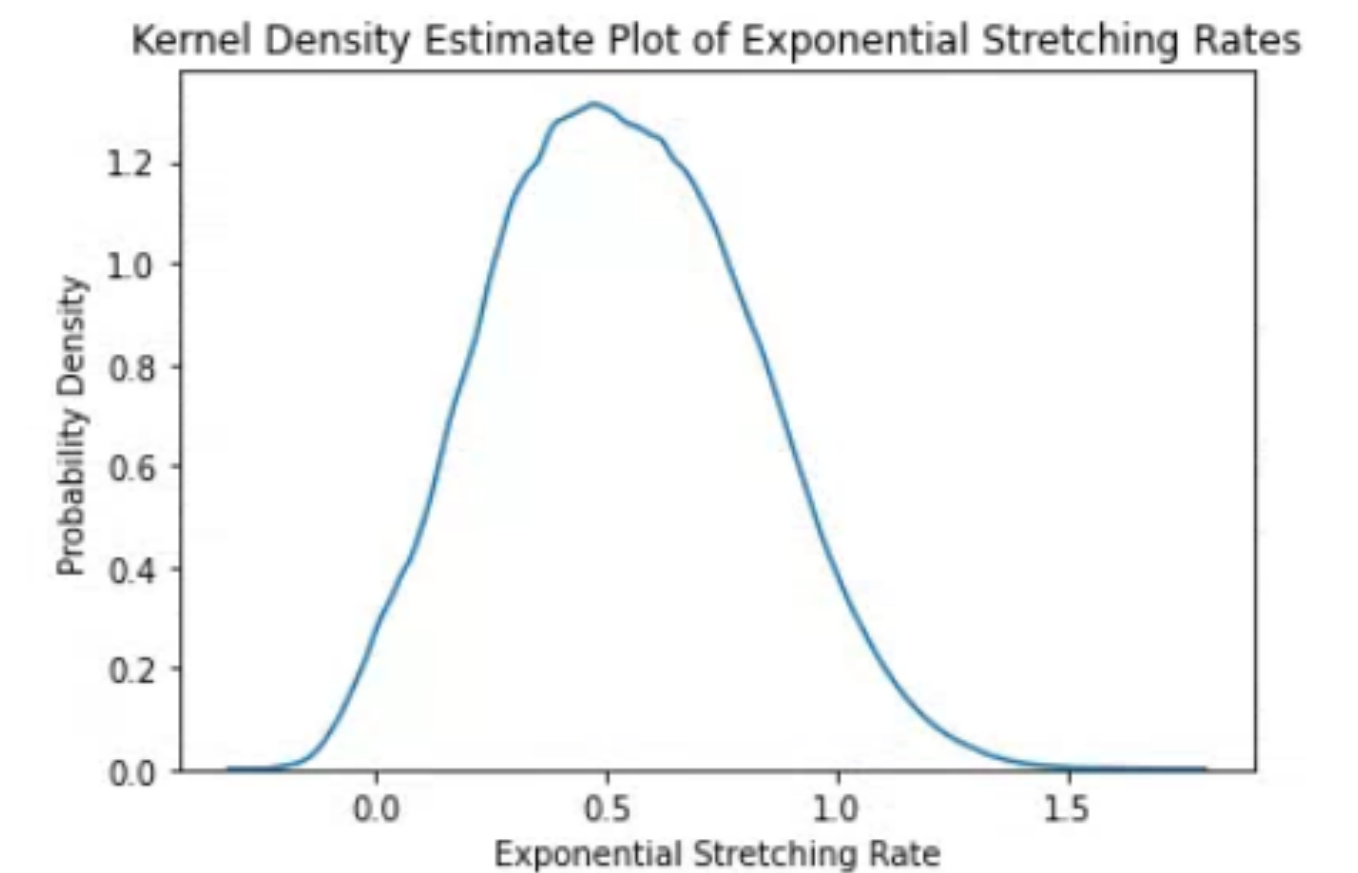
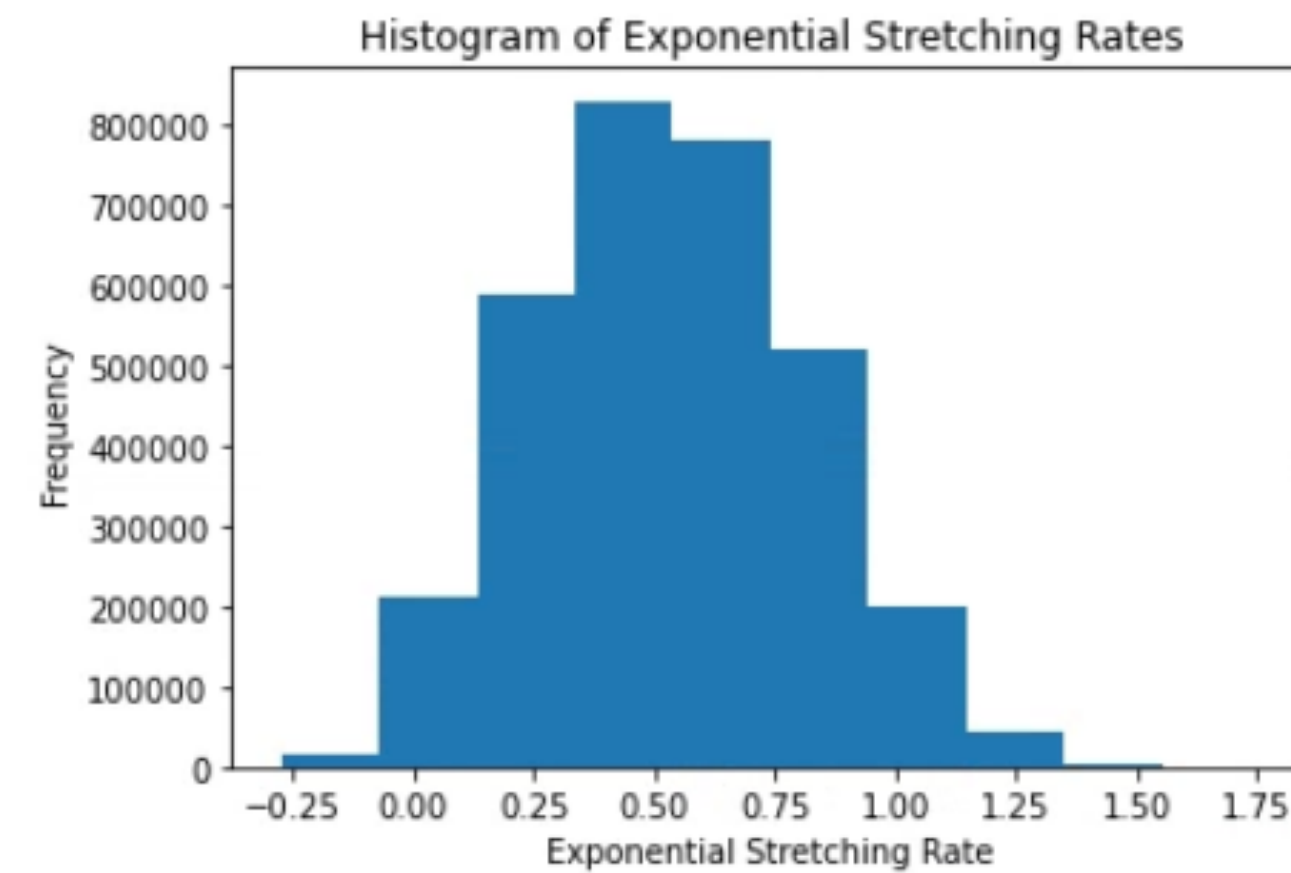
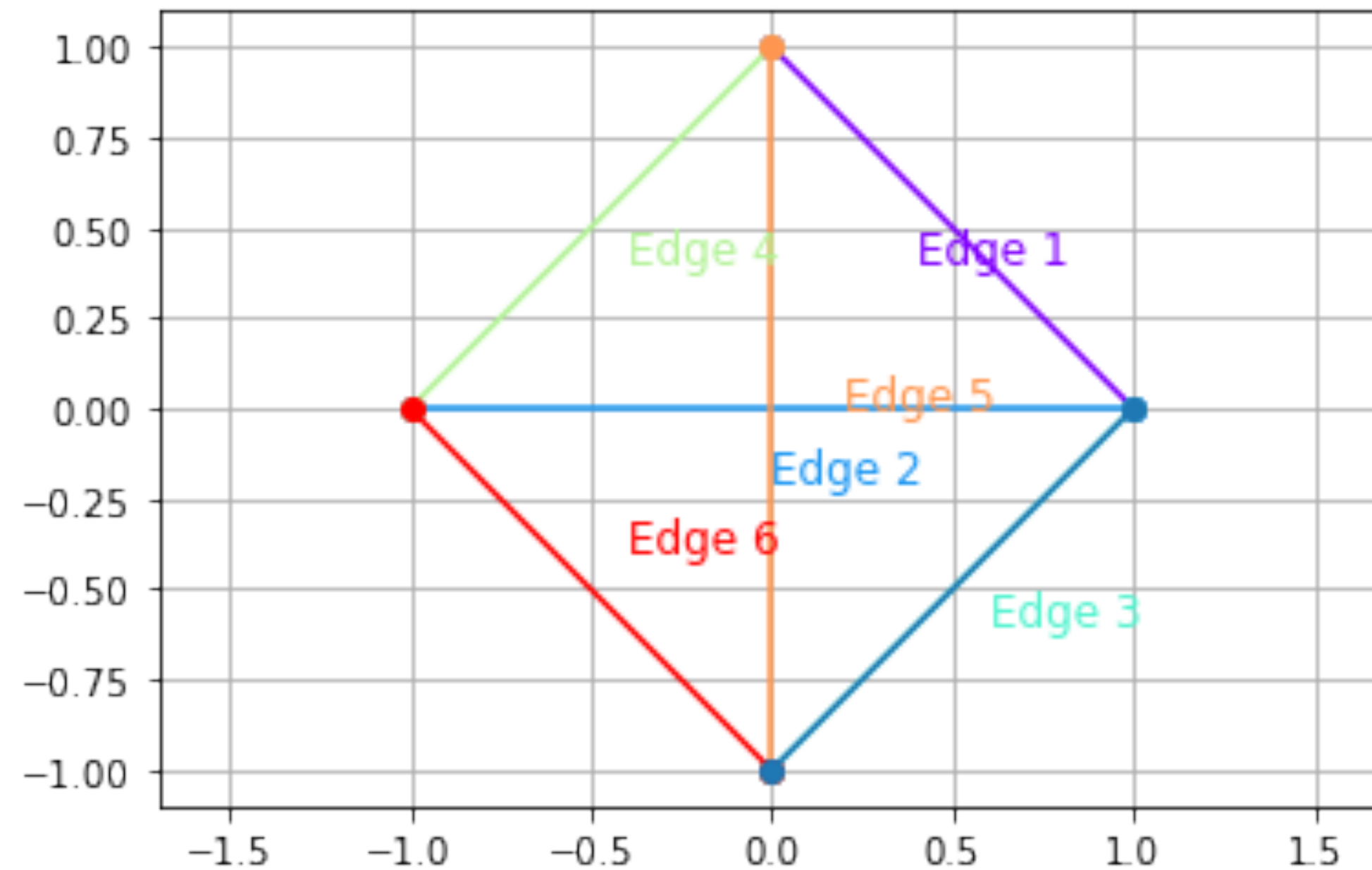
Mixing of N=4 L=4



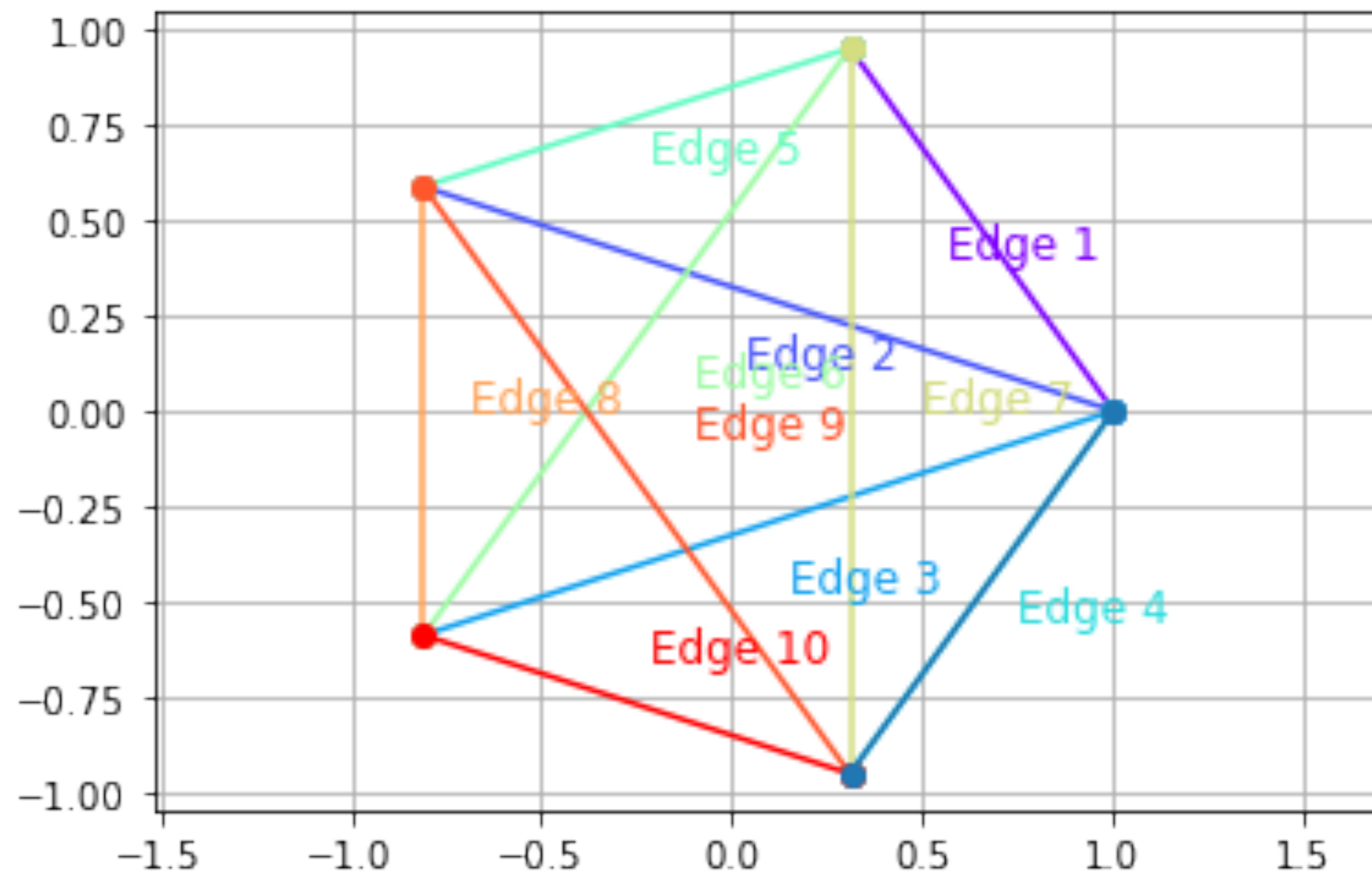
4-Rod Mixing

Length 4 -> 5 9 hrs

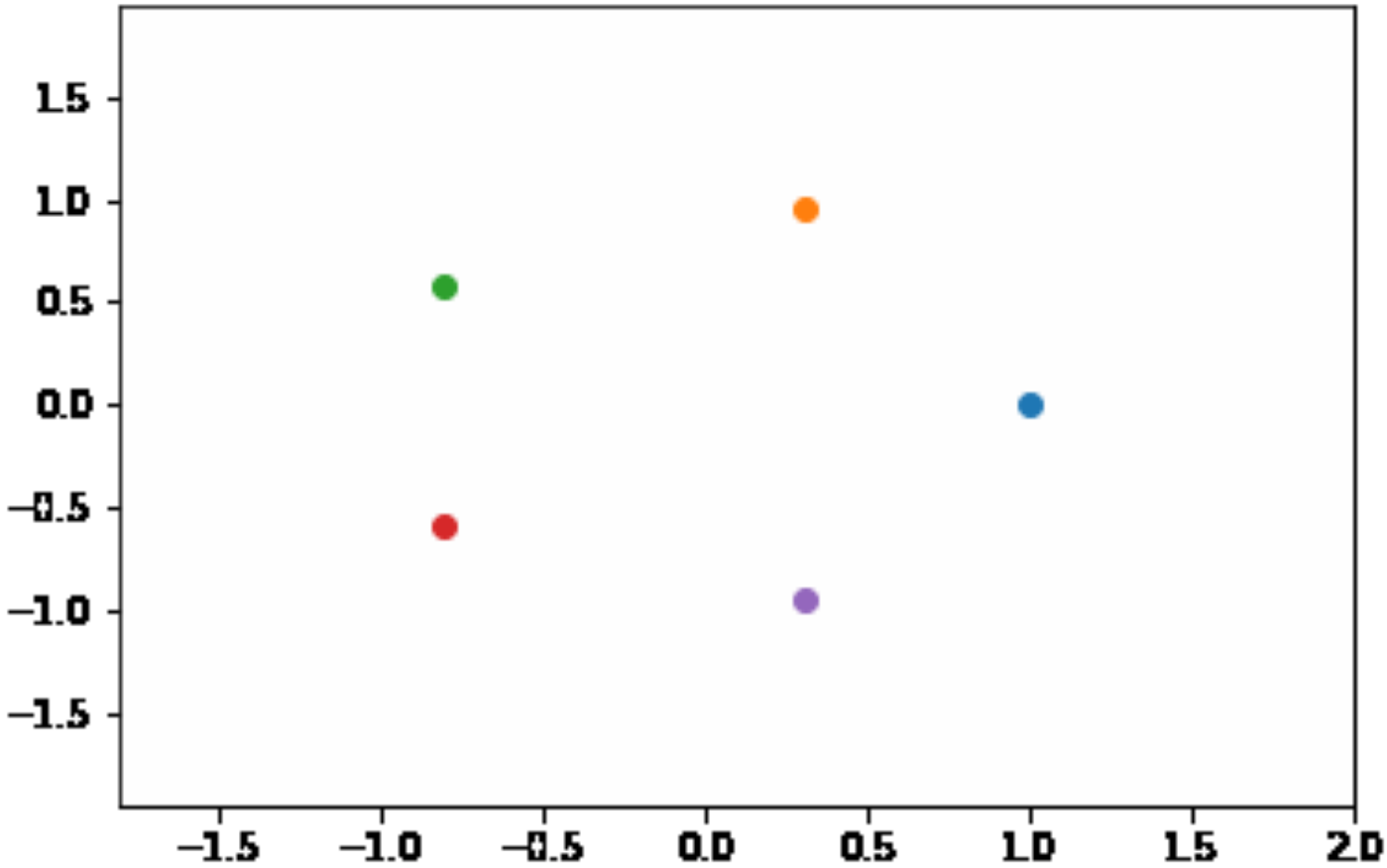
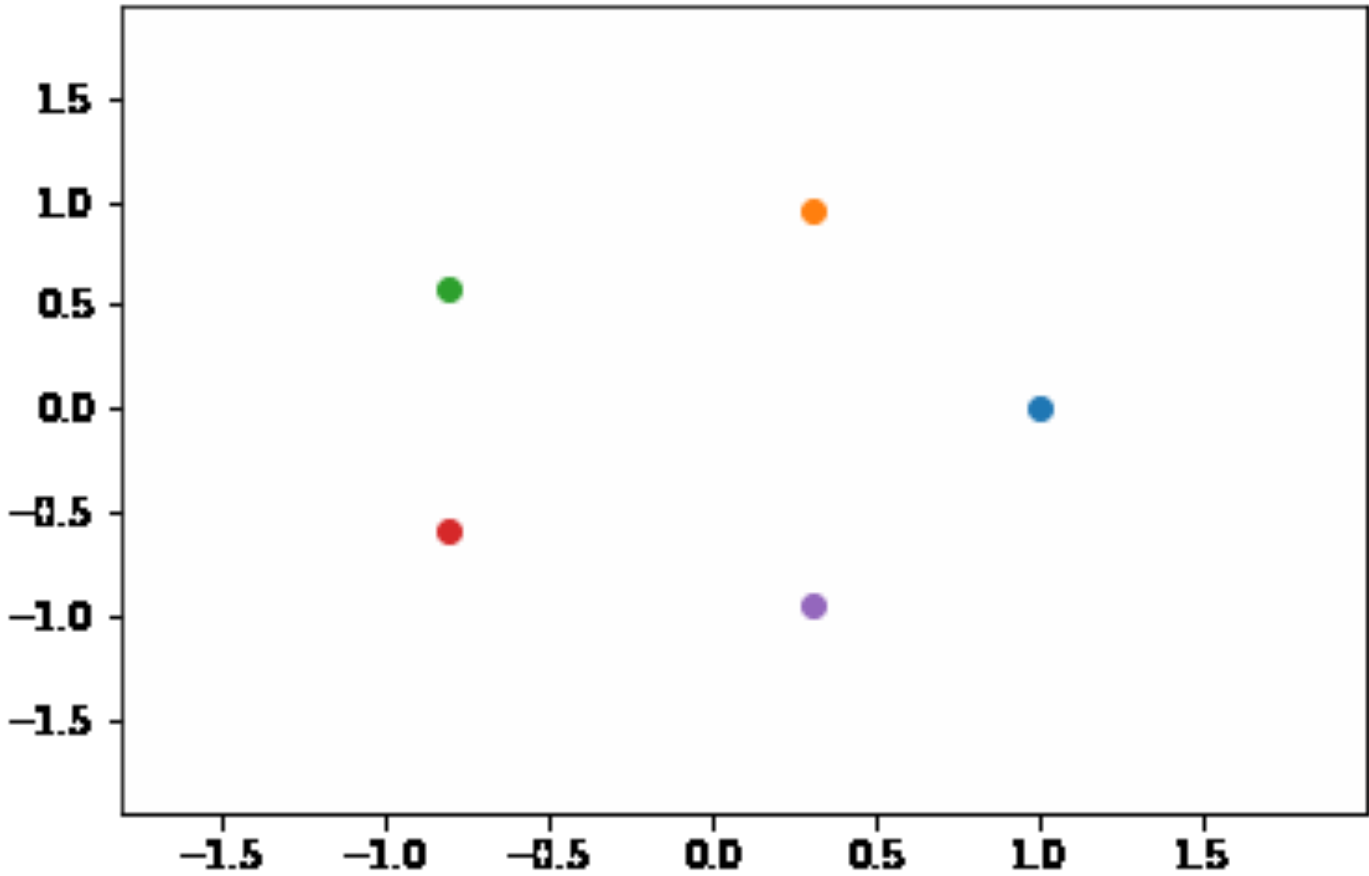
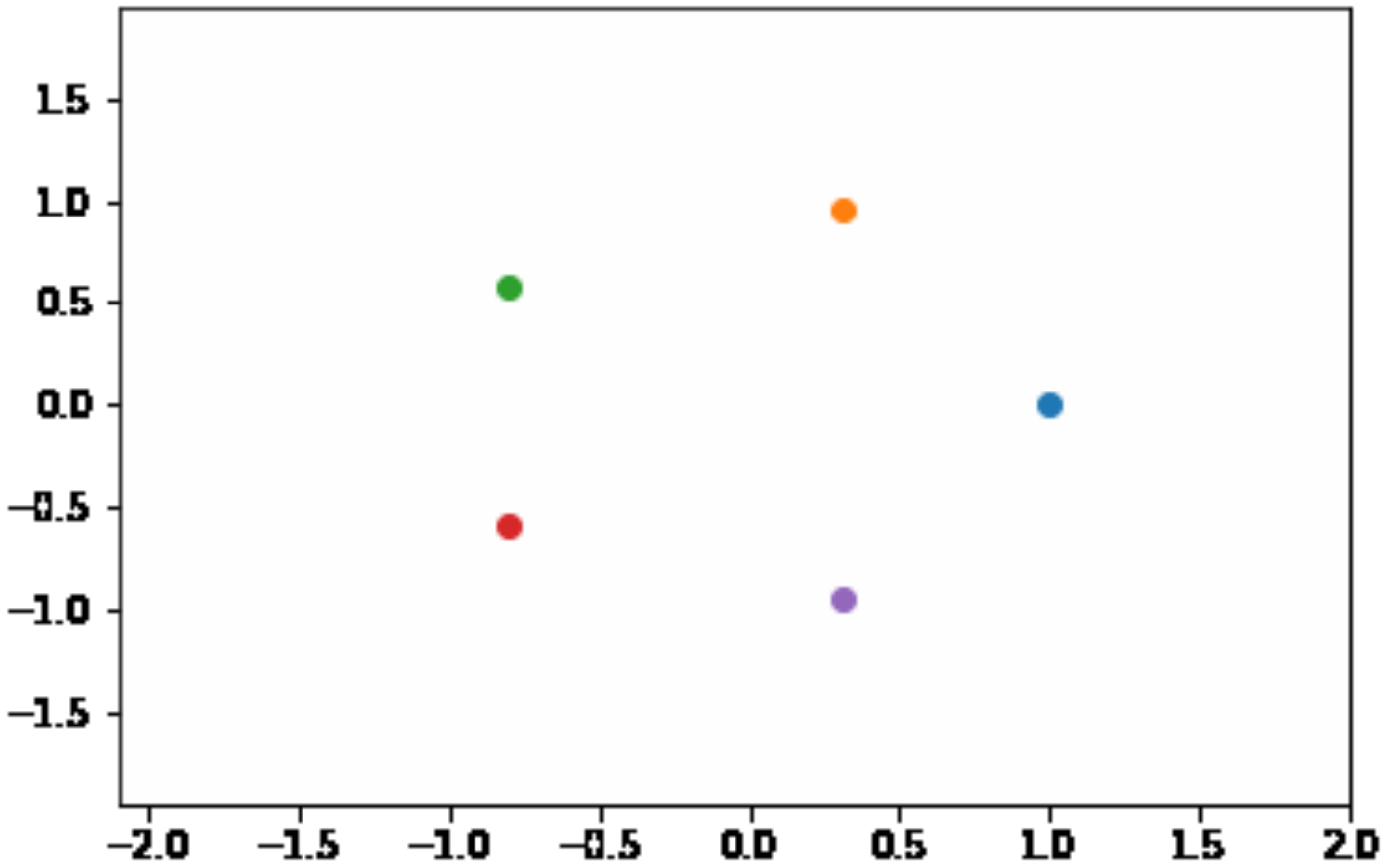
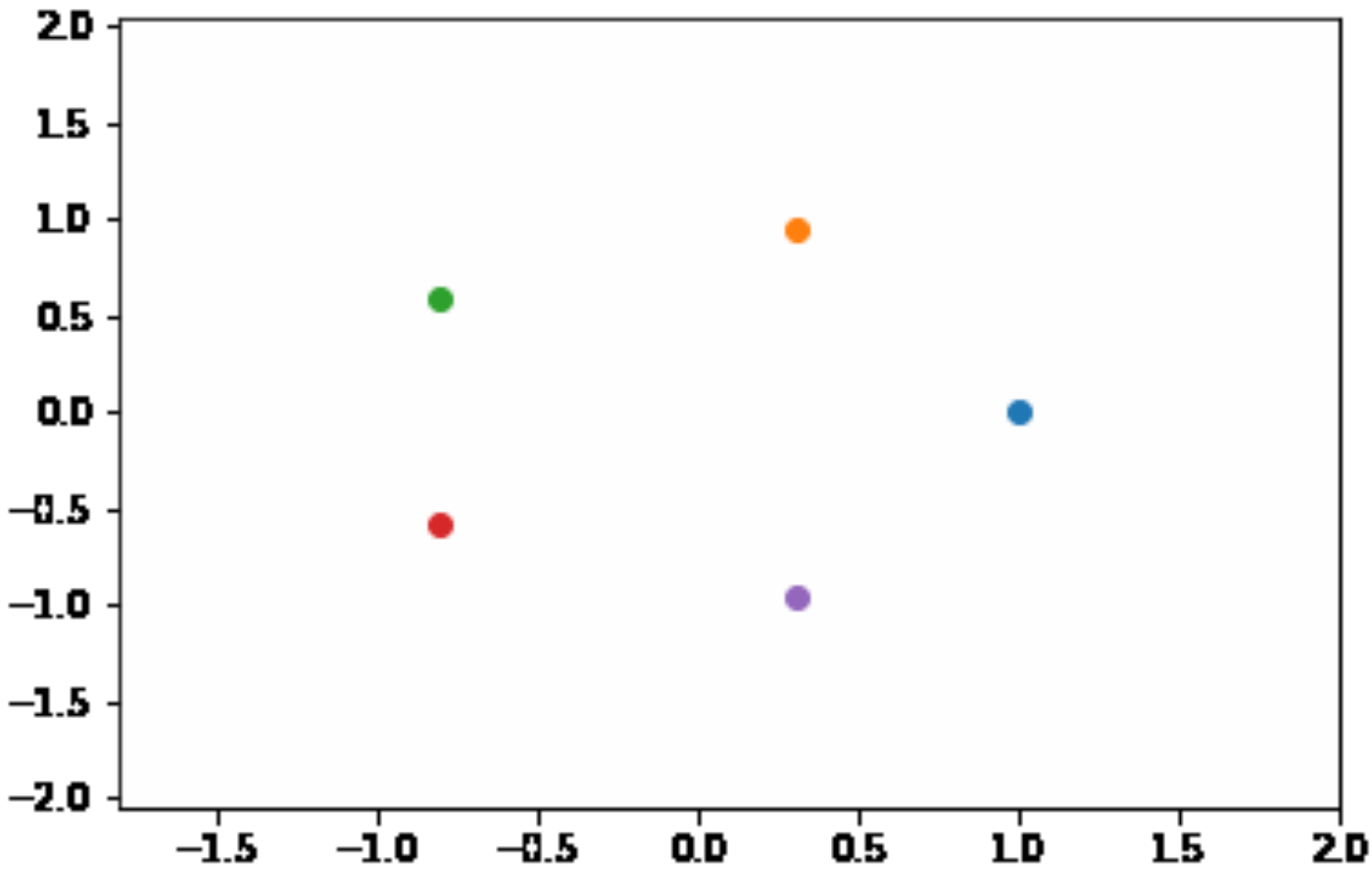
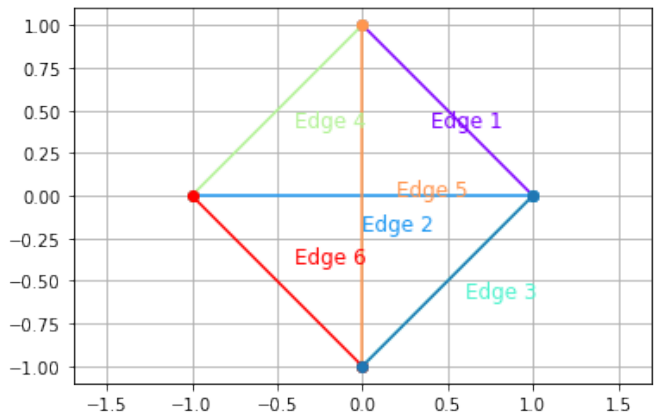
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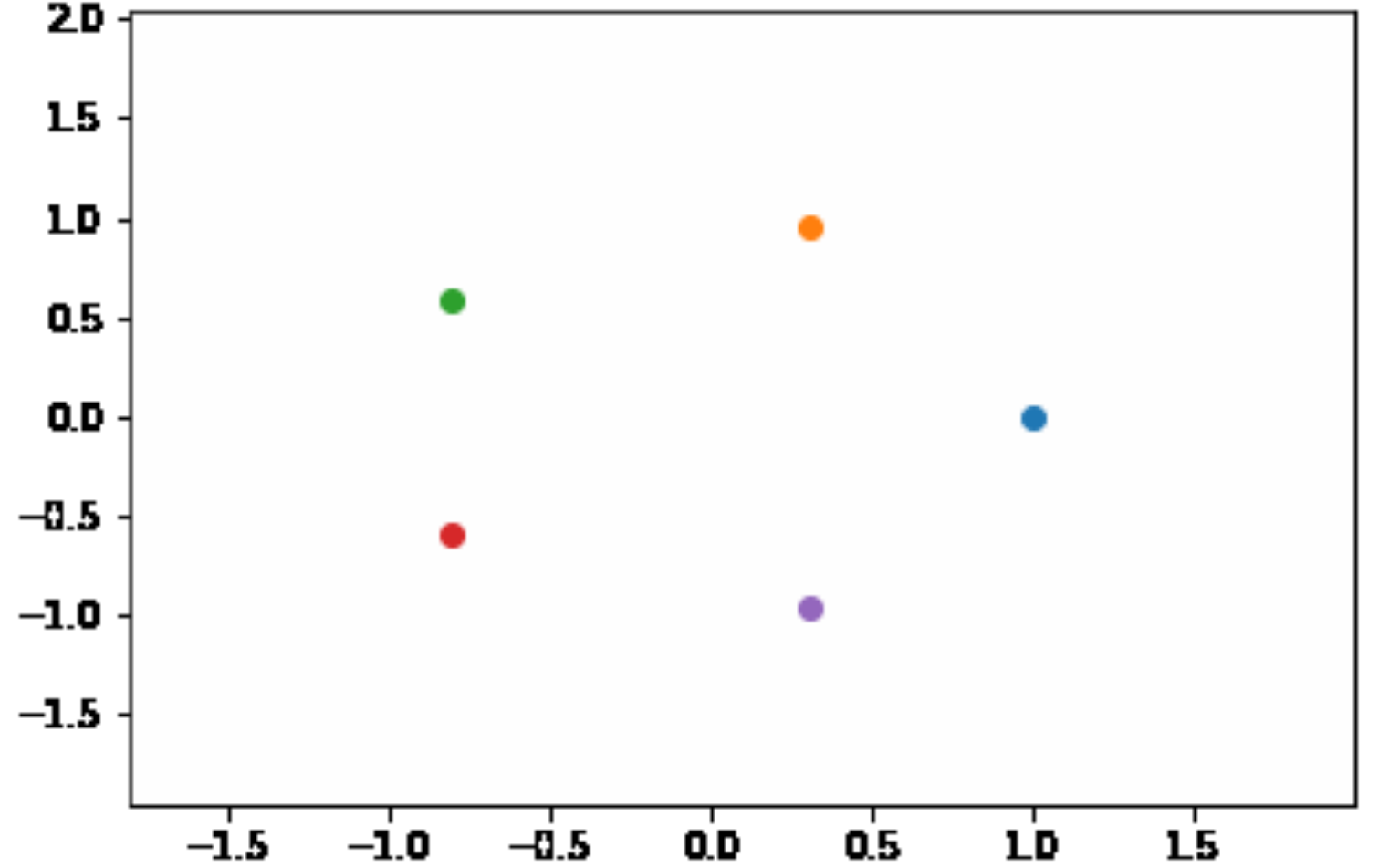
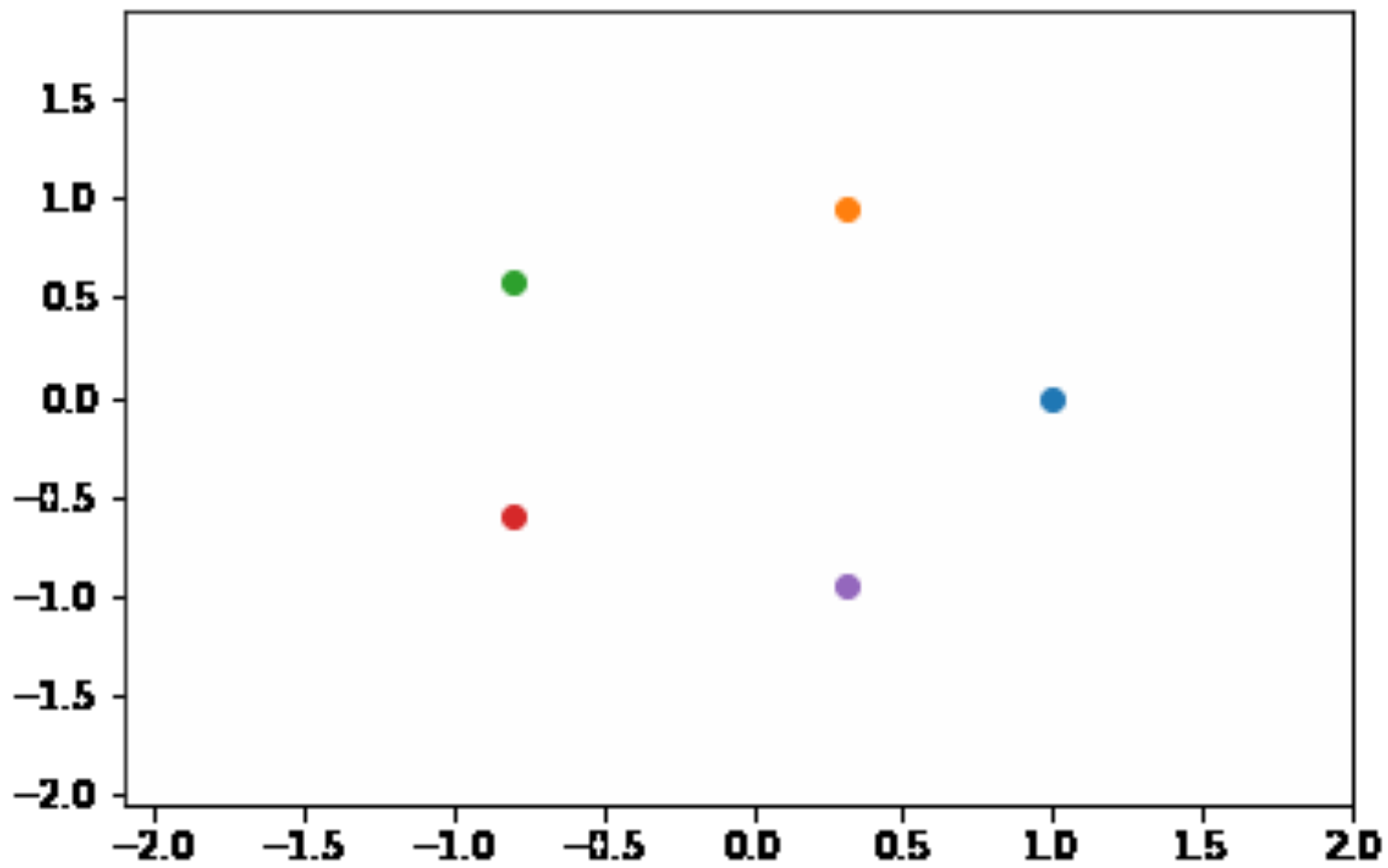
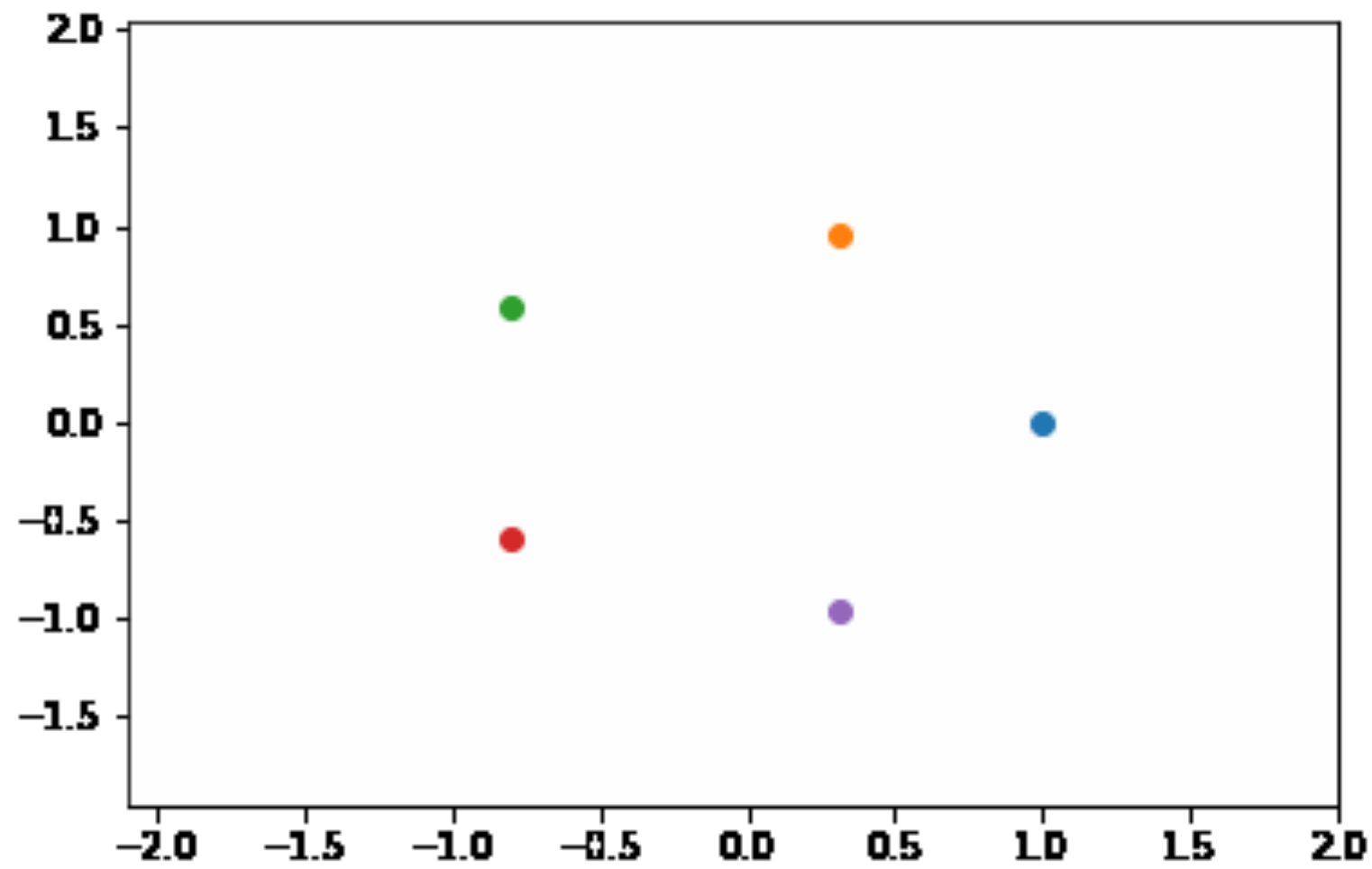
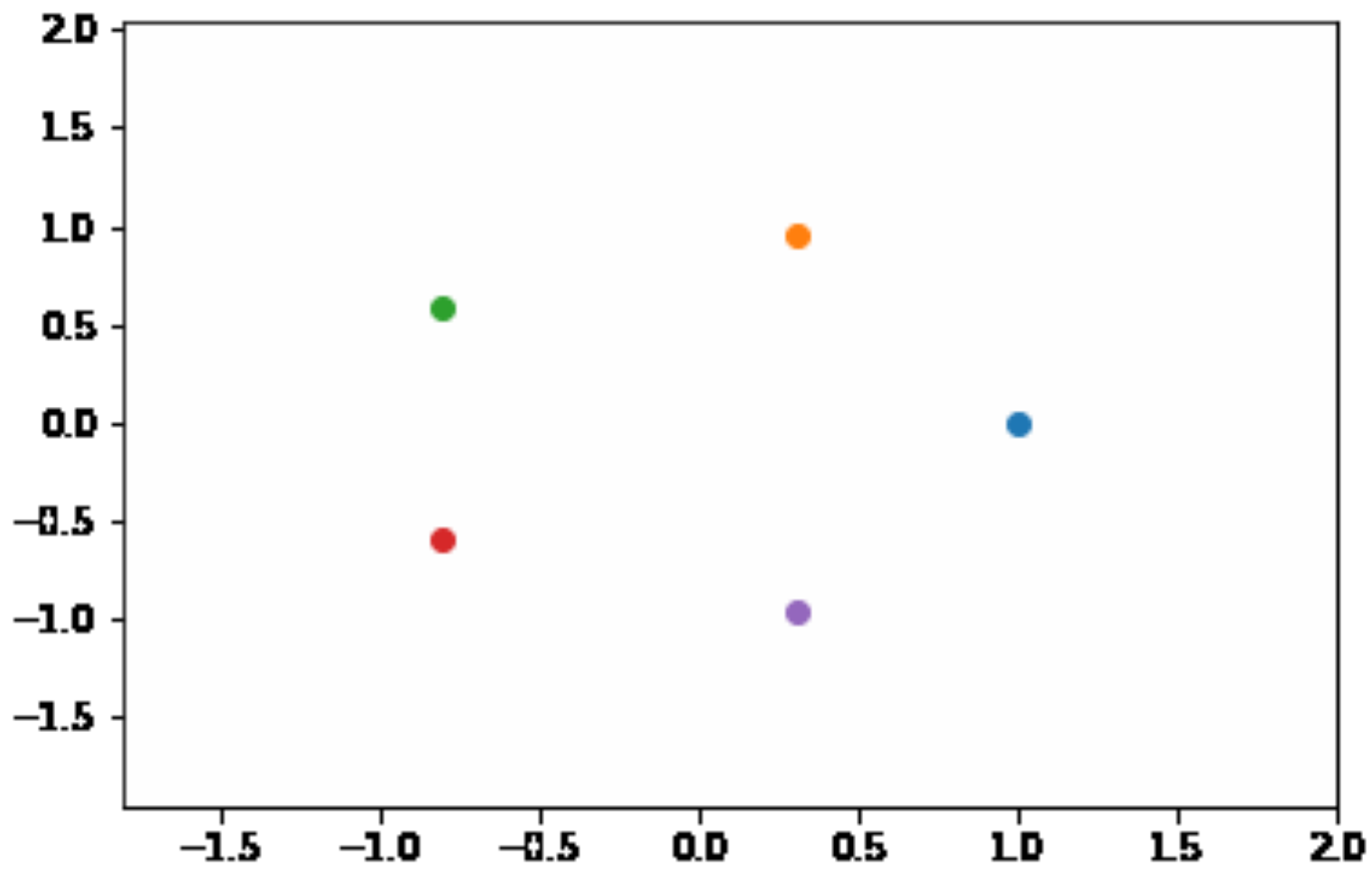
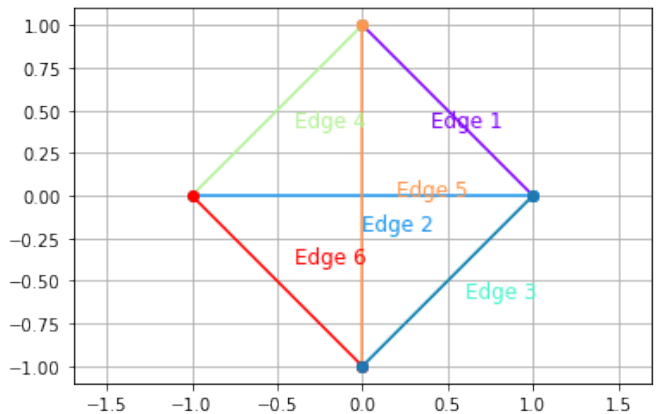
5-Rod Mixing



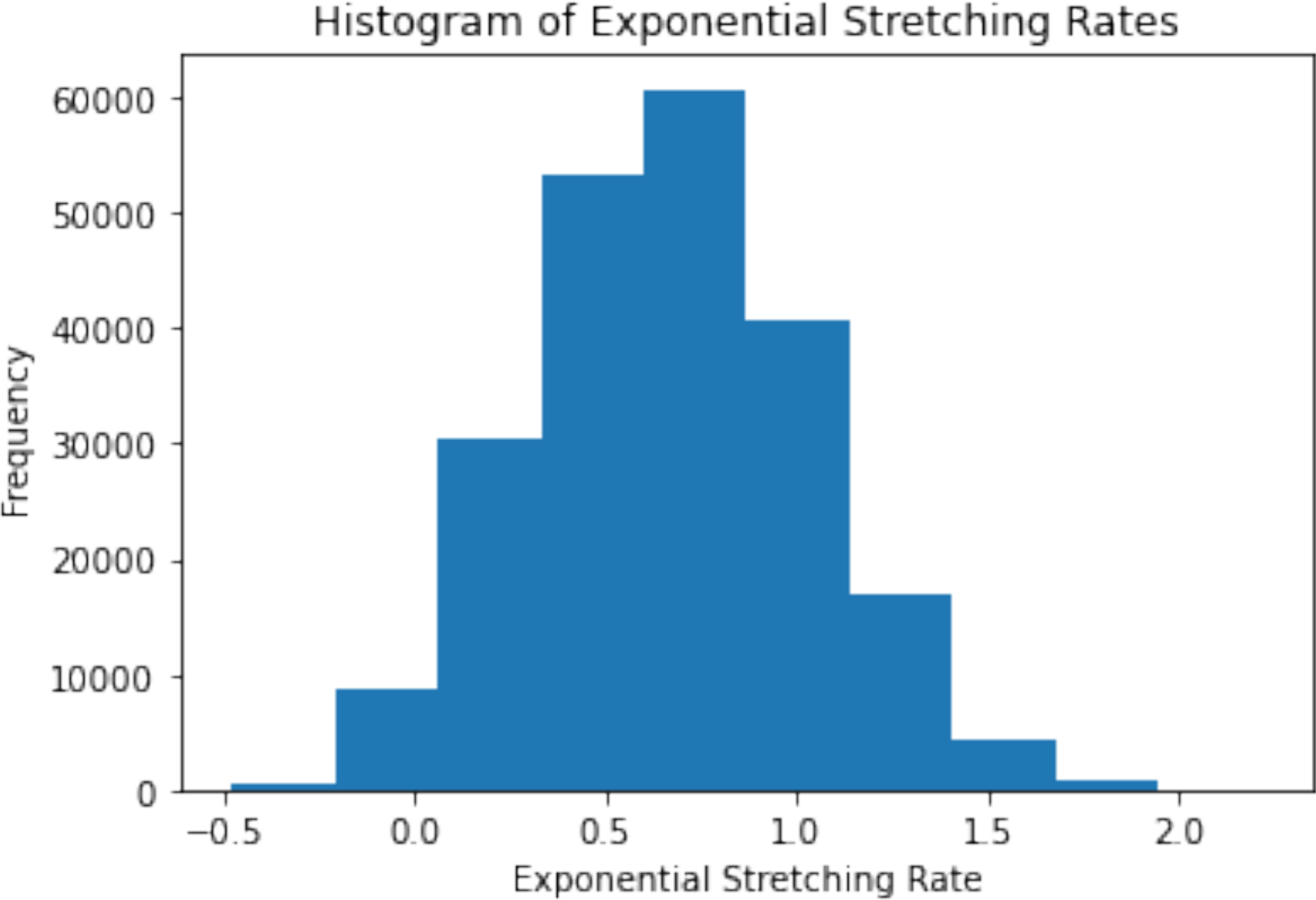
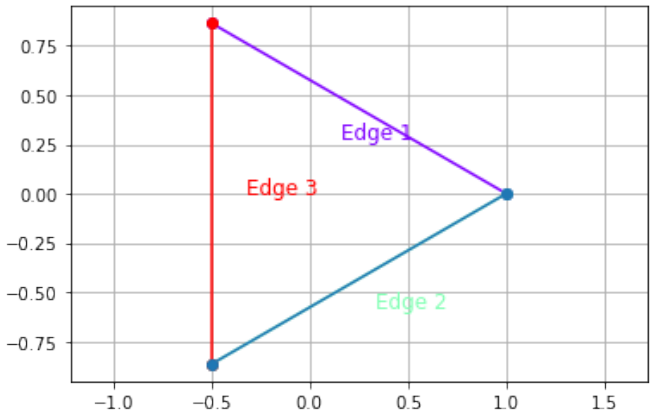
Lowest entropy mixing N=5 L=3



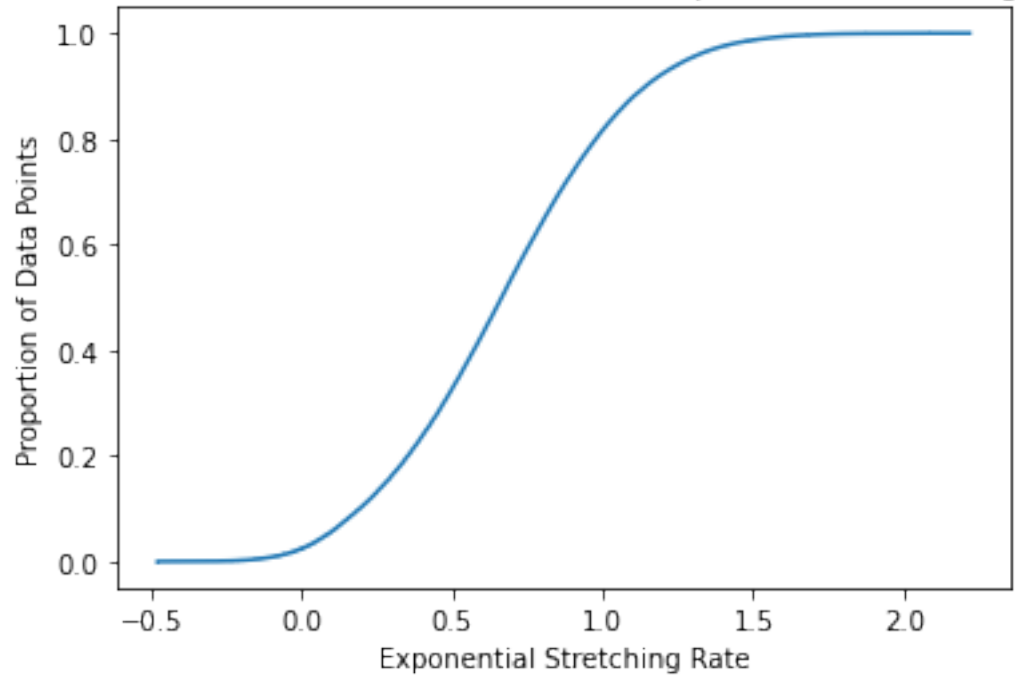
Highest entropy mixing N=5 L=3



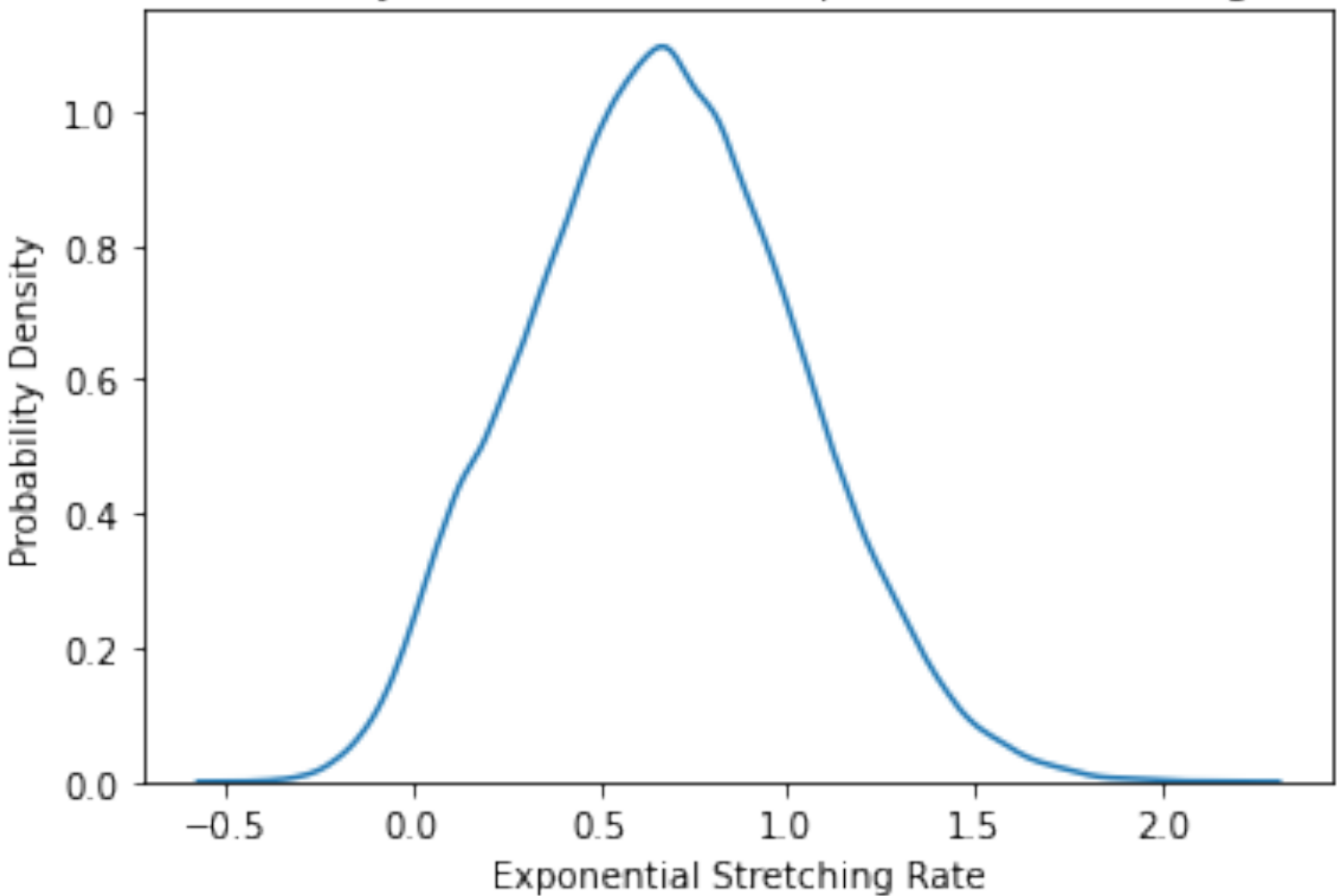
Mixing of N=5 L=3



Cumulative Distribution Function Plot of Exponential Stretching Rates

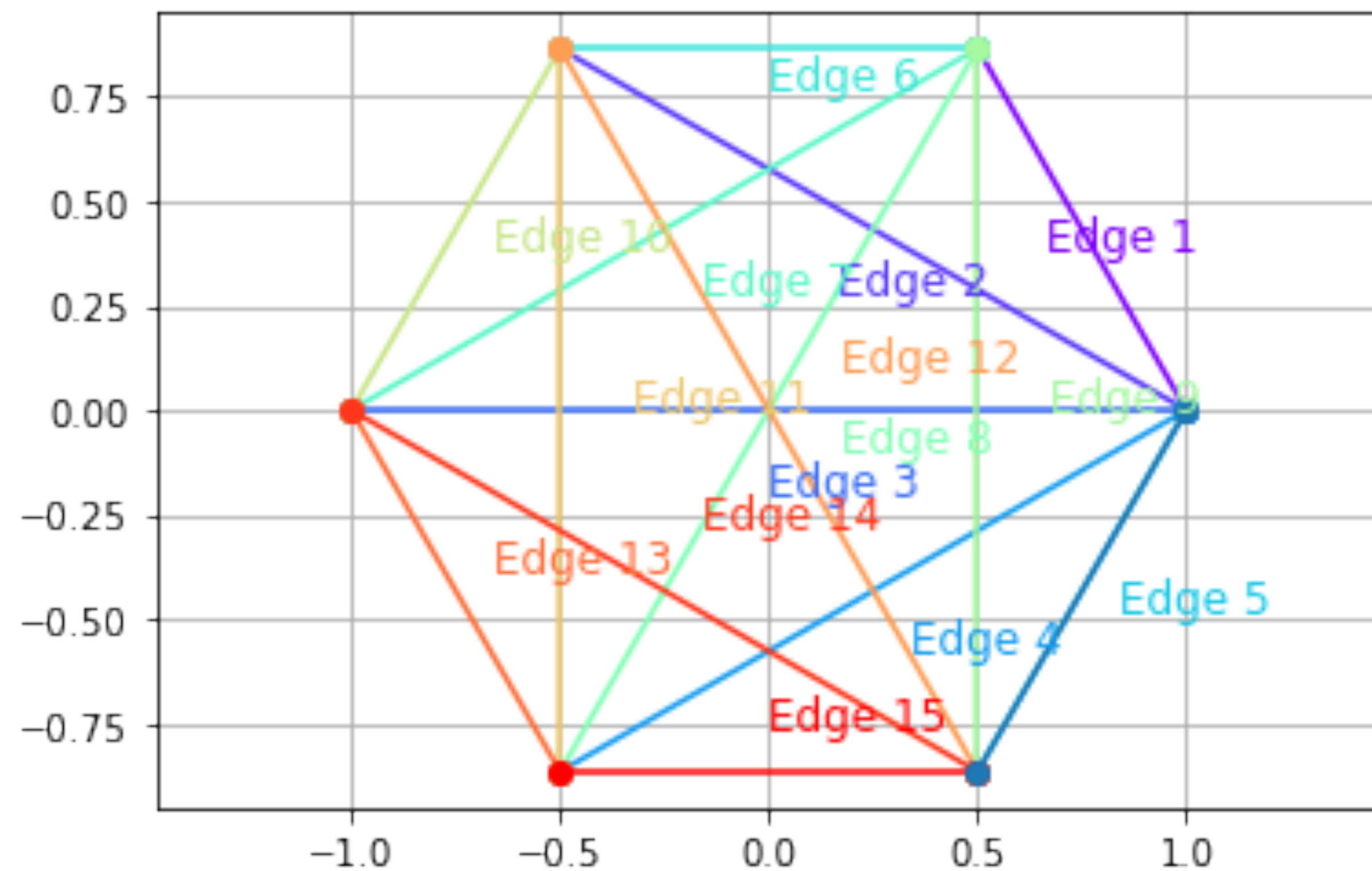


Kernel Density Estimate Plot of Exponential Stretching Rates



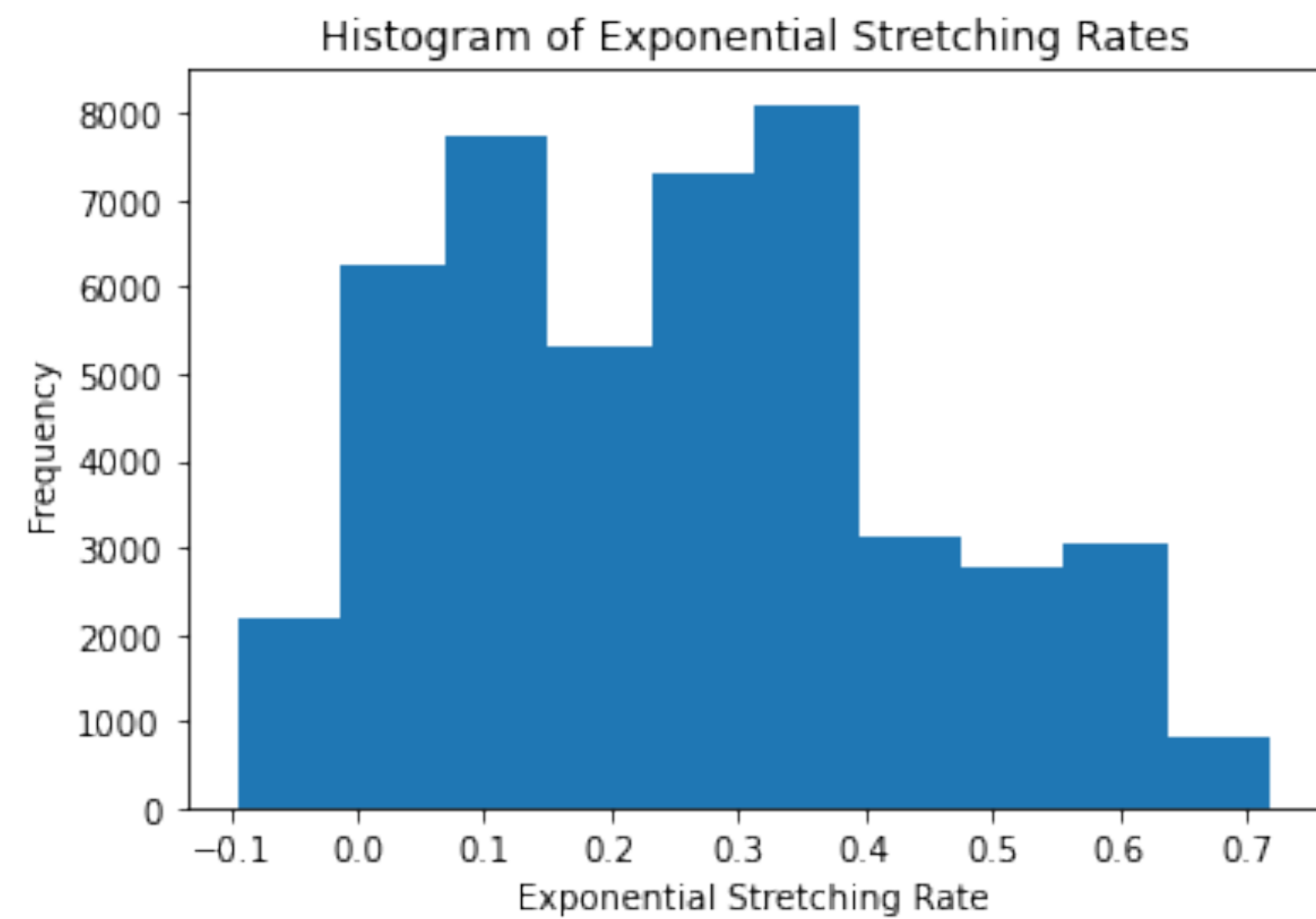
6-Rod Mixing

Computation Failed



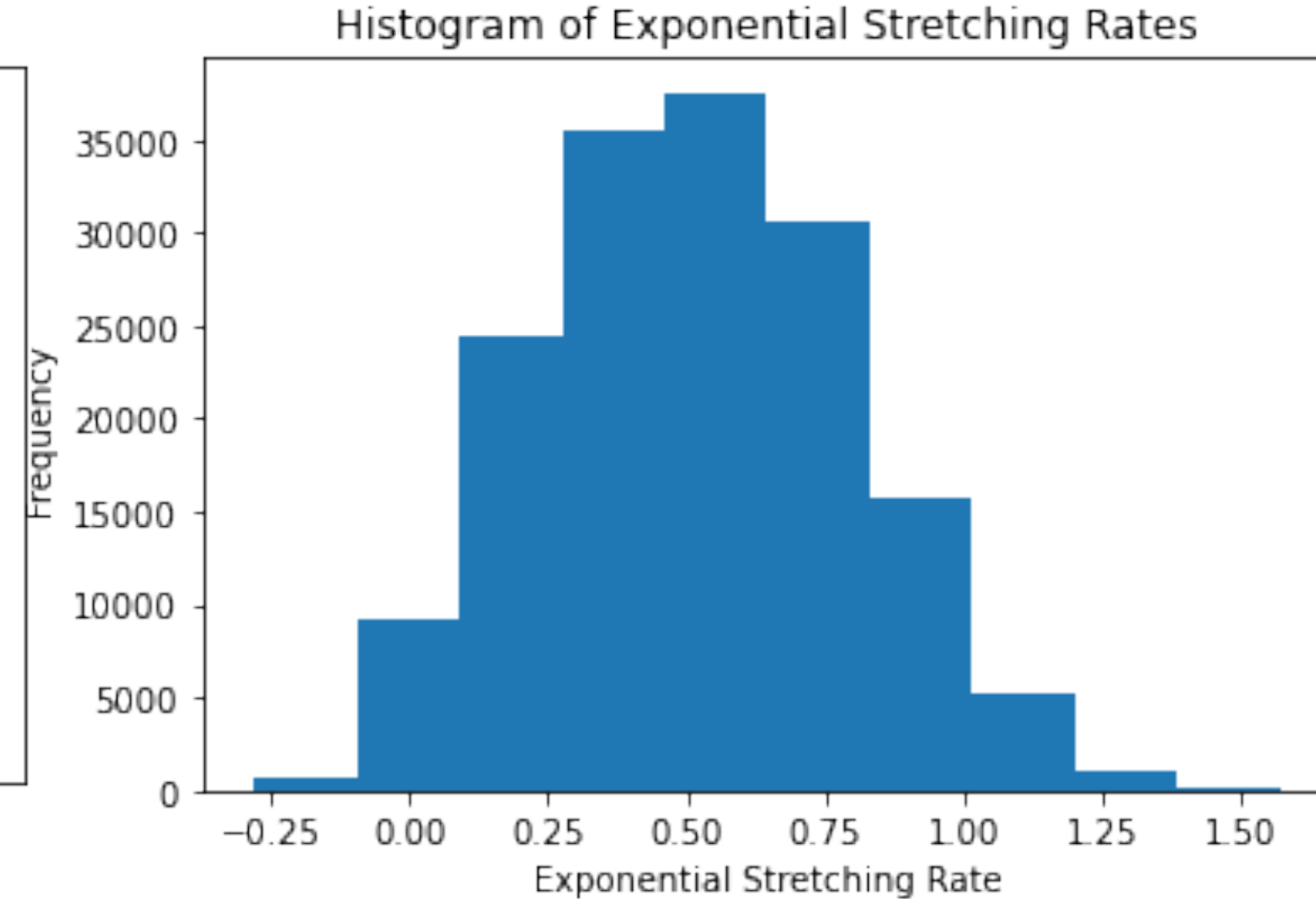
3-Rod Mixing

Length 4



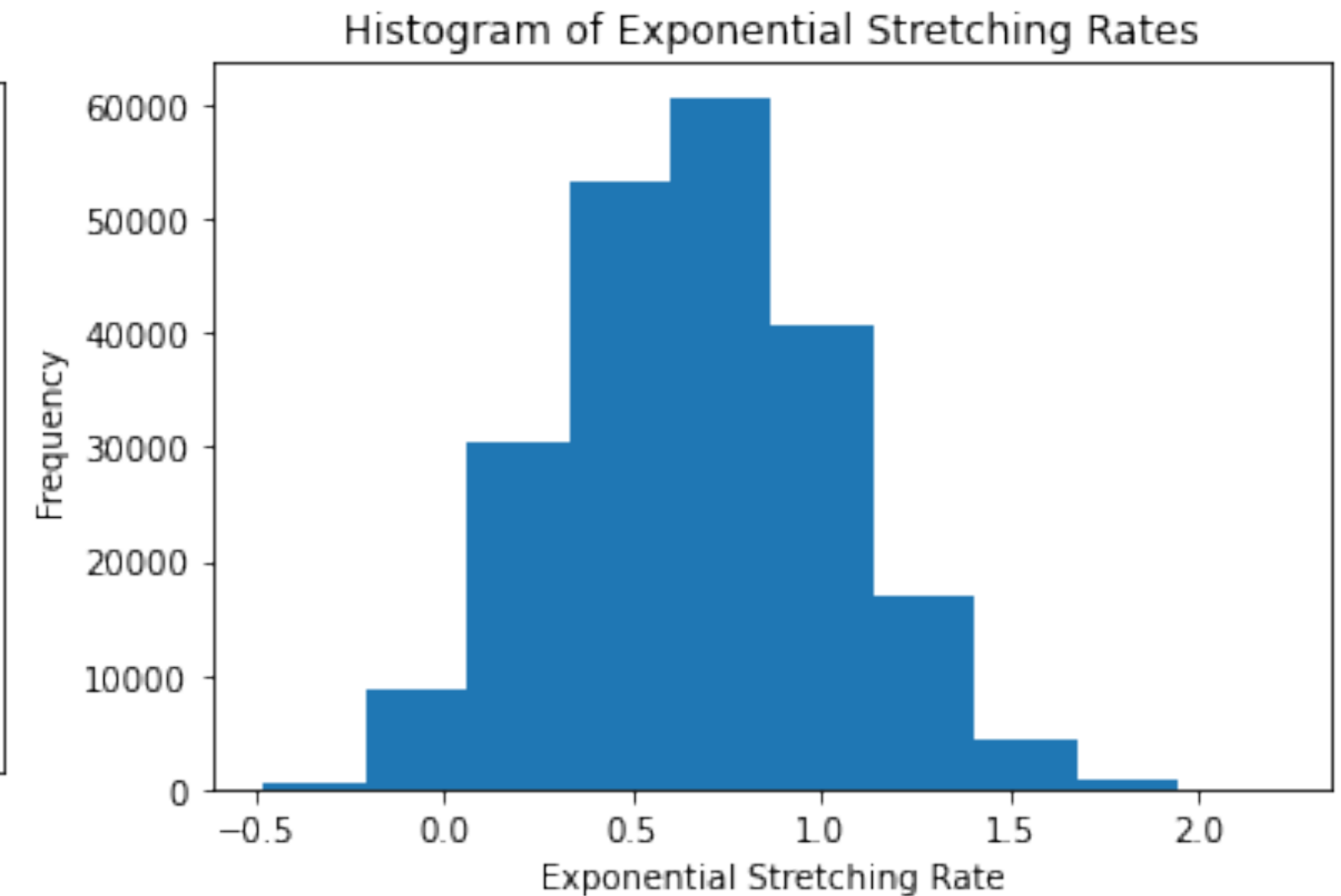
4-Rod Mixing

Length 4



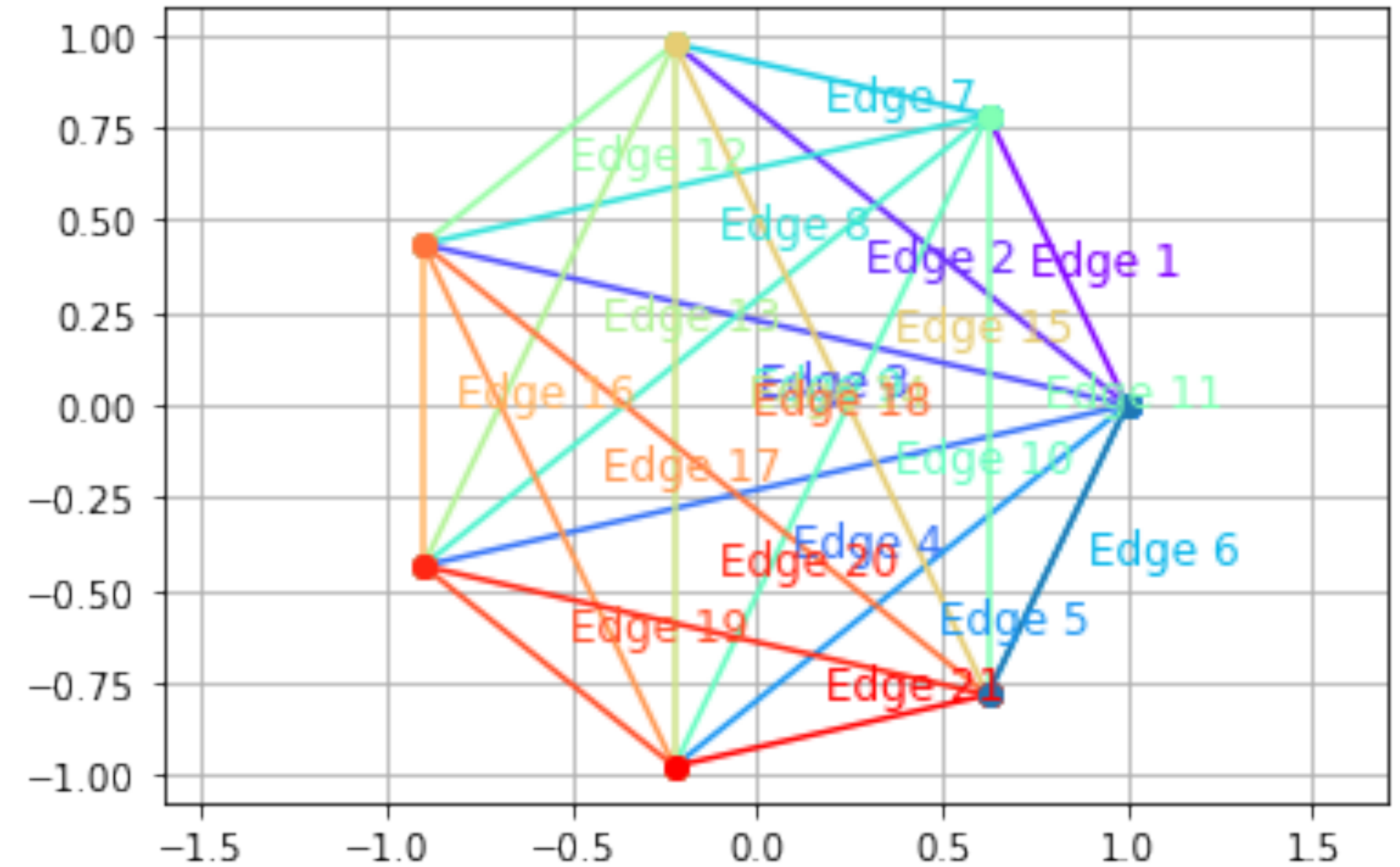
5-Rod Mixing

Length 3



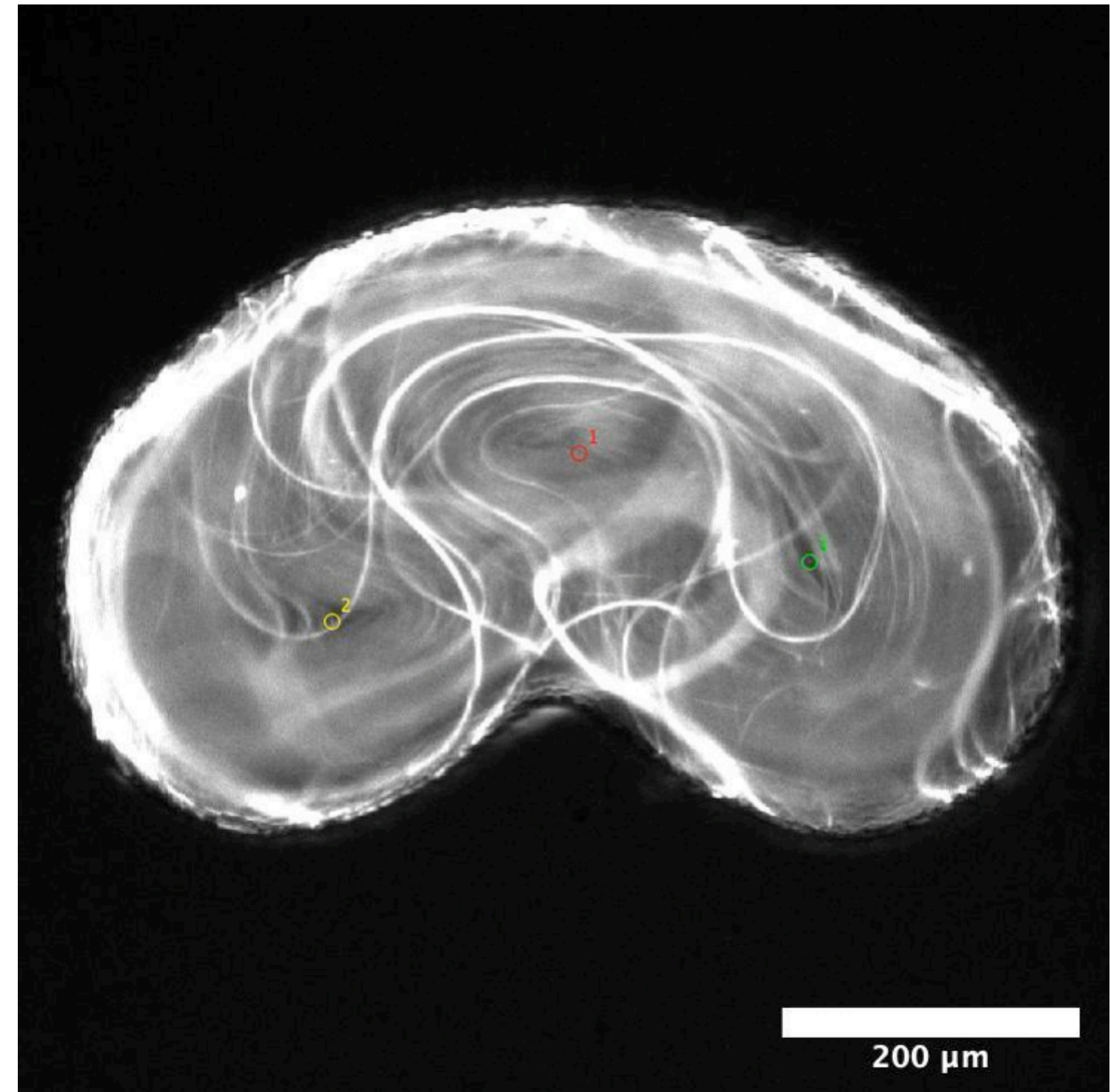
Future Steps

- Remove degeneracy
- Run for larger N and L
- Compare to experimental data



Future Steps

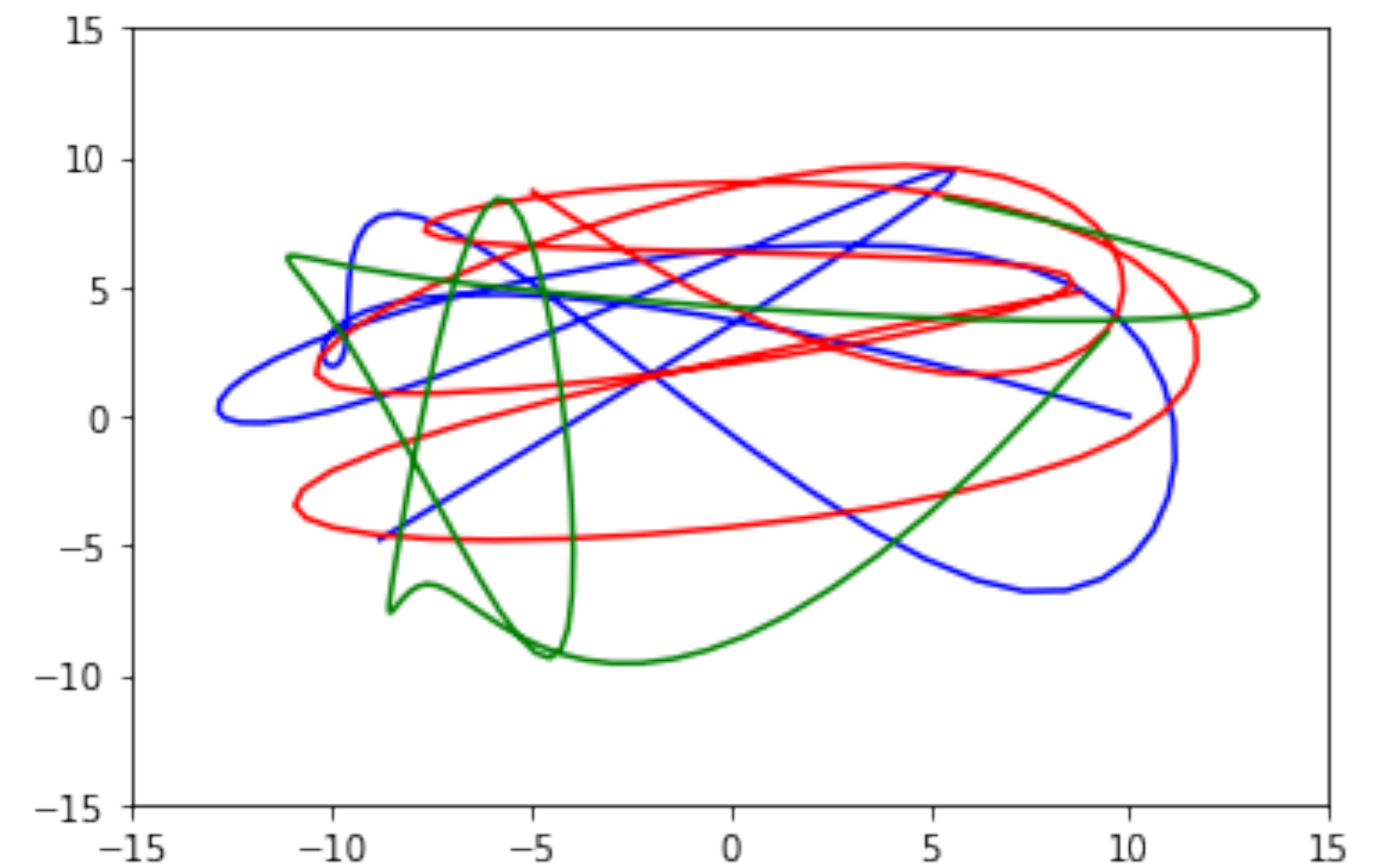
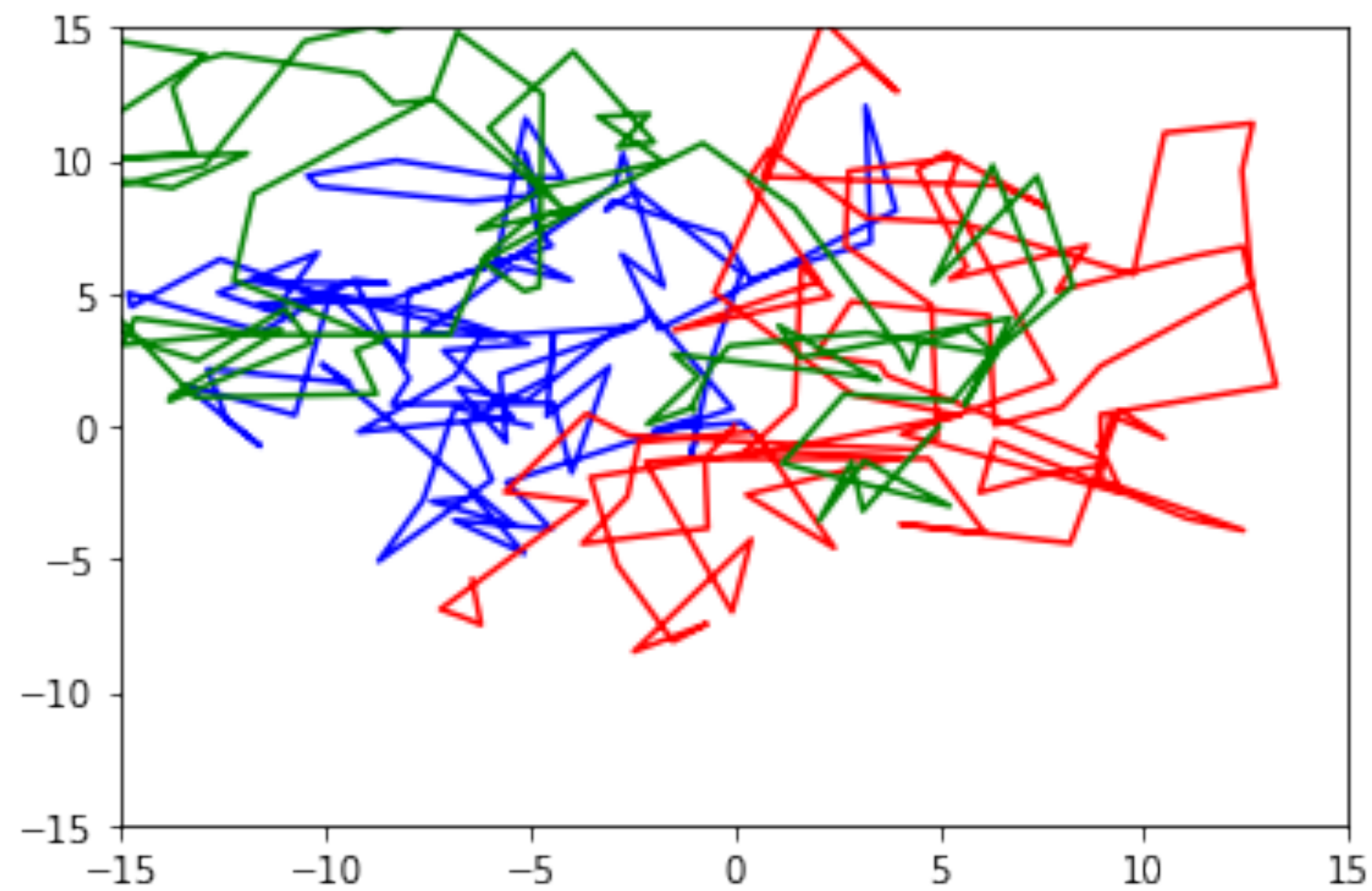
- Remove degeneracy
- Run for larger N and L
- Compare to experimental data



Brownian Motion Mixing

Complete Random Motion

Jagged vs Spline Smooth



Brownian Motion Mixing

Rotational Diffusion

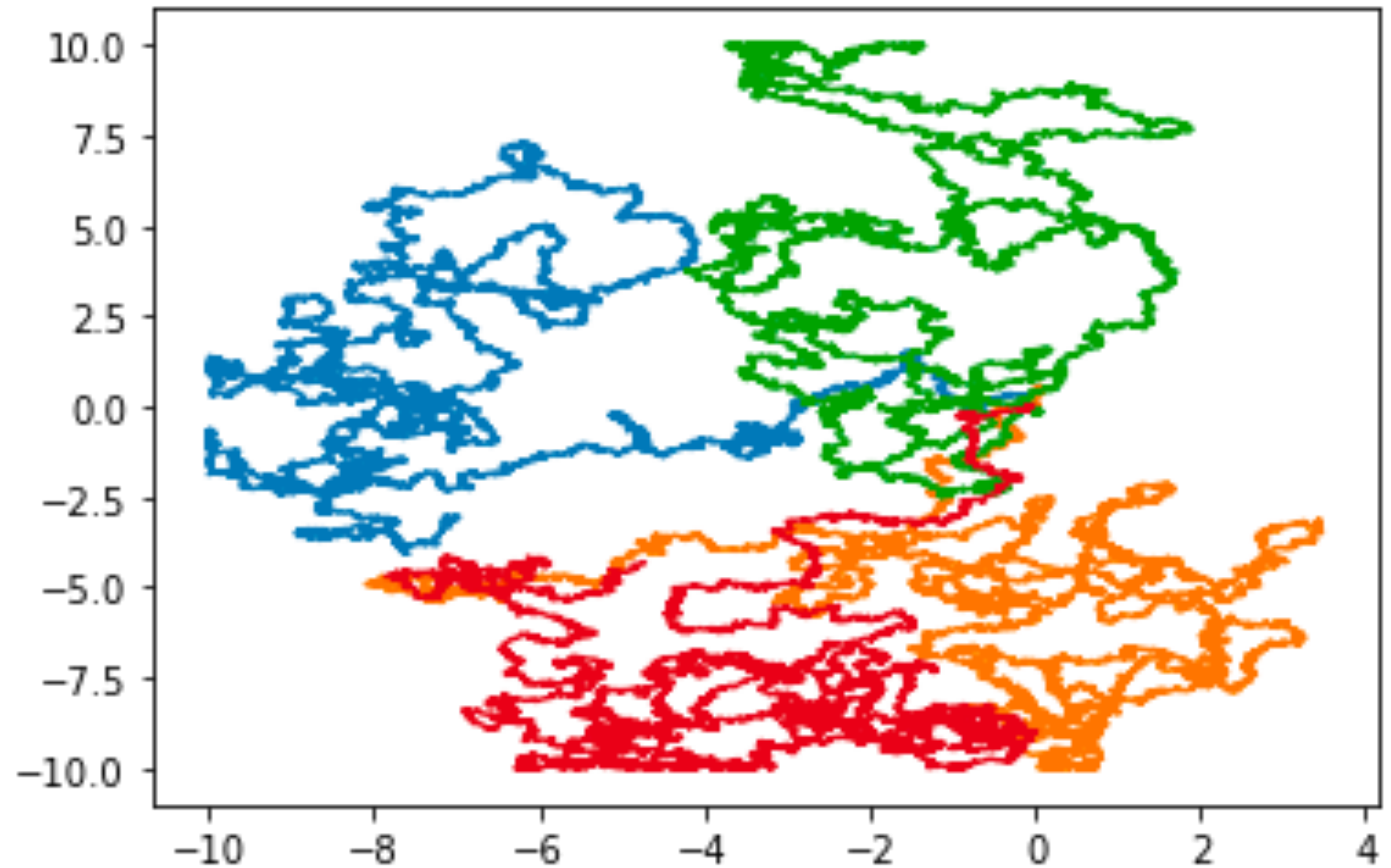
Translational Diffusion

Position

Velocity

Number of Walkers

Boundary Conditions



Brownian Motion Mixing

Rotational Diffusion

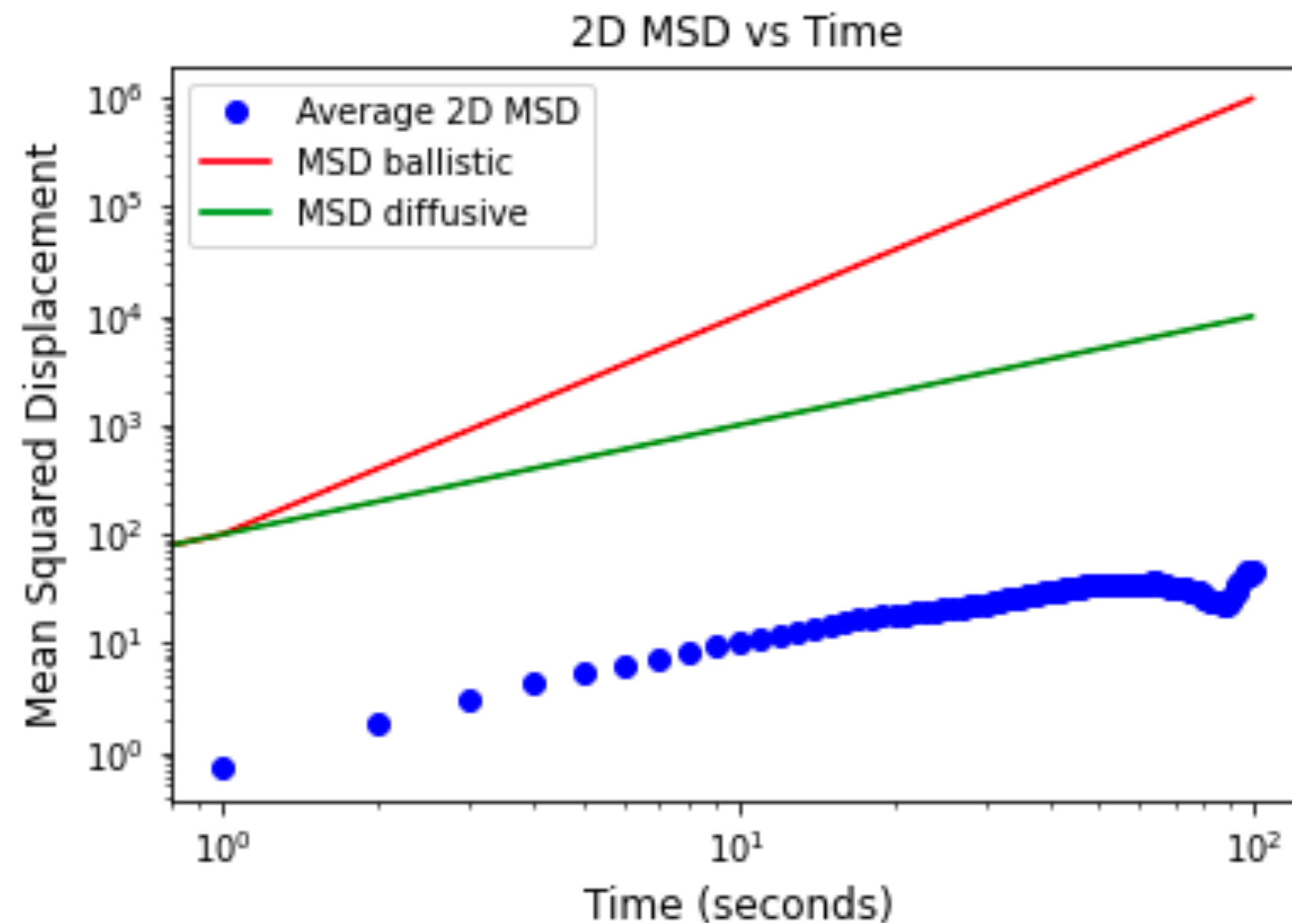
Translational Diffusion

Position

Velocity

Number of Walkers

Boundary Conditions



Brownian Motion Mixing

Rotational Diffusion

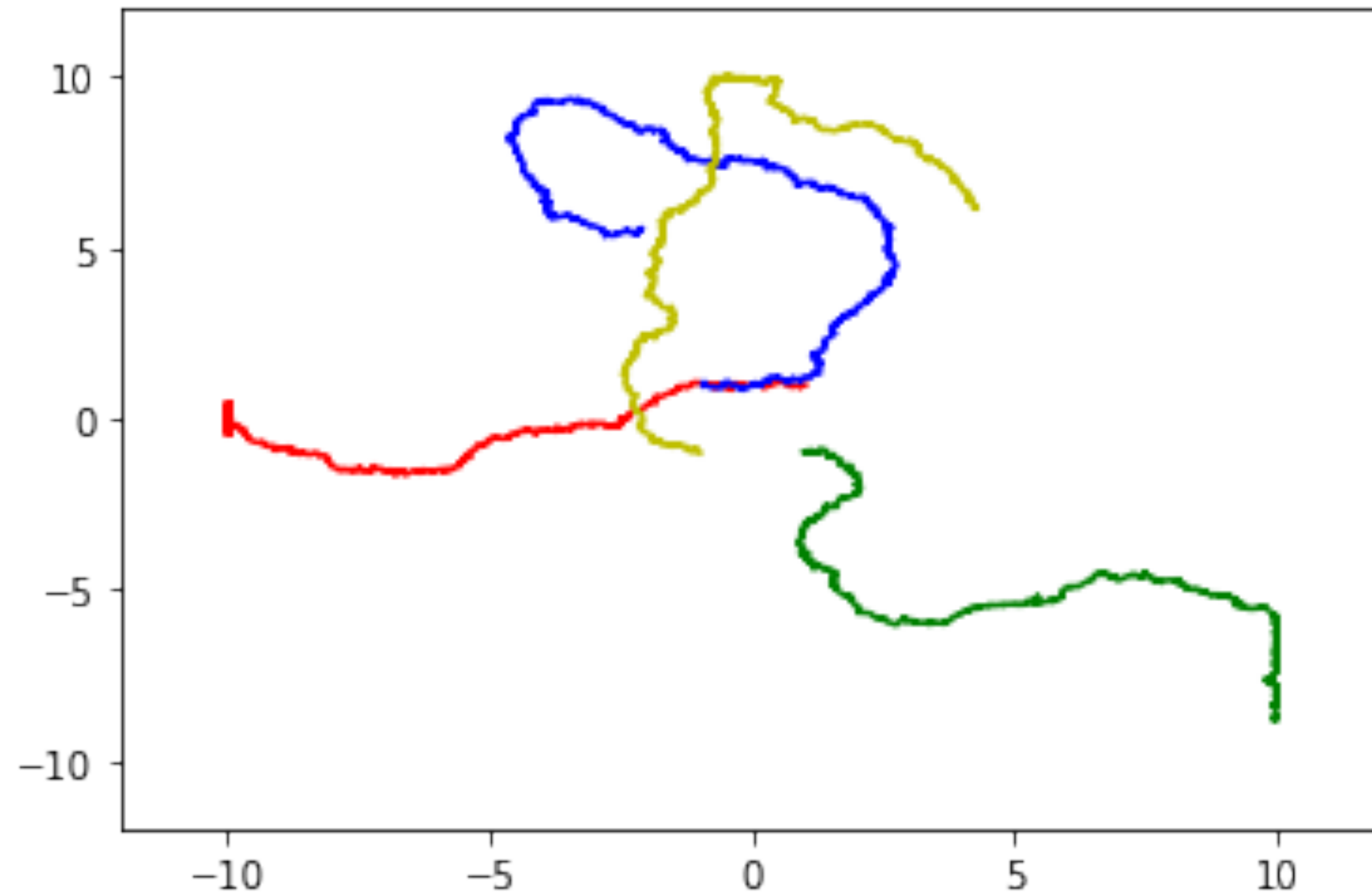
Translational Diffusion

Position

Velocity

Number of Walkers

Boundary Conditions



Future Steps

- Make Robust for Varying Walkers
- Calculate Mixing Entropy on Sets
- Determine what step count is comparable to braid length

