Dynamic Competition in Parental Investment and Child's Efforts *

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Novebmer 2024

Abstract

Competition for a limited number of seats in prestigious colleges generates a "rat-race" equilibrium effect, driving the demand for parental investment. I develop and estimate a dynamic tournament model where each household chooses the quality and hours of private tutoring and hours of student self-study. The model rationalizes persistently high educational investment despite modest effects on test scores, as the tournament structure forces students to maintain investment to avoid dropping to lower tiers. Using the estimated model, I quantify how different investment channels affect intergenerational persistence of earnings. While parental investment reinforces persistence, self-study acts as a moderating force - removing it increases the rank-rank slope by 25.3%. Counterfactual analysis on the capacity constraints shows that expanding elite college seats by 50% reduces tutoring expenditure by 25%. Demographic decline alone does not decrease investment, if there is no substantial reductions in returns to college quality.

JEL Classification Codes: D15, D64, I21,I22, I24, I26, J62
Keywords: Parental Investment, Child's Efforts, Intergenerational Mobility, Student Competition

*Institute of Economic Research, Kyoto University, Japan. This paper was previously circulated as "Parental Investment, Child's Efforts, and Intergenerational Mobility". I am immensely grateful to Steven Stern and Juan Pantano for their invaluable guidance and support. For their helpful discussions and comments, I would like to thank Meta Brown, David Wiczer, Gabriel Mihalache, Peter Arcidiacono, Michael Keane, Lance Lochner, Olivier De Groote, Michèle Tertilt, Minchul Yum, Nirav Mehta, Joseph Mullins, John Rust, Todd Stinebrickner, Michela Tincani, Heepyung Cho, Seik Kim, Sunham Kim, Changhui Kang, Minji Bang, Hugo Benitez-Silva, Elena Capatina, Juan Carlos Conesa, Samuele Centorrino, Sunghun Cho, Jihwan Do, Marcos Fernandes, Mateo Velásquez-Giraldo, Yujung Hwang, Shoya Ishimaru, Fedor Iskhakov, Chinhui Juhn, Shinhyuk Kang, Laura Karpuska, Marc Claveria, Bob Millard, Mark Montgomery, José Alfonso Muñoz, Victor Ronda, Warren Sanderson, Takashi Unayama, Greg Veramendi, Basit Zafar, Weilong Zhang, and Yiyi Zhou. All errors are my own.

1 Introduction

Student competition is a pivotal driver of parental investments. Graduating from an elite university has a significant impact on labor market outcomes (Hoekstra 2009; MacLeod *et al.* 2017; Zimmerman 2019; Jia and Li 2021), but the limited seats at prestigious colleges create intense competition. The scarcity of seats in prestigious colleges leads to a "rat-race" dynamic, which drives the demand for parental investment (Ramey and Ramey 2010). In the United States, parents frequently invest considerable time in supporting their children's extracurricular activities. In East Asia, parents frequently dedicate a significant portion of their income to private tutoring aiming to enhance their children's relative standing for admission to prestigious universities, even though evidence suggests it has only a modest effect on absolute test score improvements (Ryu and Kang 2013; Kang and Park 2021).[1] Despite the prevalence, most previous empirical work abstract from how college admission competition shapes the demand for parental investment.

As the college admission result is closely tied to future earnings of the child, it is a critical channel for parental investment's role in intergenerational income transmission. Consequently, college admission competition has significant implications for social mobility. Less well understood is the impact of child's own efforts in shaping intergenerational mobility. Recent studies report the significant impact of children's own efforts on their educational outcomes (Del Boca, Monfardini and Nicoletti 2017; Fu and Mehta 2018; Todd and Wolpin 2018; De Groote 2023; Del Boca, Flinn, Verriest and Wiswall 2023). At the same time, the self-effort of the child is not responsive to parental background as much as parental investment is affected by parental background.[2] Despite its the potential substituability or complementarity to parental investment, few studies have modeled the interdependence of parental investment and child's self-effort in shaping intergenerational mobility.

This paper investigates how college admission competition shapes the demand for parental investment and child effort, and studies the impact of these factors on intergenerational persistence of earnings. First, the intensity of household competition depends on the number of college seats with different level of quality and the number of competitors. If there are significant changes in the number of competitors,

[1] Notably, for China, South Korea, and Japan, the reported participation rates for private tutoring in secondary school are 48.7%, 61.8%, and 51.9%, respectively (Bray 2022).

[2] I show related empirical evidence in Section 3.

the decision of parental investment and child effort is likely to be affected. Relatedly, many developed countries face a drastic shift in demographic structure caused by a declining fertility rate, as shown in Figure 1, which directly influence the number of competitors. At the same time, empirical evidence suggests that colleges tend to not adjust the seats to accommodate for increasing cohort size (Bound and Turner 2007). Little is known about the impact of the number of seats in elite colleges and the shift in demographic structure on parental investment. Second, using the college admission competition set-up, this paper seeks to shed light on the role of parental investment in intergenerational persistence or earnings. The inclusion of self-effort of the child, which is often overlooked in the literature, might amplify or offset the link between the two generations.

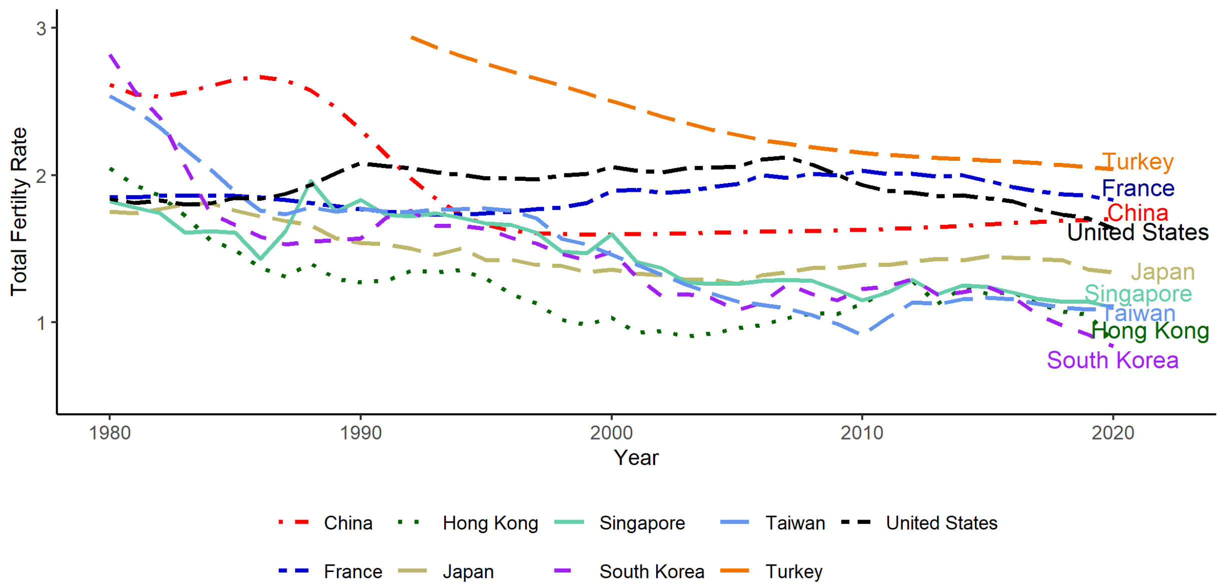
To answer these questions, this paper builds and estimates a dynamic tournament model using a unique longitudinal dataset with information on parental investment, child time allocation, and administrative test scores. Using the estimated model, I quantify how household choices affect intergenerational persistence and analyze the impact of college admission capacity on the decision of parental investment and child effort.

This study uses Korean datasets and is based on institutional features of the country.[3] Students are assigned to the middle schools within the residential education district by lottery. As the distribution of school quality of secondary school is relatively homogeneous, the private tutoring expenditure of parents stands out as a primary contribution to the child's future outcomes. The importance of the final test score in college admissions helps to link the test score measure to the child's labor market outcomes. Such institutional characteristics offer a transparent environment in which household income is translated into the educational outcome of the child.

I document descriptive evidence supporting the tournament model framework. Two empirical facts establish college admission competition as a tournament with valuable prizes. First, college ranking positively affects the growth of alumni's income. The effects are economically and statistically significant even after controlling for college entrance exam scores. Using the Korean Labor Income and Panel Study, I document substantial income differences across college-tiers, consistent with recent evidence on elite college premiums (Zimmerman 2019; Sekhri 2020; Jia and Li 2021;

[3]A number of countries share the institutional features, which I explain in Section 2.

Figure 1: Shrinking Cohort



Source: World Bank for data for China, Hongkong, Japan, Singapore and South Korea. Data for Taiwan is drawn from United Nations World Population Prospects.

Lee and Koh 2023).[4] Second, parental investment drops substantially after college admission, suggesting competition rather than human capital accumulation motivates investment.

Also, another empirical fact suggests that parental investment and the child's self-efforts potentially have different implications for intergenerational mobility. While parental investment varies substantially with household income, hours of self-study show limited correlation with parental background. At the same time, both parental investment and the child's self-efforts are expected to affect the child's outcome. If these two inputs are substitutes, income-constrained households could compensate for lower parental investment through increased self-study. This substitutability suggests that omitting self-study from analysis could overstate the role of parental investment in intergenerational persistence of earnings.

Motivated by the empirical evidence, I develop and estimate an equilibrium dynamic tournament model of college admission competition. The model builds upon the rank-order tournament model introduced by Lazear and Rosen (1981). The tournament structure is embedded into the model of altruistic households. The household

[4] Using data on students majoring in science at University of California campuses, Arcidiacono, Aucejo and Hotz (2016) show the mismatch between minority students' preparedness and higher ranked campuses decreases the likelihood of graduation.

cares about the future outcome of the child, which is the result of the college admission tournament. In every period, based on its state variable, each household makes decisions regarding parental investment and the level of the child's self-efforts, which produces a subsequent test score. The final test score's ranking and the number of available seats determine college tier assignment, which in turn determines the child's lifetime income.[5]

Two main features of the dynamic tournament model help explain household investment patterns. First, the tournament structure with limited number of seats generates strong competitive incentives, providing a framework to analyze how college capacity constraint drives parental investment demand. Second, the rich heterogeneity in state variables and choices, particularly the interaction between parental investment and self-study, provides a framework to analyze how different investment channels affect intergenerational persistence.

I estimate the model using Maximum Simulated Likelihood, incorporating the equilibrium condition of the dynamic tournament as a constraint in the estimation routine. The estimation results suggest that the marginal effects of both investments decline substantially over time, with final period effects being minimal. Despite such low marginal effects in the final period, simulation shows that removing final period efforts leads to significant tier changes for top students, highlighting how tournament structure maintains high investment incentives. Additionally, the estimate of the substitution parameter suggests that parental investments and hours of self-study are technological substitutes.

Using the estimated model, I conduct two sets of counterfactual analyses. First, I quantify the impact of different channels by alternatively shutting them down. Shutting down the channel of parental investment decreases the rank-rank slope by 49.3%. When the self-study channel is shut down, the rank-rank slope increases by 25%. These results suggest that while parental investment amplifies intergenerational persistence of earnings, self-study moderates it.

Second, I analyze how college capacity constraints affect household investment decisions. A 50% expansion of elite college seats reduces average tutoring expenditure by 24%.[6] However, simulating Korea's projected demographic decline - which

[5] This is an arguably reasonable assumption. In Section 4.2, using confidential job offers data provided by a conglomerate in South Korea, I show that the effects of college-tier on earnings are economically and statistically significant controlling for effort during the college period.

[6] The elite colleges refer to a group of colleges which I define as Tier 1 and Tier 2 in Section 2.

effectively halves the cohort-to-seat ratio by 2033 - shows that tutoring investment remains high unless returns to college quality decrease substantially. These results suggest that the combination of limited elite college capacity and substantial returns disparities fundamentally drives parental investment demand. Expanding elite college seats directly alleviates competition, while demographic decline has little effect on the relative advantage of elite college placement and thus on tutoring demand.

Related Literature and Contributions. This paper relates to three strands of literature. First is the literature on the broadly defined post-birth parental choice (Becker and Tomes 1979; Del Boca, Flinn and Wiswall 2014; Doepke and Zilibotti 2017; Bolt, French, Maccuish and O'Dea 2021), particularly recent work incorporating competition into parental choices.[7] Ramey and Ramey (2010) are the first paper that rationalizes the increase of parental time investment in the United States using a theoretical model of competition for elite colleges. Bodoh-Creed and Hickman (2019) build a static structural model of an admission contest to study returns to pre-college human capital investment in the United States and estimate their model.[8] A closely related paper on the same context is by Kim, Tertilt and Yum (2024), which studies the cause of the low fertility problem of South Korea. They propose a model of "status externality" based on the assumption that parents care about the relative position of their children's human capital compared to that of other children. The tournament model of my paper complements their study by formally modeling the dynamic competition with respect to getting into prestigious colleges, which rationalizes underlying source of the status externalities in their paper.[9] Also, the number of seats and the equilibrium cutoffs of the tournament model enable me to empirically quantify the impact of the college seats' capacity on the demand for parental investments.[10]

Second, this paper naturally relates to the literature on parental investment and its intergenerational implications (Lee and Seshadri 2019; Caucutt and Lochner 2020; Bolt, French, Maccuish and O'Dea 2021; Daruich 2022; Gayle, Golan and Soytas 2022;

[7]A group of papers associates parental choices with social interactions (Agostinelli 2018; Agostinelli, Doepke, Sorrenti and Zilibotti 2023; Boucher, Bello, Panebianco, Verdier and Zenou 2022)

[8]While abstracting from the notion of parental investment, Grau (2018) builds and estimates a static tournament model to study the college competition in Chile.

[9]Gu and Zhang (2024) is another recent paper modeling the college admission competition using heterogeneous agents framework. However, they do not model the thresholds for the college admission as equilibrium objects of the students' competition.

[10]Outside the broad literature of economics of education, a handful of papers build and estimate structural tournament models (Vukina and Zheng 2007; Chen and Shum 2010; Vukina and Zheng 2011).

Yum 2022). In particular, Del Boca, Flinn and Wiswall (2014) build and estimate a dynamic model of parental investment and cognitive development, which allows them to separately identify the different effects of parental time and monetary investments. Subsequently, emphasizing the role of child's time investment, Del Boca, Flinn, Verriest and Wiswall (2023) build a Stackelberg model of parent-child interaction and investigate the effects of conditional cash transfers on child outcomes. De Groote (2023) quantifies the role of students' efforts in the academic tracking system using a dynamic model. My paper is the first to jointly model and quantify how parental investment and child self-study affect intergenerational persistence through college competition.

Finally, this paper contributes to the literature on childhood investments and skill development by estimating the age-specific effects of parental investment and the self-efforts of the child during adolescence.[11] Most previous work focuses on estimating the effects of parental investment on child outcomes alone (for example, Cunha and Heckman 2007; Cunha, Heckman and Schennach 2010; Del Boca, Flinn and Wiswall 2014).[12] These studies find declining effects of parental time investment over age. Several studies estimate the effects of hours of self-study on academic achievements (e.g., Cooper, Robinson and Patall 2006; Stinebrickner and Stinebrickner 2008; Fu and Mehta 2018; Todd and Wolpin 2018). The production function estimates in my paper add to this literature by providing age-specific estimates of the effects of parental investment and self-efforts (Del Boca, Monfardini and Nicoletti 2017; Del Boca, Flinn, Verriest and Wiswall 2023), and offer novel evidence on their substitutability.

The rest of the paper is organized as follows. I describe the institutional features in Section 2. In Section 3, I document empirical facts that motivate the dynamic tournament model. Section 4 introduces the tournament model. Section 5 explains the estimation procedure, source of identification, and results. I present the counterfactual exercises in Section 6 and conclude in Section 7.

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[11]As college competition in reality uses actual test scores rather than unobserved skills of the student, I do not apply the factor model techniques developed in the literature (see Cunha, Heckman and Schennach (2010); Agostinelli and Wiswall (2016)).

[12]As this paper employs private tutoring expenditure as a measure of parental investment, it also complements the literature of studies on private tutoring (Stevenson and Baker 1992; Cheo and Quah 2005; Tansel and Bircan 2005; Dang 2007; Ono 2007; Ryu and Kang 2013; Hof 2014; Kang and Park 2021).

2 Key Institutional Features

The institutional features of South Korea's education system offer a useful setting for studying how college admission competition shapes parental investment and intergenerational mobility. In this section, I explain the key institutional features of the country: the high-stakes college entrance exam, hierarchical college structure, homogeneous secondary schools, and an established private tutoring market. While these features are particularly pronounced in Korea, similar characteristics exist in many other countries, making the insights from this analysis broadly relevant.

2.1 High-Stakes College Entrance Exam

The College Scholastic Ability Test (CSAT), taken at the end of 12th grade, serves as the primary determinant of college admissions in Korea.[13] The exam's centrality to academic outcomes makes it a defining feature of the Korean education system. Its importance is reflected in nationwide accommodations on exam day - aircraft takeoffs and landings are suspended during English listening tests, while firms and government offices delay their workday to help students avoid traffic congestion.

Students receive both standardized and stanine scores for subjects including Korean, Mathematics, English, and electives. Based on these scores and published college cutoffs, students apply to up to three colleges. Educational consulting firms release detailed predictions of admission cutoffs, which closely match actual outcomes, allowing students to target colleges matching their performance level.

This high-stakes examination system parallels college entrance mechanisms in many countries. China's Gaokao and France's Baccalauréat similarly serve as crucial determinants for elite college admission.[14] While the U.S. Scholastic Aptitude Test (SAT) also plays an important role, it differs in that other factors like grade-point-average and extra-curricular activities carry substantial weight in admissions decisions.

[13] In South Korea, there has been a recent increase in the quota for the holistic review process, in which test score is not the only determinant for college admission. In 2019, 24.9% of total students were admitted through the holistic admission route (Bastedo 2021).

[14] Other examples include Turkey's Yükseköğretim Kurumları Sınavı, Brazil's Exame Nacional do Ensino Médio, and Malaysia's Sijil Pelajaran Malaysia.

2.2 Hierarchical College Structure and College-Tier

College quality in Korea, as in many countries, exhibits a clear hierarchical structure with substantial implications for labor market outcomes. Empirical studies consistently show that graduating from an elite college significantly affects future earnings and career trajectories across various national contexts.[15]

Korea's college hierarchy has remained remarkably stable over time (Kim and Lee 2006; Kim 2014), enabling clear categorization of institutions into distinct tiers based on admission selectivity and outcomes. Based on the "cutoff sheet" published by Jinhak (2022), one of the major education consulting firms, I classify colleges into four ordered tiers. Tier 1 comprises the most selective universities, requiring scores in approximately the top 1% of the distribution. Tiers 2 and 3 require approximately scores in the top 5% and 15% respectively, while Tier 4 consists of the rest of four-year and two-year colleges. High school graduates form Tier 5. The specific member universities of each tier are detailed in Appendix B. The stability of these status distinctions, coupled with their significant impact on labor market outcomes (which I document in Section 4), creates strong incentives for competitive investment in college admission preparation.

2.3 Homogeneous Secondary School and Private Tutoring Market

Secondary schools in Korea are highly homogeneous in both curriculum and quality, while the private tutoring market is well developed.[16] This combination creates a unique environment for studying how household resources translate into educational outcomes.

The homogeneity of schools stems from multiple policy features. First, the curriculum is uniform and under strict government control, applying to both public and private schools. Even private schools have limited autonomy in curriculum and tuition decisions.[17] Second, consecutive school-equalization policies have led to similar edu-

[15]See, for example, Hoekstra (2009) for the United States,MacLeod *et al.* (2017) for Colombia, Zimmerman (2019) for Chile, Anelli (2020) for Italy, Sekhri (2020) for India, and Jia and Li (2021) for China.

[16]On the other hand, the competition between private and public schools is extensively studied in the US setting. See Epple and Romano (1998); Epple, Figlio and Romano (2004); Epple and Romano (2008); Epple, Romano and Urquiola (2017, 2021).

[17]One of the few decisions of private secondary schools in Korea is that they can independently hire teachers. Park, Behrman and Choi (2013) provide evidence that the difference in the quality of teachers is not significant between private and public secondary schools in Korea.

cation quality across schools.[18] Student assignment to middle schools occurs through a random lottery within residential districts, eliminating selection effects.[19] The only exception is specialized high schools, which account for just 3% of total enrollment.

Alongside this homogeneous school system exists a established private tutoring market, responsible for 2.8% of GDP (Nam 2007). Korean parents spend on average 9% of their income on private tutoring, primarily through hagwon (cram schools), one-on-one tutoring, group tutoring, and online classes.[20] With the centralized school curriculum, these tutoring options effectively substitute for parental time in academic support.

This institutional setting - combining homogeneous schools with extensive private tutoring - provides a transparent environment for studying how household resources affect educational outcomes. The limited variation in school quality helps isolate the effects of household investments, while the established tutoring market offers a clear measure of parental investment.

3 Descriptive Evidence

3.1 Data: Korean Educational Longitudinal Study 2005

The Korean Educational Longitudinal Study 2005 (KELS) provides comprehensive data for this paper's analysis, containing the key variables needed to study parental investment and student effort: private tutoring expenditure, time allocation, household characteristics, and standardized test scores, which is a rare combination for one dataset. This dataset enables me to (i) quantify how accumulated parental investment and student efforts affect intergenerational mobility and (ii) analyze the dynamic selection of household effort choices in the college admissions competition.

[18]See Section II of Kim and Lee (2010) for a description of the history of school equalization policy. As of 2010, the high school equalization policy has been adopted for all major cities in South Korea.

[19]Papers in the literature exploit this random assignment feature to estimate the effects of various independent variables of interest on educational outcomes. See, for example, Kang (2007), Park, Behrman and Choi (2013), and Park, Behrman and Choi (2018). Park, Behrman and Choi (2013) show that the issue of non-compliers to the lottery policy is a minor concern.

[20]See Bray (1999, 2021) for a comprehensive cross-country comparison of private tutoring.

Table 1: Sample Moments

(a) Sample Moments: 7th - 12th grades

School grade	7th		8th		9th	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Tutoring Expenditure	25.8	20.0	25.1	19.6	36.1	31.0
Hours of Self-Study	5.48	5.04	5.97	5.13	6.45	5.27
Hours of Tutoring	11.37	8.50	9.69	7.22	11.29	9.90
Income	370.4	161.7	369.2	151.3	400.4	169.9
Test Scores	323.03	45.63	321.50	48.72	322.65	48.45

School grade	10th		11th		12th	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Tutoring Expenditure	38.3	36.5	47.9	48.6	29.5	41.7
Hours of Self-Study	7.65	5.68	8.45	6.00	14.42	9.14
Hours of Tutoring	7.40	6.74	9.16	9.45	5.69	7.89
Income	406.9	177.0	394.4	191.1	381.4	171.4
Test Scores	-	-	-	-	415.39	62.46
N			1792			

(b) Sample Moments: Other characteristics

	Mean	Stdev
Parental Education	13.27	2.01
6th grade Academic Performance	6.52	1.70
N	1792	

Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: The unit for measuring private tutoring expenditure is 10,000 KRW, approximately 8 USD, based on the average exchange rate during the data collection periods. Hours data are weekly measures.

KELS tracks a nationally representative sample of 6,908 students (1% of the country's 703,914 7th graders) from 2005 through 2020. The survey consists of two stages: annual surveys during secondary school (2005-2012) and semi-annual surveys during college and early career (2012-2020). Household income and private tutoring expenditure are collected each year. The hours spent in tutoring activities and the hours spent for self-study are collected as a weekly average. There are five different measures of academic performance available in the dataset. Academic performance in primary school is measured as an ordered discrete measure answered by the household. For 7^{th} to 9^{th} grades, the administrative test scores are of achievement tests standardized at the national level. For 12^{th} grade, the administrative College Scholastic Ability Test

(CSAT) score is available. Table C.1 details the sample selection criteria and their effects.

Tables 1a and 1b present sample moments from KELS. A notable pattern is that while average self-study hours increase over time, tutoring hours show a decreasing trend - a pattern I examine further in Section 3.5. Household income moments remain stable throughout the sample period. Parental education, collected in the first year, is assumed constant given the relatively short time frame.[21]

To complement KELS's income data, I use the Korean Labor Income and Panel Study (KLIPS) to obtain college tier-specific lifetime income information. Details about KLIPS are provided in Appendix C.

3.2 The Lifetime Income Differential

College ranking has a strong effect on the growth of alumni's income.[22] The effect is significant even after controlling for CSAT score. Columns (1), (2), and (3) in Table C.2 provide the OLS estimates for the regression equations,

$$\ln y_{it} = \sum_{j=1}^{J} (\beta_j + \delta_j \cdot age_{it}) D_{i,j}^{Tier} + Z_{it}\gamma + \varepsilon_{it}^{y} \tag{1}$$

where $D_{i,j}^{Tier}$ is a dummy variable indicating that person i graduated from a tier j college, and Z_{it} is the set of explanatory variables including age, squared age, birth year, and gender of person i.[23]

Figure 2 shows the predicted annual income by college tier using estimates from Column (1) of Table C.2. While income differences are minimal before age 30, the gaps widen substantially with age. These effects remain significant after controlling for CSAT scores and college majors, as shown in Columns (2) and (3) of Table C.2.[24] Additional estimates including interactions between CSAT and Tier (reported in Table

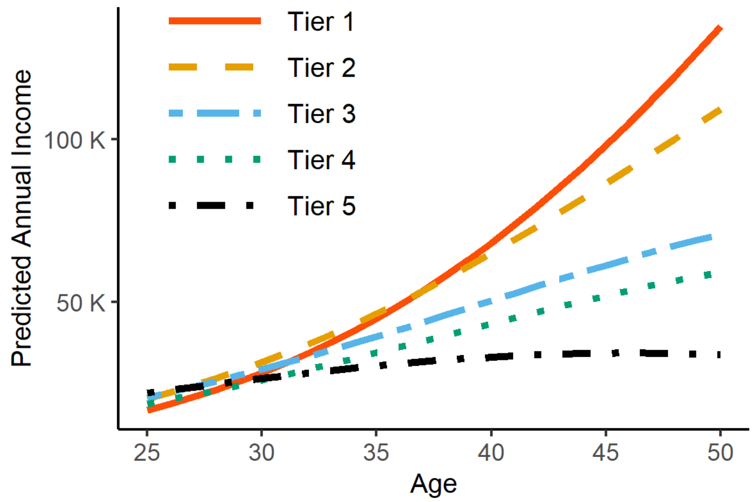
[21]This is a reasonable assumption given the relatively short period of time in the data. In fact, information on parental education is collected only in the first two years of the survey.

[22]Lee and Koh (2023) reports that the alumni of Tier 1 colleges in Korea earn 50.5% more compared to those from the bottom Tier group, based on their preferred specification and the tier definitions. The lifetime income empirical exercise in this section is consistent with their findings, but the specification differs to consistent with the structural model.

[23]The purpose of the birth year dummy variable is to capture the cohort difference in workers' income.

[24]As CSAT performance is collected as a discrete variable in KLIPS, the estimation is different with Regression Discontinuity Design.

Figure 2: Income Dynamics by College Tiers



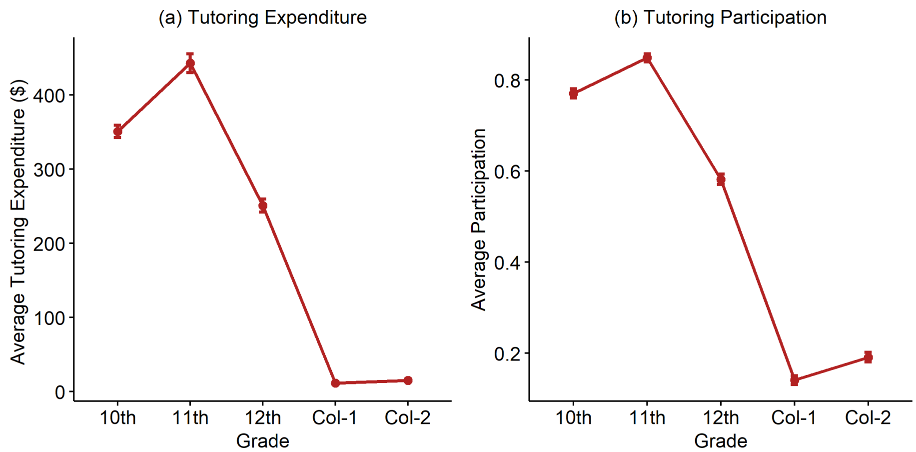
Source: Korea Labor Income and Panel Study 1998-2012, Korea Labor Institute.
Note: The sample includes workers between 25 and 65 years old who work for wages or salary. I exclude workers who are born after 1992. Unit is USD. Annual income is predicted using the Pooled-OLS estimates in column (1) of Table C.2. The income is predicted using birth year of 1992, which is the year of the KELS cohort was born.

C.3) show limited heterogeneity in tier effects by CSAT score, suggesting college quality affects earnings independently of student ability. This age pattern aligns with research emphasizing the importance of lifetime income in returns to education (Haider 2001; Tamborini *et al.* 2015; Nybom 2017). The estimates from Column (1) are used to compute college-specific lifetime income in the dynamic tournament model.

3.3 Competition Motives of Parental Investment

Competition with respect to getting into a more prestigious college is the primary motivation of parental investment. First, both tutoring expenditure and participation rates drop sharply after high school graduation, as shown in Figure 3. This abrupt decline after college admission decisions suggests tutoring primarily serves competitive rather than human capital development purposes - if the latter were true, tutoring would likely continue into college. Second, the number of seats at prestigious colleges is limited. Even with a very high final test score, students might not be able to go to a top-tier college if the seats are filled with students with higher test scores. The scarcity of seats at prestigious colleges and the fact that tutoring participation drops after the college entrance exam show that competition is the key feature determining the parental investment decision of the household.

Figure 3: Private Tutoring Expenditure and Participation in Tutoring



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

3.4 Income Gradient in Student Effort Choices

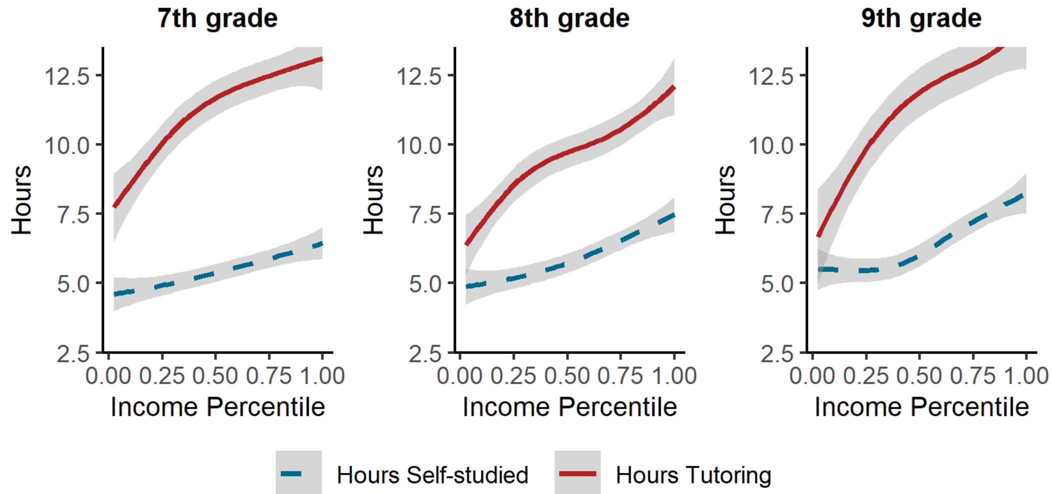
Compared to hours of tutoring, hours of self-study are less affected by parental income, which potentially has implications for intergenerational mobility. The income elasticity of hours of tutoring is higher than the income elasticity of hours of self-study. Figure 4 presents how hours of tutoring and hours of self-study vary with parental income when students are 7th, 8th, and 9th graders, using local linear regression. The slope of hours of tutoring is much steeper than the slope of hours of self-study, which shows that tutoring is an effort choice that is more responsive to parents' income.

Such empirical relationships suggest that different household backgrounds can lead to different allocations of effort choice. Thus, omitting one of the effort choices (parental investment or child effort) might result in biased estimates of intergenerational mobility, which calls for including both effort choices in the theoretical framework. Additional regression analysis examining the relationship between effort choices and parental background is presented in Appendix C.

3.5 Dynamic effort allocation of households

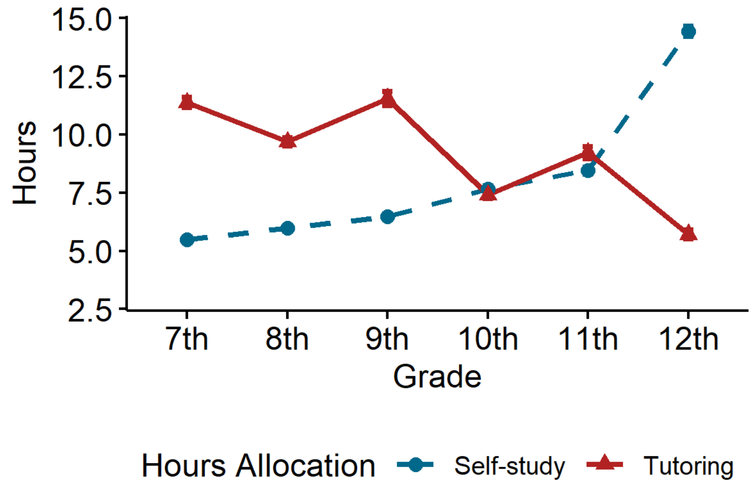
Students' time allocation of effort choices considerably changes as students proceed to the later educational stages. Figure 5 presents how the average hours of self-study and the average hours of tutoring change with students' grade level. While the average

Figure 4: Income Gradient in Effort Decision



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: The gray regions are confidence bands with a significance level of 0.05. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

Figure 5: Student's Time Allocation by Grades



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

hours of tutoring shows a decreasing trend, the average hours of self-study shows an increasing trend. In 12th grade, the average hours of self-study is almost three times the average hours spent for tutoring. Such changes in time allocation suggest that the marginal effects of hours of self-study and tutoring expenditures on academic outcomes might change over time.[25]

4 A Dynamic Model of College Admission Tournament

Motivated by the empirical evidence - particularly the competition-driven nature of parental investment and differences in lifetime earnings across college tiers - I develop a dynamic tournament model of college admissions competition. The framework builds upon the rank-order tournament literature pioneered by Lazear and Rosen (1981) and its applications to college admissions (Grau 2018; Tincani, Kosse and Miglino 2021). In the model, households make decisions about both parental investment and child effort levels, with final college placement determined by relative test score rankings and limited seat capacity.

4.1 Timeline

There exist N households in the dynamic tournament. Each household is composed of one student and the parents. I assume the household makes a unitary decision. I abstract away from the intra-household decision-making process. The students compete for the final prize against other students in the same cohort.

Figure 6 illustrates the timeline of the model. The model begins as the student of the household enters into 7th grade, which is the first year of secondary school. Each household is born with the complete income stream $\{w_{it}\}_{t=1}^{T}$, parental education m_i, and initial test score q_{i1}. Also, each household has a specific type k. Different types of households have different type-specific characteristics that are unobserved by the econometrician. I define them as λ_k^c, λ_k^x, λ_k^s and λ_k^q, which affect marginal utility from consumption, disutility from hours of tutoring, disutility from hours of self-study, and log of test score, respectively. Some households value non-academic goods such as travel more than other households conditional on the observed characteristics (Lazear 1977). Such unobserved taste for consumption is captured by λ_k^c. Some households

[25] Several studies in the literature report that the effects of parental investment decrease with children's age (Cunha *et al.* 2010; Del Boca *et al.* 2017).

prefer to encourage their child to study independently rather than send her to tutors, which is captured by the relative size of λ_k^s to λ_k^x. Some students might be particularly good or bad in taking exams, which would be captured by λ_k^q.[26]

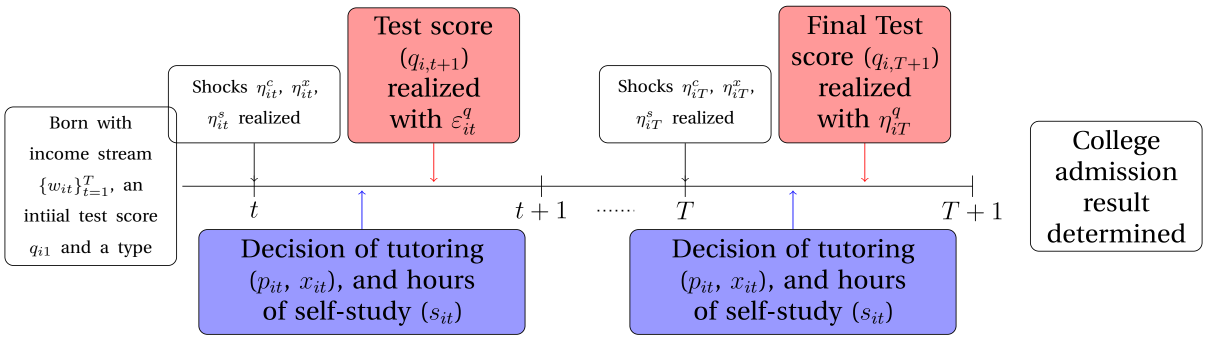


Figure 6: Model Timeline

At each time t, as the household enters into the period, the shock to the marginal utility of the consumption η_{it}^c, the shock to the marginal disutility from the tutoring activities η_{it}^x, and the shock to the marginal disutility from self-study η_{it}^s are realized. These shocks capture the unobserved time-varying components that are not accounted for by the deterministic components of the model. Based on those realized shocks and the observed state variables, each household chooses the quality of tutoring p_{it}, the hours spent on tutoring x_{it}, and the hours of self-study s_{it} to maximize its value function. The choices are subject to budget and time constraints. Subsequently, the test score $q_{i,t+1}$ is produced with the realization of the test score shock. This process repeats until the final test score $q_{i,T+1}$ is generated.

Each student is assigned to a college tier based on the ranking of the final test score and the fixed number of college seats in each tier. I denote n_j as the fixed number of seats for the j^{th} college-tier. In particular, denoting n_1 as the fixed number of seats for the first college tier, the first n_1 students are assigned to the top college tier, and the next n_2 students are assigned to the second tier. The process repeats until the $(J-1)^{th}$ college tier is filled up with n_{J-1} students so that all seats for the college tiers bind. The bottom tier is a residual tier which is composed of students whose score is below the

[26] I introduce the joint distribution of the time-specific shocks ($\eta_{it}^c, \eta_{it}^x, \eta_{it}^s$, and η_{it}^q) and the specification of type-specific unobserved heterogeneity (λ_k^c,λ_k^x,λ_k^s and λ_k^q) when I explain the flow utility component of the model.

cutoff for the $(J-1)^{th}$ college tier and the students who do not go to college.[27] The assigned college tier is the sole determinant of ex-post lifetime income.

4.2 The Preliminaries of the Tournament

Prize: Lifetime Income. The prize for going to a more prestigious college tier is a higher expected lifetime income awarded to the student, which motivates the household to exert effort. There exist J college tiers that are characterized by expected lifetime income v_j. The tier-specific lifetime income v_j is the discounted sum of the predicted income of the graduates. In particular,

$$v_j = \sum_{t=T+1}^{T^*} \beta^{t-T} \hat{y}_{jt}$$

where $\hat{y}_{jt}$ is the estimated income of the alumni of college tier j in year t, T is the age when the student graduates from college, T^* is the retirement age, and β is the discount factor fixed to 0.95.[28] I define $\hat{y}_{jt}$ as the estimated tier-specific annual income at time t, which is predicted using Pooled-OLS estimates of Column (1) in Table C.2.[29] As tier 1 is defined to be the top college tier, v decreases in j (i.e., $v_1 > v_2 > ... > v_{J-1} > v_J$).[30]

For the student of household i to obtain prize v_j, her final test score $q_{i,T+1}$ must be above the cutoff for tier j and below the cutoff for the tier $j-1$. In other words, student i is placed in college tier j iff

$$\tilde{Q}_{j-1} > q_{i,T+1} \geq \tilde{Q}_j$$

where $\tilde{Q}_j$ is the cutoff between college tier j and tier $j+1$. The cutoff $\tilde{Q}_j$ is the test score of the N_j^{th} highest student in the sample, where $N_j = \sum_{l=1}^{j} n_l$. Thus, $\{\tilde{Q}_j\}_{j=1}^{J}$ is where the competition enters the model. In order for a student to be in tier j or better, she has to be above enough competitors by at least scoring the N_j^{th} highest

[27]The implicit assumption regarding the bottom tier is that everyone graduates high school. The high school drop-out rate in South Korea is less than 2%.

[28]The average interest rate is around 5% for South Korea in 2010.

[29]I assume no earnings in the college periods.

[30]I confine the prize to pecuniary rewards and rule out other benefits from the model. One might argue that the non-pecuniary value of attending an elite college should be considered part of the reward. However, it is difficult to separately measure the non-pecuniary value of attending better colleges due to data limitations. See Gong *et al.* (2019) for an empirical quantification of the consumption value of college.

final test score. As $q_{i,T+1}$ is a function of the effort choice of each household, $\{\tilde{Q}_j\}_{j=1}^J$ is endogenously determined by the competition across households. I assume that each household can correctly predict the final test score cutoffs.[31]

Assumption 1. *Each household correctly guesses the set of final test score cutoffs* $\{\tilde{Q}_j\}_{j=1}^J$.

The facts that (i) college-tier is assigned solely using the final test score $q_{i,T+1}$ and (ii) heterogeneity in college quality is the only variation of the lifetime income in this framework imply that the final test score of a student essentially determines the lifetime income of the student. That is, under the model environment, I assume that there is no extra opportunity to improve one's lifetime income once the college entrance exam is over.

Assumption 2. *The quality of the college one graduates from is the sole determinant of one's lifetime income.*

This assumption is supported by evidence from Cho *et al.* (2024), who analyze hiring data from a major Korean conglomerate. As shown in Table C.4, graduating from a tier 1 college increases the probability of receiving a job offer by 23 percentage points (marginal effects) compared to graduating from below tier 3, while a full point increase in college GPA only raises this probability by 0.6 percentage points.

Parental Investment: One of the two modes of household effort is parental investment, which is embodied in private tutoring expenditure. Each household chooses the unit price (quality) of tutoring p_{it} and hours (quantity) of tutoring x_{it} to increase the child's test score.[32] The total amount of tutoring expenditure e_{it} is

$$e_{it} = p_{it} x_{it}.$$

The tutoring expenditure is constrained under two dimensions. A household cannot spend more tutoring expenditure than its income (i.e., $e_{it} \leq w_{it}$).[33] Also, hours of tutoring are bounded by the child's maximum available time, namely h. While the income constraint is unequal among households, available hours for the child are constant across all households.

[31] I assume away the inconsistency between the guessed cutoffs and the resulting cutoffs because the working sample did not go through significant policy shock that might cause the difference between the guessed and the resulting cutoffs. See Tincani, Kosse and Miglino (2021) for the case that resulting cutoffs significantly deviate from the guessed cutoffs.

[32] To the best of my knowledge, this is the first model to consider the quality and quantity of parental monetary investment simultaneously.

[33] I assume no borrowing.

Note that the time choice is solely about the time use of the child, which means I do not model the time allocation of parents. The data suggest that, in the secondary school periods, which the model concerns, the majority of parents do not teach their children themselves in middle school periods, and very few parents use their time to teach their child in the high school periods. A few potential explanations can be given for this empirical fact. As students grow, the test materials become more and more difficult to be taught by parents. Also, if there exists an established tutoring market, it would be a safer option for parents in terms of increasing student's test score. Note that the model concerns a regime with a high-stakes standardized test. Full-time tutors would have a comparative advantage in preparing students for exams over parents.

Child's hours of self-study: Hours of self-study is the other household's mode of effort in the tournament. Each household chooses how much time to allocate for hours of self-study s_{it} which is constrained by h. Unlike parental investment, the resource of self-study does not vary over households as time is equally granted to everyone. The taste for self-study, however, can be considerably heterogeneous across students. For example, some students might prefer studying independently rather than re-learning the same materials from the tutors. Others may prefer reviewing materials with tutors rather than studying alone. I allow the taste for hours of self-study to vary by parental education and the associated shock.

Test Score Production Function: The final test score is the result of accumulated dynamic choices of the household along with its given initial conditions. The initial academic performance q_{i1} is exogenously given and proxied by academic performance in primary school.[34] The three choices affecting test scores are quality of tutoring p_{it}, hours of tutoring x_{it}, and hours of self-study s_{it}. I allow that the quantity (hours) and quality (unit price) of the tutoring activity have different intensities in contributing to the test score production. Denoting κ as intensity of quality of tutoring, the transformed tutoring input is specified as

$$\tilde{e}_{it} = p_{it}^{\kappa} x_{it}^{1-\kappa} \tag{2}$$

[34] Although an earlier measure of the initial child's ability would be more desirable, this is the earliest time period that the academic performance data are available.

where $\kappa < 0.5$ and follows decreasing returns to scale (DRS). The DRS restriction is necessary to prevent the household from choosing an infinitesimal quantity of tutoring hours. If $\kappa \geq 0.5$, the household always has an incentive to make p_{it} greater and x_{it} smaller. The opposite case of a household choosing extremely large hours of tutoring does not occur as available time is restricted by h.

For each time $t = 1, 2, ..., T$, the test score $q_{i,t+1}$ is produced following

$$q_{i,t+1} = g(\theta_t^q, q_{it}, p_{it}, x_{it}, s_{it}, \eta_{it}^q, \lambda_k^q)$$

where η_{it}^q is the test score shock, λ_k^q is the type-specific error, and θ^q is the set of relevant parameters for the test score production. The inclusion of the test score produced in the previous period, q_{it}, allows that the previous test score has its own effects in generating subsequent test score (Cunha and Heckman 2007). Furthermore, I allow the subset of production parameters to change across periods. The effect of the combined efforts of the household is likely to change over time. As students grow older, the materials taught become more advanced, which makes it harder for students with insufficient background to catch up. Thus, private tutoring expenditure and hours of self-study can be less effective in the later stages of education. In addition, the relative importance of each investment might change over time. For example, the marginal effects of parental investment might increase (decrease) while the effects of self-study decrease (increase) over time. To reflect such changing effects, I let the marginal effects parameters ν_t, δ_{pt}, and δ_{st} be different for each period $t = 1, 2, ...T$.

For estimation, the production function g is a Constant Elasticity of Substitution (CES) production function and is specified as

$$q_{i,t+1} = A_t q_{it}^{\delta_{qt}} \left[\delta_{et}(1 + \tilde{e}_{it})^{\phi} + \delta_{st}(1 + s_{it})^{\phi} \right]^{\frac{\nu_t}{\phi}} \varepsilon_{it}^q \tag{3}$$

where A_t is total factor productivity, ν_t is the parameter of marginal effect of the combined effort choices, and ϕ is the parameter governing substitution between tutoring and self-study. The marginal effect of the total effort decision is captured by ν_t, while the relative importance of the tutoring expenditure and hours of self-study are captured by δ_{et} and δ_{st}, respectively. I define ε_{it}^q as a combined shock of λ_k^q and η_{it}^q, which is specified as $\ln \varepsilon_{it}^q = \lambda_k^q + \eta_{it}^q$.

4.3 Household

Flow Utility: The utility function of the unitary household is comprised of three parts: (i) the marginal utility from the household consumption c_{it}, (ii) the marginal disutility from hours spent on tutoring x_{it}, and (iii) the marginal disutility from hours of self-study s_{it}. I denote α_c, α_x, and α_s as taste parameters for household consumption, hours of tutoring, and hours of self-study, respectively. The taste parameters may depend on the fixed characteristics of the household. I assume additive and separable log utility, which is specified as

$$u(c_{it}, x_{it}, s_{it}, \varepsilon_{it}) = \alpha_c \varepsilon_{it}^c \log(c_{it}) + \alpha_x \varepsilon_{it}^x \log(1 + x_{it}) + \alpha_s \varepsilon_{it}^s \log(1 + s_{it}) \tag{4}$$

where ε_{it}^c is the shock to the marginal utility from consumption, ε_{it}^x is the shock to the disutility from hours of tutoring, ε_{it}^s is the shock to the disutility from hours of self-study, and $\varepsilon_{it} = \{\varepsilon_{it}^c, \varepsilon_{it}^x, \varepsilon_{it}^s\}$. The shocks are distributed joint normal and separated into the type-specific and the time-varying components. In particular, I denote λ_k^z and η_{it}^z as type-specific and time-varying components of ε_{it}^z $(z = c, x, s, q)$, respectively. The shocks are decomposed as

$$\begin{pmatrix} \ln \varepsilon_{it}^c \\ \ln \varepsilon_{it}^x \\ \ln \varepsilon_{it}^s \\ \ln \varepsilon_{it}^q \end{pmatrix} = \begin{pmatrix} \eta_{it}^c \\ \eta_{it}^x \\ \eta_{it}^s \\ \eta_{it}^q \end{pmatrix} + \begin{pmatrix} \lambda_k^c \\ \lambda_k^x \\ \lambda_k^s \\ \lambda_k^q \end{pmatrix}, \text{ and } \begin{pmatrix} \eta_{it}^c \\ \eta_{it}^x \\ \eta_{it}^s \\ \eta_{it}^q \end{pmatrix} \sim N(0, \Omega^\eta)$$

where Ω^η is the covariance matrix for the time-varying shocks.[35] I assume that the correlations between the time-varying shocks $\eta_{it}^z (z = c, x, s, q)$ are 0.

Note that I do not specify the utility flow from the current test score. Each household is concerned solely about the final outcome, and the role of the current test score is limited to the stepping stone for the final test score. That is, the current test score affects the decision of the household only through the value of the future. The specification of future value is introduced with the recursive formulation at the end of the subsection.

[35] In modeling the self-study shock, an alternative specification involves assuming that there exists unobserved heterogeneity in terms of the productivity of hours of self-study. Such an assumption, however, is computationally burdensome if the test score production function is CES.

Terminal Value: Expected lifetime income is the terminal value of the model, which drives the dynamic choices of the tournament model. With the tier-specific lifetime income v_j, the expected lifetime income is a weighted sum,

$$\sum_{j=1}^{J}\left\{ \ln(v_j) * Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT})\right\} \tag{5}$$

where $Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT})$ is the probability of getting into college tier j. The randomness of the admission probability comes from the test score shock η^q_{it}. Each student would have a different probability of going to a college tier j as they have different characteristics affecting the evolution of the test scores. The disparity among students in terms of going to each college tier leads to the discrepancies in expected lifetime income, which generates the heterogenous incentives among households. The higher expected lifetime income leads to bigger the terminal value of the household, which makes it more appealing for the parents to invest in the child.

The functional form of the expected lifetime income is determined by the test score shock ε^q_{it}. With the log-transformation, the terminal value is specified as

$$\begin{aligned}
&\sum_{j=1}^{J}\left\{ \ln(v_j) * Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT})\right\} \\
&= \sum_{j=1}^{J}\left\{ \ln(v_j) * \left\{ F_q(\frac{\ln \tilde{g}_{i,j-1}}{\sigma_q}\Big| \Gamma_{iT}) - F_q(\frac{\ln \tilde{g}_{ij-1}}{\sigma_q}\Big| \Gamma_{iT})\right\}\right\}
\end{aligned}$$

where $\ln \tilde{g}_{ij}$ is the distance between the deterministic components of log final test score of student i and the log cutoff of the college tier j (i.e. $\ln \bar{g}_{ij} = \ln \tilde{Q}_{j-1} - \ln \widehat{q_{i,T+1}} - \lambda^q_k$), and F is the distribution of η^q_{it}. I assume F follows normal distribution in the spirit of rank-order tournament model (Lazear and Rosen 1981; Han, Kang and Lee 2016; Grau 2018; Tincani, Kosse and Miglino 2021).[36]

Budget and Time Constraints: The choices of the household are restricted by the budget and the time constraints. The budget constraint is given by

$$c_{it} + p_{it}x_{it} \leq w_{it} \tag{6}$$

[36] One can also adopt a functional form that η^q_{it} follows Generalized Extreme Value distribution which results in a Tullock (2001) contest.

where w_{it} is household income, and the time constraint is

$$x_{it} + s_{it} \leq h \tag{7}$$

where h is student's disposable time. I define h as the maximum time each student can use every week, which is assumed to be 63.[37]

State Variables: There are observed and unobserved state variables in the dynamic model. The set of observed state variables Z_{it} includes the previous test score q_{it}, parental education m_i, and the complete income stream $\{w_{it}\}_{t=1}^{T}$. The set of unobserved state variables Ψ_{it} includes the set of unobserved shocks and the type specific heterogeneity. Based on the timeline, the time-varying shock regarding test score is not an unobserved state variables. (i.e., $\Psi_{it} = \{\eta_{it}^c, \eta_{it}^x, \eta_{it}^s, \lambda_k^c, \lambda_k^x, \lambda_k^s, \lambda_k^q\}$).

Information and Uncertainty: I assume a continuum of households. The continuum assumption is useful in that the information of other households can be summed up as a distribution of households.

Assumption 3. *The distribution of household is common knowledge.*

As stated in Assumption 1, each household correctly anticipates the set of college tier cutoffs $\{\tilde{Q}_j\}_{j=1}^{J}$.[38] They know the distribution of the final test scores in advance and make dynamic choices based upon the perfect guess.

Assumption 4. *Each household knows its complete wage stream.*

There is no uncertainty in the income process. In fact, each household is assumed to know its complete wage stream as the model begins. As this is a markov model, past wages are irrelevant after conditioning on the remaining state variables. As depicted in Figure 6, each household learns about the realization of the consumption shock η_{it}^c, the disutility shock to hours of tutoring η_{it}^x, and disutility shock to hours of self-study η_{it}^s at the beginning of each period. However, it does not know about the test score shock η_{it}^q before it makes a decision. Therefore, it makes a set of choices based on the expectation over η_{it}^q, $\eta_{i,t+1}^c$, $\eta_{i,t+1}^x$, and $\eta_{i,t+1}^s$, conditional on observed state variables and type-specific unobserved heterogeneity.

[37] I assume each student can use 9 hours everyday for non-leisure activities other than hours spent in regular school

[38] In the static model of Grau (2018), Assumption 3 implies that the tournament participants can correctly guess the cutoffs. In my dynamic model, however, Assumption 3 does not guarantee the perfect foresight due to the presence of future shocks that each individual cannot predict.

Household Value Function: Building upon the model components, I describe the value function of the household. As stated earlier, each household chooses the unit price (quality) of tutoring p_{it}, hours of tutoring x_{it}, and hours of self-study s_{it} based on the anticipation of future values. In particular, at each time t, the household i solves

$$V_{it}(Z_{it}, \Psi_{it}) = \max_{p_{it}, x_{it}, s_{it}} \left\{ u(c_{it}, x_{it}, s_{it}, \varepsilon_{it}) + \beta \underset{\eta^q_{it}, \eta_{it}}{E} \left[V_{i,t+1}(Z_{i,t+1}, \Psi_{i,t+1} \middle| \Gamma_{it}) \right] \right\}, \tag{8}$$

subject to equation (3) and constraints (6) and (7), where $\Gamma_{it} = \{Z_{it}, \Psi_{it}, \{\bar{Q}_j\}_{j=1}^J\}$ is the set of information before making the decision and $\eta_{it} = \{\eta^c_{it}, \eta^x_{it}, \eta^s_{it}\}$ is the set of unobserved time-varying shocks. Each household faces a tradeoff between current flow utility and future payoffs. Each choice variable incurs costs associated with the choice. In particular, investing more in parental investment (i.e., increasing p_{it} or x_{it}) requires suffering more from the disutility from hours of tutoring and sacrificing current consumption. Spending more time on hours of self-study leads to an increase in the disutility from hours of self-study. This dynamic incentive structure governs the decision of the household.

At the final test stage ($t = T$), where the tournament term appears, the value function is

$$\begin{aligned} V_{iT}(Z_{iT}, \Psi_{iT}) = \max_{p_{iT}, x_{iT}, s_{iT}} & \Bigg\{ u(c_{iT}, x_{iT}, s_{iT}, \varepsilon_{iT}) \\ & + \alpha_v \sum_{j=1}^{J} \ln(v_j) \times Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT}) \Bigg\} \end{aligned} \tag{9}$$

where α_v is an altruism parameter. The altruism parameter measures the "exchange rate" between the current household utility and the child's future lifetime income. All-in-all, each household makes a choice between the child's lifetime income and its flow utility. If the marginal value to the household is greater than the marginal loss of flow utility of the household, it exerts more efforts using either parental investment, the child's self-efforts, or both.

4.4 Equilibrium of the Tournament

In this section, I define the dynamic equilibrium of the tournament model. Then I prove the existence of the equilibrium using the Schauder Fixed-Point Theorem (Amir 1996; Fey 2008; Mertens and Judd 2018; Engers, Hartmann and Stern 2022). I define a set $k = \{\{V_t(p_t, x_t, s_t; Z_t, \Psi_t)\}_{t=1}^{T}, \{\tilde{Q}_j\}_{j=1}^{J}\}$, where $\{V_t(p_t, x_t, s_t; Z_t, \Psi_t)\}_{t=1}^{T}$ is a set of value functions that are specified in equations (8) and (9), and $\{\tilde{Q}_j\}_{j=1}^{J}$ is the set of college-tier cutoffs. I define $\mathcal{K}$ as a set of all possible k.

Definition 1. *Given the set of initial conditions and Assumptions 1, 2, and 3, a Markovian equilibrium of the model is a vector* $k^* = \left\{\{V_t^*(p_t, x_t, s_t; Z_t, \Psi_t)\}_{t=1}^{T}, \{\tilde{Q}_j^*\}_{j=1}^{J}\right\}$, *which is generated by the following process:*

1. I define $\mathcal{K}_a$ as a set of all possible combinations of choice variables $\{p_t, x_t, s_t\}_{t=1}^{T}$. Given the set of initial conditions $\{q_{i1}, \{w_{it}\}_{t=1}^{T}, m_i\}$, a mapping $\aleph_a$ maps $\mathcal{K}$ into $\mathcal{K}_a$ ($\aleph_a : \mathcal{K} \to \mathcal{K}_a$), based on the value functions specified in equations (8) and (9). [39]
2. I define $\mathcal{K}_b$ as possible distributions of the final test score q_{T+1}. A mapping $\aleph_b$ maps $\mathcal{K}_a$ into $\mathcal{K}_b$ ($\aleph_b : \mathcal{K}_a \to \mathcal{K}_b$), based on the test score production function specified in equation (3) .
3. I define $\mathcal{K}_c$ as possible sets of resulting cutoffs $\{\check{Q}_j\}_{j=1}^{J}$. Given the number of seats for each college tier $\{n_j\}_{j=1}^{J}$, a mapping $\aleph_c$ maps the distribution of the final test score q_{T+1} and $\{n_j\}_{j=1}^{J}$ into the set of cutoffs, $\{\check{Q}_j\}_{j=1}^{J}$ ($\aleph_c : \mathcal{K}_b \to \mathcal{K}_c$). The mapping $\aleph_c$ is based on the rules of college admission.
4. A mapping $\aleph_d$ maps $\{\check{Q}_j\}_{j=1}^{J}$ into $\mathcal{K}$ ($\aleph_d : \mathcal{K}_c \to \mathcal{K}$).
5. In equilibrium, the set of guessed cutoffs $\{\tilde{Q}_j\}_{j=1}^{J}$ match the set of realized cutoffs $\{\check{Q}_j\}_{j=1}^{J}$.

Finally, I define a mapping $\aleph : \mathcal{K} \to \mathcal{K}$. The mapping $\aleph$ is the composition of submappings. In particular,

$$\begin{aligned}\aleph =& \aleph_a \circ \aleph_b \circ \aleph_c \circ \aleph_d \\ =& \aleph_a(\aleph_b(\aleph_c(\aleph_d(k)))).\end{aligned}$$

Lemma 2. *The mapping $\aleph$ is compact*

Proof. [In Appendix A.1] □

Lemma 3. *The mapping $\aleph$ is continuous*

Proof. [In Appendix A.2] □

[39]The mapping $\aleph_a$ involves backward recursion.

Theorem 4. *A Markovian equilibrium exists.*

Proof. Previous results establish that $\mathcal{K}$ is a nonempty, compact, and closed subset of a locally convex Hausdorff space. The map $\aleph$ is continuous. Therefore, the set of fixed points of $\aleph$ is nonempty and compact. The mapping satisfies all the requirements of Schauder Fixed-Point Theorem. Hence a fixed point exists. □

5 Estimation Strategy

I estimate the model parameters using Maximum Simulated Likelihood to leverage the individual-level longitudinal data. The likelihood function is maximized subject to the constraint that model-predicted college tier cutoffs match the observed cutoffs, ensuring the estimated parameters are consistent with tournament equilibrium.[40]

5.1 The likelihood function

I denote θ as the set of parameters, Z_{it} as the set of observed state variables, and λ_k as the set of type-specific unobserved heterogeneity. The individual likelihood contribution of household i is

$$\mathcal{L}_i(\theta|q_{i1}, \{w_{it}\}_{t=1}^T, m_i) = \sum_{k=1}^{K} \left\{ \left(\Pi_{t=1}^T \mathcal{L}_{it}(\theta|Z_{it}, \lambda_k) \right) \Pr(type = k) \right\}, \tag{10}$$

which is conditional on the initial test score q_{i1}, the household income stream $\{w_{it}\}_{t=1}^T$, and parental education m_i. The time-specific likelihood contribution $\mathcal{L}_{it}(\theta|Z_{it}, \lambda_k)$ can be characterized in four different ways depending on the combination of the tutoring and self-study participation. In particular,

$$\begin{aligned}
\mathcal{L}_{it}(\theta|Z_{it}, \lambda_k) = &\Big[f(p_{it}, x_{it}, s_{it}, q_{it}) \Big]^{d_{it}^x d_{it}^s} \\
&\times \Big[\Pr(p_{it}, x_{it}, s_{it} = 0) \cdot f_{q_{it}}(q_{it}|x_{it}, s_{it} = 0) \Big]^{d_{it}^x (1-d_{it}^s)} \\
&\times \Big[\Pr(x_{it} = 0, s_{it}) \cdot f_{q_{it}}(q_{it}|x_{it} = 0, s_{it}) \Big]^{(1-d_{it}^x) d_{it}^s} \\
&\times \Big[\Pr(x_{it} = 0, s_{it} = 0) \cdot f_{q_{it}}(q_{it}|x_{it} = 0, s_{it} = 0) \Big]^{(1-d_{it}^x)(1-d_{it}^s)}
\end{aligned}$$

[40]The idea of using the equilibrium of game as a constraint of the estimation routine is in line with Su (2014) and Egesdal, Lai and Su (2015)

where $d^x_{it} = 1$ means that household participates in tutoring at time t, and $d^s_{it} = 1$ means that the student of the household i has non-zero hours of self-study at time t.

To ensure that the simulated choices of the households follow the equilibrium of the dynamic tournament, the likelihood function is maximized under the constraint that the simulated cutoffs ($\{\tilde{Q}_j\}_{j=1}^J$) match the cutoffs given by the data ($\{\widehat{Q}_j\}_{j=1}^J$).

Finally, the structural parameters are estimated by solving

$$\max_\theta \log \mathcal{L}(\theta)$$
$$s.t.\ \{\tilde{Q}_j\}_{j=1}^J = \{\widehat{Q}_j\}_{j=1}^J$$

where

$$\log \mathcal{L}(\theta) = \sum_{i=1}^N \log \mathcal{L}_i(\theta|q_{i1}, \{w_{it}\}_{t=1}^T, m_i).$$

5.2 Identification

Parameters of the model can be classified into the productivity parameters associated with the test score function and the taste parameters that directly affect the value function. The productivity parameters in the test score production function are identified by the covariation between the subsequent test score $q_{i,t+1}$ and the inputs (q_{it},p_{it}, x_{it}, and s_{it}). As data on the inputs are available for each period, I can separately identify the productivity parameters for each time t.

The taste parameters α_c, α_x, α_s, and the altruism parameter α_v affect the value function, and do not directly affect the test score function. These parameters are the constants for the likelihood contribution of the corresponding choice variables. I do not differentiate the taste parameters for each period.

The longitudinal structure of the data allows me to identify time-invariant type-specific unobserved heterogeneity regarding consumption (λ^c), disutility from hours of tutoring (λ^x), self-study (λ^s), and test score (λ^q).

The time-varying shocks $\eta^z_{it}(z = c, x, s, q)$ capture the parts that are not explained by the observable characteristics and the type-specific unobserved heterogeneity. The elements of the covariance matrix of the shocks are identified by the residuals of the structural model.

Finally, the identifying assumption is the time-varying unobserved shocks are orthogonal to the initial conditions, which are (i) the academic performance in primary school q_{i1}, (ii) the parental education m_i, and (iii) the income stream of parents $\{w_{it}\}_{t=1}^{T}$. Formally,

$$\{\eta_{it}^{c}, \eta_{it}^{x}, \eta_{it}^{s}, \eta_{it}^{q}\}_{t=1}^{T} \perp \left\{q_{i1}, m_i, \{w_{it}\}_{t=1}^{T}\right\}.$$

6 Estimation Results

6.1 Test score function parameters

Table 2: Parameter Estimates: Test score production function

Time-varying Parameters	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
Previous Test Score (δ_{qt})	0.176 (0.003)	0.964 (0.017)	0.796 (0.001)	0.809 (0.001)	0.464 (0.001)	0.457 (0.001)
Effort Parameters (ν_t)	0.799 (0.001)	0.776 (0.007)	0.611 (0.009)	0.481 (0.004)	0.215 (0.003)	0.029 (0.001)
Share of tutoring Expenditure (δ_{et})	0.332 (0.001)	0.357 (0.001)	0.376 (0.012)	0.412 (0.016)	0.421 (0.001)	0.536 (0.001)
Constants (δ_{0t})	3.620 (0.032)	-1.097 (0.001)	0.060 (0.001)	0.457 (0.001)	0.344 (0.003)	4.260 (0.003)
Time Invariant Parameters						
Substitution Parameter (ϕ)	0.428 (0.003)	Intensity of quality (κ)			0.086 (0.002)	

Note: Standard errors are computed using delta method and are in parentheses below estimates. Based on the CES test score function, share of hours of self-study is implied by share of tutoring expenditure. (i.e., $\delta_{st} = 1 - \delta_{et}$).

Table 2 presents the estimates of test score production function. The estimated effort parameters show patterns of declining marginal effects over time, with the final period effect plummeting to 0.03. Figure 7 presents the computed average marginal effects for hours of tutoring and self-study. For both investemnts, the average marignal effects to log test score in the final period are about 0.002. Despite such minimal impact in the final period, households maintain substantial investment for both tutoring and hours of self-study.[41] This seemingly puzzling pattern can be rationalized

[41] As I show in Section 6.3, this data pattern is captured by the model

by the tournament nature of college admissions. Table 3 presents simulation results showing that removing final period efforts for Tier 1 students leads to significant tier changes: about 44.4% of Tier 1 students drop to Tier 2 if they stop spending on tutoring. As demonstrated in Table F.8, these slots would be filled by the next best students who were originally in Tier 2. These results suggest that even small advantages matter greatly in maintaining elite college admission chances, driving continued investment despite low marginal returns.

Table 3: Proportion of Tier 1 Students Dropping Tiers With Reduced Investments

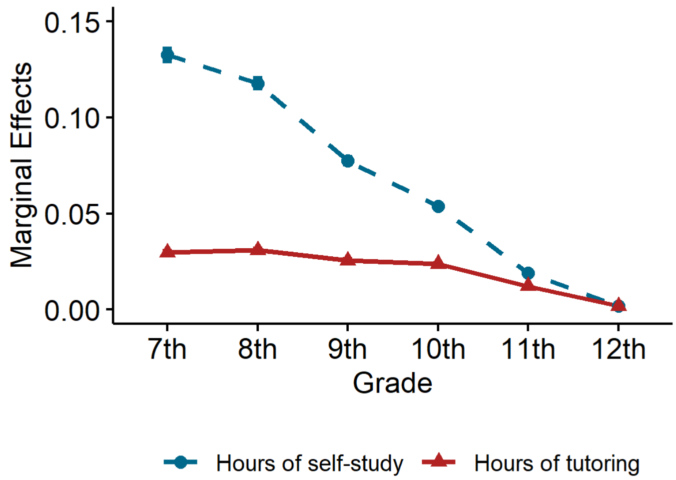
	Type of Effort Restriction		
Outcome	Tutoring	Self-Study	Both
Panel B: Half Effort			
Drop to Tier 2	0.167	0.222	0.333
Drop to Tier 3	0.000	0.000	0.000
Panel A: Zero Effort			
Drop to Tier 2	0.444	0.722	0.167
Drop to Tier 3	0.000	0.056	0.833

Notes: This table shows the proportion of initial Tier 1 students who drop to lower tiers under different counterfactual scenarios. Panel A shows results when effort is completely eliminated, while Panel B shows results when effort is reduced by 50%.

The marginal effects shown in Figure 7 reveal the importance of early-period investments. The effects of both tutoring and self-study are substantially larger in earlier periods compared to later periods. While the difference between tutoring and self-study effects becomes negligible in the final period, the accumulated advantages from early investments persist through the production function's dynamic structure, where previous test scores strongly influence subsequent test scores. The persistence parameter estimates (ranging from 0.464 to 0.964) suggest that earlier investments have persisting effects through the dynamic structure.

Finally, the production function estimates suggest that tutoring and self-study are gross substitutes, with a substitution parameter of 0.43. This high substitutability has important implications for understanding intergenerational mobility. Without accounting for self-study as a substitute for tutoring, models would likely overstate the role of parental investment in perpetuating inequality. The availability of self-study as an alternative means of effort provides income-constrained households a channel to compete in the college admission tournament, potentially mitigating the advantages of high income families who can afford extensive tutoring.

Figure 7: Average Marginal Effects of Hours Allocation



Note: This figure presents the average of marginal effects of hours of self-study and hours of tutoring over time. Due to the functional form of the test score function, the marginal effects differ by each individual. The marginal effects are computed using the first order derivative with respect to hours of self-study (s_{it}) or hours of tutoring (x_{it}) and the estimated parameters. The vertical interval at each point indicates the standard deviation of the marginal effects.

6.2 Preference and shock parameters

Table F.9 presents the estimates of the preference parameters and the shock parameters. The preference parameters are components of equations (4), (8), and (9). For the preference parameters, the estimates are relative estimates of utility from hours of leisure, of which parameter is fixed to 0. The altruism parameter is estimated as 1.090. To capture the observed heterogeneity of the household, I allow the disutility parameters to vary by parental education. In particular, $\exp(\tau_x D_i^{pedu})$ is multiplied to the disutility from hours of tutoring α_x and $\exp(\tau_s D_i^{pedu})$ is multiplied to the disutility from hours of self-study, where D_i^{pedu} is 1 for household whose average years of parental education is strictly greater than 12. Table F.9 (a) includes the estimates of the effects of parental education on the preference parameters. Based on the estimates, parental education alleviates the disutility to hours of tutoring. Specifically, a child of a household whose average education of parents is greater than 12 years feels less disutility of tutoring by 0.127. In contrast, the effect of parental education on mitigating disutility from hours of study is not statisticially different from 0.

The estimated standard deviations of unobserved shocks are overall modest, suggesting that observed characteristics and the structural model capture a considerable proportion of heterogeneity in the data. At the same time, Table F.10 presents unobserved heterogeneity across household types. Notably, the type probability estimate of Type 2 household is 0.389, which suggests considerable unobserved heterogeneity in consumption and the disutility from tutoring hours. This finding suggests an unignorable, time-invariant heterogeneity associated with tutoring quality (linked to the consumption shock) and tutoring hours, which appear to be highly correlated.

6.3 Model Fit

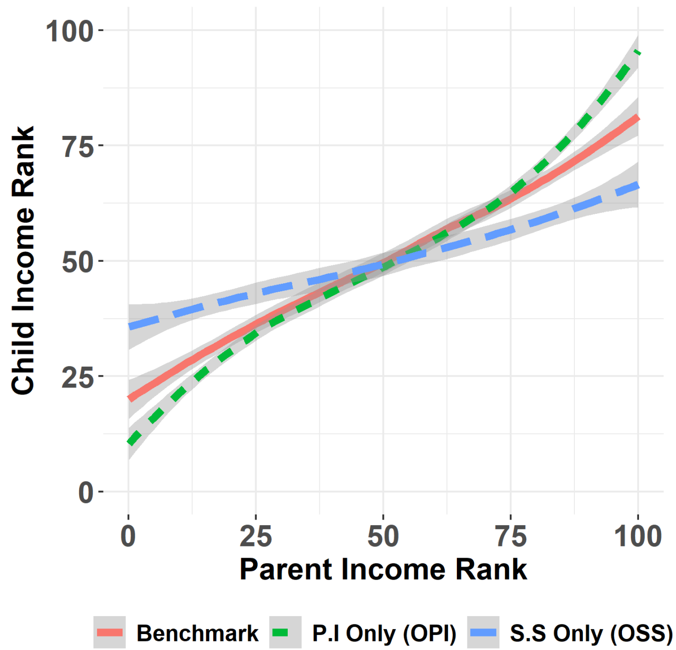
The model exhibits a very good fit to both key empirical moments and overall distributions of the data. Figure G.1 compares model predictions with data for choice variables and log test scores over time, showing the model captures dynamic patterns in tutoring hours, self-study time, and test score evolution. Table F.7 confirms precise matching of the college tier cutoffs, which were targeted as a constraint in the maximum likelihood estimation. To evaluate distributional fit beyond aggregate moments, I employ local linear regression to examine how well model predictions $\widehat{y}$ match actual values y across the full distribution of each outcome $(y = e, p, x, s, q)$.[42] Figures G.2 through G.6 demonstrate strong distributional fit for tutoring expenditure, hours of tutoring, quality of tutoring, hours of self-study, and test scores, indicating the model captures the outcome distributions very well.

7 Counterfactual Analyses

[42]Appendix G includes the specification of local linear regression used to judge the fit.

7.1 Decomposition of Intergenerational Persistence of Earnings

Figure 8: Intergenerational Persistence of Earnings by Scenarios



Note: This graph presents local linear curves that fit child income rank and parental income rank under different counterfactual scenarios. Parent Income Rank is the average income over six years. Child Income Rank is computed based on the simulated results of each scenario. BCF is the benchmark counterfactual; OPI is a simulation where each household can use only parental investment; and OSS is a simulation where each household can use only hours of self-study.

The purpose of the quantification exercise is to decompose the role of channels affecting intergenerational persistence of earnings. I simulate the model under the counterfactual environments that help quantify the impact of the investment decisions and the household characteristics. As the cutoffs are unknown for these hypothetical cases, I simulate the household behavior until their guess of the cutoffs becomes identical with the resulting cutoffs.[43] Each simulation produces a different distribution of final test scores, leading to a different distribution of predicted child income.

Each counterfactual scenario is defined in the following way.

- BCF is the status quo where the model is simulated without a counterfactual modification.

[43] I use this algorithm again in the college constraint counterfactual analyses in the subsequent subsection.

- OPI is the counterfactual where only parental investment is the means of the tournament model, and hours of self-study are excluded from the choice of the household and fixed to 0.
- OSS is the counterfactual where only child's self-study is the means of the tournament, and parental investment is excluded from the choice and fixed to 0.[44]

I adopt the rank-rank slope (Chetty *et al.* 2014) as the measure of the intergenerational persistence of the earnings. In particular, it is the slope estimate of the regression equation,

$$R_i = \delta_{01} + \delta_{RR} P_i + v_i, \tag{11}$$

where R_i is the percentile rank of the child income within the generation, and P_i is the rank of the parental income within the generation.[45]

Table 4 presents the estimates of intergenerational persistence under different counterfactual scenarios. The benchmark simulation (BCF) yields a rank-rank slope of 0.574, while counterfactual simulations suggest the contrasting roles of different effort choices. When self-study is removed as an option, the rank-rank slope increases by 25.3% to 0.719 (Column 2). Conversely, removing parental investment reduces the slope by 49.3% to 0.291 (Column 3). These results suggest that while parental investment reinforces intergenerational persistence of earnings, self-study serves as a moderating force. This finding aligns with the substitution parameter estimate in Table 2, which indicates substantial substitutability between tutoring and self-study. Additionally, comparison of BCF with NST and NED show the role of initial conditions. The initial test score and the parental education are responsible for 16% and 10.2% of the intergenerational persistence of earnings, respectively. Table H.0 shows that the slope changes of Table 4 are not sensitive to the initial conditions.

[44] The OSS simulation is equivalent to China's tutoring ban policy in that it completely prohibits private tutoring activities. Gu and Zhang (2024) analyze this policy's macroeconomic impacts.

[45] Intergenerational elasticity of earnings (IGE) is an alternative measure. I focus on rank-rank slopes as they are more robust to differences in income variance across generations compared to IGE estimates. The IGE results are reported in Appendix G.

Table 4: Rank-Rank Slope Estimates

	(1) BCF	(2) OPI	(3) OSS	(4) NST	(5) NED
Rank-Rank Slope	0.574***	0.719***	0.291***	0.482***	0.516***
	(0.019)	(0.016)	(0.023)	(0.021)	(0.020)
R-squared	0.329	0.517	0.085	0.233	0.266

Notes: This table presents rank-rank slope estimates under different counterfactual scenarios. BCF is the benchmark counterfactual. OPI allows only parental investment. OSS allows only hours of self-study. Standard errors in parentheses.

7.2 Relaxing College Constraints

Structural estimates and the quantification exercise of Tier 1 students in Section 6.1 suggest that the competition for limited seats drives high investment despite low marginal increase in the test scores. This raises important policy questions: Could expanding elite college access reduce excessive private tutoring? How will demographic changes affect investment incentives? These questions are particularly relevant as many countries face dramatic demographic shifts. Korea, for instance, projects its high school cohort size to shrink by nearly 50% by 2033 due to declining fertility.

I leverage the equilibrium structure of the tournament model to analyze these policy-relevant scenarios through two counterfactual simulations: (i) a 50% increase in seats at top-tier colleges, mimicking potential expansion of elite higher education, and (ii) a 50% decrease in the cohort-to-seat ratio, reflecting projected demographic changes. These simulations shed light on how institutional constraints and demographic shifts shape the competitive dynamics of educational investment.

Changes in college capacity affect tutoring demand through two opposing effects. First, expanding seats increases the marginal return to tutoring for students who were relatively farther below from the admission cutoffs, as their chances of admission become more responsive to score improvements. Second, higher admission probability reduces the competitive pressure for students near-above the original cutoffs, potentially lowering their tutoring investment. The net effect on average tutoring expenditure depends on which effect dominates - the increased investment from pre-

Table 5: Description of College Constraint Simulation

	Tier 1		Tier 2		Tier 3		Tier 4		Tier 5	
	# Seats	Prize	# Seats	Prize	# Seats	Prize	# Seats	Prize	# Seats	Prize
Status Quo	n_1	v_1	n_2	v_2	n_3	v_3	n_4	v_4	n_5	v_5
Simulation I	$1.5n_1$	$v_1 r$	n_2	$v_2 - \frac{O_1}{2n_2}$	n_3	$v_3 - \frac{O_1}{4n_3}$	n_4	$v_4 - \frac{O_1}{4n_4}$	n_5	v_5
Simulation II	$2n_1$	$v_1 r$	$2n_2$	$v_2 r$	$2n_3$	$v_3 r$	$2n_4$	$v_4 r$	n_5	$v_5 - \frac{O_2}{n_5}$

Note: For Simulation I and II, the overflow of lifetime income is computed as $O_I = \sum_{j=1}^{2} v_j r(n_j + n'_j) - v_1 n_1 - v_2 n_2$ and $O_{II} = 2r \sum_{j=1}^{4} v_j n_j - \sum_{j=1}^{4} v_j n_j$, respectively.

viously marginal students or the decreased investment from previously competitive ones - across different tier thresholds.

Simulation I: Relaxation of Elite College Constraints I investigate the consequences of expanding seats in elite universities. Let n'_j denote the increase in seats for Tier j. In this simulation, I expand the number of seats for Tier 1 and Tier 2 universities by 50% while keeping other tiers unchanged ($n'_1 = n'_2 = 1.5$, and $n'_j = 0$ for other js). This expansion increases the aggregate lifetime income in the tournament by $\sum_{j=1}^{J} v_j n'_j$. To maintain the zero-sum nature of the college admission competition, this "overflow" of lifetime income must be offset through adjustments in college tier returns.[46] I assume the adjacent tier (Tier 3) bears half of this overflow, reflecting its closest substitutability with Tiers 1 and 2, while Tiers 4 and 5 equally share the remaining burden. The expansion might also reduce the alumni returns for expanded tiers from admitting more students, captured by a return adjustment parameter r_j for each tier j. Table 5 summarizes these changes in seats and returns across college tiers.

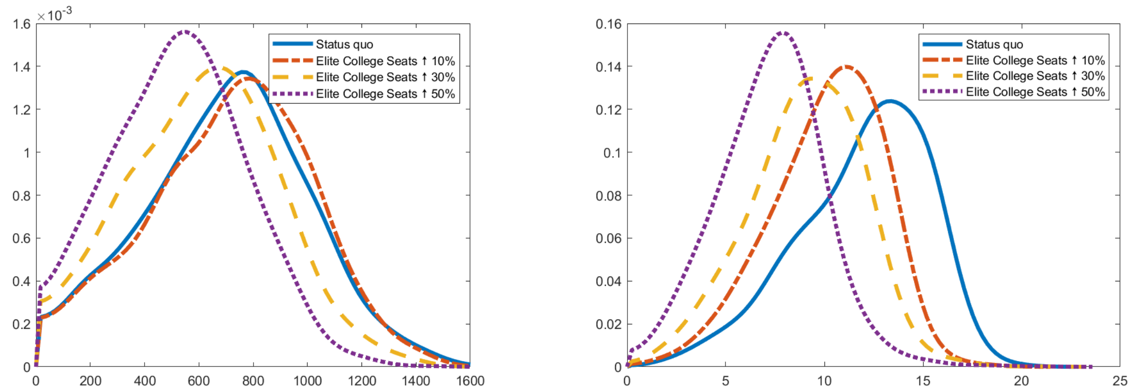
Table 6 presents the results of the college constraint simulations. A 50% expansion of seats in elite colleges (Tiers 1 and 2) leads to a 25% decrease in average private tutoring expenditure. Figures 9a and 9b show the distributions of monthly tutoring expenditure and weekly self-study hours under different expansion scenarios. Both distributions shift leftward compared to the status quo, indicating reduced investment in both types of effort.

The effects vary by the size and scope of expansion. A modest 10% increase in elite college seats actually raises tutoring expenditure by 1.6%, as the increased admission chances strengthen investment incentives. For self-study hours, larger expansions produce monotonically greater leftward shifts in the distribution. When the expansion

[46] This adjustment is analogous to assuming the signaling role of colleges in the labor market.

Figure 9: Simulation I: Simulated Household Choices under Elite College Expansion

(a) Private Tutoring Expenditure (b) Hours of Self-study



Note: Graph (a) depicts the distribution of the simulated monthly private tutoring expenditure. Graph (b) depicts the distribution of the simulated weekly hours of self-study. "Elite College" refers to Tier 1 and Tier 2 colleges in the model

is limited to Tier 1 alone, the effects differ markedly: a 50% increase in Tier 1 seats raises average tutoring expenditure by 17%, suggesting that concentrated expansion of the highest tier increases overall incentives for private tutoring investment. The results of Tier 1 expansion are presented in Figure H.1.

These results demonstrate how the limited capacity of elite colleges drives the underlying demand for parental investment. The simulations show that broad expansion of elite college access can significantly affect household investment decisions, but the response depends crucially on how the expansion is implemented. Expanding only the highest tier increases the marginal return to investment and raises tutoring demand, while simultaneous expansion of both top tiers reduces overall tutoring investment through decreased competition intensity. This pattern suggests that the demand for private tutoring responds systematically to changes in college capacity, with different expansion policies generating heterogeneous equilibrium responses in household investment decisions.

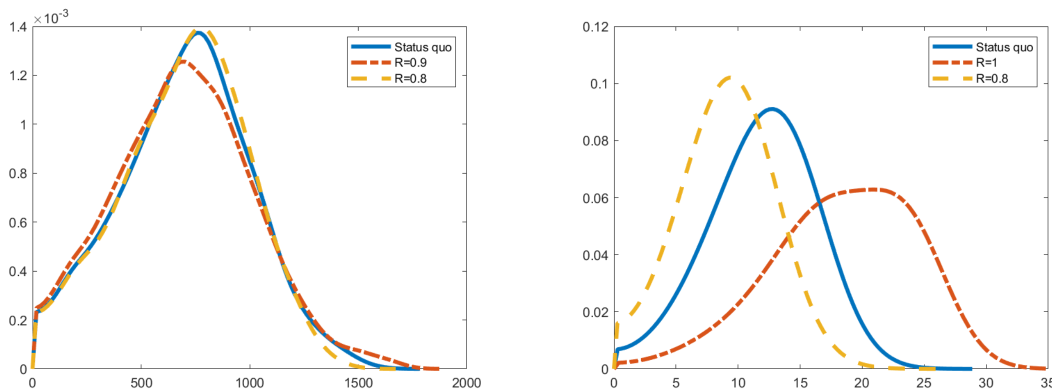
Simulation II: Cohort Size Reduction Motivated by South Korea's projected demographic changes, I simulate how parental investment responds to a 50% reduction in cohort size. This demographic shift effectively halves the cohort-to-seats ratio across all tiers, as the fixed number of college seats must now accommodate a smaller cohort.

In terms of competitive pressure, this is equivalent to doubling the number of seats while maintaining the original cohort size.

Accordingly, in this simulation, I increase the number of seats in Tier 1 to 4 by 100% ($n'_j = n_j$ and $n_5 = 0$). Tier 1 to 4 burdens the cost of increasing seats, thus the tier-specific lifetime income is decreased to $v_j r$ $(r < 1)$ for $j = 1, 2, 3, 4$. Then, the overflow of Simulation II is defined as $O_{II} = 2r\sum_{j=1}^{4} n_j v_j - \sum_{j=1}^{4} n_j v_j$. The overflow of the future lifetime income is subtracted from the future lifetime income of Tier 5. That is, v_5 is changed to $v_5 - \frac{O_{II}}{n_5}$.

Figure 10: Simulation II: Shrinking Cohort by half

(a) Private Tutoring Expenditure (b) Hours of Self-study



Note: Graph (a) depicts the distribution of the simulated monthly private tutoring expenditure when the assumed return R_1 is 0.75, compared to the status quo. Graph (b) shows the distribution of the simulated monthly private tutoring expenditure for $R_1 = 0.9, 0.8, 0.75$ and the status quo.

The simulation results suggest that a decrease in cohort size does not necessarily reduce private tutoring expenditure without substantial declines in college-tier returns. Figure 10b presents average tutoring expenditure for different return adjustments $(r = 0.9, 0.8, 0.75)$.[47] A modest 10% return decline $(r = 0.9)$ reduces average tutoring expenditure by less than 1%. As shown in Figure 10a, substantial reductions in tutoring only occur with larger return declines. When returns fall by 25% $(r = 0.75)$, tutoring expenditure drops by 13%, reflecting the significantly reduced incentives on parental investment.

[47]If r falls below 0.72, Tier 4 returns fall below Tier 5, violating the tournament structure.

Table 6: Changes in choice variables under the college-constraint simulation

		Tutoring Expenditure	Hours of Self-study
Status quo		100	100
Simulation I	Increasing Elite College Seats		
	10% Expansion	101.6	84.8
	30% Expansion	87.2	75.7
	50% Expansion	75.0	61.2
Simulation II	Halving Cohort-to-Seat Ratio		
	r=0.9	99.1	154.6
	r=0.8	99.2	76.0
	r=0.75	87.1	107.5

Note: Both tutoring expenditure and hours of self-study are the aggregated values through all time periods. I standardize the value by setting the status quo values as 100. Results of Simulation II are presented using different assumptions on the decreasing returns r.

8 Conclusion

I develop and estimate a dynamic tournament model of college admissions in which households compete using private tutoring and self-study. With students clustering at the boundaries of top tiers, the model demonstrates that high levels of investment are rationalized by the need to avoid dropping to a lower tier, despite their modest impact on the level of the test scores.

Using the estimated model, I quantify how different channels affect intergenerational persistence of earnings. Parental investment significantly contributes to this persistence even controlling for child effort, while excluding child effort from the model increases persistence by 25.3%, highlighting self-study's moderating role.

The model also demonstrates how limited number of college seats drive the demand for parental investment. Expansion of elite college seats reduces average tutoring expenditure by 24%. Moreover, declining cohort size alone does not reduce tutoring investment without accompanying decreases in college-tier returns.

These findings suggest two directions for future research. First, incorporating wealth transmission within households could provide a more complete picture of intergenerational links, as suggested by Becker and Tomes (1979). Second, endogenizing higher education market structure could help understand how parental investment competition is affected by the student-college equilibrium. I leave these for future research.

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Appendices

Appendix A: Equilibrium Proofs

Appendix A.1

Proof of Lemma 2: Compactness

Proof. A value function is the sum of flow utility and the discounted future value. The flow utility term $u(c_{it}, x_{it}, s_{it}, \varepsilon_{it})$ is monotone in its arguments. Also, u is defined at the lower and upper bounds of c_{it}, x_{it}, s_{it}. Thus, $u(c_{it}, x_{it}, s_{it}, \varepsilon_{it})$ is closed and bounded. The expected future value EV_{t+1} is closed and bounded. For the final period, the tournament term described in equation (9) is closed and bounded because (i) the v_j term is finite and greater than 0, and (ii) $Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT}) \in [0,1]$. Therefore, the choice-specific value function of the final period, $V_{it}(Z_{it}, \Psi_{it})$ is closed and bounded for $t = T$. Following the backward recursion, $V_{it}(Z_{it}, \Psi_{it})$ is closed and bounded. □

Appendix A.2

Proof of Lemma 3: Continuity

Proof. I start by showing that the value function V_{it} is continuous. To show V_{it} is continuous, It suffices to show that both $u(c_{it}, x_{it}, s_{it}, \varepsilon_{it})$ and $\int_\eta V_{t+1}(Z_{t+1}, \mathit{\Psi}_{t+1}) f(\eta) d\eta$ are continuous. □

- Start from the final period and show that the final term is continous: Bounded right hand side. Left hand side is continous in its arguemnt. Use Dominated Convergence Theorem.
- Previous period same
- Then move on to the continuity of the mapping

Proof. One way to show the continuity of the expected value function is show that it is sequentially continuous. For any sequence of the arguments of the value function,

$$\{Z_t^n, \mathit{\Psi}_t^n\} \to \{Z_t^0, \mathit{\Psi}_t^0\},$$

we have

$$\int_{\Psi} V_{t+1}(Z^n_{t+1}, \Psi^n_{t+1}) d\Psi \rightarrow \int_{\Psi} V_{t+1}(Z^0_{t+1}, \Psi^0_{t+1}) d\Psi.$$

Recall that $\Psi_{t+1} = \{\eta^c_{i,t+1}, \eta^x_{i,t+1}, \eta^s_{i,t+1}, \eta^q_{it}\}$. As the expectation of the unobserved shocks has finite expectation, the expected value term has finite expectation as well. $\int_{\Psi} V_{t+1}(Z^0_{t+1}, \Psi^0_{t+1}) d\Psi$ is continuous by the Dominated Convergence. Each supmapping is continuous as its elements are continuous. As each submapping is continuous. By induction, the composition of mapping is continuous. Therefore, $\aleph$ is continuous. $\square$

Appendix B: Member colleges for each college tier

	List of the member colleges
First Tier	Seoul National, Yonsei, Korea , Sogang, SKKU, Hanyang, KAIST, Pusan, Ewha, Postech
Second Tier	Choongang, Kyunghee, HUFS, University of Seoul, KU, Dongguk,
	Kyongpook, Sookmyung, Ajou, Honggik, Inha, Hangkong, Kookmin
Third Tier	Soongsil, Sejong, Dankook, Kwangwoon, Cheonnam, Seoul Industrial University
	Myongji, Sangmyeong, Catholic, Choongam, Choongbook, Seongshin, Kyeongki
	Kyongwon, Deoksong women, Dongdeok women, Dong-A, Bookyeong
Fourth Tier	The rest of the colleges and the 2 year colleges
Fifth Tier	High school graduates

Appendix C: Descriptive Evidence

Korean Labor Income and Panel Study

The college tier-specific lifetime income is inferred from the Korean Labor Income and Panel Study (KLIPS). KLIPS is a panel dataset of representative Korean households from 1998 to 2021. The dataset provides information on which college each worker graduated from, her major, income history, and other demographic characteristics. Using KLIPS, I generate the average lifetime income of the alumni for each college tier and complement the labor market information of KELS. In fact, KELS also provides individual information on the early labor market outcomes of the sample. Still, both the income data and the participation data have a substantial proportion of missing data compared to KLIPS. Employing KLIPS is more useful in predicting alumni's lifetime income as it contains data on workers of age between 20 and 65.[48]

Selection Rules and Effects

Table C.1: Data Selection

Original Sample Size	6,908
Cause of Exclusion	
Missing CSAT	3,310
Missing at least one period of Income	1,576
Zero Income	16
Missing Initial Test Score	40
Missing one of the parental education	59
Tutoring Expenditure greater than income	6
All choice variables missing	62
Implausible unit price of tutoring	47
Remaining Sample Size	1,792

The proportion of observations lost to missing the final test score is 0.48. Meanwhile, 99.9% of the students in the dataset report that they applied for the final exam, which suggests that the missing final exam score is not caused by the selection to take the final exam. I include households with missing choice variables (tutoring expenditure, tutoring hours, or self-study hours).

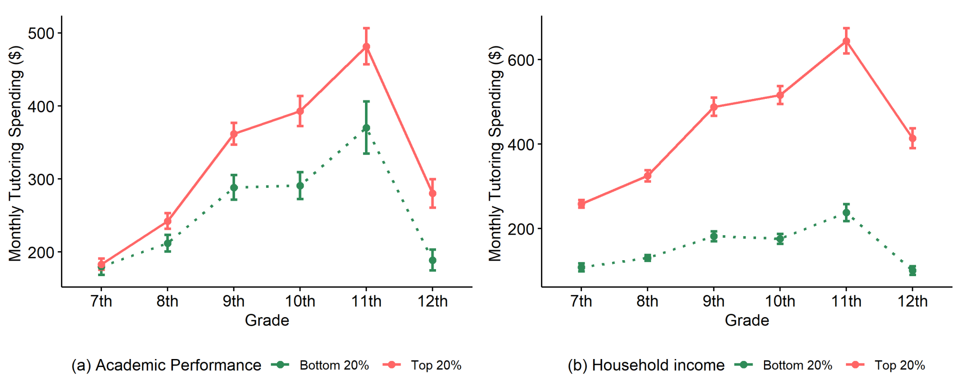
Additional Descriptive Evidence: Dynamic Allocation of Effort choices

The initial conditions of the household persistently affect the parental investment decisions throughout the secondary school periods. Figure A.1 presents changes in the average hours of tutoring expenditure over time differentiated by two of households' pre-conditions: the initial academic performance and the initial parents' income. To see how these initial conditions affect the investment decision of households, I present the changes in average tutoring expenditure of two sub-groups: the top 20% and the bottom 20% of the ordered initial conditions. In particular, the solid lines of

[48]The Lifelong Career Survey (LCS) by the Korea Research Institute for Vocational Education & Training (KRIVET) is an alternative dataset that could be used to generate the proxy of the prize of the tournament (Han, Kang and Lee 2016). For the purpose of this paper, KLIPS is preferred because it can recover the age-specific income profile.

Figure A.1 connect the average tutoring expenditure of the highest 20% of households classified by the two initial conditions. In the same manner, the dotted lines connect the average tutoring expenditure of the bottom 20% of households. Figure A.1 (a) shows the increasing gap in tutoring expenditure between those who were in the top 20% of the test score in 6th grade and who were in the bottom 20% of the test score in 6th grade over time. In 7th grade, there is no significant difference between the two groups in terms of tutoring expenditure. From 8th grade on, there is an evident gap in tutoring expenditure between these two groups. Based on the average tutoring expenditure in 12th grade, students who were in the top 20% of the test score in 7th grade increased their tutoring expenditure compared to when they were in 7th grade. In comparison, the students who were in the lowest 20% of the test score in 7th grade decreased their tutoring expenditure compared to when they were in 7th grade. Figure A.1 (b) presents the average tutoring expenditure of high-income and low-income groups. The gap is significant in 7th grade and becomes greater over time. On average, high-income households' tutoring expenditure increases in 12th grade compared to when the students were in 7th grade. On the other hand, low-income households' tutoring expenditure decreases on average compared to when the students were in 7th grade.

Figure A.1: Dynamic of Parental Investment by Initial Conditions



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: In this figure, academic performance is measured in 6th grade and used for subsequent years. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

Lifetime Income Estimates

Table C.2: Log Income Regression

	(1) loginc	(2) loginc	(3) loginc	(4) loginc
Tier Level				
College_Tier=1	-1.906***	-1.649***	-2.932***	-2.906***
	(0.347)	(0.307)	(0.498)	(0.475)
College_Tier=2	-1.325***	-1.172***	-2.413	-2.163
	(0.350)	(0.302)	(1.669)	(1.315)
College_Tier=3	-0.862**	-0.715**	-1.123**	-0.130
	(0.315)	(0.275)	(0.414)	(0.642)
College_Tier=4	-0.894***	-0.604**	-2.053***	-1.465**
	(0.231)	(0.171)	(0.413)	(0.487)
age	0.092***	0.092***	0.171***	0.179***
	(0.001)	(0.001)	(0.018)	(0.026)
Age*Tier				
College_Tier=1 × age	0.066***	0.065***	0.111***	0.124***
	(0.010)	(0.010)	(0.015)	(0.012)
College_Tier=2 × age	0.049***	0.051***	0.097	0.101*
	(0.011)	(0.010)	(0.054)	(0.047)
College_Tier=3 × age	0.031**	0.033**	0.044**	0.017
	(0.009)	(0.009)	(0.016)	(0.022)
College_Tier=4 × age	0.029***	0.027***	0.076***	0.068***
	(0.007)	(0.007)	(0.017)	(0.014)
N	29599	29599	685	685
Major	No	Yes	No	Yes
CSAT	No	No	Yes	Yes

Source: Korea Labor Income and Panel Study 1998-2012, Korea Labor Institute.
Note: The sample includes workers between 25 and 65 years old who work for wages or salary. I exclude workers who are born after 1992. Unit is USD. Annual income is predicted using the Pooled-OLS estimates in column (1) of Table C.2. The income is predicted using birth year of 1992, which is the year of the KELS cohort was born. Birth year is controlled, and standard errors are clustered at the college major level.

Table C.3: Log Income Regression: Interaction with CSAT low dummy

	(1) loginc	(2) loginc
Tier Level		
College_Tier=1	-2.701***	-2.679***
	(0.310)	(0.448)
College_Tier=2	-2.358	-1.981*
	(1.279)	(0.905)
College_Tier=3	-1.288**	-0.309
	(0.400)	(0.519)
College_Tier=4	-1.614***	-1.116***
	(0.287)	(0.279)
Tier-age Interaction		
College_Tier=1 × age	0.102***	0.118***
	(0.006)	(0.010)
College_Tier=2 × age	0.085	0.087*
	(0.055)	(0.043)
College_Tier=3 × age	0.057***	0.040*
	(0.015)	(0.019)
College_Tier=4 × age	0.076***	0.069***
	(0.012)	(0.009)
Tier-(Low Csat) Interaction		
College_Tier=1 × csat_low=1	0.225	0.150
	(0.192)	(0.179)
College_Tier=2 × csat_low=1	0.305	0.292
	(0.519)	(0.411)
College_Tier=3 × csat_low=1	-0.225	-0.452**
	(0.128)	(0.147)
College_Tier=4 × csat_low=1	-0.458***	-0.321*
	(0.097)	(0.138)
N	685	685
Major	No	Yes
CSAT	Yes	Yes

Source: Korea Labor Income and Panel Study 1998-2012, Korea Labor Institute.
Note: CSAT Low is a dummy variable that equals one if CSAT performance is less than 12 out of 12 ordered discrete variables. The sample includes workers between 25 and 65 years old who work for wages or salary. I exclude workers who are born after 1992. Unit is USD. Annual income is predicted using the Pooled-OLS estimates in column (1) of Table C.2. The income is predicted using birth year of 1992, which is the year of the KELS cohort was born. Birth year is controlled, and standard errors are clustered at the college major level.

Table C.4: Final Interview Request Regression (Probit)

	(1)	(2)	(3)
Tier=1	0.327***	0.737***	0.760***
	(0.038)	(0.103)	(0.113)
Tier=2	0.029	0.140***	0.161***
	(0.044)	(0.030)	(0.035)
Tier=3	-0.123	-0.018	-0.036
	(0.096)	(0.127)	(0.109)
ColGPA	0.006**	0.021***	0.022***
	(0.002)	(0.007)	(0.006)
N	9138	9132	9132
Major	No	Yes	Yes
Company	Yes	No	Yes
ClusterSE	No	Yes	Yes

Source: Confidential data of the conglomerate in late 2010s.
Note: The data are on the applicants to the subsidiary firms of the conglomerate for the latest three years. Other explanatory variables include the subsidiary firm's information and the applicants' information such as college major, age, and gender. The college GPA measured is scaled 0 to 100. ColGPA refers to the average of standardized college GPA.

Hours Allocation Regression

Table C.5: The Effects of Parental Background on the Hours Allocation

	(1)	(2)	(3)
	log(1+Study)	log(1+Study)	log(1+Study)
log(Income)	0.238***	0.152***	0.036
	(0.022)	(0.025)	(0.027)
Parental Edu		0.055***	
		(0.007)	
N	10454	10454	10454
Year	Yes	Yes	Yes
FE	No	No	Yes

Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: log(1+Study) and log(1+Tutoring) refer to log of hours of self-study plus one and hours of tutoring plus one, respectively. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income. Parental Educ indicates average years of parents' education.

Table C.6: The Effects of Parental Background on the Hours Allocation

	(4) log(1+Tutoring)	(5) log(1+Tutoring)	(6) log(1+Tutoring)
log(Income)	0.677***	0.616***	0.269***
	(0.027)	(0.030)	(0.037)
Parental Edu		0.038***	
		(0.008)	
N	9431	9431	9423
Year	Yes	Yes	Yes
FE	No	No	Yes

Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: log(1+Study) refer to log of hours of self-study plus one and hours of tutoring plus one, respectively. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income. Parental Educ indicates average years of parents' education.

Parental education soaks up significant variation in hours of self-study, which leaves a relatively small variation with parental income. Tables C.5 and C.6 presents the pooled OLS estimates of the regression equation,

$$\ln(1 + y_{it}) = \beta_0 + \beta_1 \log(hhinc_{it}) + \beta_2 m_i + \epsilon_{it} \tag{12}$$

where $hhinc_{it}$ is the income and m_i is parental education of household i. Columns (1) through (3) present the results where y_{it} is hours of self-study, and columns (4) through (6) present the results where y_{it} is hours of tutoring. Columns (1) and (4) provide the estimates without including the average years of parents' education, and Columns (2) and (5) provide the estimates with including the average years of parents' education to equation (12). Overall, hours of tutoring are explained more by parents' income than hours of self-study. Moreover, much of the covariation between hours of self-study and income is absorbed after controlling for the average years of parents' education.

Appendix D: Model Derivation Details

Define the first order conditions as

$$
\begin{aligned}
V_p &= \alpha_c \varepsilon^c_{it} u^c_p(c_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \frac{\partial \ln q_{i,t+1}}{\partial p_{it}} \\
V_x &= \alpha_c \varepsilon^c_{it} u^c_x(c_{it}) + \alpha_x \varepsilon^x_{it} u^x_x(x_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \frac{\partial \ln q_{i,t+1}}{\partial x_{it}} \\
V_s &= \alpha_s \varepsilon^s_{it} u^s_s(s_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \frac{\partial \ln q_{i,t+1}}{\partial s_{it}}
\end{aligned}
$$

$$
\frac{\partial p}{\partial w} = -\frac{\frac{\partial V_p}{\partial w}}{\frac{\partial V_p}{\partial p}}
$$

$$
\frac{\partial V_p}{\partial w} = \alpha_c \varepsilon^c_{it} \frac{x_{it}}{(w_{it} - p_{it} x_{it})^2}
$$

$$
\begin{aligned}
\frac{\partial V_p}{\partial p} =& \frac{\partial}{\partial p} \varepsilon^c_{it} \frac{-x_{it}}{(w_{it} - p_{it} x_{it})} + \beta \frac{\partial}{\partial p} \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \left(\nu_t \frac{\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1 + s_{it})^{\phi}]} (\kappa {p_{it}}^{\kappa-1} {x_{it}}^{1-\kappa}) \right) \\
=& -2\alpha_c \varepsilon^c_{it} \frac{x^3_{it}}{(w_{it} - p_{it} x_{it})^3} + \beta \left[\frac{\partial^2}{\partial^2 \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \left(\nu_t \frac{\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1 + s_{it})^{\phi}]} (\kappa {p_{it}}^{\kappa-1} {x_{it}}^{1-\kappa}) \right)^2 \\
&+ \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \nu_t \Bigg(- \frac{\delta^2_{2t} \kappa^2 p^{(2\kappa-2)}_{it} \phi x^{(2-2\kappa)}_{it} (1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{2\phi-2}}{[\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1 + s_{it})^{\phi}]^2} (\kappa {p_{it}}^{\kappa-1} {x_{it}}^{1-\kappa}) \\
&+ \frac{\delta_{2t} \kappa^2 p^{(2\kappa-2)}_{it} (\phi - 1) x^{(2-2\kappa)} (1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi-2}}{[\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1 + s_{it})^{\phi}]} \\
&+ \frac{\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{2t}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1 + s_{it})^{\phi}]} ((\kappa - 1) \kappa p^{\kappa-2}_{it} x^{1-\kappa}_{it}) \Bigg)
\end{aligned}
$$

As $\phi < 1$, $\kappa < 0.5$, and $\frac{\partial^2}{\partial^2 \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) < 0$, $\frac{\partial V_p}{\partial p} < 0$ and $\frac{\partial V_p}{\partial w} > 0$, $\frac{\partial p}{\partial w} > 0$.

Appendix E: Likelihood Function Details

Details of the Likelihood Function

The likelihood contributions of the choice variables are computed by transforming the characterized expression of the shocks, using the Jacobian-transformation. In particular, the time-specific likelihood contribution can be expressed as

$$\begin{aligned}\mathcal{L}_{it}(\theta|S_{it},\lambda_k) = &\left[f_{\eta^c_{it}}(\tilde{\eta}^c_{it})\cdot f_{\eta^x_{it}}(\tilde{\eta}^x_{it})\cdot f_{\eta^s_{it}}(\tilde{\eta}^s_{it})\cdot f_{\eta^q_{it}}(\tilde{\eta}^q_{it})|det\left(\frac{\partial(\tilde{\eta}^c_{it},\tilde{\eta}^x_{it},\tilde{\eta}^s_{it},\tilde{\eta}^q_{it})}{\partial(p_{it},x_{it},s_{it},q_{it})}|\right]^{d^x_{it}}\right.\\ &\times\left[\int_{\tilde{\eta}^s_{it}}\left(f_{\eta^c_{it}}(\tilde{\eta}^c_{it})\cdot f_{\eta^x_{it}}(\tilde{\eta}^x_{it})\cdot f_{\eta^s_{it}}(\eta^s_{it})\cdot f_{\eta^q_{it}}(\tilde{\eta}^q_{it})\right)d\eta^s_{it}|\det\frac{\partial(\tilde{\eta}^c_{it},\tilde{\eta}^x_{it},\tilde{\eta}^q_{it})}{\partial(p_{it},x_{it},q_{it})}|\right]^{d^x_{it}(1-d^s_{it})}\\ &\times\left[\int_{-\infty}^{\infty}\int_{\tilde{\eta}^x_{it}(\eta^c_{it})}\left[f_{\eta^c_{it}}(\eta^c_{it})\cdot f_{\eta^l_{it}}(\eta^x_{it}|\eta^c_{it})\cdot f_{\eta^s_{it}}(\tilde{\eta}^s_{it})\cdot f_{\eta^q_{it}}(\tilde{\eta}^q_{it})\right]d\eta^x_{it}d\eta^c_{it}|\det\frac{\partial(\tilde{\eta}^s_{it},\tilde{\eta}^q_{it})}{\partial(s_{it},q_{it})}|\right]^{(1-d^x_{it})d^s_{it}}\\ &\times\left[\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\Pr\{V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it})>V_{x0}(\eta^s_{it}),V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it})>V_{0s}(\eta^c_{it},\eta^x_{it}),V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it})>V_{xs})\}\right.\right.\right.\\ &\left.\left.\left.\times\, d\eta^c_{it}d\eta^x_{it}d\eta^s_{it}\right]f(\tilde{\eta}^q_{it})|\det\frac{\partial\tilde{\eta}^q_{it}}{\partial q_{i,t+1}}|\right]^{(1-d^x_{it})(1-d^s_{it})}\end{aligned}$$

where V_{00} is the value when $x_{it}=s_{it}=0$, V_{x0} is the value when $x_{it}>0$ and $s_{it}=0$, and V_{0s} is the value when $x_{it}=0$ and $s_{it}>0$.[49]

To evaluate the integrals in the likelihood function, I use the Montecarlo simulation. Borsch-Supan, Hajivassiliou and Kotlikoff (1992) show that the MSL estimates perform well under a moderate number of draws, such as 20, with an adoption of a good simulation method. To reduce the variance of simulation error, I use antithetic acceleration (Geweke 1988; Stern 1997).

About 8.3% of the household-year observations are missing, creating "holes" in the household data. I simulate the unobserved choice variables using the value function of the model (Lavy, Palumbo and Stern 1998; Stinebrickner 1999; Sullivan 2009). In particular, for each draw of the set of errors, I replace the unobserved choice variables with the optimized choices that maximize the value function of the model. Also, for periods 4 and 5, the test score data are unobserved. I simulate the unobserved test scores for each draw of test score error η^q_{it} using equation (3). In the next subsection, I show the derivation of the density and probability I use for computing the likelihood function, and I explain the simulation of unobserved variables.

First-order conditions used for likelihood contribution

The goal of this section is to get a closed form expression of the shocks, which are the building blocks of the likelihood function. I denote $u^c_p(c_{it})$ and $u^c_x(c_{it})$ as the first order derivatives of $u^c(c_{it})$ with respect to x_{it} and p_{it} respectively, and $u^l_x(l_{it})$ and $u^l_s(l_{it})$ as the

[49]For the case $d^x_{it}=d^s_{it}=0$, I am working on a G.H.K type of simulation to reduce the variance of simulation error.

first order derivatives with respect to x_{it} and s_{it} respectively. The first order conditions of the value function in equation (8) are

$$\begin{aligned}
\frac{\partial}{\partial p_{it}} &: \alpha_c \exp(\eta^c_{it} + \lambda^c_k) + \beta \frac{1}{u^c_p(c_{it})} \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1}) \right] \frac{\partial \ln q_{i,t+1}}{\partial p_{it}} = 0; \\
\frac{\partial}{\partial x_{it}} &: \alpha_c \exp(\eta^c_{it} + \lambda^c_k) u^c_x(c_{it}) + \alpha_x \exp(\eta^k_{it} + \lambda^c_k) u^x_x(x_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1}) \right] \frac{\partial \ln q_{i,t+1}}{\partial x_{it}} = 0; \\
\frac{\partial}{\partial s_{it}} &: \exp(\eta^s_{it} + \lambda^s_k) u^s_s(s_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1}) \right] \frac{\partial \ln q_{i,t+1}}{\partial s_{it}} = 0.
\end{aligned}$$

With the functional form assumptions of log utility,

$$\begin{aligned}
u^c_x(c_{it}) &= -\frac{p_{it}}{w_{it} - p_{it} x_{it}}; \\
u^c_p(c_{it}) &= -\frac{x_{it}}{w_{it} - p_{it} x_{it}}; \\
u^s_s(s_{it}) &= \frac{1}{1 + s_{it}}; \\
u^x_x(x_{it}) &= \frac{1}{1 + x_{it}}
\end{aligned}$$

And with the functional form of the test score function,

$$q_{i,t+1} = A_{it} q_{it}^{\delta_{1t}} \left[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi} \right]^{\frac{\nu_t}{\phi}} \exp(\lambda^q_k + \eta^q_{it})$$
$$\ln q_{i,t+1} = \ln A_{it} + \delta_{1t} \ln q_{it} + \frac{\nu}{\phi} \ln[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}] + \lambda^q_k + \eta^q_{it}$$

$$\begin{aligned}
\frac{\partial \ln q_{i,t+1}}{\partial p_{it}} &= \nu_t \frac{\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}]} (\kappa {p_{it}}^{\kappa-1} {x_{it}}^{1-\kappa}); \\
\frac{\partial \ln q_{i,t+1}}{\partial x_{it}} &= \nu_t \frac{\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}]} ((1-\kappa) {p_{it}}^{\kappa} {x_{it}}^{-\kappa}); \\
\frac{\partial \ln q_{i,t+1}}{\partial s_{it}} &= \nu_t \frac{\delta_{st}(1 + s_{it})^{\phi-1}}{[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}]}.
\end{aligned}$$

The first order conditions with respect to p_{it} is characterized as

$$\alpha_c \exp(\eta_{it}^c + \lambda_k^c) - \beta \frac{w_{it} - p_{it}x_{it}}{x_{it}} \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it})), \Psi_{i,t+1}) \right]$$
$$\times \nu_t \frac{\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi - 1}}{[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}]} \times (\kappa {p_{it}}^{\kappa - 1} {x_{it}}^{1-\kappa}) = 0. \tag{13}$$

The first order conditions with respect to x_{it} is characterized as

$$-\alpha_c \exp(\eta_{it}^c + \lambda_k^c) \frac{p_{it}}{w_{it} - p_{it}x_{it}} + \alpha_x \exp(\eta_{it}^x + \lambda_k^x) \frac{1}{1 + x_{it}} + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1}) \right]$$
$$\times \nu_t \frac{\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi - 1}}{[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}]} \times (1 - \kappa) {p_{it}}^{\kappa} {x_{it}}^{-\kappa} = 0. \tag{14}$$

The first order conditions with respect to s_{it} is characterized as

$$\alpha_s \exp(\eta_{it}^s + \lambda_k^s) \frac{1}{1 + s_{it}} + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it})), \Psi_{i,t+1}) \right]$$
$$\times \nu_t \frac{\delta_{st}(1 + s_{it})^{\phi - 1}}{[\delta_{et}(1 + {p_{it}}^{\kappa} {x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1 + s_{it})^{\phi}]} = 0. \tag{15}$$

This difference between the previous period and the final period can be confusing. For the final period,

$$EV_{i,T+1} = v_1 - \sum_{j=1}^{J} \Big(\ln(v_j) - \ln(v_{j+1}) \Big) \Phi\Big(\frac{\ln \bar{q}_j - \ln q_{iT+1} - \lambda_i^q}{\sigma_q} \Big);$$
$$\frac{\partial}{\partial \ln q_{i,T+1}} EV_{i,T+1}(\cdot, \cdot, \cdot) = \sum_{j=1}^{J} \Big(\ln(v_j) - \ln(v_{j+1}) \Big) \frac{1}{\sigma_q} \phi\Big(\frac{\ln \bar{q}_j - \ln q_{iT+1} - \lambda_i^q}{\sigma_q} \Big),$$

while for $t < T$, EV_{it} is an interpolated value function.

Computation of Likelihood Contribution

(Case 1) ($x_{it} > 0$ and $s_{it} > 0$)

I define $\tilde{\eta}_{it}^z$ for $z = c, x, s$ as the particular realization of η_{it}^z that satisfies the first order conditions. The likelihood contribution for all-positive case is

$$
\begin{aligned}
f(p_{it}, x_{it}, s_{it}, q_{it}) =& f(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it}, \tilde{\eta}^s_{it}, \tilde{\eta}^q_{it}) \cdot |det\left(\frac{\partial(\widetilde{\eta}^c_{it}, \widetilde{\eta}^x_{it}, \widetilde{\eta}^s_{it}, \widetilde{\eta}^q_{it})}{\partial(p_{it}, x_{it,} s_{it}, q_{it})}\right)| \\
=& \phi(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it}, \tilde{\eta}^s_{it}, \tilde{\eta}^q_{it}) \cdot |det\left(\frac{\partial(\widetilde{\eta}^c_{it}, \widetilde{\eta}^x_{it}, \widetilde{\eta}^s_{it}, \widetilde{\eta}^q_{it})}{\partial(p_{it}, x_{it,} s_{it}, q_{it})}\right)| \\
=& (2\pi)^{-4/2}|det(\Omega)|^{-1/2} \exp\Bigg[-0.5 \underbrace{\begin{pmatrix} \widetilde{\eta}^c_{it} \\ \widetilde{\eta}^x_{it} \\ \widetilde{\eta}^s_{it} \\ \widetilde{\eta}^q_{it} \end{pmatrix}'}_{1\times 4} \underset{4\times 4}{\Omega^{-1}} \underbrace{\begin{pmatrix} \widetilde{\eta}^c_{it} \\ \widetilde{\eta}^x_{it} \\ \widetilde{\eta}^s_{it} \\ \widetilde{\eta}^q_{it} \end{pmatrix}}_{4\times 1}]\Bigg| det\left(\frac{\partial(\widetilde{\eta}^c_{it}, \widetilde{\eta}^x_{it}, \widetilde{\eta}^s_{it}, \widetilde{\eta}^q_{it})}{\partial(p_{it}, x_{it,} s_{it}, q_{it})}\right)\Bigg|
\end{aligned}
$$

(Case 2) ($x_{it} > 0$ and $s_{it} = 0$)

This is the case where household participate in tutoring, but have zero hours of self-study. First, I define the joint probability of such case, and separate the density of η^c_{it} and η^x_{it} out using Bayes' theorem. I denote $A_{x_{it}, s_{it}=0}$ as the corresponding region that the joint integration of η^c_{it}, η^x_{it}, and η^s_{it} needs to be made.

$$
\begin{aligned}
\Pr(p_{it}, x_{it}, s_{it} = 0) =& \Pr(s_{it} = 0 | p_{it}, x_{it}) f(p_{it}, x_{it}) \\
&= \Pr(\eta^s_{it} > \underline{\eta}^s_{it} | \tilde{\eta}^c_{it}, \tilde{\eta}^x_{it}) f_\eta(\tilde{\eta}^x_{it}, \tilde{\eta}^c_{it}) | \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,} x_{it})}|,
\end{aligned}
$$

where $\underline{\eta}^s_{it}$ is the minimum value of η^s_{it} that leads to zero hours of self-study. I use the first order condition with respect to s_{it}, equation (15), in computing the critical value.

With the zero correlation assumption between eta,

$$
\begin{aligned}
&\Pr(\eta^s_{it} > \underline{\eta}^s_{it} | \tilde{\eta}^c_{it}, \tilde{\eta}^x_{it}) f_\eta(\tilde{\eta}^x_{it}, \tilde{\eta}^c_{it}) | \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,} x_{it})}| \\
&= \Pr(\eta^s_{it} > \underline{\eta}^s_{it}) f(\tilde{\eta}^x_{it}) f(\tilde{\eta}^c_{it}) | \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,} x_{it})}| \\
&= \left(1 - \Phi(\underline{\eta}^s_{it})\right) \frac{1}{\sigma_x} \phi(\frac{\tilde{\eta}^x_{it}}{\sigma_x}) \frac{1}{\sigma_c} \phi(\frac{\tilde{\eta}^c_{it}}{\sigma_x}) | \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,} x_{it})}|,
\end{aligned}
$$

which is what I use for computing the likelihood contribution for (Case 2).

(Case 3) ($x_{it} = 0$ **and** $s_{it} > 0$)

This is the case where household do not participate in tutoring, but do positive hours of self-study. Since $p_{it} > 0$ for all households, $p_{it}x_{it} = 0$ is equivalent to $x_{it} = 0$. For people who have $x_{it} = 0$, I let them consider minimum quality of tutoring, $\bar{p}$, which is equivalent the minimum market price.

Denote $A_{x_{it}=0}$ as the corresponding region that the joint integration of η^c_{it} and η^x_{it} needs to be made. First, I separate out the marginal density of η^s_{it} using Bayes' theorem, which gives me

$$\begin{aligned}\Pr(x_{it} = 0, s_{it}) &= \Pr(x_{it} = 0|s_{it})f(s_{it})\\ &= \Pr(\eta^c_{it}, \eta^x_{it} \in A_{x_{it}=0,s_{it}}|\tilde{\eta}^s_{it}) \cdot \frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|.\end{aligned}$$

As I assume there is no correlation between η_{it},

$$\begin{aligned}&\Pr(\eta^c_{it}, \eta^x_{it} \in A_{x_{it}=0,s_{it}}|\tilde{\eta}^s_{it}) \cdot \frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|\\ &= \Pr(\eta^c_{it}, \eta^x_{it} \in A_{x_{it}=0,s_{it}}) \cdot \frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|.\end{aligned}$$

Here, I use the first order condition, equation (15), in characterizing the unique values of $\tilde{\eta}^s_{it}$. Define $\underline{\eta}^x_{it}$ as a minimum amount of shock that makes individual start doing zero hours of tutoring. Again, with the assumption of no correlation between η_{it},

$$\begin{aligned}&\Pr(\eta^c_{it}, \eta^x_{it} \in A_{x_{it}=0,s_{it}}) \cdot \frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|\\ &= \Pr(\eta^c_{it}, \eta^x_{it} > \underline{\eta}^s_{it}|\tilde{\eta}^s_{it})d\eta^x_{it}d\eta^c_{it}\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|\\ &= \Bigg[\int_{-\infty}^{\infty}\Bigg\{\int_{\underline{\eta}^x_{it}(\eta^c_{it})}^{\infty}\frac{1}{\sigma_x}\phi(\frac{\eta^x_{it}}{\sigma_x})d\eta^x_{it}\Bigg\}\frac{1}{\sigma_c}\phi(\frac{\eta^c_{it}}{\sigma_c})d\eta^c_{it}\Bigg]\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|\\ &= \Bigg[\int_{-\infty}^{\infty}\Bigg\{1 - \Phi(\frac{\underline{\eta}^x_{it}(\eta^c_{it})}{\sigma_x})\Bigg\}\frac{1}{\sigma_c}\phi(\frac{\eta^c_{it}}{\sigma_c})d\eta^c_{it}\Bigg]\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|\end{aligned}$$

(Case 4) ($x_{it} = 0$ and $s_{it} = 0$)

This is the case where $x_{it} = 0$ and $s_{it} = 0$. To make the notation concise, I denote V_{00} as the value when $x_{it} = s_{it} = 0$. V_{x0} denotes the case $x > 0$ and $s = 0$. V_{0s} denotes the case $x = 0$ and $s > 0$.

$$\begin{aligned}\Pr(x_{it} = 0, s_{it} = 0) &= \Pr(\eta^c_{it}, \eta^x_{it}, \eta^s_{it} \in A_{x_{it}=0, s_{it}=0}) \\ &= \Pr(V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{x0}(\eta^s_{it}), V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{0s}(\eta^c_{it}, \eta^x_{it}), V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{xs}).\end{aligned}$$

$$\begin{aligned}\Pr(x_{it} = 0, s_{it} = 0) =& \Pr(\eta^c_{it}, \eta^x_{it}, \eta^s_{it} \in A_{x_{it}=0, s_{it}=0}) \\ =& \Big(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1\{V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{x0}(\eta^s_{it}), V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{0s}(\eta^c_{it}, \eta^x_{it}), V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{xs})\} \\ =& f(\eta^s_{it}) f(\eta^x_{it}) f(\eta^c_{it}) d\eta^s_{it} d\eta^x_{it} d\eta^c_{it} \Big)\end{aligned}$$

The integral does not have an analytical solution and needs to be simulated.
Simulation algorithm is
(1) I draw an unconditional set of $\eta^r_{it} = \{\eta^{cr}_{it}, \eta^{xr}_{it}, \eta^{sr}_{it}\}$
(2) Let household optimize their choices.
(3) Count the proportion of cases that household chooses $x_{it} = 0$ and $s_{it} = 0$
In particular define x^r and s^r such that

$$(x^r, s^r) = \arg\max_{x_{it}, s_{it}} V_{it}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it})$$

Compute

$$\frac{1}{R} \sum_{r=1}^{R} \mathbb{1}(x*, s* = 0).$$

So

$$\Pr(x_{it} = 0, s_{it} = 0) \approx \frac{1}{R} \sum_{r=1}^{R} \mathbb{1}(x*, s* = 0).$$

Simulation of unobserved variables

For each missing choice variables, I draw a set of corresponding error. For example, if x_{it} is missing for person i, the simulation algorithm is

(1) I draw a simulation for the corresponding error. In this example, it is η_{it}^{xr}

(2) Let household optimize their choice

$$x^r = \frac{1}{R} \sum_{r=1}^{R} \left\{ \arg\max_{x_{it}, s_{it}} V_{it}(\eta_{it}^c, \eta_{it}^{xr}, \eta_{it}^s) \right\}.$$

The optimized choice is used for computing likelihood function.

For missing test score, I draw a set of errors for η_{it}^q. Then the unobserved test score is simulated using equation (3).

Appendix F: Estimation

Table F.7: College Tier Cutoffs (Log Test Scores)

	Model	Data
Tier 1	6.2989	6.2916
Tier 2	6.2519	6.2442
Tier 3	6.2226	6.2146
Tier 4	6.1841	6.1738

Notes: This table compares model-predicted and actual log test score cutoffs for each college tier. The model closely matches the observed tier cutoffs, with differences of less than 0.01 log points across all tiers. This suggests the model successfully replicates the equilibrium sorting of students into different college tiers.

Table F.8: Equilibrium change in Tier 1 Threshold

Student ID	Baseline Log Score	Baseline Tier	Counterfactual College Tier No Tutor	No Study	Neither
*98	6.376	Tier 1	Tier 1	Tier 1	Tier 2
*77	6.336	Tier 1	Tier 2	Tier 2	Tier 3
*4	6.334	Tier 1	Tier 2	Tier 2	Tier 3
*4	6.326	Tier 1	Tier 2	Tier 2	Tier 3
*2	6.326	Tier 1	Tier 2	Tier 2	Tier 3
Tier 1 Cutoff					
*39	6.322	Tier 2	Tier 1	Tier 1	Tier 1
*55	6.321	Tier 2	Tier 1	Tier 1	Tier 1
*150	6.321	Tier 2	Tier 1	Tier 1	Tier 1
*17	6.319	Tier 2	Tier 1	Tier 1	Tier 1
*291	6.319	Tier 2	Tier 1	Tier 1	Tier 1

Notes: This table shows changes in college tier assignment under different counterfactual scenarios where top students' effort choices are restricted in Period 6. Log Score indicates the student's test score in log form from the baseline simulation. The middle row indicates the cutoff between Tier 1 and Tier 2 in the baseline simulation. Student IDs are masked for privacy protection.

Table F.9: Parameter Estimates: Preference and Shock Parameters

(a) Preference parameters

		Estimate	Standard error
Preference Parameters			
Taste for consumption	α_c	0.006	(1e-4)
Altruism for the child's future	α_ν	1.090	(0.008)
Disutility from hours of tutoring	α_x	-0.007	(1e-4)
Disutility from hours of self-study	α_s	-0.009	(1e-4)
Parental education parameters			
disutility from hours of tutoring	τ_x	-0.127	(2e-4)
disutility from hours of self-study	τ_s	-4e-5	(1e-4)

(b) Shock parameters

Standard Deviation of		Estimate	Standard Error
Test score shock	$\sigma_{\eta q}$	0.315	(0.001)
Consumption shock	$\sigma_{\eta c}$	0.716	(0.011)
Study disutility shock	$\sigma_{\eta s}$	0.204	(0.001)
Tutoring disutility shock	$\sigma_{\eta x}$	0.512	(0.001)

Note: Standard errors are computed using delta method and are in parentheses below estimates.
$\frac{1}{N\times T\times 6}(\sum logL_i - Jacob) = -0.848.$

Table F.10: Parameter Estimates: Unobserved Heterogeneity Parameters

		Estimate	Standard error
Type Probability			
Type 2		0.002	(1e-4)
Type 3		0.929	(0.023)
Type 4		0.060	(0.003)
Type-specific unobserved shocks (λ^{type})			
Type 2			
Type-specific shock to consumption	λ_c^2	-0.910	(0.004)
Type-specific shock to tutoring disutility	λ_x^2	0.044	(0.002)
Type-specific shock to self-study disutility	λ_s^2	0.054	(0.002)
Type-specific shock to log test-score	λ_q^2	-0.023	(0.001)
Type 3			
Type-specific shock to consumption	λ_c^3	-1.127	(0.037)
Type-specific shock to tutoring disutility	λ_x^3	-0.244	(0.004)
Type-specific shock to self-study disutility	λ_s^3	0.005	(0.001)
Type-specific shock to log test-score	λ_q^3	-0.034	(0.001)
Type 4			
Type-specific shock to consumption	λ_c^4	-1.465	(0.020)
Type-specific shock to tutoring disutility	λ_x^4	-0.132	(0.001)
Type-specific shock to self-study disutility	λ_s^4	-0.022	(0.002)
Type-specific shock to log test-score	λ_q^4	-0.045	(0.001)

Note: Standard errors are computed using delta method and are in parentheses below estimates. For identification, Type 1 shocks are normalized to 0.

Appendix G: Model Fit

The expected value of data y conditional on the model predicted value $\widehat{y}$ is $E(y|\widehat{y}) = \widehat{\kappa}_0(\widehat{y})$, where

$$\begin{pmatrix} \widehat{\kappa}_0(y) \\ \widehat{\kappa}_1(y) \end{pmatrix} = \sum_i \left[K(\frac{y_i - \widehat{y}_i}{b}) \begin{pmatrix} 1 \\ y_i - \widehat{y}_i \end{pmatrix} \begin{pmatrix} 1 & y_i - \widehat{y}_i \end{pmatrix} \right]^{-1} \cdot \left[K(\frac{y_i - \widehat{y}_i}{b}) \begin{pmatrix} 1 \\ y_i - \widehat{y}_i \end{pmatrix} y_i \right],$$

and

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp(-0.5x^2)$$

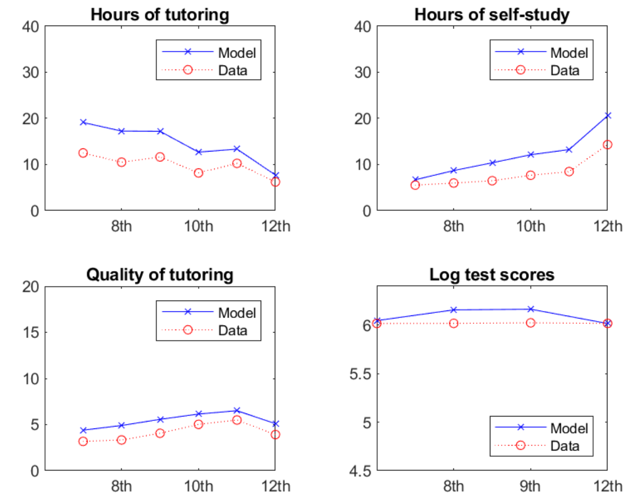


Figure G.1: Model Fit of Household Choices

Notes: This figure compares the average of model-predicted choices with the average of observed choices.

is the kernel function with bandwidth b. The farther the kernel curve deviates from the 45 degree line, the less the model is successful in fitting the data.

Figure G.2: Sample Fit: Private Tutoring Expenditure

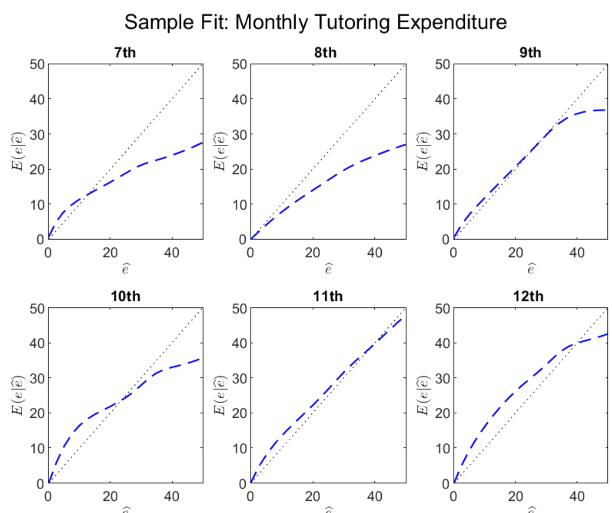


Figure G.3: Hours of Tutoring

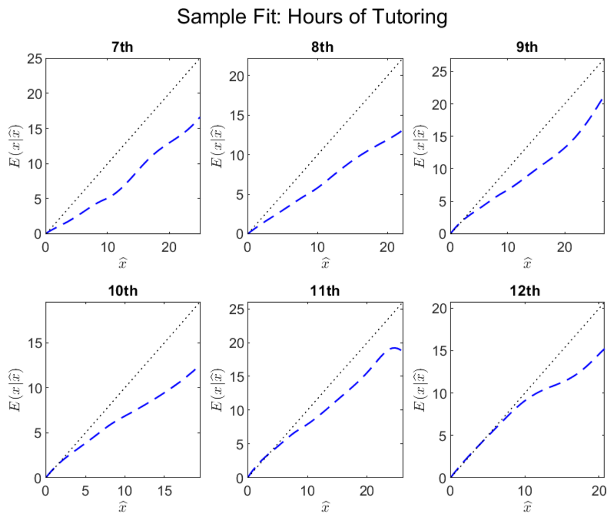


Figure G.4: Sample Fit: Quality of Tutoring

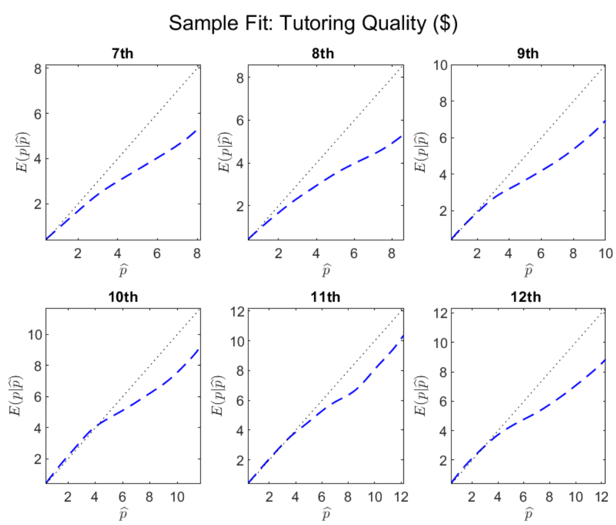


Figure G.5: Sample Fit: Hours of self-study

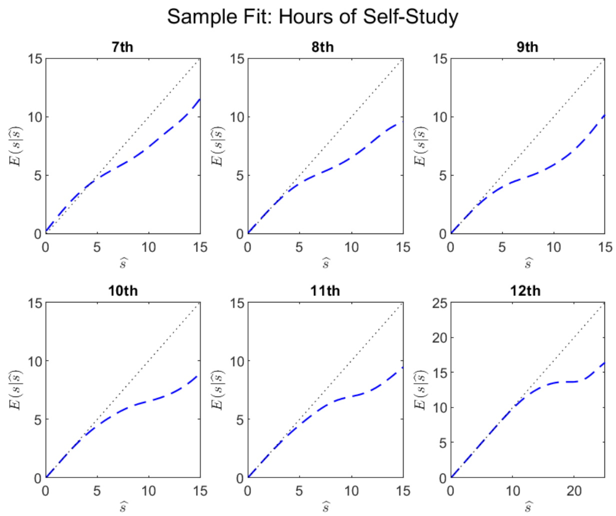
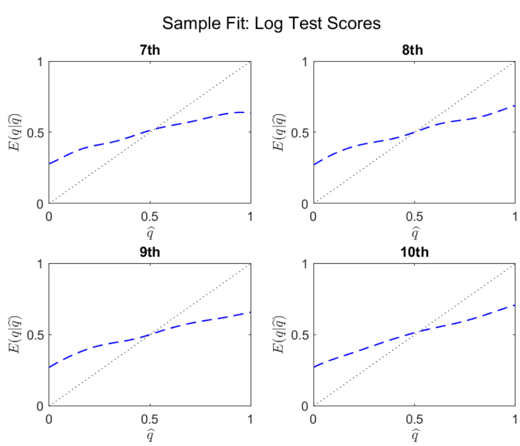
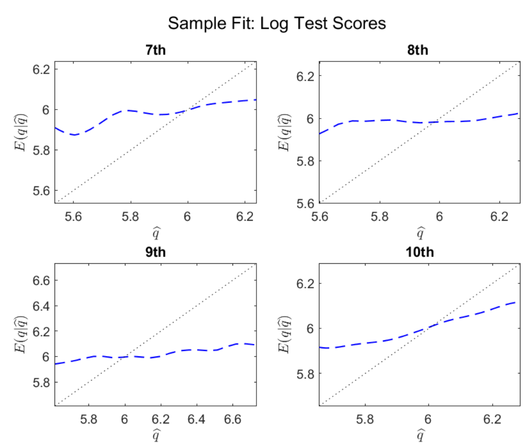


Figure G.6: Sample Fit: Log Test Scores

(a) Fit by distribution



(b) Fit by level



Appendix H: Additional Counterfactual Analyses

Intergenerational persistence of earnings

Table H.0: Intergenerational Persistence of Earnings Fixing Initial Conditions

(a) Rank-rank Slope: Fixing Initial test score

	(1) BCF	(2) OPI	(3) OSS
Rank-Rank Slope	0.482*** (0.021)	0.614*** (0.019)	0.451*** (0.021)
R-squared	0.233	0.377	0.203

(b) Rank-Rank Slope: Fixing Parental Education

	(1) BCF	(2) OPI	(3) OSS
Rank-Rank Slope	0.516*** (0.020)	0.778*** (0.015)	0.389*** (0.022)
R-squared	0.266	0.606	0.151

(c) Rank-Rank Slope: Fixing Parental Income

	(1) BCF	(2) OSS
Rank-Rank Slope	0.274*** (0.023)	0.260*** (0.023)
R-squared	0.075	0.067

Note: Table (a) to (c) provide the estimates of rank-rank slope fixing different initial conditions. For (c), simulation results for Only Parental Investment (OPI) scenario is not included, as it is not clear whether the fixed income or the parental income data should be the explanatory variable.

Table H.0: Intergenerational Persistence of Earnings

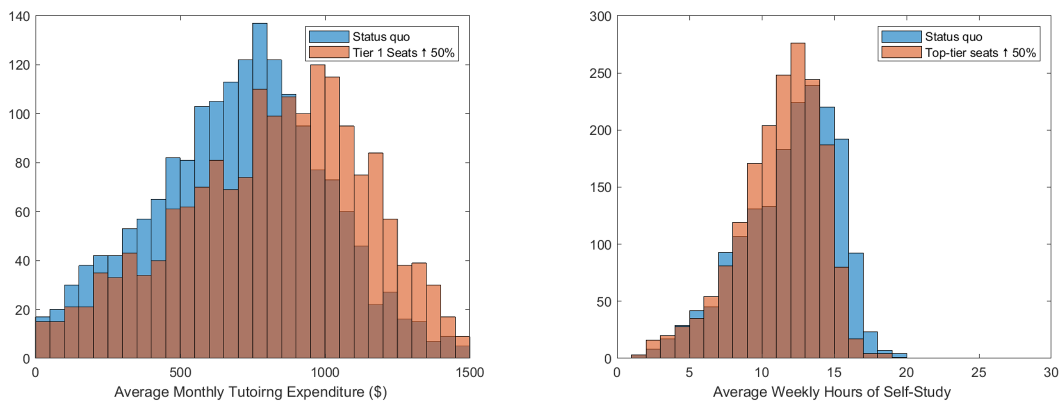
	(1) BCF	(2) OPI	(3) OSS	(4) NST	(5) NED
logpinc	0.298***	0.182***	0.123***	0.211***	0.210***
	(0.010)	(0.004)	(0.010)	(0.009)	(0.009)
R-squared	0.312	0.557	0.085	0.217	0.239

Note: This table report the results using the alternative measure, Intergenerational elasticity of earnings. The decrease of IGE in OPI simulation does not necessarily imply the decrease in the intergenerational persistence of eanrings, as it can be seen that the R-squared increase for the OPI simulation.

Figure H.1: Simulation 1A: Simulated Household Choices under Tier 1 Expansion

(a) Private Tutoring Expenditure

(b) Hours of Self-study



Note: Graph (a) depicts the distribution of the simulated monthly private tutoring expenditure. Graph (b) depicts the distribution of the simulated weekly hours of self-study.