Dynamic Competition in Parental Investment and Child's Efforts

Implications for Intergenerational Mobility and Population Decline *

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First Version: November 2022
Current Version: April 2024

Abstract

Competition for a limited number of seats in prestigious college generates a "rat-race" equilibrium effects, leading to increased household investment. I construct and estimate a dynamic tournament model where each household chooses the quality and hours of private tutoring as well as hours of student self-study. The model rationalizes the high amount of parental investments in secondary school despite their lower effects on academic achievement. The estimated model is first used to evaluate the impact of household investments on intergenerational persistence of earnings. I find that heterogeneity in parental investment significantly contributes to intergenerational earnings persistence. Removing child efforts amplifies this persistence by 30%, emphasizing the role of self-effort in moderating the intergenerational link. Secondly, by leveraging the number of competitors in the framework, I assess the heterogeneous impact of the rapidly shrinking cohort size on the demand for parental investment.

JEL Classification Codes: D15, D64, I21,I22, I24, I26, J62
Keywords: Parental Investment, Child's Efforts, Intergenerational Mobility, Student Competition

*Institute of Economic Research, Kyoto University, Japan. I am immensely grateful to Steven Stern and Juan Pantano for invaluable guidance and support. I would also like to thank Meta Brown, David Wiczer, Gabriel Mihalache, Peter Arcidiacono, Michael Keane, Lance Lochner, John Rust, Todd Stinebrickner, Michèle Tertilt, Alexis Anagnostopoulos, Arda Aktas, Hugo Benitez-Silva, Juan Carlos Conesa, Samuele Centorrino, Miguel Poblete-Cazenave, Sunghun Cho, Jihwan Do, Marcos Fernandes, Jeremy Fox, Mateo Velásquez-Giraldo, Yujung Hwang, Fedor Iskhakov, Chinhui Juhn, Laura Karpuska, Jiyeon Kim, Sunham Kim, Alfonso Flores Lagunes, Marc Claveria Mayol, Bob Millard, Mark Montgomery, José Alfonso Muñoz-Alvarado, Victor Ronda, Warren Sanderson, Eran Shmaya, Takashi Unayama, Greg Veramendi, Jiayi Wen, Sangha Yoon, Basit Zafar, Weilong Zhang, and Yiyi Zhou for helpful comments. All errors are my own.

1. Introduction

Competition motive is a pivotal driver of time investments by parents and the child. Graduating from an elite university has a sizeable impact on labor market outcomes (Hoekstra 2009; MacLeod *et al.* 2017; Zimmerman 2019; Anelli 2020; Sekhri 2020; Guo and Leung 2021; Jia and Li 2021; Lee and Koh 2023), but seats for such prestigious colleges are limited, and evidence suggests that colleges tend to not adjust the seats to accommodate for increasing cohort size (Bound and Turner 2007). The scarcity of seats in prestigious colleges leads to competition, potentially creating a "rat-race" scenario that drives an increasing trend in parental investment (Ramey and Ramey 2010). In the United States, parents frequently invest considerable time in supporting their children's extracurricular activities. In East-Asian countries, parents often allocate a significant portion of their income to private tutoring, primarily aimed at securing admission to prestigious universities, (Bray 1999, 2022), despite evidence suggesting minimal impact on test score improvement (Ryu and Kang 2013; Kang and Park 2021). Most previous work on parental investment does not include this competition aspect into their framework.[1,2]

On the other hand, parental investment significantly contributes to the intergenerational transmission of earnings (Caucutt and Lochner 2020; Bolt *et al.* 2021a,b; Gayle, Golan and Soytas 2022; Yum 2022). It is natural to conjecture that children who have received more investment from parents are likely to achieve better future outcomes such as better performance in the labor market (Becker and Tomes 1979; Guryan, Hurst and Kearney 2008). Thus, parental investment potentially has important consequences on social mobility. Meanwhile, previous studies report the significant impact of children's own efforts on their educational outcomes (Stinebrickner and Stinebrickner 2004; Del Boca, Monfardini and Nicoletti 2017). The self-effort of the child is not responsive to parental background as much as parental investment is affected by parental background.[3] Despite the potential relevance, few studies have modeled the interdependence of parental investment and children's self-effort in shaping intergenerational mobility.

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[1]On the other hand, the competition between private and public schools is extensively studied in the literature. See Epple and Romano (1998); Epple, Figlio and Romano (2004); Epple and Romano (2008); Epple, Romano and Urquiola (2017, 2021).

[2]In Section 3.3, I state the reasons why I do not model competition between secondary schools.

[3]I show related empirical evidence in Section 4.

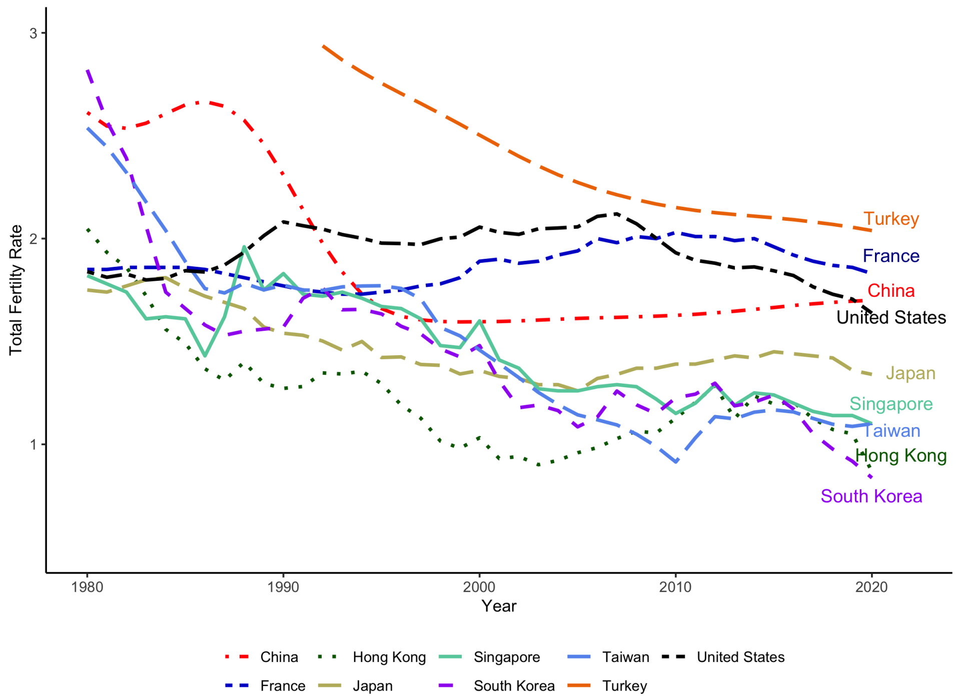
This paper investigates these two interrelated aspects of parental investment. First, it seeks to shed light on the role of parental investment on intergenerational persistence or earnings. The inclusion of self-effort of the child, which is often ignored in the literature, might amplify or offset the link between the two generations. Second, the intensity of household competition is affected by the size of the cohort for the limited number of college seats with different level of quality. If there are significant changes in the number of competitors, the decision of parental investment and child effort is likely to be affected. In fact, many developed countries face a drastic shift in demographic structure caused by a declining fertility rate, as shown in Figure 1. Little is known about the consequence of the shift in the demographic structure on parental investment.

To answer these questions, this paper builds and estimates a dynamic tournament model using a unique longitudinal dataset that contains information on parental investment, the child's time allocation, and administrative test scores. I first document the descriptive evidence regarding parental investments and the self-efforts of the child. Second, motivated by empirical evidence, I build a dynamic model of a tournament which approximates the college admission competition among households. I estimate the tournament model using Maximum Simulated Likelihood. In a series of graphs, I show that the model fits reasonably well with the data. Finally, I perform counterfactual exercises using the estimated structural model. I quantify the impact of parental investment and the child's self-efforts on intergenerational mobility. Then I simulate the model to measure the effects of the shrinking cohort size on parental investment.

This study uses Korean datasets and is based on institutional features of the country.[4] Students are assigned to the middle schools within the residential education district by lottery. As the distribution of school quality of secondary school is relatively homogeneous, the private tutoring expenditure of parents stands out as a primary contribution to the child's future outcomes. The importance of the final test score in college admissions helps to link the test score measure to the child's labor market outcomes. Such institutional characteristics offer a transparent environment in which household income is translated into the educational outcome of the child.

[4]A number of countries share the institutional features, which I explain in Section 3.

Figure 1: Shrinking Cohort



Source: World Bank for data for China, Hongkong, Japan, Singapore and South Korea. Data for Taiwan is drawn from United Nations World Population Prospects.

I start by documenting the descriptive evidence that provides the empirical basis of the dynamic tournament model. Two empirical facts show that competition with respect to getting into a more prestigious college is the primary motivation for parental investment. First, college ranking positively affects the growth of alumni's income. Using the Korean Labor Income and Panel Study, I estimate the effects of college-tier, a categorization of colleges in Korea based on their quality measured by alumni's income growth. Pooled OLS results suggest that there is a significant variation in lifetime income based on tier of the college from which workers graduate. The effects are economically and statistically significant, controlling for CSAT score. This evidence is consistent with the empirical studies on the effects of elite colleges on labor market outcomes (Zimmerman 2019; Sekhri 2020; Jia and Li 2021).[5] Second, the amount of parental investment drops substantially as students finish the college admission process. This suggests that the purpose of parental investment is for their child to do

[5] Using data on students majoring in science at University of California campuses, Arcidiacono, Aucejo and Hotz (2016) show the mismatch between minority students' preparedness and higher ranked campuses decreases the likelihood of graduation.

well in the college admission competition rather than enhancing the child's human capital.

Also, another empirical fact suggests that parental investment and the child's self-efforts potentially have different implications for intergenerational mobility. As expected, data show that parental background, especially household income, generates a significant variation in parental investment. On the other hand, self-efforts of the child, measured by hours of self-study, do not vary as much as parental investment with different levels of parental income. At the same time, both parental investment and the child's self-efforts are expected to affect the child's outcome. If parental investment and self-efforts are technological substitutes, an income-constrained household can compensate for the lack of parental investment by increasing hours of self-study. Thus, omitting self-efforts of the child might result in an exaggeration of intergenerational persistence of earnings. This suggests the importance of modeling both parental investment and the child's self-efforts in understanding the contribution of educational investments to the intergenerational persistence of earnings.

Finally, the data show that households select the different levels of parental investment and child efforts over time based on their preconditions. Students' time allocations change considerably as they proceed to the later periods in secondary school. Also, the exogenous characteristics of the household persistently affect the parental investment decisions throughout the secondary school periods.

Motivated by the empirical evidence, I develop and estimate an equilibrium dynamic tournament model of college admission competition. The model builds upon the rank-order tournament model introduced by Lazear and Rosen (1981). The tournament structure is embedded into the model of altruistic households. The household cares about the future outcome of the child, which is the result of the college admission tournament. In every period, each household makes decisions of parental investment and the level of the child's self-efforts, and these two are inputs of the test score. To capture the student's persistence in test-taking skills, I allow the previous test score to have its own direct effect in the production of test score (Cunha and Heckman 2007). The model structure repeats until the final test score is produced, and students are assigned to the college tiers based on their final test score. The college tier is the sole determinant of the child's lifetime income. This is an arguably reasonable assumption. Using confidential job offers data provided by a conglomerate in South Korea, I show that the effects of college-tier on earnings is economically and statistically significant controlling for effort during the college period.

The dynamic tournament model offers several features that help answer the research question of this paper. First, the rich heterogeneity of state variables and the choice set and the specification of the test score production function help disentangle the source of intergenerational persistence of earnings. Each household can simultaneously choose the quality of tutoring, hours of tutoring, and hours of self-study in the model based on its state variables. Second, the rank-order feature of the tournament model enables me to study the effects of the changes in cohort size and the role of disparity in college quality. Third, as I allow for the time-varying effects of the choices, I can compare the effects of hours of self-study and hours of parental investment.

I estimate the model using Maximum Simulated Likelihood. The estimation results suggest that hours of self-study have stronger average marginal effects on the subsequent test score than hours of tutoring. Both the marginal effects of parental investments and self-study of the child decline over time. Also, the estimate of the substitution parameter of the production function suggests that parental investments and hours of self-study are close to perfect substitutes. Compared to hours of tutoring and hours of self-study, there exists sizeable unobserved heterogeneity in the quality of tutoring. Using local linear regression, I show that the estimated model fits the sample reasonably well.

Using the estimated structural model, I first quantify the role of heterogeneity in household income. I use the rank-rank slope, the slope between income percentiles of two generations, as the measure of intergenerational persistence of earnings (Chetty *et al.* 2014). Removing heterogeneity in the parental income during the adolescent period decreases the rank-rank slope by 47.2%. Next, I quantify the role of parental investments and the self-efforts of the child on intergenerational persistence of earnings. I simulate the model by shutting down one of the choices. In particular, I compare the changes in the rank-rank slope by shutting down the choice of (i) self-study of the child or (ii) the parental investments for the child. Relative to the estimated model, the rank-rank slope increases by 30.2% when the channel of self-study is shut down. Also, the rank-rank slope decreases by 79.5% when the channel of parental investments is shut down. The result of the quantification suggests that parental investment reinforces the intergenerational persistence of earnings and the self-study of the child mitigates it.

Next, to understand the effects of the shrinking cohort size on the choices of households, I simulate the structural model using the projected number of high school graduates and the assumptions on changes in the number of seats in colleges and changes in the distribution of college quality. College admission competition is about winning a limited seat within the cohort. The tournament model enables studying the effects of changes in cohort size and the distribution of college quality. Based on the model projection, as the size of the cohort shrinks by 52.5%, low-income households spend more on private tutoring expenditure as cohort size decreases, while there is virtually no change in the private tutoring expenditure of high-income households. As the intensity of competition becomes weaker due to the decrease in the cohort-to-seat ratio, low-income households have a higher probability of going to a better college tier. With a higher probability of going to a better college tier, low-income households have incentive to spend more on private tutoring.

The rest of the paper is organized as follows. I discuss the related literature and contributions of this paper in Section 2. I describe the institutional features in Section 3. In Section 4, I document empirical facts that motivate the dynamic tournament model. Section 5 introduces the tournament model. Section 6 explains the estimation procedure, source of identification, and results. I present the counterfactual exercises in Section 7 and conclude in Section 8.

2. Related Literature and Contributions

This paper contributes to the burgeoning empirical literature seeking to understand the source of intergenerational mobility. The literature of intergenerational mobility has been focusing on reporting estimates of intergenerational persistence in earnings (Solon 1999; Mazumder 2005; Chetty, Hendren, Kline and Saez 2014; Adermon, Lindahl and Palme 2021). Only recently, there have appeared a few papers empirically investigating the mechanism that generates the intergenerational correlations in earnings (Lee and Seshadri 2019; Caucutt and Lochner 2020; Bolt, French, Maccuish and O'Dea 2021b; Daruich 2022; Gayle, Golan and Soytas 2022; Yum 2022).[6] I contribute to this literature by building and estimating a dynamic model of a tournament that incorporates parental investment and the self-efforts of the child. Previous studies

[6]Relatedly, Björklund, Lindahl and Plug (2006) estimate the contribution of pre-natal factors on intergenerational transmission of earnings utilizing data on adoptees in Sweden.

quantifying intergenerational mobility do not consider the self-efforts of the child in their framework. Also, the novel feature of my paper is to incorporate the competition among households to the dynamic structural model.

This paper relates to the large body of literature modeling post-birth parental choice.[7] Since Becker and Tomes (1979), economists have sought to understand how parents allocate resources to their children and how such decisions affect the child's outcomes such as cognitive development (Doepke, Sorrenti and Zilibotti 2019). Del Boca, Flinn and Wiswall (2014) build and estimate a dynamic model in which parents jointly choose the amount of time investment, the amount of monetary investment, and the decision of labor supply participation. Doepke and Zilibotti (2017) formulate a model of parenting styles. Agostinelli, Doepke, Sorrenti and Zilibotti (2020) extend their work by combining the choice of parenting with the child's peer formation. Papers in this literature have recently started incorporating externalities into parental choices.[8] In particular, papers also model externalities by incorporating competition among students. Ramey and Ramey (2010) are the first paper that rationalizes the increase of parental time investment in the United States using a theoretical model of competition for elite colleges. The two closest papers modeling student competition are Bodoh-Creed and Hickman (2019) and Grau (2018). Bodoh-Creed and Hickman (2019) build a static structural model of an admission contest to study returns to pre-college human capital investment in the United States and estimate their model. Also, Grau (2018) builds a static tournament model, estimates it's parameters and applies the estimated model to the college competition in Chile.

The theoretical model in this paper is different from theirs in several ways. First, I model the dynamic tournament allowing for uncertainties in the test score generation and the household choices.[9] This helps estimate the effects of household investments and capture how households self-select into the high and low level of investments. Second, I model the channel of monetary and time investments, which has direct implications for intergenerational transmission of earnings. Previous static models abstract away from the measure of the resources and the efforts used for the college admission competition. I propose a dynamic tournament model and suggest plausi-

[7]See Chiappori, Salanié and Weiss (2017) for a model of joint decision of marriage and parental investment.

[8]a group of papers associates parental choices with social interactions (Agostinelli 2018; Agostinelli *et al.* 2020; Boucher *et al.* 2022)

[9]Outside the broad literature of economics of education, a handful of papers build and estimate structural tournament models (Vukina and Zheng 2007; Chen and Shum 2010; Vukina and Zheng 2011).

ble measures for the resources (household income) and the efforts (private tutoring expenditure and hours of self-study) of the competition.

Another closely related paper is by Kim, Tertilt and Yum (2022), which studies the cause of the low fertility problem of South Korea. They propose a heterogeneous-agents model of "status externality" based on the assumption that parents care about the relative position of their children's human capital compared to that of other children. The tournament model of this article complements their study by formally modeling the dynamic competition with respect to getting into prestigious colleges. The tournament structure can rationalize the underlying source of the status externalities in their paper.

Finally, this paper contributes to the literature on childhood investments and skill development by estimating the effects of parental investment and the self-efforts of the child in the adolescent period. Most previous work focuses on estimating the effects of parental investment on child outcomes alone (for example, Cunha and Heckman 2007; Cunha, Heckman and Schennach 2010; Del Boca, Flinn and Wiswall 2014).[10] These studies find declining effects of parental time investment over age. Several studies estimate the effects of hours of self-study on academic achievements (e.g., Cooper, Robinson and Patall 2006; Stinebrickner and Stinebrickner 2008), but they do not jointly estimate the effects of parental investments. Only recently, a few papers have estimated models incorporating both parental investment and self-efforts of the child. Del Boca, Monfardini and Nicoletti (2017) find that the effect of self-effort of the child is stronger than the effect of the mother's time investment during adolescence, and the effect of self-effort of the child increases over time. Del Boca, Flinn, Verriest and Wiswall (2019) build a Stackelberg model of parent-child interaction and study the effects of conditional-cash-transfers on child outcomes. Such a line of research suggests the importance of modeling both parental investment and self-efforts of the child in studying the source of intergenerational mobility, which is the focus of this paper.[11,12]

[10]As this paper employs private tutoring expenditure as a measure of parental investment, it also complements the literature of studies on private tutoring (Stevenson and Baker 1992; Cheo and Quah 2005; Tansel and Bircan Bodur 2005; Dang 2007; Ono 2007; Ryu and Kang 2013; Hof 2014; Kang and Park 2021).

[11]Agostinelli and Sorrenti (2021) find that the trade-off between more household income from labor supply and parental time investment is significant for disadvantaged families (mothers).

[12]As college competition in reality uses actual test scores rather than unobserved skills of the student, I do not apply the factor model techniques developed in the literature (see Cunha, Heckman and Schennach (2010); Agostinelli and Wiswall (2016)).

3. Key Institutional Features

As this paper utilizes Korean datasets, the theoretical framework and the identification strategy are based on the country's institutional features. In this section, I explain the key institutional features of the country: the high-stakes college entrance exam, hierarchical college structure, homogeneous secondary schools, and an established private tutoring market. While these institutional characteristics offer several advantages in studying the research questions, a number of countries share these features. As I describe the characteristics of the system, I explain the possibility of generalization for other countries.

3.1 High-Stakes College Entrance Exam

In Korea, the College Scholastic Ability Test (CSAT), the college entrance exam taking place at the end of 12$^{\text{th}}$ grade, is the single most important factor for college admission.[13] Students take Korean, Mathematics, English, and elective subjects. The exam starts at 8:40 am and finishes at 5:45 pm. For this exam, take-offs and landings of airplanes are suspended for 35 minutes during the English listening test. Firms and government offices are encouraged to delay their workday by an hour to help students avoid heavy traffic. All these suggest that the taking of the CSAT is a huge national event. After the exam, students receive a scoresheet that contains a standardized score and a stanine score for each subject.[14,15] Many educational consulting firms publish the "cutoff sheet" that contains the firm's prediction for the cutoffs for all colleges. The predictions are largely consistent across the firms and are close to the actual cutoffs. Based on the CSAT score and the predicted cutoffs, each student chooses up to three colleges in which to apply. Based on the CSAT score and the quota, colleges determine admission results for students. Several countries have their own high-stakes college entrance exam. *Gaokao* of China is a representative example in that the ranking in the exam is the most crucial factor in college admission. Other examples include *Yükseköğretim Kurumları Sınavı* of Turkey, Exame Nacional do Ensino Médio of Brazil,

[13] In South Korea, there has been a recent increase in the quota for the holistic review process, in which test score is not the only determinant for college admission. In 2019, 24.9% of total students were admitted through the holistic admission route (Bastedo 2021).

[14] There was one exception in 2007 in which only stanine scores were available for the college admission process. The original standardized score system was restored in 2008. Han, Kang and Lee (2016) estimate the changes in aggregate effort level of the students due to the grade scheme shift.

[15] A stanine score is nine discrete scales ranging from a low of 1 to a high of 9.

Sijil Pelajaran Malaysia of Malayasia, and Ulttyq Biryńǵaı Testileý of Kazakhstan are highly similar in terms of their importance in the college admission process. Baccalauréat of France is highly important for getting into grandes écoles, the group of elite colleges of the country. The Scholastic Aptitude Test (SAT) of the United States is also utilized as an important factor in college admission, but other components such as high school grade-point-average and extra-curricular activities also matter.

3.2 Hierarchical College Structure and College-Tier

The institutional feature also prevalent in other countries is a hierarchical college structure. In many countries including Korea, college quality is unequal in terms of alumni outcomes. Empirical studies report that graduating from an elite college significantly affects a student's future labor market outcomes.[16] In South Korea, the college hierarchy has changed little (Kim and Lee 2006; Kim 2014). Starting from the top institution, Seoul National University, the applicants' preferences have been stable for decades, and "SKY" is a well-known acronym that refers to the top three universities in the country. In the 1980s, as the demand for elite colleges have increased, the SKY universities have become too far of a reach for many people.[17] Kim and Lee (2006) study this hierarchical market structure of universities in Korea and show that a strong university hierarchy is present in the country. They report that universities in the first three deciles strictly dominate the rest in terms of their measure of labor market outcomes, private donations, quality of faculties, and physical facilities.

Motivated by the college hierarchy, I categorize colleges in Korea into four ordered tiers based on the "cutoff sheet" published by Jinhak (2022), one of the major education consulting firms. Tier 1 includes the most prestigious universities. The cutoff of Tier 1 is around the top 1% of CSAT scores. Successively, the cutoffs of Tier 2 and 3 are approximately the top 5%, and top 15% of the CSAT score distribution, respectively.[18] Tier 4 is composed of graduates from 2 year colleges. Tier 5 is the residual tier that absorbs the rest of the students in the cohort. The member universities of each tier are specifically reported in Appendix B. I use this categorization of college tiers throughout

[16] See, for example, Hoekstra (2009) for the United States, MacLeod *et al.* (2017) for Colombia, Zimmerman (2019) for Chile, Anelli (2020) for Italy, Sekhri (2020) for India, and Jia and Li (2021) for China.

[17] In the late 1990s, the term "In Seoul" has appeared, which refers to a group of all universities in Seoul. Anecdotally, Korean parents often say that they hope their children go to one of these "In Seoul" universities.

[18] The top 15% score is the cutoff for the "In Seoul" universities previously mentioned.

this paper. In Section 4, I present empirical evidence suggesting the significant effects of the college tier on post-graduation labor market outcomes.

3.3 Homogeneous Secondary School and Private Tutoring Market

Secondary schools in South Korea are homogeneous, which provides a transparent environment where private expenditure translates into students' academic performance. First, the curriculum of secondary school is uniform and under the strict control of the Korean government. In addition to public schools, even private schools do not have autonomy in terms of the curriculum and tuition.[19] Second, as a result of the consecutive school-equalization policies, the quality of education provided by schools is similar.[20] No schools are allowed to select students independently.[21] In fact, school assignments for middle school and high school are random within the residential district for most regions. After graduating from primary school, students are assigned to the middle schools within the residential education district by lottery.[22]

At the same time, 2.8% of GDP is spent on private tutoring activities for students by households in South Korea (Nam 2007). Parents spend 9% of their income on private tutoring activities for their children, which is a significant amount of expenditure.[23] The form of private tutoring varies. The most common form of private tutoring is *hagwon* (or cram school), the private academic institutions students go to after regular school hours. There are also one-on-one tutoring, group tutoring, and online classes. The country has an established private tutoring market. With the centralized school curriculum, private tutoring institutes are an effective substitute for parental time in teaching their kids. I use private tutoring as a measure of parental investment throughout the paper.

[19] One of the few decisions of private secondary schools in Korea is that they can independently hire teachers. Park, Behrman and Choi (2013) provide evidence that the difference in the quality of teachers is not significant between private and public secondary schools in Korea.

[20] See Section II of Kim and Lee (2010) for a description of the history of school equalization policy. As of 2010, the high school equalization policy has been adopted for all major cities in South Korea.

[21] One exception is specialized high schools, which are not subject to the equalization policy. However, not like private schools in the United States, admission to specialized schools is mostly merit-based. The enrollment for the specialized schools accounts for only 3% of total enrollment. I expect the disparities due to the specialized high schools are captured by the household characteristics of the dataset.

[22] Papers in the literature exploit this random assignment feature to estimate the effects of various independent variables of interest on educational outcomes. See, for example, Kang (2007), Park, Behrman and Choi (2013), and Park, Behrman and Choi (2018). Park, Behrman and Choi (2013) show that the issue of non-compliers to the lottery policy is a minor concern.

[23] See Bray (1999, 2021) for a comprehensive cross-country comparison of private tutoring.

Two main features highlighting the education system of Korea are the homogeneous secondary schools and the fact that college admission relies heavily on the final exam. This feature provides a transparent environment in which the household income is translated into the educational outcome of the child.

4. Empirical Evidence

4.1 Data

I use the Korean Educational Longitudinal Study 2005 (KELS) for the main estimation procedure. To supplement the income information of KELS, I use the Korean Labor Income and Panel Study (KLIPS) to supplement the college tier-specific lifetime income.

4.1.1 Korean Educational Longitudinal Study

The choice of data is motivated by the main goals of the paper: (i) to quantify the role of accumulated parental investment and student efforts on intergenerational mobility, and (ii) to account for the dynamic selection of the effort choices of the household. Estimating the marginal effects of parental investment and hours of self-study in each period is necessary to achieve the goals. The Korean Educational Longitudinal Study 2005 provides a rare combination of relevant data. The dataset includes information on private tutoring expenditure, hours spent for private tutoring, hours spent for self-study, income of the household, standardized test scores, and parental education. Household income and private tutoring expenditure are collected each year. The hours spent in tutoring activities and the hours spent for self-study are collected as a weekly average. There are five different measures of academic performance available in the dataset. Academic performance in primary school is measured as an ordered discrete measure answered by the household. For 7^{th} to 9^{th} grades, the administrative test scores are of achievement tests standardized at the national level. For 12^{th} grade, the administrative College Scholastic Ability Test (CSAT) score is available. The actual scores are available for the three achievement tests and the CSAT, which I treat as continuous variables.

Table 1: Data Selection

Original Sample Size	6,908
Cause of Exclusion	
Missing CSAT	3,310
Missing at least one period of Income	1,576
Zero Income	16
Missing Initial Test Score	40
Missing one of the parental education	59
Tutoring Expenditure greater than income	6
All choice variables missing	62
Implausible unit price of tutoring	47
Remaining Sample Size	1,792

Table 2: Sample Moments

(a) Sample Moments: Other characteristics

	Mean	Stdev
Parental Education	13.27	2.01
6th grade Academic Performance	6.52	1.70
N	1792	

Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.

The nationally representative dataset tracks 6,908 students (1^{st} -year middle-school students) sampled from the country's 703,914 7^{th} grade students. The students are tracked starting from 2005 when they are 7th graders. In the first stage of the survey, the cohort is surveyed yearly up to 2012. In the second stage of the survey, namely the college and the labor market period, the cohort is surveyed semi-annually up to 2020, which is ten years after the cohort graduates from high school. The rules of selection and their effects are reported in Table 1. The proportion of observations lost to missing the final test score is 0.48. Meanwhile, 99.9% of the students in the dataset report that they applied for the final exam, which suggests that the missing final exam score is not caused by the selection to take the final exam. In Appendix 9, I show that the effects of the selection do not result in severe differences in the sample moments. The observations lost to missing income selection tend to have missing CSAT scores as well. Importantly, I include households missing one of the choice variables: tutoring expenditure, hours of tutoring, and hours of self-study. In the estimation section, I explain the rules to simulate the missing choice variables.

Table 3: Sample Moments (Continued)

(a) Sample Moments: 7th - 9th grades

School grade	7th		8th		9th	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Tutoring Expenditure	25.8	20.0	25.1	19.6	36.1	31.0
Hours of Self-Study	5.48	5.04	5.97	5.13	6.45	5.27
Hours of Tutoring	11.37	8.50	9.69	7.22	11.29	9.90
Income	370.4	161.7	369.2	151.3	400.4	169.9
Test Scores	323.03	45.63	321.50	48.72	322.65	48.45
N	1792					

(b) Sample Moments: 10th - 12th grades

School grade	10th		11th		12th	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Tutoring Expenditure	38.3	36.5	47.9	48.6	29.5	41.7
Hours of Self-Study	7.65	5.68	8.45	6.00	14.42	9.14
Hours of Tutoring	7.40	6.74	9.16	9.45	5.69	7.89
Income	406.9	177.0	394.4	191.1	381.4	171.4
Test Scores	-	-	-	-	415.39	62.46
N	1792					

Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.

Table 2 and 3 present sample moments of KELS. While the average hours of self-study increase over time, the average hours of tutoring overall show a decreasing trend. I revisit the implications of such changes in hours allocation in Section 4.5. The moments of household income are stable over time. I use parental education data collected in the first year of the survey, and I assume that parental education does not change within the model period. This is a reasonable assumption given the relatively short period of time in the data. In fact, information on parental education is collected only in the first two years of the survey.

4.1.2 Korean Labor Income and Panel Study

The college tier-specific lifetime income is inferred from the Korean Labor Income and Panel Study (KLIPS). KLIPS is a panel dataset of representative Korean households from 1998 to 2021. The dataset provides information on which college each worker graduated from, her major, income history, and other demographic characteristics. Using KLIPS, I generate the average lifetime income of the alumni for each college tier and complement the labor market information of KELS. In fact, KELS also provides individual information on the early labor market outcomes of the sample. Still, both the income data and the participation data have a substantial proportion of missing data compared to KLIPS. Employing KLIPS is more useful in predicting alumni's lifetime income as it contains data on workers of age between 20 and 65.[24]

4.2 The Lifetime Income Differential

College ranking has a strong effect on the growth of alumni's income.[25] The effect is significant controlling for CSAT score. Columns (1), (2), and (3) in Table 4 provide the OLS estimates for the regression equations,

$$\ln y_{it} = \sum_{j=1}^{J} (\beta_j + \delta_j \cdot age_{it}) D_{i,j}^{Tier} + Z_{it}\gamma + \varepsilon_{it}^{y} \tag{1}$$

where $D_{i,j}^{Tier}$ is a dummy variable indicating that person i graduated from a tier j college, and Z_{it} is the set of explanatory variables including age, squared age, birth year, and gender of person i.[26]

[24]The Lifelong Career Survey (LCS) by the Korea Research Institute for Vocational Education & Training (KRIVET) is an alternative dataset that could be used to generate the proxy of the prize of the tournament (Han, Kang and Lee 2016). For the purpose of this paper, KLIPS is preferred because it can recover the age-specific income profile.

[25]Lee and Koh (2023) reports that the alumni of Tier 1 colleges in Korea earn 50.5% more compared to those from the bottom Tier group, based on their preferred specification and the tier definitions. The lifetime income empirical exercise in this section is consistent with their findings, but the specification differs to consistent with the structural model.

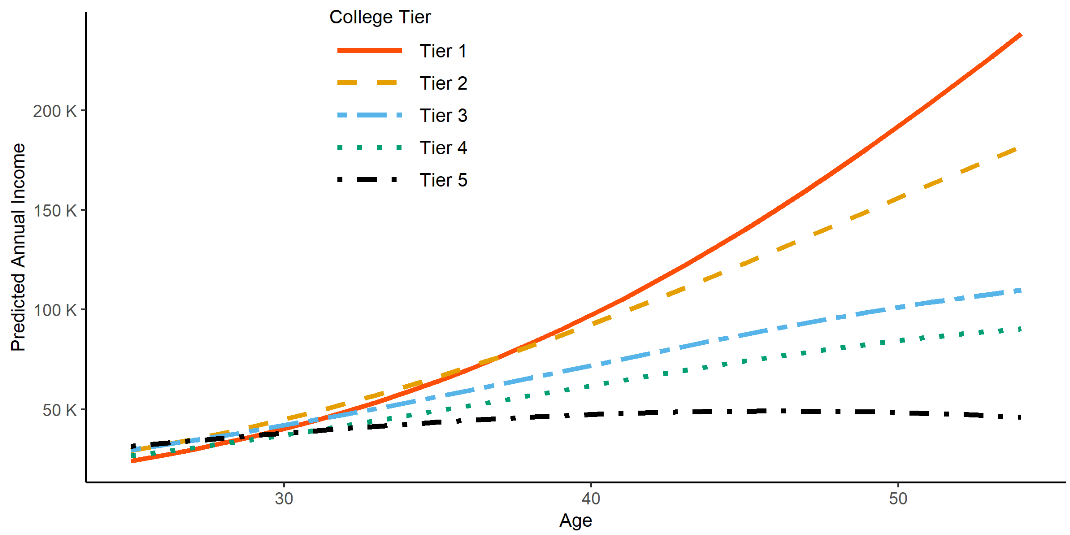
[26]The purpose of the birth year dummy variable is to capture the cohort difference in workers' income.

Table 4: Log Income Regression

	(1) Pooled OLS	(2) Pooled OLS	(3) Pooled OLS	(4) RE	(5) RE	(6) RE
College Tier						
Top tier	-1.931***	-1.684***	-2.958***	-1.671***	-1.491***	-2.194***
	(0.347)	(0.325)	(0.494)	(0.279)	(0.288)	(0.723)
Second Tier	-1.332***	-1.195***	-2.460	-1.409***	-1.364***	-1.908**
	(0.350)	(0.296)	(1.683)	(0.321)	(0.323)	(0.820)
Third Tier	-0.864**	-0.817***	-1.549**	-0.958***	-1.075***	0.190
	(0.269)	(0.196)	(0.507)	(0.363)	(0.364)	(1.249)
Fourth Tier	-0.895***	-0.618**	-1.954***	-0.727***	-0.524***	-0.425
	(0.232)	(0.186)	(0.361)	(0.079)	(0.123)	(0.436)
age	0.092***	0.092***	0.167***	0.095***	0.095***	0.168***
	(0.001)	(0.001)	(0.018)	(0.002)	(0.002)	(0.040)
Interactions						
Top tier × age	0.067***	0.065***	0.111***	0.058***	0.058***	0.105***
	(0.009)	(0.009)	(0.014)	(0.009)	(0.009)	(0.024)
Second Tier × age	0.050***	0.052***	0.098	0.051***	0.055***	0.095***
	(0.011)	(0.010)	(0.054)	(0.010)	(0.011)	(0.031)
Third Tier × age	0.032***	0.037***	0.058**	0.034***	0.044***	0.011
	(0.008)	(0.007)	(0.019)	(0.012)	(0.013)	(0.050)
Fourth Tier × age	0.029***	0.027***	0.073***	0.022***	0.021***	0.038***
	(0.007)	(0.007)	(0.015)	(0.002)	(0.002)	(0.013)
N	29599	29599	685	29599	29599	752
Major	No	Yes	No	No	Yes	Yes
RE	No	No	No	Yes	Yes	Yes
CSAT	No	No	Yes	No	No	No

Source: Korea Labor Income and Panel Study 1998-2012, Korea Labor Institute.
Note: RE refers to "Random Effects." Explanatory variables used in the regressions such as squared age, birth year, and gender are excluded from the table for brevity. The sample includes workers between 25 and 65 years old who work for wages or salary. I exclude workers who are born after 1992.

Figure 2: Income Dynamics by College Tiers



Source: Korea Labor Income and Panel Study 1998-2012, Korea Labor Institute.
Note: The sample includes workers between 25 and 65 years old who work for wages or salary. I exclude workers who are born after 1992. The figure has units of 1,000 KRW, which is about 0.85 USD. Annual income is predicted using the Pooled-OLS estimates in column (1) of Table 4.

Note that the regression equation captures both the effect of graduating from a tier j college on the level and the growth of an alumnus's income, respectively by β_j and δ_j. Columns (3) and (4) provide the estimates of the random effects model,

$$\ln y_{it} = \sum_{j=1}^{J} (\beta_j + \delta_j \cdot age_{it}) D_{i,j}^{Tier} + Z_{it}\gamma + \lambda_i^y + \eta_{it}^y$$

where λ_i^y and η_{it}^y are the individual-specific and the idiosyncratic errors respectively.[27] Columns (2) and (5) include the dummy variables of college-major, showing that the inclusion of major does not critically affect the main results of Columns (1) and (4), respectively. The Tier 1 dummy has the smallest estimate of intercept but the largest estimate of age differential. Figure 2 presents the predicted annual income of alumni using the estimates in Column (1) of Table 4. Before age 30, there is no economically significant difference in terms of annual income. On the other hand, the gap becomes significantly larger as people age. The effects are significant controlling for CSAT

[27] Since the focus of the regression is the college tier, which is time-invariant, I do not consider the fixed effects model.

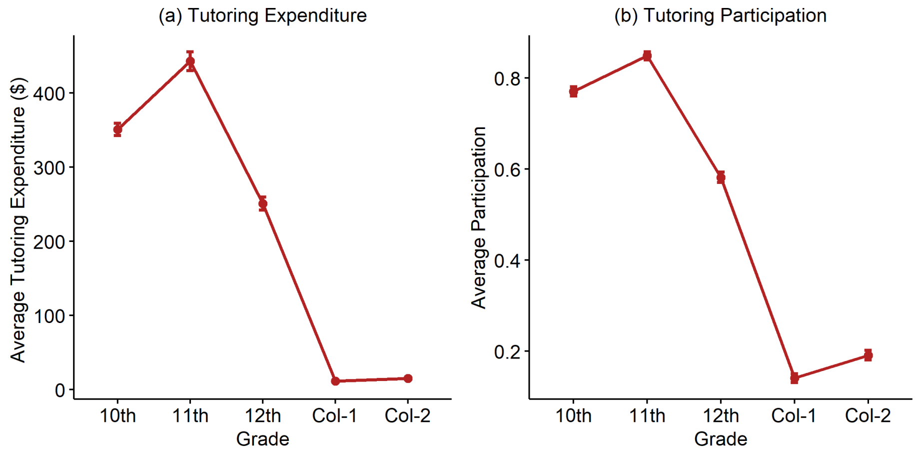
score, as can be seen in Columns (3) and (5) of Table 4.[28] The estimation results are consistent with the studies stressing the importance of using lifetime income in the returns to schooling literature (Haider 2001; Tamborini *et al.* 2015; Nybom 2017). The effects of parental investment on labor market outcomes through college reputation would be underestimated if researchers narrow their focus to the early labor market outcomes. The Pooled-OLS estimates in Column (1) of Table 1 are used in computing college-specific lifetime income, which is a component of the dynamic tournament model.

4.3 Competition Motives of Parental Investment

Competition with respect to getting into a more prestigious college is the primary motivation of parental investment. First, data suggest that the demand for private tutoring expenditure significantly drops as students finish the college admission process. Figure 3 presents the change of tutoring expenditure and participation rate over time for the sample cohort of KELS. Both expenditure and participation of private tutoring rapidly drop as soon as students graduate from high school, which suggests that the primary purpose of tutoring expenditure is associated with college admission. If the purpose of tutoring expenditure was for enhancing the student's human capital, it is unlikely that most students would completely stop private tutoring activities upon graduating from high school. Second, the number of seats at prestigious colleges is limited. Even with a very high final test score, students might not be able to go to a top-tier college if the seats are filled with students with higher test scores. The scarcity of seats at prestigious colleges and the fact that tutoring participation drops after the college entrance exam show that competition is the key feature determining the parental investment decision of the household.

[28] As CSAT performance is collected as a discrete variable in KLIPS, the estimation is different with Regression Discontinuity Design.

Figure 3: Private Tutoring Expenditure and Participation in Tutoring

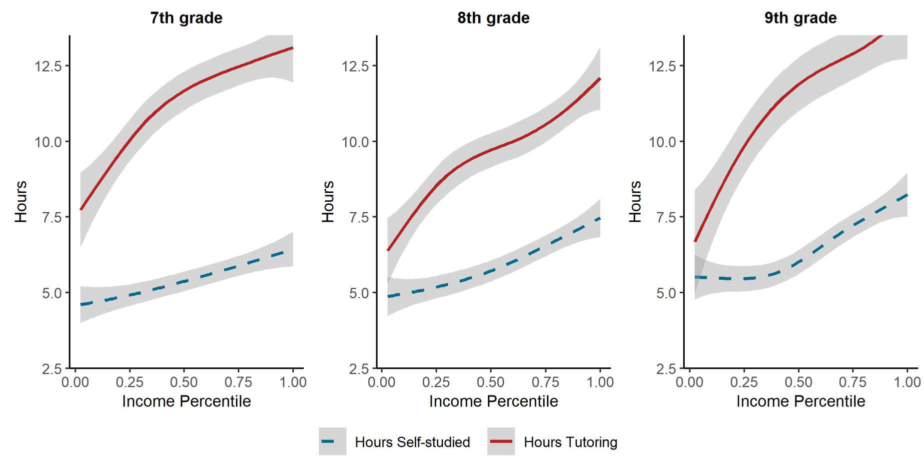


Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

4.4 Parental background and child's hours allocation

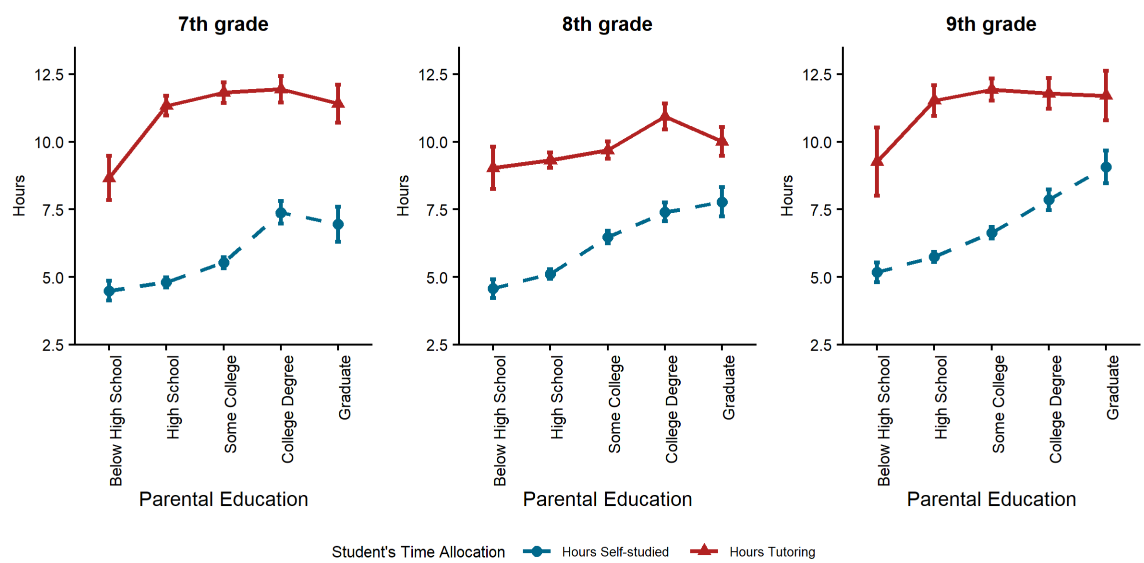
Compared to hours of tutoring, hours of self-study are less affected by parental income, which potentially has implications for intergenerational mobility. On the one hand, the income elasticity of hours of tutoring is higher than the income elasticity of hours of self-study. Figure 4 presents how hours of tutoring and hours of self-study vary with parental income when students are 7th, 8th, and 9th graders, using local linear regression. The slope of hours of tutoring is much steeper than the slope of hours of self-study, which shows that tutoring is an effort choice that is more responsive to parents' income. On the other hand, the covariation between hours of self-study and parental education is higher than the covariation between hours of tutoring and parental education, conditional on other household characteristics. Figure 5 presents how hours of tutoring and hours of self-study vary with parental education when students are 7th, 8th, and 9th graders. Unlike household income, the effect of parental education is higher on hours of self-study than the effect of parental education on hours of tutoring.

Figure 4: Income Gradient in Effort Decision



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: The gray regions are confidence bands with a significance level of 0.05. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

Figure 5: Parental Education and Efforts Allocation



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: In this graph, parental education is a categorical variable and based on the average years of parents educationm_i, which is defined as follows: Below High School if $m_i < 12$, High School if $m_i = 12$, Some College if $12 < m_i < 16$, College Degree if $m_i = 16$, and Graduate if $m_i > 16$. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, household income, and parental education.

Parental education soaks up significant variation in hours of self-study, which leaves a relatively small variation with parental income. Tables 5 and 6 presents the pooled OLS estimates of the regression equation,

$$\ln(1 + y_{it}) = \beta_0 + \beta_1 \log(hhinc_{it}) + \beta_2 m_i + \epsilon_{it} \tag{2}$$

where $hhinc_{it}$ is the income and m_i is parental education of household i. Columns (1) through (3) present the results where y_{it} is hours of self-study, and columns (4) through (6) present the results where y_{it} is hours of tutoring. Columns (1) and (4) provide the estimates without including the average years of parents' education, and Columns (2) and (5) provide the estimates with including the average years of parents' education to equation (2). Overall, hours of tutoring are explained more by parents' income than hours of self-study. Moreover, much of the covariation between hours of self-study and income is absorbed after controlling for the average years of parents' education.

Such empirical relationships suggest that different household backgrounds can lead to different allocations of effort choice. Thus, omitting one of the effort choices (parental investment or child effort) might result in biased estimates of intergenerational mobility, which calls for including both effort choices in the theoretical framework.

Table 5: The Effects of Parental Background on the Hours Allocation

	(1) log(1+Study)	(2) log(1+Study)	(3) log(1+Study)
log(Income)	0.238*** (0.022)	0.152*** (0.025)	0.036 (0.027)
Parental Edu		0.055*** (0.007)	
N	10454	10454	10454
Year	Yes	Yes	Yes
FE	No	No	Yes

Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: log(1+Study) and log(1+Tutoring) refer to log of hours of self-study plus one and hours of tutoring plus one, respectively. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income. Parental Educ indicates average years of parents' education.

Table 6: The Effects of Parental Background on the Hours Allocation

	(4) log(1+Tutoring)	(5) log(1+Tutoring)	(6) log(1+Tutoring)
log(Income)	0.677***	0.616***	0.269***
	(0.027)	(0.030)	(0.037)
Parental Edu		0.038***	
		(0.008)	
N	9431	9431	9423
Year	Yes	Yes	Yes
FE	No	No	Yes

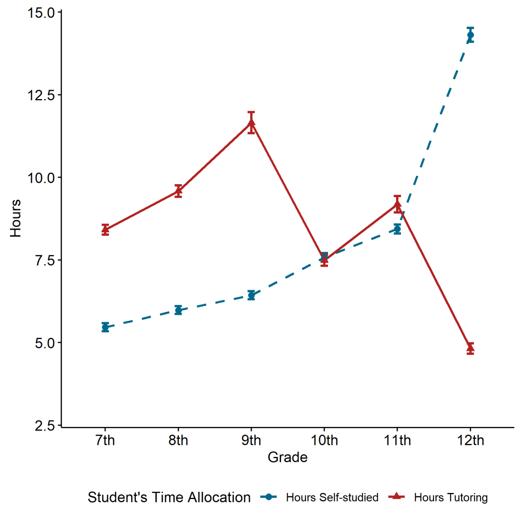
Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: log(1+Study) refer to log of hours of self-study plus one and hours of tutoring plus one, respectively. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income. Parental Educ indicates average years of parents' education.

4.5 Dynamic effort allocation of households

Students' time allocation of effort choices considerably changes as students proceed to the later educational stages. Figure 6 presents how the average hours of self-study and the average hours of tutoring change with students' grade level. While the average hours of tutoring shows a decreasing trend, the average hours of self-study shows an increasing trend. In 12th grade, the average hours of self-study is almost three times the average hours spent for tutoring. Such changes in time allocation suggest that the marginal effects of hours of self-study and tutoring expenditures on academic outcomes might change over time.[29]

[29] Several studies in the literature report that the effects of parental investment decrease with children's age (Cunha *et al.* 2010; Del Boca *et al.* 2017). To the best of my knowledge, there is no study reporting the changing effects of self-study over time.

Figure 6: Dynamic Allocation of Efforts

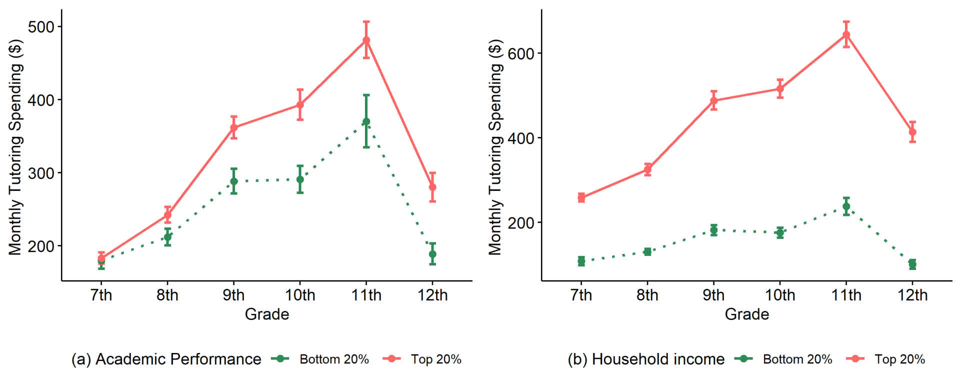


Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

The initial conditions of the household persistently affect the parental investment decisions throughout the secondary school periods. Figure 7 presents changes in the average hours of tutoring expenditure over time differentiated by two of households' pre-conditions: the initial academic performance and the initial parents' income. To see how these initial conditions affect the investment decision of households, I present the changes in average tutoring expenditure of two sub-groups: the top 20% and the bottom 20% of the ordered initial conditions. In particular, the solid lines of Figure 7 connect the average tutoring expenditure of the highest 20% of households classified by the two initial conditions. In the same manner, the dotted lines connect the average tutoring expenditure of the bottom 20% of households. Figure 7 (a) shows the increasing gap in tutoring expenditure between those who were in the top 20% of the test score in 6th grade and who were in the bottom 20% of the test score in 6th grade over time. In 7th grade, there is no significant difference between the two groups in terms of tutoring expenditure. From 8th grade on, there is an evident gap in tutoring expenditure between these two groups. Based on the average tutoring expenditure in 12th grade, students who were in the top 20% of the test score in 7th grade increased their tutoring expenditure compared to when they were in 7th grade. In comparison, the students who were in the lowest 20% of the test score in 7th grade decreased their tutoring expenditure compared to when they were in 7th grade.

Figure 7 (b) presents the average tutoring expenditure of high-income and low-income groups. The gap is significant in 7th grade and becomes greater over time. On average, high-income households' tutoring expenditure increases in 12th grade compared to when the students were in 7th grade. On the other hand, low-income households' tutoring expenditure decreases on average compared to when the students were in 7th grade.

Figure 7: Dynamic of Parental Investment by Initial Conditions



Source: Korea Educational Longitudinal Study 2005, Korean Educational Development Institute.
Note: In this figure, academic performance is measured in 6th grade and used for subsequent years. I only include households who do not have missing information on the following variables: tutoring expenditure, CSAT scores, and household income.

The evidence suggests that households self-select into the different effort levels based on their preconditions, and the allocation of the two efforts changes over time. As suggested earlier, different effort choices might have different implications for intergenerational mobility. Capturing the changing behavior of the households is crucial to get the correct quantification of the statistics of interest.

5. A Dynamic Model of College Admission Tournament

Motivated by the empirical evidence, I build and estimate a dynamic model of competition where each household chooses the amount of parental investment and the level of child's efforts. The dynamic model is built upon the rank-order tournament first described by Lazear and Rosen (1981) and related to its applications in college admission competition (Han, Kang and Lee 2016; Grau 2018; Tincani, Kosse and Miglino 2021).

5.1 Timeline

There exist N households in the dynamic tournament. Each household is composed of one student and the parents. I assume the household makes a unitary decision. I abstract away from the intra-household decision-making process. The students compete for the final prize against other students in the same cohort.

Figure 8 illustrates the timeline of the model. The model begins as the student of the household enters into 7th grade, which is the first year of secondary school. Each household is born with the complete income stream $\{w_{it}\}_{t=1}^{T}$, parental education m_i, and initial test score q_{i0}. Also, each household has a specific type k. Different types of households have different person-specific characteristics that are unobserved by the econometrician. I define them as λ_k^c, λ_k^x, λ_k^s and λ_k^q, which affect marginal utility from consumption, disutility from hours of tutoring, disutility from hours of self-study, and log of test score, respectively. Some households value non-academic goods such as travel more than other households conditional on the observed characteristics (Lazear 1977). Such unobserved taste for consumption is captured by λ_k^c. Some households prefer to encourage their child to study independently rather than send her to tutors, which is captured by the relative size of λ_k^s to λ_k^x. Some students might be particularly good or bad in taking exams, which would be captured by λ_k^q.[30]

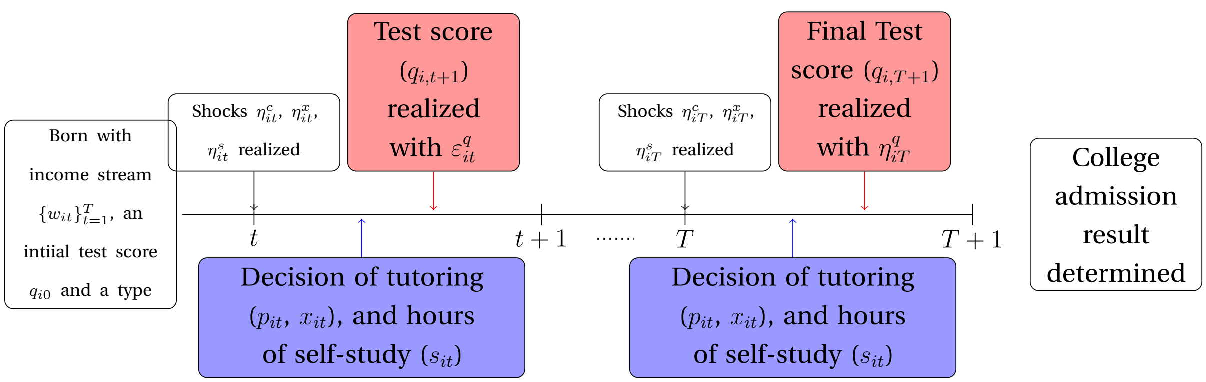


Figure 8: Model Timeline

At each time t, as the household enters into the period, the shock to the marginal utility of the consumption η_{it}^c, the shock to the marginal disutility from the tutoring activities η_{it}^x, and the shock to the marginal disutility from self-study η_{it}^s are realized. These shocks capture the unobserved time-varying components that are not

[30] I introduce the joint distribution of the time-specific shocks ($\eta_{it}^c, \eta_{it}^x, \eta_{it}^s$, and η_{it}^q) and the specification of type-specific unobserved heterogeneity (λ_k^c,λ_k^x,λ_k^s and λ_k^q) when I explain the flow utility component of the model.

accounted for by the deterministic components of the model. Based on those realized shocks and the observed state variables, each household chooses the quality of tutoring p_{it}, the hours spent on tutoring x_{it}, and the hours of self-study s_{it} to maximize its value function. The choices are subject to budget and time constraints. Subsequently, the test score $q_{i,t+1}$ is produced with the realization of the test score shock. This process repeats until the final test score $q_{i,T+1}$ is generated.

Each student is assigned to a college tier based on the ranking of the final test score and the fixed number of college seats in each tier. I denote n_j as the fixed number of seats for the j^{th} college-tier. In particular, denoting n_1 as the fixed number of seats for the first college tier, the first n_1 students are assigned to the top college tier, and the next n_2 students are assigned to the second tier. The process repeats until the $(J-1)^{th}$ college tier is filled up with n_{J-1} students so that all seats for the college tiers bind. The bottom tier is a residual tier which is composed of students whose score is below the cutoff for the $(J-1)^{th}$ college tier and the students who do not go to college.[31] The assigned college tier is the sole determinant of ex-post lifetime income.

5.2 The Preliminaries of the Tournament

Prize: Lifetime Income. The prize for going to a more prestigious college tier is a higher expected lifetime income awarded to the student, which motivates the household to exert effort. There exist J college tiers that are characterized by expected lifetime income v_j. The tier-specific lifetime income v_j is the discounted sum of the predicted income of the graduates. In particular,

$$v_j = \sum_{t=T+1}^{T^*} \beta^{t-T} \hat{y}_{jt}$$

where $\hat{y}_{jt}$ is the estimated income of the alumni of college tier j in year t, T is the age when the student graduates from college, T^* is the retirement age, and β is the discount factor fixed to 0.95.[32] I define $\hat{y}_{jt}$ as the estimated tier-specific annual income at time t,

[31]The implicit assumption regarding the bottom tier is that everyone graduates high school. The high school drop-out rate in South Korea is less than 2%.

[32]The average interest rate is around 5% for South Korea in 2010.

which is predicted using Pooled-OLS estimates of Column (1) in Table 4.[33] As tier 1 is defined to be the top college tier, v decreases in j (i.e., $v_1 > v_2 > ... > v_{J-1} > v_J$).[34]

For the student of household i to obtain prize v_j, her final test score $q_{i,T+1}$ must be above the cutoff for tier j and below the cutoff for the tier $j-1$. In other words, student i is placed in college tier j iff

$$\tilde{Q}_{j-1} > q_{i,T+1} \geq \tilde{Q}_j$$

where $\tilde{Q}_j$ is the cutoff between college tier j and tier $j+1$. The cutoff $\tilde{Q}_j$ is the test score of the N_j^{th} highest student in the sample, where $N_j = \sum_{l=1}^{j} n_l$. Thus, $\{\tilde{Q}_j\}_{j=1}^{J}$ is where the competition enters the model. In order for a student to be in tier j or better, she has to be above enough competitors by at least scoring the N_j^{th} highest final test score. As $q_{i,T+1}$ is a function of the effort choice of each household, $\{\tilde{Q}_j\}_{j=1}^{J}$ is endogenously determined by the competition across households. I assume that each household can correctly predict the final test score cutoffs.[35]

Assumption 1. *Each household correctly guesses the set of final test score cutoffs* $\{\tilde{Q}_j\}_{j=1}^{J}$.

The facts that (i) college-tier is assigned solely using the final test score $q_{i,T+1}$ and (ii) heterogeneity in college quality is the only variation of the lifetime income in this framework imply that the final test score of a student essentially determines the lifetime income of the student. That is, under the model environment, I assume that there is no extra opportunity to improve one's lifetime income once the college entrance exam is over.

Assumption 2. *The quality of the college one graduates from is the sole determinant of one's lifetime income.*

This is an arguably reasonable assumption under the institutional setting of the interest. I borrow the results of Kang, Kang, and Kim (2022) as supporting evidence for

[33] I assume no earnings in the college periods.

[34] I confine the prize to pecuniary rewards and rule out other benefits from the model. One might argue that the non-pecuniary value of attending an elite college should be considered part of the reward. However, it is difficult to separately measure the non-pecuniary value of attending better colleges due to data limitations. See Gong *et al.* (2019) for an empirical quantification of the consumption value of college.

[35] I assume away the inconsistency between the guessed cutoffs and the resulting cutoffs because the working sample did not go through significant policy shock that might cause the difference between the guessed and the resulting cutoffs. See Tincani, Kosse and Miglino (2021) for the case that resulting cutoffs significantly deviate from the guessed cutoffs.

Assumption 2. Using a dataset of one of the big 5 companies of Korea, they find that the effect of college reputation dominates the effect of college GPA on receiving an offer from the firm. Table 7 presents their estimates. Based on their probit estimates, increases in college GPA by 10 leads to a 6% increase in getting an offer from one of the subsidiary firms of the conglomerate. Meanwhile, graduating from one of the tier 1 colleges increases probability of getting an offer by 23% relative to graduating from a college below tier 3.

Table 7: Job Offer Regression

	(1) probit
Tier=1	0.221***
	(0.039)
Tier=2	0.056***
	(0.005)
Tier=3	0.002
	(0.028)
ColGPA	0.006***
	(0.002)
N	9132

Source: Confidential data of the conglomerate in late 2010s.
Note: The data are on the applicants to the subsidiary firms of the conglomerate for the latest three years. Other explanatory variables include the subsidiary firm's information and the applicants' information such as college major, age, and gender. The college GPA measured is scaled 0 to 100. ColGPA refers to the average of standardized college GPA.

Parental Investment: One of the two modes of household effort is parental investment, which is embodied in private tutoring expenditure. Each household chooses the unit price (quality) of tutoring p_{it} and hours (quantity) of tutoring x_{it} to increase the child's test score.[36] The total amount of tutoring expenditure e_{it} is

$$e_{it} = p_{it} x_{it}.$$

[36] To the best of my knowledge, this is the first model to consider the quality and quantity of parental monetary investment simultaneously.

The tutoring expenditure is constrained under two dimensions. A household cannot spend more tutoring expenditure than its income (i.e., $e_{it} \leq w_{it}$).[37] Also, hours of tutoring are bounded by the child's maximum available time, namely h. While the income constraint is unequal among households, available hours for the child are constant across all households.

Note that the time choice is solely about the time use of the child, which means I do not model the time allocation of parents. The data suggest that, in the secondary school periods, which the model concerns, the majority of parents do not teach their children themselves in middle school periods, and very few parents use their time to teach their child in the high school periods. A few potential explanations can be given for this empirical fact. As students grow, the test materials become more and more difficult to be taught by parents. Also, if there exists an established tutoring market, it would be a safer option for parents in terms of increasing student's test score. Note that the model concerns a regime with a high-stakes standardized test. Full-time tutors would have a comparative advantage in preparing students for exams over parents.

Child's hours of self-study: Hours of self-study is the other household's mode of effort in the tournament. Each household chooses how much time to allocate for hours of self-study s_{it} which is constrained by h. Unlike parental investment, the resource of self-study does not vary over households as time is equally granted to everyone. The taste for self-study, however, can be considerably heterogeneous across students. For example, some students might prefer studying independently rather than re-learning the same materials from the tutors. Others may prefer reviewing materials with tutors rather than studying alone. I allow the taste for hours of self-study to vary by parental education and the associated shock.

Test Score Production Function: The final test score is the result of accumulated dynamic choices of the household along with its given initial conditions. The initial academic performance q_{i1} is exogenously given and proxied by academic performance in primary school.[38] The three choices affecting test scores are quality of tutoring p_{it}, hours of tutoring x_{it}, and hours of self-study s_{it}. I allow that the quantity (hours) and quality (unit price) of the tutoring activity have different intensities in contributing to the test score production. Denoting κ as intensity of quality of tutoring, the

[37] I assume no borrowing.

[38] Although an earlier measure of the initial child's ability would be more desirable, this is the earliest time period that the academic performance data are available.

transformed tutoring input is specified as

$$\tilde{e}_{it} = p_{it}^{\kappa} x_{it}^{1-\kappa} \tag{3}$$

where $\kappa < 0.5$ and follows decreasing returns to scale (DRS). The DRS restriction is necessary to prevent the household from choosing an infinitesimal quantity of tutoring hours. If $\kappa \geq 0.5$, the household always has an incentive to make p_{it} greater and x_{it} smaller. The opposite case of a household choosing extremely large hours of tutoring does not occur as available time is restricted by h.

For each time $t = 1, 2, ..., T$, the test score $q_{i,t+1}$ is produced following

$$q_{i,t+1} = g(\theta_t^q, q_{it}, p_{it}, x_{it}, s_{it}, \eta_{it}^q, \lambda_k^q)$$

where η_{it}^q is the test score shock, λ_k^q is the type-specific error, and θ^q is the set of relevant parameters for the test score production. The inclusion of the test score produced in the previous period, q_{it}, allows that the previous test score has its own effects in generating subsequent test score (Cunha and Heckman 2007). Furthermore, I allow the subset of production parameters to change across periods. The effect of the combined efforts of the household is likely to change over time. As students grow older, the materials taught become more advanced, which makes it harder for students with insufficient background to catch up. Thus, private tutoring expenditure and hours of self-study can be less effective in the later stages of education. In addition, the relative importance of each investment might change over time. For example, the marginal effects of parental investment might increase (decrease) while the effects of self-study decrease (increase) over time. To reflect such changing effects, I let the marginal effects parameters ν_t, δ_{pt}, and δ_{st} be different for each period $t = 1, 2, ...T$.

For estimation, the production function g is a Constant Elasticity of Substitution (CES) production function and is specified as

$$q_{i,t+1} = A_t q_{it}^{\delta_{qt}} \left[\delta_{et}(1 + \tilde{e}_{it})^{\phi} + \delta_{st}(1 + s_{it})^{\phi} \right]^{\frac{\nu_t}{\phi}} \varepsilon_{it}^q \tag{4}$$

where A_t is total factor productivity, ν_t is the parameter of marginal effect of the combined effort choices, and ϕ is the parameter governing substitution between tutoring and self-study. The marginal effect of the total effort decision is captured by ν_t, while the relative importance of the tutoring expenditure and hours of self-study are captured by δ_{et} and δ_{st}, respectively. I define ε^q_{it} as a combined shock of λ^q_k and η^q_{it}, which is specified as $\ln \varepsilon^q_{it} = \lambda^q_k + \eta^q_{it}$.

5.3 Household

Flow Utility: The utility function of the unitary household is comprised of three parts: (i) the marginal utility from the household consumption c_{it}, (ii) the marginal disutility from hours spent on tutoring x_{it}, and (iii) the marginal disutility from hours of self-study s_{it}. I denote α_c, α_x, and α_s as taste parameters for household consumption, hours of tutoring, and hours of self-study, respectively. The taste parameters may depend on the fixed characteristics of the household. I assume additive and separable log utility, which is specified as

$$u(c_{it}, x_{it}, s_{it}, \varepsilon_{it}) = \alpha_c \varepsilon^c_{it} \log(c_{it}) + \alpha_x \varepsilon^x_{it} \log(1 + x_{it}) + \alpha_s \varepsilon^s_{it} \log(1 + s_{it}) \tag{5}$$

where ε^c_{it} is the shock to the marginal utility from consumption, ε^x_{it} is the shock to the disutility from hours of tutoring, ε^s_{it} is the shock to the disutility from hours of self-study, and $\varepsilon_{it} = \{\varepsilon^c_{it}, \varepsilon^x_{it}, \varepsilon^s_{it}\}$. The shocks are distributed joint normal and separated into the type-specific and the time-varying components. In particular, I denote λ^z_k and η^z_{it} as type-specific and time-varying components of ε^z_{it} $(z = c, x, s)$, respectively. The shocks are decomposed as

$$\begin{pmatrix} \ln \varepsilon^c_{it} \\ \ln \varepsilon^x_{it} \\ \ln \varepsilon^s_{it} \\ \ln \varepsilon^q_{it} \end{pmatrix} = \begin{pmatrix} \eta^c_{it} \\ \eta^x_{it} \\ \eta^s_{it} \\ \eta^q_{it} \end{pmatrix} + \begin{pmatrix} \lambda^c_k \\ \lambda^x_k \\ \lambda^s_k \\ \lambda^q_k \end{pmatrix}, \text{ and } \begin{pmatrix} \eta^c_{it} \\ \eta^x_{it} \\ \eta^s_{it} \\ \eta^q_{it} \end{pmatrix} \sim N(0, \Omega^\eta)$$

where Ω^η is the covariance matrix for the time-varying shocks.[39] I assume that the correlations between the time-varying shocks η^z_{it} $(z = c, x, s, q)$ are 0.

Note that I do not specify the utility flow from the current test score. Each household is concerned solely about the final outcome, and the role of the current test score is limited to the stepping stone for the final test score. That is, the current test score affects the decision of the household only through the value of the future. The

[39] In modeling the self-study shock, an alternative specification involves assuming that there exists unobserved heterogeneity in terms of the productivity of hours of self-study. Such an assumption, however, is computationally burdensome if the test score production function is CES.

specification of future value is introduced with the recursive formulation at the end of the subsection.

Terminal Value: Expected lifetime income is the terminal value of the model, which drives the dynamic choices of the tournament model. With the tier-specific lifetime income v_j, the expected lifetime income is a weighted sum,

$$\sum_{j=1}^{J}\left\{ \ln(v_j) * Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \middle| \Gamma_{iT})\right\} \tag{6}$$

where $Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \middle| \Gamma_{iT})$ is the probability of getting into college tier j. The randomness of the admission probability comes from the test score shock η^q_{it}. Each student would have a different probability of going to a college tier j as they have different characteristics affecting the evolution of the test scores. The disparity among students in terms of going to each college tier leads to the discrepancies in expected lifetime income, which generates the heterogenous incentives among households. The higher expected lifetime income leads to bigger the terminal value of the household, which makes it more appealing for the parents to invest in the child.

The functional form of the expected lifetime income is determined by the test score shock ε^q_{it}. With the log-transformation, the terminal value is specified as

$$\begin{aligned}&\sum_{j=1}^{J}\left\{ \ln(v_j) * Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \middle| \Gamma_{iT})\right\}\\ &= \sum_{j=1}^{J}\left\{ \ln(v_j) * \left\{ F_q(\frac{\ln \tilde{g}_{i,j-1}}{\sigma_q} \middle| \Gamma_{iT}) - F_q(\frac{\ln \tilde{g}_{ij-1}}{\sigma_q} \middle| \Gamma_{iT})\right\}\right\}\end{aligned}$$

where $\ln \tilde{g}_{ij}$ is the distance between the deterministic components of log final test score of student i and the log cutoff of the college tier j (i.e. $\ln \bar{g}_{ij} = \ln \tilde{Q}_{j-1} - \ln \widehat{q_{i,T+1}} - \lambda^q_k$), and F is the distribution of η^q_{it}. I assume F follows normal distribution in the spirit of rank-order tournament model (Lazear and Rosen 1981; Han *et al.* 2016; Grau 2018; Tincani *et al.* 2021).[40]

Budget and Time Constraints: The choices of the household are restricted by the budget and the time constraints. The budget constraint is given by

[40]One can also adopt a functional form that η^q_{it} follows Generalized Extreme Value distribution which results in a Tullock (2001) contest.

$$c_{it} + p_{it}x_{it} \leq w_{it} \tag{7}$$

where w_{it} is household income, and the time constraint is

$$x_{it} + s_{it} \leq h \tag{8}$$

where h is student's disposable time. I define h as the maximum time each student can use every week, which is assumed to be 63.[41]

State Variables: There are observed and unobserved state variables in the dynamic model. The set of observed state variables Z_{it} includes the previous test score q_{it}, parental education m_i, and the complete income stream $\{w_{it}\}_{t=1}^{T}$. The set of unobserved state variables Ψ_{it} includes the set of unobserved shocks and the type specific heterogeneity. Based on the timeline, the time-varying shock regarding test score is not an unobserved state variables. (i.e., $\Psi_{it} = \{\eta_{it}^c, \eta_{it}^x, \eta_{it}^s, \lambda_k^c, \lambda_k^x, \lambda_k^s, \lambda_k^q\}$).

Information and Uncertainty: I assume a continuum of households. The continuum assumption is useful in that the information of other households can be summed up as a distribution of households.

Assumption 3. *The distribution of household is common knowledge.*

As stated in Assumption 1, each household correctly anticipates the set of college tier cutoffs $\{\tilde{Q}_j\}_{j=1}^{J}$.[42] They know the distribution of the final test scores in advance and make dynamic choices based upon the perfect guess.

Assumption 4. *Each household knows its complete wage stream.*

There is no uncertainty in the income process. In fact, each household is assumed to know its complete wage stream as the model begins. As this is a markov model, past wages are irrelevant after conditioning on the remaining state variables. As depicted in Figure 8, each household learns about the realization of the consumption shock η_{it}^c, the disutility shock to hours of tutoring η_{it}^x, and disutility shock to hours of self-study

[41] I assume each student can use 9 hours everyday for non-leisure activities other than hours spent in regular school

[42] In the static model of Grau (2018), Assumption 3 implies that the tournament participants can correctly guess the cutoffs. In my dynamic model, however, Assumption 3 does not guarantee the perfect foresight due to the presence of future shocks that each individual cannot predict.

η^s_{it} at the beginning of each period. However, it does not know about the test score shock η^q_{it} before it makes a decision. Therefore, it makes a set of choices based on the expectation over η^q_{it}, $\eta^c_{i,t+1}$, $\eta^x_{i,t+1}$, and $\eta^s_{i,t+1}$, conditional on observed state variables and type-specific unobserved heterogeneity.

Household Value Function: Building upon the model components, I describe the value function of the household. As stated earlier, each household chooses the unit price (quality) of tutoring p_{it}, hours of tutoring x_{it}, and hours of self-study s_{it} based on the anticipation of future values. In particular, at each time t, the household i solves

$$V_{it}(Z_{it}, \varPsi_{it}) = \max_{p_{it}, x_{it}, s_{it}} \left\{ u(c_{it}, x_{it}, s_{it}, \varepsilon_{it}) + \beta \underset{\eta^q_{it}, \eta_{it}}{E} \left[V_{i,t+1}(Z_{i,t+1}, \varPsi_{i,t+1} \middle| \Gamma_{it}) \right] \right\}, \tag{9}$$

subject to equation (4) and constraints (7) and (8), where $\Gamma_{it} = \{Z_{it}, \Psi_{it}, \{\bar{Q}_j\}_{j=1}^J\}$ is the set of information before making the decision and $\eta_{it} = \{\eta^c_{it}, \eta^x_{it}, \eta^s_{it}\}$ is the set of unobserved time-varying shocks. Each household faces a tradeoff between current flow utility and future payoffs. Each choice variable incurs costs associated with the choice. In particular, investing more in parental investment (i.e., increasing p_{it} or x_{it}) requires suffering more from the disutility from hours of tutoring and sacrificing current consumption. Spending more time on hours of self-study leads to an increase in the disutility from hours of self-study. This dynamic incentive structure governs the decision of the household.

At the final test stage ($t = T$), where the tournament of the final score occurs, the value function is

$$\begin{aligned} V_{iT}(Z_{iT}, \varPsi_{iT}) = \max_{p_{iT}, x_{iT}, s_{iT}} & \Bigg\{ u(c_{iT}, x_{iT}, s_{iT}, \varepsilon_{iT}) \\ & + \alpha_v \sum_{j=1}^{J} \ln(v_j) \times Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT}) \Bigg\} \end{aligned} \tag{10}$$

where α_v is an altruism parameter. The altruism parameter measures the "exchange rate" between the current household utility and the child's future lifetime income. All-in-all, each household makes a choice between the child's lifetime income and its flow utility. If the marginal value to the household is greater than the marginal loss of flow utility of the household, it exerts more efforts using either parental investment, the child's self-efforts, or both.

5.4 Equilibrium of the Tournament

In this section, I define the dynamic equilibrium of the tournament model. Then I prove the existence of the equilibrium using the Schauder Fixed-Point Theorem (Amir 1996; Fey 2008; Mertens and Judd 2018; Engers, Hartmann and Stern 2022). I define a set $k = \{\{V_t(p_t, x_t, s_t; Z_t, \Psi_t)\}_{t=1}^{T}, \{\tilde{Q}_j\}_{j=1}^{J}\}$, where $\{V_t(p_t, x_t, s_t; Z_t, \Psi_t)\}_{t=1}^{T}$ is a set of value functions that are specified in equations (9) and (10), and $\{\tilde{Q}_j\}_{j=1}^{J}$ is the set of college-tier cutoffs. I define $\mathcal{K}$ as a set of all possible k.

Definition 1. *Given the set of initial conditions and Assumptions 1, 2, and 3, a Markovian equilibrium of the model is a vector* $k^* = \left\{ \{V_t^*(p_t, x_t, s_t; Z_t, \Psi_t)\}_{t=1}^{T}, \{\tilde{Q}_j^*\}_{j=1}^{J} \right\}$*, which is generated by the following process:*

1. I define $\mathcal{K}_a$ as a set of all possible combinations of choice variables $\{p_t, x_t, s_t\}_{t=1}^{T}$. Given the set of initial conditions $\{q_{i1}, \{w_{it}\}_{t=1}^{T}, m_i\}$, a mapping $\aleph_a$ maps $\mathcal{K}$ into $\mathcal{K}_a$ ($\aleph_a : \mathcal{K} \to \mathcal{K}_a$), based on the value functions specified in equations (9) and (10). [43]

2. I define $\mathcal{K}_b$ as possible distributions of the final test score q_{T+1}. A mapping $\aleph_b$ maps $\mathcal{K}_a$ into $\mathcal{K}_b$ ($\aleph_b : \mathcal{K}_a \to \mathcal{K}_b$), based on the test score production function specified in equation (4) .

3. I define $\mathcal{K}_c$ as possible sets of resulting cutoffs $\{\check{Q}_j\}_{j=1}^{J}$. Given the number of seats for each college tier $\{n_j\}_{j=1}^{J}$, a mapping $\aleph_c$ maps the distribution of the final test score q_{T+1} and $\{n_j\}_{j=1}^{J}$ into the set of cutoffs, $\{\check{Q}_j\}_{j=1}^{J}$ ($\aleph_c : \mathcal{K}_b \to \mathcal{K}_c$). The mapping $\aleph_c$ is based on the rules of college admission.

4. A mapping $\aleph_d$ maps $\{\check{Q}_j\}_{j=1}^{J}$ into $\mathcal{K}$ ($\aleph_d : \mathcal{K}_c \to \mathcal{K}$).

5. In equilibrium, the set of guessed cutoffs $\{\tilde{Q}_j\}_{j=1}^{J}$ match the set of realized cutoffs $\{\check{Q}_j\}_{j=1}^{J}$.

[43] The mapping $\aleph_a$ involves backward recursion.

Finally, I define a mapping $\aleph : \mathcal{K} \to \mathcal{K}$. The mapping $\aleph$ is the composition of submappings. In particular,

$$\begin{aligned}\aleph =& \aleph_a \circ \aleph_b \circ \aleph_c \circ \aleph_d \\ =& \aleph_a(\aleph_b(\aleph_c(\aleph_d(k)))).\end{aligned}$$

Lemma 2. *The mapping $\aleph$ is compact*

Proof. [In Appendix A.1] □

Lemma 3. *The mapping $\aleph$ is continuous*

Proof. [In Appendix A.2] □

Theorem 4. *A Markovian equilibrium exists.*

Proof. Previous results establish that $\mathcal{K}$ is a nonempty, compact, and closed subset of a locally convex Hausdorff space. The map $\aleph$ is continuous. Therefore, the set of fixed points of $\aleph$ is nonempty and compact. The mapping satisfies all the requirements of Schauder Fixed-Point Theorem. Hence a fixed point exists.

□

5.5 Features of the Model

The dynamic tournament model offers several features that help answer the research question of this paper. First, the rich heterogeneity of state variables and the choice set and the specification of the test score production function help disentangle the source of intergenerational persistence of earnings. Each household can simultaneously choose the quality of tutoring, hours of tutoring, and hours of self-study in the model based on its state variables. The test score production function allows for a variety of inputs: previous test score q_{it}, quality of tutoring p_{it}, hours of tutoring x_{it}, and hours of self-study s_{it}. This specification enables me to separately quantify the impact of parental investment, self-effort of the child, and other household characteristics on intergenerational persistence of earnings. In the model, each household can choose the other mode of investment even if they are not allowed to use one of the options. Thus, the model provides an opportunity to simulate the reaction of the household when one of the choices is restricted. In Section 8, I quantify the role of each choice by simulating the model with shutting down that particular choice from the model. Then I compare the simulation results with when all choices are allowed.

Second, the rank-order feature of the tournament model enables me to study the effects of the changes in cohort size and the role of disparity in college quality. Since the tournament model is about obtaining a limited number of seats in the better colleges, the model can be used to evaluate the exogenous changes in the number of competitors. As described in Section 2, countries with high private tutoring expenditure face a sharp decrease in total fertility rate. The reduction of cohort size due to declining fertility means that there is a less number of competitors for higher college tiers.[44] The model provides an opportunity to simulate parental investment of households if the cohort size is reduced. Additionally, the tournament model also can be used in assessing the role of disparity in college quality on parental investment. The well-known feature of the tournament model is that the size of the prize differential affects the effort choice of the agents (Lazear and Rosen 1981). The prize differential in my model is the income differential of the higher college-tier. Such a feature of the tournament model captures the role of the distribution of college qualities on the investment decision of the household.

Third, as I allow for the time-varying effects of the choices, I can compare the effects of hours of self-study and hours of parental investment.

6. Estimation Strategy

The touranment term at the final period T generates heterogenous incentives for households with different state variabels. These features make the policy function highly non-linear. The choice variables of the structural model involves the pairs of interior-interior, interior-corner, and corner-corner. In addition, the structural model involves endogenous regressors. To this end, I estimate the parameters of the model using Maximum Simulated Likelihood. I describe the likelihood function and discuss the sources of identification underlying the estimation procedure.

[44]In Section 8 I show changes in the number of seats in colleges and the number of high school graduates using administrative data of South Korea.

6.1 The likelihood function

I denote θ as the set of parameters, Z_{it} as the set of observed state variables, and λ_k as the set of unobserved type-specific characteristics. The individual likelihood contribution of household i is

$$\mathcal{L}_i(\theta | q_{i0}, \{w_{it}\}_{t=1}^{T}, m_i) = \sum_{k=1}^{K} \left\{ \left(\Pi_{t=1}^{T} \mathcal{L}_{it}(\theta | Z_{it}, \lambda_k) \right) \Pr(type = k) \right\} \tag{11}$$

which is conditional on the initial test score q_{i0}, the income stream $\{w_{it}\}_{t=1}^{T}$, and parental education m_i. The time-specific likelihood contribution $\mathcal{L}_{it}(\theta | Z_{it}, \lambda_k)$ can be characterized in four different ways depending on the combination of the tutoring-participation dummy variable d_{it}^{x} and self-study participation dummy variable d_{it}^{s}. In particular,

$$\begin{aligned}
\mathcal{L}_{it}(\theta | Z_{it}, \lambda_k) = & \Big[f(p_{it}, x_{it}, s_{it}, q_{it}) \Big]^{d_{it}^{x} d_{it}^{s}} \\
& \times \Big[\Pr(p_{it}, x_{it}, s_{it} = 0) \cdot f_{q_{it}}(q_{it} | x_{it}, s_{it} = 0) \Big]^{d_{it}^{x}(1-d_{it}^{s})} \\
& \times \Big[\Pr(x_{it} = 0, s_{it}) \cdot f_{q_{it}}(q_{it} | x_{it} = 0, s_{it}) \Big]^{(1-d_{it}^{x}) d_{it}^{s}} \\
& \times \Big[\Pr(x_{it} = 0, s_{it} = 0) \cdot f_{q_{it}}(q_{it} | x_{it} = 0, s_{it} = 0) \Big]^{(1-d_{it}^{x})(1-d_{it}^{s})}
\end{aligned}$$

where $d_{it}^{x} = 1$ means that household participates in tutoring at time t, and $d_{it}^{s} = 1$ means that the student of the household i has non-zero hours of self-study at time t.

The final form of the likelihood function is a sum of the log likelihood contributions,

$$\log \mathcal{L}(\theta) = \sum_{i=1}^{N} \log \mathcal{L}_i(\theta | q_{i0}, \{w_{it}\}_{t=1}^{T}, m_i).$$

The likelihood contributions of the choice variables are computed by transforming the characterized expression of the shocks, using the Jacobian-transformation. In particular, the time-specific likelihood contribution can be expressed as

$$
\begin{aligned}
\mathcal{L}_{it}(\theta|S_{it},\lambda_k) = & \left[f_{\eta^c_{it}}(\tilde{\eta}^c_{it}) \cdot f_{\eta^x_{it}}(\tilde{\eta}^x_{it}) \cdot f_{\eta^s_{it}}(\tilde{\eta}^s_{it}) \cdot f_{\eta^q_{it}}(\tilde{\eta}^q_{it}) |det\left(\frac{\partial(\tilde{\eta}^c_{it},\tilde{\eta}^x_{it},\tilde{\eta}^s_{it},\tilde{\eta}^q_{it})}{\partial(p_{it},x_{it},s_{it},q_{it})}| \right]^{d^x_{it}} \\
& \times \left[\int_{\tilde{\eta}^s_{it}} \left(f_{\eta^c_{it}}(\tilde{\eta}^c_{it}) \cdot f_{\eta^x_{it}}(\tilde{\eta}^x_{it}) \cdot f_{\eta^s_{it}}(\eta^s_{it}) \cdot f_{\eta^q_{it}}(\tilde{\eta}^q_{it}) \right) d\eta^s_{it} | \det \frac{\partial(\tilde{\eta}^c_{it},\tilde{\eta}^x_{it},\tilde{\eta}^q_{it})}{\partial(p_{it},x_{it},q_{it})}| \right]^{d^x_{it}(1-d^s_{it})} \\
& \times \left[\int_{-\infty}^{\infty} \int_{\tilde{\eta}^x_{it}(\eta^c_{it})} \left[f_{\eta^c_{it}}(\eta^c_{it}) \cdot f_{\eta^l_{it}}(\eta^x_{it}|\eta^c_{it}) \cdot f_{\eta^s_{it}}(\tilde{\eta}^s_{it}) \cdot f_{\eta^q_{it}}(\tilde{\eta}^q_{it}) \right] d\eta^x_{it} d\eta^c_{it} | \det \frac{\partial(\tilde{\eta}^s_{it},\tilde{\eta}^q_{it})}{\partial(s_{it},q_{it})}| \right]^{(1-d^x_{it})d^s_{it}} \\
& \times \Bigg[\Bigg(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Big[\Pr\{V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it}) > V_{x0}(\eta^s_{it}), V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it}) > V_{0s}(\eta^c_{it},\eta^x_{it}), V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it}) > V_{xs})\} \\
& \quad \times d\eta^c_{it} d\eta^x_{it} d\eta^s_{it} \Big] f(\tilde{\eta}^q_{it}) | \det \frac{\partial \tilde{\eta}^q_{it}}{\partial q_{i,t+1}} | \Bigg]^{(1-d^x_{it})(1-d^s_{it})}
\end{aligned}
$$

where V_{00} is the value when $x_{it} = s_{it} = 0$, V_{x0} is the value when $x_{it} > 0$ and $s_{it} = 0$, and V_{0s} is the value when $x_{it} = 0$ and $s_{it} > 0$.[45]

To evaluate the integrals in the likelihood function, I use the Montecarlo simulation. Borsch-Supan, Hajivassiliou and Kotlikoff (1992) show that the MSL estimates perform well under a moderate number of draws, such as 20, with an adoption of a good simulation method. To reduce the variance of simulation error, I use antithetic acceleration (Geweke 1988; Stern 1997).

About 8.3% of the household-year observations are missing, creating "holes" in the household data. I simulate the unobserved choice variables using the value function of the model (Lavy, Palumbo and Stern 1998; Stinebrickner 1999; Sullivan 2009). In particular, for each draw of the set of errors, I replace the unobserved choice variables with the optimized choices that maximize the value function of the model. Also, for periods 4 and 5, the test score data are unobserved. I simulate the unobserved test scores for each draw of test score error η^q_{it} using equation (4). In Appendix D, I show the derivation of the density and probability I use for computing the likelihood function, and I explain the simulation of unobserved variables.

6.2 Identification

Parameters of the model can be classified into the productivity parameters associated with the test score function and the taste parameters that directly affect value function. The productivity parameters in the test score production function are identified by

[45] For the case $d^x_{it} = d^s_{it} = 0$, I am working on a G.H.K type of simulation to reduce the variance of simulation error.

the covariation between the subsequent test score $q_{i,t+1}$ and the inputs (q_{it}, p_{it}, x_{it}, and s_{it}). As data of the inputs are available for each period, I can separately identify the productivity parameters for each time t.

The taste parameters α_c, α_x, α_s, and the altruism parameter α_v affect the value function, and do not directly affect the test score function. These parameters are the constants for the likelihood contribution of the corresponding choice variables. I do not differentiate the taste parameters for each period. The element of the covariance matrix of the shocks are identified in maximizing the log-likelihood contribution of the associated shocks.

The exogenous variables in the model are the academic performance in primary school q_{i1}, the parental education m_i, and the complete income stream of parents $\{w_{it}\}_{t=1}^{T}$. The identifying assumption is the time varying shocks are orthogonal to the initial conditions. In particular,

$$\{\eta_{it}^{c}, \eta_{it}^{x}, \eta_{it}^{s}, \eta_{it}^{q}\}_{t=1}^{T} \perp \left\{ q_{i1}, m_i, \{w_{it}\}_{t=1}^{T} \right\}.$$

7. Estimation Results

7.1 Test score function parameters

Table 8 presents the estimates of test score production function specified in equation (4). The interpretation of the previous test score parameter is the same as the log-log case of a linear regression equation. For example, for $t = 2$, a 1% increase in the previous test score leads to a 0.98% increase in the subsequent test score controlling for other inputs. The marginal effects of the effort parameters ν_t largely decline over time. Especially in the final period, the effect plummets to 0.02. This estimate suggests that, in the final period, the marginal effects of both the parental investment and hours of self-study are significantly lower and it is more difficult to increase a test score with the same amount of monetary or time investments.

Hours of self-study have stronger average marginal effects on the subsequent test score than hours of tutoring. The marginal effects are computed by the partial derivative of test score $q_{i,t+1}$ with respect to either hours of tutoring (x_{it}) or hours of self-study (s_{it}). I present the average marginal effects, which could vary by households because the calculation of marginal effects involves the data of p_{it}, x_{it}, and s_{it}. Figure

9 presents the comparison of the average marginal effects of hours of self-study and hours of tutoring. In almost all periods, hours of self-study have greater marginal effects than hours of tutoring. Only in the final period, the average marginal effect of hours of tutoring is slightly larger than the average marginal effects of hours of self-study. However, as seen in Figure 9, the difference of the marginal effects in the final period is neither statistically nor economically significant.

Table 8: Parameter Estimates: Test score production function

Time-varying Parameters	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
Previous	0.163	0.980	0.754	0.731	0.474	0.426
Test Score (δ_{qt})	(0.001)	(0.002)	(0.001)	(0.001)	(0.009)	(0.001)
Effort Parameters (ν_t)	0.556	0.659	0.420	0.360	0.150	0.020
	(0.001)	(0.014)	(0.009)	(0.001)	(0.002)	(0.001)
Share of tutoring	0.472	0.485	0.480	0.526	0.592	0.737
Expenditure (δ_{et})	(0.007)	(0.005)	(0.011)	(0.011)	(0.012)	(0.015)
Constants (δ_{0t})	4.030	-1.124	0.627	1.205	1.022	4.077
	(0.006)	(0.001)	(0.001)	(0.001)	(0.001)	(0.004)
Time Invariant Parameters						
Substitution Parameter (ϕ)	0.880					
	(0.001)					
Intensity of Private tutoring	0.145					
Quality (κ)	(0.002)					

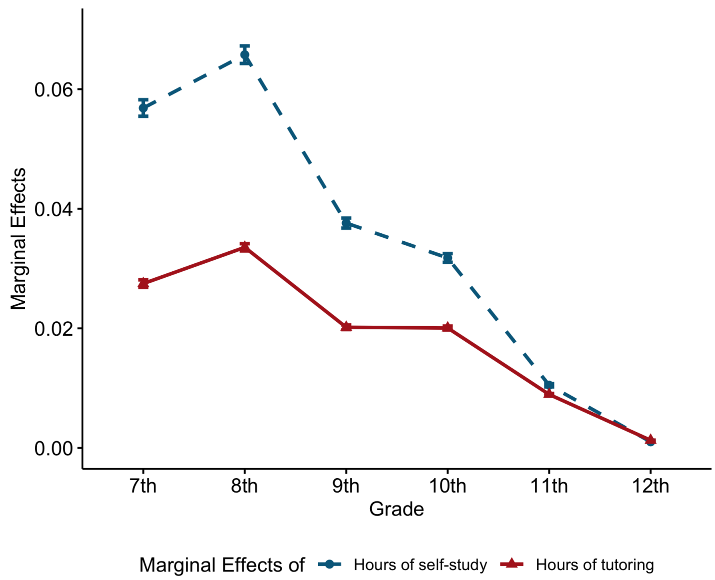
Note: Standard errors are in parentheses below estimates. Based on the CES test score function, share of hours of self-study is implied by share of tutoring expenditure. (i.e., $\delta_{st} = 1 - \delta_{et}$).

The early period investments have delayed effects through evolving test scores. As seen in Figure 9, the marginal effect of hours of self-study are already stronger in the earlier periods. Considering the delayed effects of hours of self-study through the evolving test scores, hours of self-study have a strong effect on the final test score.

The estimate of the substitution parameter shows that the impact of parental investments could be exaggerated in simulating the model if the child's self-study is not incorporated into the mechanism. The hours of self-study and hours of tutoring are nearly perfect substitutes for each other based on the structural estimate. Table 8 includes the estimate of the substitution parameter ϕ, which is approximately 0.88.

Suppose a researcher wants to conduct a counterfactual experiment of restricting parental investment using a structural model without other options of investing in the child. Each household must accept the restriction as given. Such an omission of mechanism might result in an exaggeration of the effects of parental investment on outcomes such as intergenerational persistence of earnings. This substitutability plays an important role because the channel of hours of self-study provides a household with a restrictive income constraint an opportunity to exert efforts in the tournament.

Figure 9: Average Marginal Effects of Hours Allocation



Note: This figure presents the average of marginal effects of hours of self-study and hours of tutoring over time. Due to the functional form of the test score function, the marginal effects differ by each individual. The marginal effects are computed using the first order derivative with respect to hours of self-study (s_{it}) or hours of tutoring (x_{it}) and the estimated parameters. The vertical interval at each point indicates the standard deviation of the marginal effects.

7.2 Preference and shock parameters

Table 9 presents the estimates of the preference parameters and the shock parameters. The preference parameters are components of equations (5), (9), and (10). For the preference parameters, the estimates are relative estimates of the other preference parameters. The altruism parameter is estimated as 1.018. To capture the observed heterogeneity of the household, I allow the preference parameters to vary by parental education. In particular, $\exp(\tau_x D_i^{pedu})$ is multiplied to the disutility from hours of tutoring α_x and $\exp(\tau_s D_i^{pedu})$ is multiplied to the disutility from hours of self-study, where D_i^{pedu} is 1 for household whose average years of parental education is strictly greater than 12. Table 9 (a) includes the estimates of the effects of parental education on the preference parameters. Based on the estimates, parental education alleviates the disutility to hours of self-study. Specifically, a child of a household whose average education of parents is greater than 12 years feels more disutility of study by 0.005. In contrast, the effect of parental education on mitigating disutility from hours of tutoring is not statisticially different from 0.

Table 9: Parameter Estimates: Preference and Shock Parameters

		Estimate	Standard error
Preference Parameters			
Taste for consumption	α_c	0.028	(0.000)
Altruism for the child's future	α_ν	1.018	(0.001)
Disutility from hours of tutoring	α_x	-0.006	(0.000)
Disutility from hours of self-study	α_s	-0.005	(0.006)
Parental education parameters			
disutility from hours of tutoring	τ_x	-0.001	(0.002)
disutility from hours of self-study	τ_s	-0.005	(0.002)

(a) Preference parameters

Standard Deviation of		Estimate	Standard Error
Test score shock	$\sigma_{\eta q}$	0.230	(0.000)
Consumption shock	$\sigma_{\eta c}$	0.742	(0.014)
Study shock	$\sigma_{\eta s}$	0.549	(0.001)
Tutoring shock	$\sigma_{\eta l}$	0.472	(0.005)

(b) Shock parameters

Note: Standard errors are in parentheses. $\frac{1}{N\times T\times 6}(\sum logL_i - Jacob) = -0.848$.

The estimated standard deviations of unobserved shocks are overall modest, which suggests that the observed characteristics and the structural model capture a considerable proportion of heterogeneity in the data. The unobserved heterogeneity from consumption is considerably different among the different types of households. This could be due to the fact that I do not model the supply side of private tutoring, and the price differences across regions are not captured by the deterministic parts of the model.

7.3 Model Fit

To examine the goodness-of-fit of the structural model, I use a local linear regression estimator to see how well the model prediction $\widehat{y}$ fits the actual data value y, for dependent variables $y = p, x, s, q$. Specifically, the expected value of data y conditional on the model predicted value $\widehat{y}$ is $E(y|\widehat{y}) = \widehat{\kappa}_0(\widehat{y})$, where

$$\begin{pmatrix} \widehat{\kappa}_0(y) \\ \widehat{\kappa}_1(y) \end{pmatrix} = \sum_i \left[K(\frac{y_i - \widehat{y}_i}{b}) \begin{pmatrix} 1 \\ y_i - \widehat{y}_i \end{pmatrix} \begin{pmatrix} 1 & y_i - \widehat{y}_i \end{pmatrix} \right]^{-1} \cdot \left[K(\frac{y_i - \widehat{y}_i}{b}) \begin{pmatrix} 1 \\ y_i - \widehat{y}_i \end{pmatrix} y_i \right],$$

and

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp(-0.5x^2)$$

is the kernel function with bandwidth b. The farther the kernel curve deviates from the 45 degree line, the less the model is successful in fitting the data.

Figures E.1 to E.5 present the sample fit of quality of tutoring, hours of tutoring, hours of self-study, and test scores, respectively. Overall, the fits are very good. While the quality of tutoring and the hours of self-study show excellent fits, hours of tutoring are somewhat a little bit overpredicted. The level of the final test score, as depicted in Figure E.5b, is somewhat overpredicted as well. This is due to the fact that the constants of the test scores are part of the tournament model as specified in the tournament component of equation (10). Thus, the constant of the test scores cannot be separated from the constants of all the other dependent variables' likelihood contributions. Nevertheless, the model fits the distribution of the test scores very well, as shown in Figure E.5a. As the tournament model is about the ranking of the final test score, the distributions of the test scores are the major concern in simulating the model, which is captured by the tournament model.

8. Counterfactual Analyses

8.1 Quantification and Decomposition of Intergenerational Persistence of Earnings

The purpose of the quantification is to decompose the role of channels affecting intergenerational persistence of earnings. Using the structural estimates, I simulate the model under the counterfactual environments that help quantify the role of the relevant channels. Each simulation produces a different distribution of the final test scores, which leads to a different distribution of the predicted income of the child. I define the predicted income of the child of household i as $childinc_i$. Two measures of the intergenerational persistence of earnings are intergenerational elasticity of earnings (IGE) and the rank-rank slope. In particular, the measure of IGE is the coefficient of the regression equation,

$$\ln childinc_i = \delta_{00} + \delta_{IGE} \ln hhinc_i + \varepsilon_i \tag{12}$$

where $hhinc_i$ is the average of household i's income over the periods. The estimate of IGE is $\widehat{\delta}_{IGE}$, and (1-$\widehat{\delta}_{IGE}$) is the measure of intergenerational mobility (Black and Devereux 2010). On the other hand, the rank-rank slope (Chetty *et al.* 2014) is the estimate of the regression equation,

$$R_i = \delta_{01} + \delta_{RR} P_i + \upsilon_i \tag{13}$$

where R_i is the percentile rank of the child income within the generation, and P_i is the rank of the parental income within the generation. Although I present both IGE and the rank-rank slope for each counterfactual simulation, the preferred estimate of intergenerational persistence of earnings is the rank-rank slope. The IGE is sensitive to the ratio of the income inequalities of the two generations.[46] To minimize this issue, the discussion is based on the results of estimates of the rank-rank slope, which is more robust to the difference in the income variance across the generations.

Table 10 provides definitions of the counterfactual simulations. In particular,

[46] In particular, $\delta_{IGE} = \rho_{ch} \frac{\sigma_{\ln childinc}}{\sigma_{\ln hhinc}}$, where σ_x is a standard deviation of data x and ρ_{ch} is a correlation between $\ln childinc_i$ and $\ln hhinc_i$.

- BCF is the baseline of the counterfactuals where the model is simulated without a counterfactual modification
- OPI is the counterfactual where only parental investment is the means of the tournament model, and hours of self-study are excluded from the choice of the household and fixed to 0.
- OSS is the counterfactual where only child's self-study is the means of the tournament, and parental investment is excluded from the choice and fixed to 0.
- NIN is the counterfactual where all monthly net household income is fixed to 4,000,000 KRW (approximately 2800 USD).

Table 10: Definitions of Counterfactual Simulations

	Household Inc	6th grade Score	Parental Educ	Parental Investments	Child's Self-Study
Benchmark Counterfactual (BCF)	O	O	O	O	O
Parental Investments (OPI)	O	O	O	O	X
Self-Study (OSS)	O	O	O	X	O
Without 6th grade Test (NST)	O	X	O	O	O
Without Household Inc (NIN)	X	O	O	O	O

Note: For each simulation definition in the first column, the channel is either allowed (O) or shut down (X). For example, in OPI, all channels of intergenerational transmission are allowed except for hours of self-study.

Table 11 presents the estimates of the rank-rank slope and the intergenerational elasticity of earnings under five different simulations. The estimated rank-rank slope for the benchmark counterfactual (BCF) is 0.635.[47] The OSS simulation is interesting on its own because it provides implications for the tutoring ban policy of China.

[47]The estimated IGE with the benchmark model (BCF) is 0.269.

Table 11: Intergenerational Persistence of Earnings under the Counterfactual Simulations

(a) Rank-rank Slope Estimates

	(1) BCF	(2) OPI	(3) OSS	(4) NIN
pincprctile	0.635***	0.827***	0.130***	0.341***
	(0.018)	(0.013)	(0.023)	(0.022)
R-squared	0.403	0.684	0.017	0.116

(b) Intergenerational Elasticity of Earnings Estimates

	(1) BCF	(2) OPI	(3) OSS	(4) NIN
log(hhinc)	0.269***	0.313***	0.068***	0.173***
	(0.008)	(0.005)	(0.009)	(0.011)
R-squared	0.398	0.708	0.031	0.119

Note: Table (a) and (b) provide the estimates of equation (13) and (12), respectively. BCF is a benchmark counterfactual. NIN is a simulation where household income is fixed to the mean. OPI is a simulation where household can only use parental investment. OSS is a simulation where household can only use hours of self-study.

The quantification exercise highlights several findings. First, removing heterogeneity in parental income decreases the rank-rank slope by 46.2%, which can be found in the result of the NIN simulation in Column (5) in Table 11a. The result suggests that heterogeneity in parental income is responsible for a substantial part of the intergenerational persistence of earnings. Second, omitting hours of self-study leads to significant increases in the intergenerational persistence of earnings. The estimated rank-rank slope under the OPI simulation is 0.827, which is greater than the benchmark simulation by 30.2%, as shown in Column (2) in Table 11a. At the same time, when the channel of parental investment is removed, the rank-rank slope decreases by 79.5%, as shown in Column (3) in Table 11a. Such results suggest that while parental investment reinforces the intergeneration persistence of earnings, the self-study of the child mitigates it. Third, the effect of heterogeneity in the academic performance in primary school on the intergenerational persistence of earnings is modest. To control for the difference among students before 7th grade, I run the counterfactual simulations with fixing the academic performance in primary school and parental education, which can be found in Table 12. For example, BCF' is the same simulation as BCF except that 6th-grade academic performance and parental education are fixed across households. The results are consistent with the original counterfactual simulations that are conducted without fixing the household characteristics.

Table 12: Intergenerational Persistence of Earnings Fixing Initial Conditions

(a) Rank-rank Slope Estimates

	(1) BCF'	(2) OPI	(3) OSS'
pincprctile	0.641*** (0.018)	0.834*** (0.013)	0.323*** (0.022)
R-squared	0.411	0.695	0.104

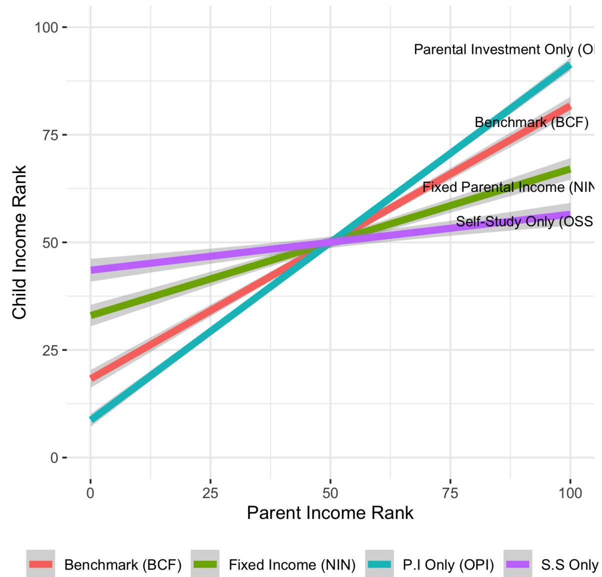
(b) Intergenerational Elasticity of Earnings Estimates

	(1) BCF'	(2) OPI'	(3) OSS'
logpinc	0.393*** (0.011)	0.312*** (0.005)	0.100*** (0.007)
R-squared	0.401	0.719	0.097

Note: Table (a) and (b) provide the estimates of equation (13) and (12), respectively. In order to assess the importance of initial conditions, all simulations are conducted fixing the initial test score and parental education. Parental income is not fixed except for the NIN' simualtion. BCF' is the benchmark counterfactual; NIN' is a simulation where each household income is fixed to the mean; OPI' is a simulation where each household can use only parental investment; and OSS' is a simulation where each household can use only hours of self-study. The linearity of the lines follows from the linearity of the rank-rank equation.

Based on the simulation results of Section 8.1, which can be found in Table 11a, the private tutoring ban policy would decrease intergenerational persistence of earnings. In other words, such a policy would increase intergenerational mobility. However, the increase in mobility would come with the expense of an increase in consumption inequality.

Figure 10: Intergenerational Persistence of Earnings by Scenarios

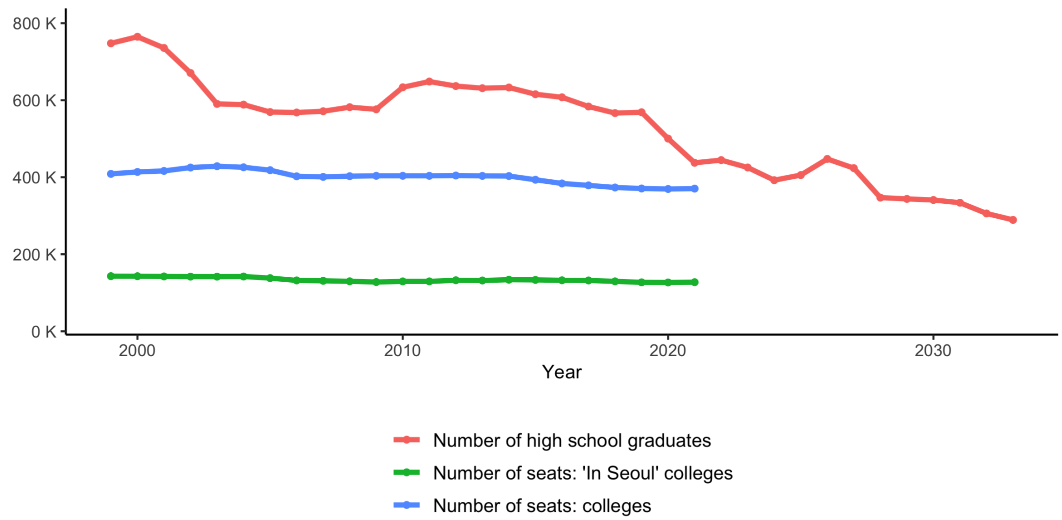


Note: This graph is a visual representation of the rank-rank slope estimates described in equation (13). Parent Income Rank is the average income over six years. Child Income Rank is computed based on the simulated results of each scenario. BCF is the benchmark counterfactual; NIN is a simulation where each household income is fixed to the mean; OPI is a simulation where each household can use only parental investment; and OSS is a simulation where each household can use only hours of self-study. The linearity of the lines follows from the linearity of the rank-rank equation.

8.2 Parental Investment Demand in Shrinking Cohorts

Exploiting the number of competitors in the tournament model, I simulate the demand of parental investment amidst a sharp cohort size reduction. Countries with a high amount of average parental monetary investments tend to experience extremely low fertility rates. South Korea, China, Turkey, Singapore, and Taiwan are countries where the demand for private tutoring is high (Bray 1999, 2021), and they are experiencing a drastic reduction in the size of the cohort as can be seen in Figure 1. The reduction in the cohort size is equivalent to the reduction in the number of competitors in the college admission tournament. If there is a radical reduction in the number of competitors, the degree of competition would be less fiercer given that the number of seats for colleges do not change.

Figure 11: Number of seats in colleges



Source: (1) Number of students by year, (2) number of graduates by year, and (3) number of entance quota in college all by Korean Educational Development Institute
Note: The number of high school graduates after 2022 is projected by the number of lower graders assuming that drop out rates do not change.

Colleges have not decreased the number of seats amid the demographic shift. Figure 11 presents the number of high school graduates up to 2033, the number of seats of colleges in Korea, and colleges in Seoul up to 2022. The number of high school graduates after 2022 is projected using the average dropout rates and the number of graduates from younger cohorts in 2022. The number of projected high school graduates in 2033 is 289,216, which is only 44.5% of the number of high school graduates in 2011. Meanwhile, colleges do not adjust the number of seats. Cohort size has been decreasing since around 2000, but colleges have not changed the number of seats, as can be seen in Figure 11. Colleges in Seoul, which is the equivalent of Tier 1 to Tier 3 colleges of the earlier classification, do not decrease the number of seats either.

The simulation is conducted under various scenarios. The three key exogenous factors that characterize each simulation are: (i) the number of cohort sizes, (ii) the exit of existing colleges, and (iii) changes in the premium of upper college tiers. The first factor can be plausibly projected using fertility rates and dropout rates. As it is empirically challenging to pin down the second and third factors, I evaluate the impact of each factor under the most plausible scenarios.

Simulation I: Cohort Size Impact on Parental Investment Motivated by the cohort reduction in South Korea, I simulate changes in the amount of parental investment when the size of the cohort decreases by 50%. The effects of the cohort changes are reflected through the increased seats of college tiers. As there are half of the competitors relative to the unchanged number of college seats, it is equivalent to the number of seats for each tier doubling. For this simulation, I assume that the inequality of the college qualities remain same.

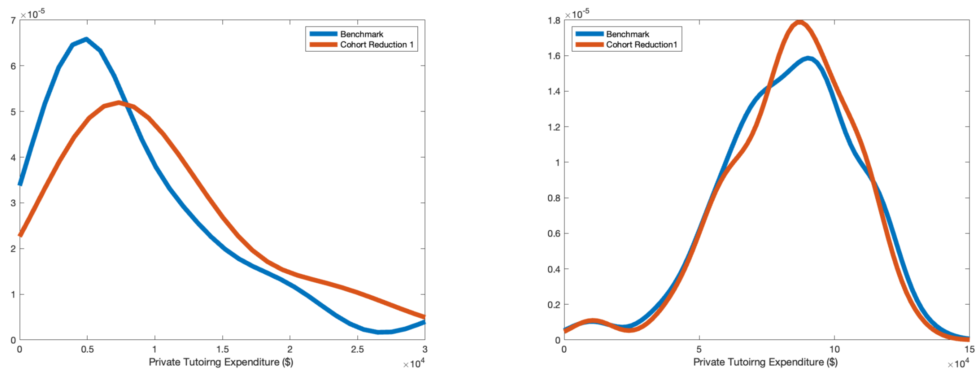
According to the simulation results, the amount of private tutoring expenditure slightly increases assuming the disparities in college quality do not change. Figures 12 presents the density of private tutoring expenditures of low income households (lowest 5%) and high income households (highest 5%) respectively. High income households increase their total spending by 0.05% while low income households increase their spending by 18.04%. The increase of parental investment might be driven by the assumption that college inequality does not change over time. As a result of cohort reduction, students have a higher probability of going to a better college tier with less fiercer competition. In turn, people who previously had no chance of getting into an upper college tiers would have better chance. For low-income households, fewer competitor leads to an increase in the probability of going to higher tier college. Low income households have a higher marginal values from investing in the child, which makes them spend more.

Simulation II: Low-Tier Colleges Exit (TBU)

Simulation III: Reduced College Inequality To evaluate the effects of college inequality on parental investment, I change the values of $\{v_j\}_{j=1}^{J}$, the earning prospects of alumni for each tier. The relative size of v_1,v_2,v_3 and v_4 relative to v_5 are 2.22, 1.95, 1.45, 1.25 respectively. In this simulation, I change them to 1.5, 1.4, 1.3, and 1.2 respectively. I set the percentiles that students need to make for each tier as the same with the benchmark simulation. Figure 13 presents the resulting changes on parental investment for the households. The decrease in college inequality leads to a substantial decrease in private tutoring expenditure of the entire households. Table 13 presents the quantification of the changes in private tutoring expenditure. I report the changes amount of total private tutoring expenditure compared to the benchmark simulation where I normalize private tutoring expenditure as 100. It can be seen that the changes in college hierarchy results in a significant decrease in total private tutoring expenditure.

Figure 12: Cohort Simulation

(a) Low Income Households (b) High Income Households



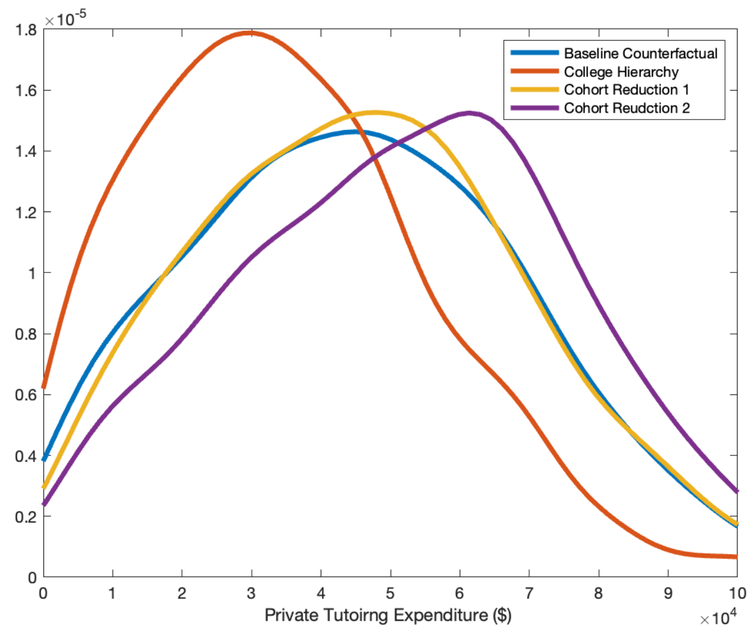
Note: Figure 13 displays simulated private tutoring expenditure for low income households (lowest 5%) for each counterfactual scenario. Cohort Reduction 1 simulation is based on the scenario where the cohort size decrease by 50%.

Table 13: Changes in Simulated Private Tutoring Expenditure

	Benchmark	Cohort Reduction 1	Cohort Reduction 2	College Hierarchy
Entire HH	100	101.57	114.06	76.13
Low Income HH	100	114.38	137.44	68.63
High Income HH	100	101.07	102.77	83.98

Note: Table 12 presents the average simulated private tutoring expenditure for each counterfactual scenario. To emphasize the change, expenditure of the benchmark case is standardized to 100. Cohort Reduction 1 simulation is based on the scenario that the cohort size decreases by 50%. Cohort Reduction 2 simulation is based on the scenario that the cohort size decrease by 50% and the number of seats for elite college increases. College Hierarchy simulation is based on the scenario that the relative size of v_1,v_2,v_3 and v_4 relative to v_5 are decreased to 1.5, 1.4, 1.3, and 1.2 respectively. The number of seats are the same as the benchmark counterfactual in the College Hierarchy simulation.

Figure 13: College Inequality Simulation



Note: Figure presents the average simulated private tutoring expenditure for each counterfactual scenario. To emphasize the change, expenditure of the benchmark case is standardized to 100. Cohort Reduction 1 simulation is based on the scenario that the cohort size decreases by 50%. Cohort Reduction 2 simulation is based on the scenario that the cohort size decrease by 50% and the number of seats for elite college increases. College Hierarchy simulation is based on the scenario that the relative size of v_1,v_2,v_3 and v_4 relative to v_5 are decreased to 1.5, 1.4, 1.3, and 1.2 respectively. The number of seats are the same as the benchmark counterfactual in the College Hierarchy simulation.

9. Conclusion

College admission is a pivotal event in terms of lifetime income. A college one graduates from significantly affects lifetime income controlling for the results of the college entrance exam. Each household competes for the limited seats in prestigious colleges with excellent lifetime income prospects. Such competition for prestigious colleges is a channel where the income of parents translates into the future disparity in a child's income. Parental investment and a child's self-efforts are two important means of the household in this college admission competition.

The first goal of this paper is to quantify the role of parental investment in intergenerational persistence of earnings. While parental investment is highly responsive to parental income, self-efforts of the child is not as responsive to parental income as parental investment. Such an empirical relationship suggests the potential importance of incorporating self-efforts of the child into the mechanism. I develop a dynamic tournament model of college admission in which each household uses both private tutoring expenditure and hours of self-study by the child. I estimate the structural model using Maximum Simulated Likelihood. Using the estimated model, I quantify the role of private tutoring expenditure, hours of self-study, and other household characteristics. I find that heterogeneity in parental income in adolescent periods accounts for 46% of intergenerational persistence of earnings. Parental investment is responsible for a substantial portion of intergenerational persistence of earnings controlling for the child's efforts. Ignoring child's efforts from the mechanism leads to a significant increase in intergenerational persistence of earnings by 30%, which suggests the role of the self-effort as a mitigator of intergenerational persistence of earnings.

Finally, in light of the recent fertility declines in developed countries, I assess the impact of the rapidly shrinking cohort size on parental investment. Based on the model projection, low income households spend money on private tutoring expenditure as cohort size decreases, while there is virtually no change in the parental investment spending of high income households. Additionally, I evaluate the impact of college inequality on parental investment. A counterfactual simulation suggests that a decrease in inequality in college income prospects leads to a significant decrease in parental investment.

The findings of this paper suggest two avenues for future research. First, this paper does not allow the possibility of wealth transmission within the household. As Becker and Tomes (1979) suggest, the transmission of capital can be an alternative way of inheriting the income of the parents, especially when the child does not perform well academically. Incorporating the channel of capital transmission within a family requires at least decent data on wealth for more than one generation, which is not an easy data requirement. Second, this paper does not allow for unobserved heterogeneity of labor income conditional on college quality, mainly due to the data limitations. An efficiency analysis on the rat-race nature of the college admission competition would be feasible with the addition of the channel. I leave this for future research.

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Appendix A

Appendix A.1

Proof of Lemma 2: Compactness

Proof. A value function is the sum of flow utility and the discounted future value. The flow utility term $u(c_{it}, x_{it}, s_{it}, \varepsilon_{it})$ is monotone in its arguments. Also, u is defined at the lower and upper bounds of c_{it}, x_{it}, s_{it}. Thus, $u(c_{it}, x_{it}, s_{it}, \varepsilon_{it})$ is closed and bounded. The expected future value EV_{t+1} is closed and bounded. For the final period, the tournament term described in equation (10) is closed and bounded because (i) the v_j term is finite and greater than 0, and (ii) $Prob(\ln \tilde{Q}_{j-1} \geq \ln q_{i,T+1} \geq \ln \tilde{Q}_j \Big| \Gamma_{iT}) \in [0, 1]$. Therefore, the choice-specific value function of the final period, $V_{it}(Z_{it}, \Psi_{it})$ is closed and bounded for $t = T$. Following the backward recursion, $V_{it}(Z_{it}, \Psi_{it})$ is closed and bounded. □

Appendix A.2

Proof of Lemma 3: Continuity

Proof. I start by showing that the value function V_{it} is continuous. To show V_{it} is continuous, It suffices to show that both $u(c_{it}, x_{it}, s_{it}, \varepsilon_{it})$ and $\int_\eta V_{t+1}(Z_{t+1}, \mathit{\Psi}_{t+1}) f(\eta) d\eta$ are continuous. □

- Start from the final period and show that the final term is continous: Bounded right hand side. Left hand side is continous in its arguemnt. Use Dominated Convergence Theorem.
- Previous period same
- Then move on to the continuity of the mapping

Proof. One way to show the continuity of the expected value function is show that it is sequentially continuous. For any sequence of the arguments of the value function,

$$\{Z_t^n, \mathit{\Psi}_t^n\} \to \{Z_t^0, \mathit{\Psi}_t^0\},$$

we have

$$\int_{\Psi} V_{t+1}(Z^n_{t+1}, \Psi^n_{t+1}) d\Psi \to \int_{\Psi} V_{t+1}(Z^0_{t+1}, \Psi^0_{t+1}) d\Psi.$$

Recall that $\Psi_{t+1} = \{\eta^c_{i,t+1}, \eta^x_{i,t+1}, \eta^s_{i,t+1}, \eta^q_{it}\}$. As the expectation of the unobserved shocks has finite expectation, the expected value term has finite expectation as well. $\int_{\Psi} V_{t+1}(Z^0_{t+1}, \Psi^0_{t+1}) d\Psi$ is continuous by the Dominated Convergence. Each supmapping is continuous as its elements are continuous. As each submapping is continuous. By induction, the composition of mapping is continuous. Therefore, $\aleph$ is continuous. $\square$

Appendix B

	List of the member colleges
First Tier	Seoul National, Yonsei, Korea , Sogang, SKKU, Hanyang, KAIST, Pusan, Ewha, Postec
Second Tier	Choongang, Kyunghee, HUFS, University of Seoul, KU, Dongguk, Kyongpook, Sookmyung, Ajou, Honggik, Inha, Hangkong, Kookmin, Soongsil, Sejong, Dankook, Kwangwoon, Cheonnam, Seoul Industrial University
Third Tier	Myongji, Sangmyeong, Catholic, Choongam, Choongbook, Seongshin, Kyeongki Kyongwon, Deoksong women, Dongdeok women, Dong-A, Bookyeong
Fourth Tier	The rest of the 2 year colleges
Fifth Tier	High school graduates

Appendix C

Define the first order conditions as

$$\begin{aligned}
V_p &= \alpha_c \varepsilon^c_{it} u^c_p(c_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \frac{\partial \ln q_{i,t+1}}{\partial p_{it}} \\
V_x &= \alpha_c \varepsilon^c_{it} u^c_x(c_{it}) + \alpha_x \varepsilon^x_{it} u^x_x(x_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \frac{\partial \ln q_{i,t+1}}{\partial x_{it}} \\
V_s &= \alpha_s \varepsilon^s_{it} u^s_s(s_{it}) + \beta \left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) \right] \frac{\partial \ln q_{i,t+1}}{\partial s_{it}}
\end{aligned}$$

$$\frac{\partial p}{\partial w} = -\frac{\frac{\partial V_p}{\partial w}}{\frac{\partial V_p}{\partial p}}$$

$$\frac{\partial V_p}{\partial w} = \alpha_c \varepsilon_{it}^c \frac{x_{it}}{(w_{it} - p_{it}x_{it})^2}$$

$$\begin{aligned}
\frac{\partial V_p}{\partial p} =& \frac{\partial}{\partial p}\varepsilon_{it}^c \frac{-x_{it}}{(w_{it} - p_{it}x_{it})} + \beta \frac{\partial}{\partial p}\left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot)\right]\left(\nu_t \frac{\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1+s_{it})^{\phi}]}(\kappa {p_{it}}^{\kappa-1}{x_{it}}^{1-\kappa})\right) \\
=& -2\alpha_c\varepsilon_{it}^c \frac{x_{it}^3}{(w_{it}-p_{it}x_{it})^3} + \beta\left[\frac{\partial^2}{\partial^2 \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot)\right]\left(\nu_t \frac{\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1+s_{it})^{\phi}]}(\kappa {p_{it}}^{\kappa-1}{x_{it}}^{1-\kappa})\right)^2 \\
&+ \beta\left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot)\right]\nu_t\Bigg(- \frac{\delta_{2t}^2\kappa^2 p_{it}^{(2\kappa-2)}\phi x_{it}^{(2-2\kappa)}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{2\phi-2}}{[\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1+s_{it})^{\phi}]^2}(\kappa {p_{it}}^{\kappa-1}{x_{it}}^{1-\kappa}) \\
&+ \frac{\delta_{2t}\kappa^2 p_{it}^{(2\kappa-2)}(\phi-1)x^{(2-2\kappa)}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-2}}{[\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1+s_{it})^{\phi}]} \\
&+ \frac{\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{2t}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi} + \delta_{3t}(1+s_{it})^{\phi}]}((\kappa-1)\kappa p_{it}^{\kappa-2}x_{it}^{1-\kappa})\Bigg)
\end{aligned}$$

As $\phi < 1$, $\kappa < 0.5$, and $\frac{\partial^2}{\partial^2 \ln q_{i,t+1}} EV_{i,t+1}(\cdot,\cdot,\cdot) < 0$, $\frac{\partial V_p}{\partial p} < 0$ and $\frac{\partial V_p}{\partial w} > 0$, $\frac{\partial p}{\partial w} > 0$.

Appendix D

First-order conditions used for likelihood contribution

The goal of this section is to get a closed form expression of the shocks, which are the building blocks of the likelihood function. I denote $u_p^c(c_{it})$ and $u_x^c(c_{it})$ as the first order derivatives of $u^c(c_{it})$ with respect to x_{it} and p_{it} respectively, and $u_x^l(l_{it})$ and $u_s^l(l_{it})$ as the first order derivatives with respect to x_{it} and s_{it} respectively. The first order conditions of the value function in equation (9) are

$$\begin{aligned}
\frac{\partial}{\partial p_{it}} &: \alpha_c \exp(\eta_{it}^c + \lambda_k^c) + \beta \frac{1}{u_p^c(c_{it})}\left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1})\right]\frac{\partial \ln q_{i,t+1}}{\partial p_{it}} = 0; \\
\frac{\partial}{\partial x_{it}} &: \alpha_c \exp(\eta_{it}^c + \lambda_k^c)u_x^c(c_{it}) + \alpha_x \exp(\eta_{it}^k + \lambda_k^c)u_x^x(x_{it}) + \beta\left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1})\right]\frac{\partial \ln q_{i,t+1}}{\partial x_{it}} = 0; \\
\frac{\partial}{\partial s_{it}} &: \exp(\eta_{it}^s + \lambda_k^s)u_s^s(s_{it}) + \beta\left[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it}), \Psi_{i,t+1})\right]\frac{\partial \ln q_{i,t+1}}{\partial s_{it}} = 0.
\end{aligned}$$

With the functional form assumptions of log utility,

$$
\begin{aligned}
u_x^c(c_{it}) =& -\frac{p_{it}}{w_{it}-p_{it}x_{it}};\\
u_p^c(c_{it}) =& -\frac{x_{it}}{w_{it}-p_{it}x_{it}};\\
u_s^s(s_{it}) =& \frac{1}{1+s_{it}};\\
u_x^x(x_{it}) =& \frac{1}{1+x_{it}}
\end{aligned}
$$

And with the functional form of the test score function,

$$
q_{i,t+1} = A_{it}q_{it}^{\delta_{1t}}\left[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}\right]^{\frac{\nu_t}{\phi}}\exp(\lambda_k^q+\eta_{it}^q)
$$
$$
\ln q_{i,t+1} = \ln A_{it} + \delta_{1t}\ln q_{it} + \frac{\nu}{\phi}\ln[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}] + \lambda_k^q+\eta_{it}^q
$$

$$
\begin{aligned}
\frac{\partial \ln q_{i,t+1}}{\partial p_{it}} =& \nu_t\frac{\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}]}(\kappa{p_{it}}^{\kappa-1}{x_{it}}^{1-\kappa});\\
\frac{\partial \ln q_{i,t+1}}{\partial x_{it}} =& \nu_t\frac{\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}]}((1-\kappa){p_{it}}^{\kappa}{x_{it}}^{-\kappa});\\
\frac{\partial \ln q_{i,t+1}}{\partial s_{it}} =& \nu_t\frac{\delta_{st}(1+s_{it})^{\phi-1}}{[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}]}.
\end{aligned}
$$

The first order conditions with respect to p_{it} is characterized as

$$
\begin{aligned}
&\alpha_c\exp(\eta_{it}^c+\lambda_k^c) - \beta\frac{w_{it}-p_{it}x_{it}}{x_{it}}\left[\frac{\partial}{\partial \ln q_{i,t+1}}EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it},x_{it},s_{it})),\Psi_{i,t+1})\right]\\
&\times\nu_t\frac{\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}]}\times(\kappa{p_{it}}^{\kappa-1}{x_{it}}^{1-\kappa}) = 0. \qquad (14)
\end{aligned}
$$

The first order conditions with respect to x_{it} is characterized as

$$
\begin{aligned}
&-\alpha_c\exp(\eta_{it}^c+\lambda_k^c)\frac{p_{it}}{w_{it}-p_{it}x_{it}} + \alpha_x\exp(\eta_{it}^x+\lambda_k^x)\frac{1}{1+x_{it}} + \beta\left[\frac{\partial}{\partial \ln q_{i,t+1}}EV_{i,t+1}(\ln q_{i,t+1}(p_{it},x_{it},s_{it}),\Psi_{i,t+1})\right]\\
&\times\nu_t\frac{\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi-1}}{[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi}+\delta_{st}(1+s_{it})^{\phi}]}\times(1-\kappa){p_{it}}^{\kappa}{x_{it}}^{-\kappa} = 0. \qquad (15)
\end{aligned}
$$

The first order conditions with respect to s_{it} is characterized as

$$\alpha_s \exp(\eta_{it}^s + \lambda_k^s)\frac{1}{1+s_{it}} + \beta\Big[\frac{\partial}{\partial \ln q_{i,t+1}} EV_{i,t+1}(Z_{i,t+1}(\ln q_{i,t+1}(p_{it}, x_{it}, s_{it})), \Psi_{i,t+1})\Big]$$
$$\times \nu_t \frac{\delta_{st}(1+s_{it})^{\phi-1}}{[\delta_{et}(1+{p_{it}}^{\kappa}{x_{it}}^{1-\kappa})^{\phi} + \delta_{st}(1+s_{it})^{\phi}]} = 0. \tag{16}$$

This difference between the previous period and the final period can be confusing. For the final period,

$$EV_{i,T+1} = v_1 - \sum_{j=1}^{J}\Big(\ln(v_j) - \ln(v_{j+1})\Big)\Phi(\frac{\ln \bar{q}_j - \ln q_{iT+1} - \lambda_i^q}{\sigma_q});$$
$$\frac{\partial}{\partial \ln q_{i,T+1}} EV_{i,T+1}(\cdot,\cdot,\cdot) = \sum_{j=1}^{J}\Big(\ln(v_j) - \ln(v_{j+1})\Big)\frac{1}{\sigma_q}\phi(\frac{\ln \bar{q}_j - \ln q_{iT+1} - \lambda_i^q}{\sigma_q}),$$

while for $t < T$, EV_{it} is an interpolated value function.

Computation of Likelihood Contribution

(Case 1) $(x_{it} > 0$ and $s_{it} > 0)$

I define $\tilde{\eta}_{it}^z$ for $z = c, x, s$ as the particular realization of η_{it}^z that satisfies the first order conditions. The likelihood contribution for all-positive case is

$$\begin{aligned}
f(p_{it}, x_{it}, s_{it}, q_{it}) =& f(\tilde{\eta}_{it}^c, \tilde{\eta}_{it}^x, \tilde{\eta}_{it}^s, \tilde{\eta}_{it}^q) \cdot |det\left(\frac{\partial(\widetilde{\eta}_{it}^c, \widetilde{\eta}_{it}^x, \widetilde{\eta}_{it}^s, \widetilde{\eta}_{it}^q)}{\partial(p_{it}, x_{it,} s_{it}, q_{it})}\right)| \\
=& \phi(\tilde{\eta}_{it}^c, \tilde{\eta}_{it}^x, \tilde{\eta}_{it}^s, \tilde{\eta}_{it}^q) \cdot |det\left(\frac{\partial(\widetilde{\eta}_{it}^c, \widetilde{\eta}_{it}^x, \widetilde{\eta}_{it}^s, \widetilde{\eta}_{it}^q)}{\partial(p_{it}, x_{it,} s_{it}, q_{it})}\right)| \\
=& (2\pi)^{-4/2}|det(\Omega)|^{-1/2}\exp\Big[-0.5\underbrace{\begin{pmatrix}\widetilde{\eta}_{it}^c\\ \widetilde{\eta}_{it}^x\\ \widetilde{\eta}_{it}^s\\ \widetilde{\eta}_{it}^q\end{pmatrix}'}_{1\times 4}\underbrace{\Omega^{-1}}_{4\times 4}\underbrace{\begin{pmatrix}\widetilde{\eta}_{it}^c\\ \widetilde{\eta}_{it}^x\\ \widetilde{\eta}_{it}^s\\ \widetilde{\eta}_{it}^q\end{pmatrix}}_{4\times 1}]\Big|det\left(\frac{\partial(\widetilde{\eta}_{it}^c, \widetilde{\eta}_{it}^x, \widetilde{\eta}_{it}^s, \widetilde{\eta}_{it}^q)}{\partial(p_{it}, x_{it}, s_{it}, q_{it})}\right)\Big|
\end{aligned}$$

(Case 2) ($x_{it} > 0$ and $s_{it} = 0$)

This is the case where household participate in tutoring, but have zero hours of self-study. First, I define the joint probability of such case, and separate the density of η^c_{it} and η^x_{it} out using Bayes' theorem. I denote $A_{x_{it},s_{it}=0}$ as the corresponding region that the joint integration of η^c_{it}, η^x_{it}, and η^s_{it} needs to be made.

$$\begin{aligned}
\Pr(p_{it}, x_{it}, s_{it} = 0) =& \Pr(s_{it} = 0|p_{it}, x_{it}) f(p_{it}, x_{it}) \\
&= \Pr(\eta^s_{it} > \underline{\eta}^s_{it}|\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it}) f_\eta(\tilde{\eta}^x_{it}, \tilde{\eta}^c_{it})| \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,}x_{it})}|,
\end{aligned}$$

where $\underline{\eta}^s_{it}$ is the minimum value of η^s_{it} that leads to zero hours of self-study. I use the first order condition with respect to s_{it}, equation (16), in computing the critical value.

With the zero correlation assumption between eta,

$$\begin{aligned}
&\Pr(\eta^s_{it} > \underline{\eta}^s_{it}|\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it}) f_\eta(\tilde{\eta}^x_{it}, \tilde{\eta}^c_{it})| \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,}x_{it})}| \\
=& \Pr(\eta^s_{it} > \underline{\eta}^s_{it}) f(\tilde{\eta}^x_{it}) f(\tilde{\eta}^c_{it})| \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,}x_{it})}| \\
=& \left(1 - \Phi(\underline{\eta}^s_{it})\right) \frac{1}{\sigma_x} \phi(\frac{\tilde{\eta}^x_{it}}{\sigma_x}) \frac{1}{\sigma_c} \phi(\frac{\tilde{\eta}^c_{it}}{\sigma_x})| \det \frac{\partial(\tilde{\eta}^c_{it}, \tilde{\eta}^x_{it})}{\partial(p_{it,}x_{it})}|,
\end{aligned}$$

which is what I use for computing the likelihood contribution for (Case 2).

(Case 3) ($x_{it} = 0$ and $s_{it} > 0$)

This is the case where household do not participate in tutoring, but do positive hours of self-study. Since $p_{it} > 0$ for all households, $p_{it}x_{it} = 0$ is equivalent to $x_{it} = 0$. For people who have $x_{it} = 0$, I let them consider minimum quality of tutoring, $\bar{p}$, which is equivalent the minimum market price.

Denote $A_{x_{it}=0}$ as the corresponding region that the joint integration of η^c_{it} and η^x_{it} needs to be made. First, I separate out the marginal density of η^s_{it} using Bayes' theorem, which gives me

$$
\begin{aligned}
\Pr(x_{it}=0,s_{it}) &= \Pr(x_{it}=0|s_{it})f(s_{it}) \\
&= \Pr(\eta^c_{it},\eta^x_{it}\in A_{x_{it}=0,s_{it}}|\tilde{\eta}^s_{it})\cdot\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|.
\end{aligned}
$$

As I assume there is no correlation between η_{it},

$$
\begin{aligned}
&\Pr(\eta^c_{it},\eta^x_{it}\in A_{x_{it}=0,s_{it}}|\tilde{\eta}^s_{it})\cdot\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}| \\
&= \Pr(\eta^c_{it},\eta^x_{it}\in A_{x_{it}=0,s_{it}})\cdot\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|.
\end{aligned}
$$

Here, I use the first order condition, equation (16), in characterizing the unique values of $\tilde{\eta}^s_{it}$. Define $\underline{\eta}^x_{it}$ as a minimum amount of shock that makes individual start doing zero hours of tutoring. Again, with the assumption of no correlation between η_{it},

$$
\begin{aligned}
&\Pr(\eta^c_{it},\eta^x_{it}\in A_{x_{it}=0,s_{it}})\cdot\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}| \\
=&\Pr(\eta^c_{it},\eta^x_{it}>\underline{\eta}^s_{it}|\tilde{\eta}^s_{it})d\eta^x_{it}d\eta^c_{it}\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}| \\
=&\Bigg[\int_{-\infty}^{\infty}\Bigg\{\int_{\underline{\eta}^x_{it}(\eta^c_{it})}^{\infty}\frac{1}{\sigma_x}\phi(\frac{\eta^x_{it}}{\sigma_x})d\eta^x_{it}\Bigg\}\frac{1}{\sigma_c}\phi(\frac{\eta^c_{it}}{\sigma_c})d\eta^c_{it}\Bigg]\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}| \\
=&\Bigg[\int_{-\infty}^{\infty}\Bigg\{1-\Phi(\frac{\underline{\eta}^x_{it}(\eta^c_{it})}{\sigma_x})\Bigg\}\frac{1}{\sigma_c}\phi(\frac{\eta^c_{it}}{\sigma_c})d\eta^c_{it}\Bigg]\frac{1}{\sigma_s}\phi(\frac{\tilde{\eta}^s_{it}}{\sigma_s})|\frac{\partial\tilde{\eta}^s_{it}}{\partial(s_{it})}|
\end{aligned}
$$

(Case 4) ($x_{it}=0$ and $s_{it}=0$)

This is the case where $x_{it}=0$ and $s_{it}=0$. To make the notation concise, I denote V_{00} as the value when $x_{it}=s_{it}=0$. V_{x0} denotes the case $x>0$ and $s=0$. V_{0s} denotes the case $x=0$ and $s>0$.

$$
\begin{aligned}
\Pr(x_{it}=0,s_{it}=0) &= \Pr(\eta^c_{it},\eta^x_{it},\eta^s_{it}\in A_{x_{it}=0,s_{it}=0}) \\
&= \Pr(V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it})>V_{x0}(\eta^s_{it}),V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it})>V_{0s}(\eta^c_{it},\eta^x_{it}),V_{00}(\eta^c_{it},\eta^x_{it},\eta^s_{it})>V_{xs}).
\end{aligned}
$$

$$
\begin{aligned}
\Pr(x_{it}=0, s_{it}=0) =& \Pr(\eta^c_{it}, \eta^x_{it}, \eta^s_{it} \in A_{x_{it}=0, s_{it}=0}) \\
=& \Big(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1\{V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{x0}(\eta^s_{it}), V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{0s}(\eta^c_{it}, \eta^x_{it}), V_{00}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it}) > V_{xs})\} \\
=& f(\eta^s_{it}) f(\eta^x_{it}) f(\eta^c_{it}) d\eta^s_{it} d\eta^x_{it} d\eta^c_{it} \Big)
\end{aligned}
$$

The integral does not have an analytical solution and needs to be simulated.
Simulation algorithm is
(1) I draw an unconditional set of $\eta^r_{it} = \{\eta^{cr}_{it}, \eta^{xr}_{it}, \eta^{sr}_{it}\}$
(2) Let household optimize their choices.
(3) Count the proportion of cases that household chooses $x_{it} = 0$ and $s_{it} = 0$
In particular define x^r and s^r such that

$$(x^r, s^r) = \arg\max_{x_{it}, s_{it}} V_{it}(\eta^c_{it}, \eta^x_{it}, \eta^s_{it})$$

Compute

$$\frac{1}{R} \sum_{r=1}^{R} \mathbb{1}(x*, s* = 0).$$

So

$$\Pr(x_{it}=0, s_{it}=0) \approx \frac{1}{R} \sum_{r=1}^{R} \mathbb{1}(x*, s* = 0).$$

Simulation of unobserved variables

For each missing choice variables, I draw a set of corresponding error. For example, if x_{it} is missing for person i, the simulation algorithm is
(1) I draw a simulation for the corresponding error. In this example, it is η^{xr}_{it}
(2) Let household optimize their choice

$$x^r = \frac{1}{R}\sum_{r=1}^{R}\left\{\arg\max_{x_{it},s_{it}} V_{it}(\eta^c_{it}, \eta^{xr}_{it}, \eta^s_{it})\right\}.$$

The optimized choice is used for computing likelihood function.

For missing test score, I draw a set of errors for η^q_{it}. Then the unobserved test score is simulated using equation (4).

Appendix E

Figure E.1: Sample Fit: Quality of Tutoring

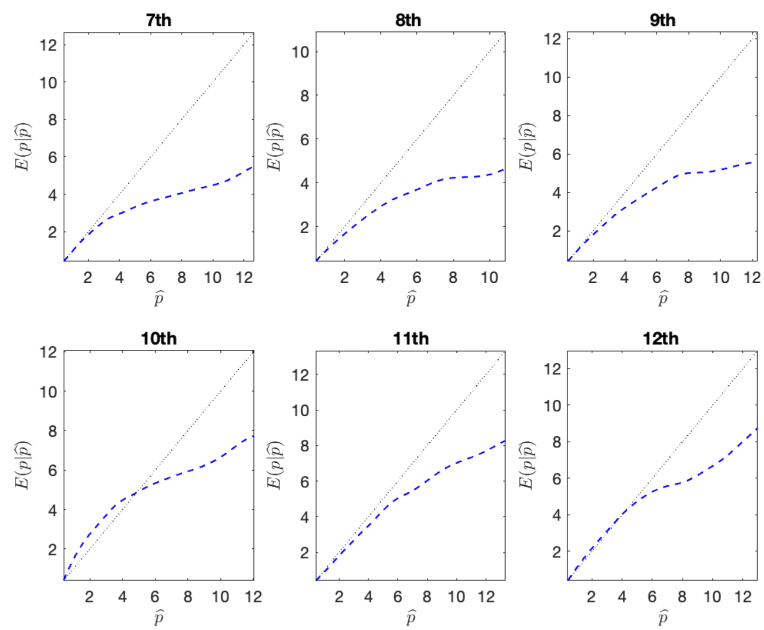


Figure E.2: Hours of Tutoring

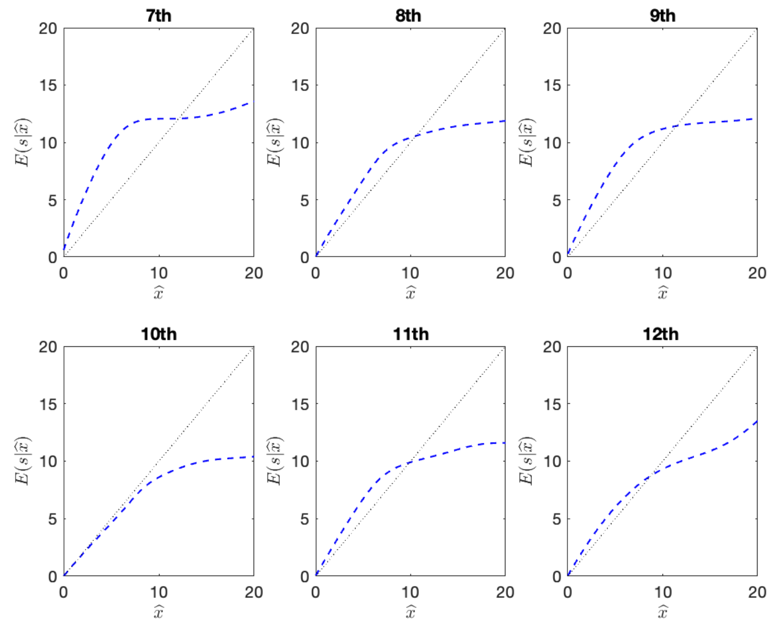


Figure E.3: Sample Fit: Private Tutoring Expenditure

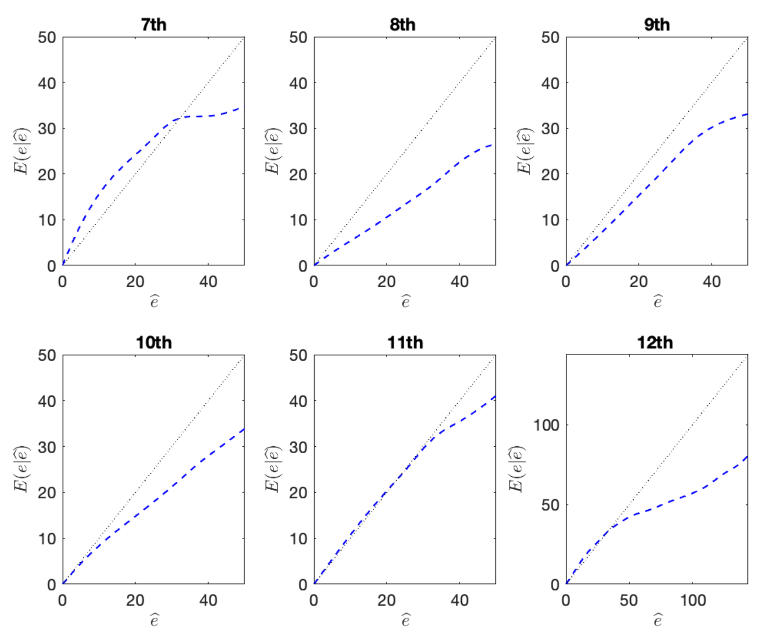


Figure E.4: Sample Fit: Hours of self-study

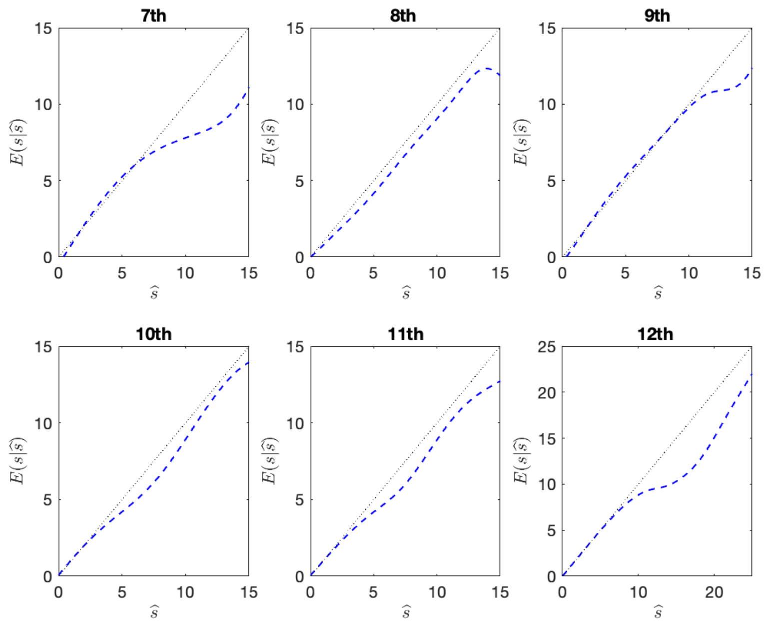
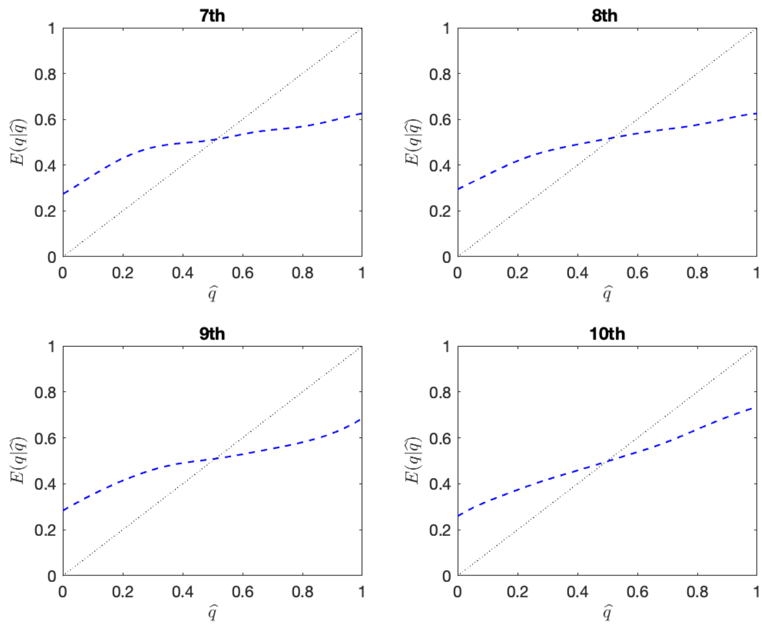


Figure E.5: Sample Fit: Log Test Scores

(a) Fit by distribution



(b) Fit by level