Alternative Methods to Test for Positive Assortative Mating

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Abstract

We propose easy econometric methods to test for positive assortative mating. Compared to existing methods such as by Siow (2015), our procedure avoids the cost associated with imposing positive assortative mating restrictions, and it allows for more general tests. The test does not require numerical optimization, and the test statistics can be calculated easily. We compare the power of the tests using simulation. The empirical application suggests that the null hypothesis of no assortative mating is rejected for selected years of CPS data that range from 1962 to 2019.

1 Introduction

This paper proposes easy methods to test for positive assortative mating. A number of empirical studies on sorting in the marriage market have investigated the intensity of assortative mating and how the assortativeness of mating has changed over time. Only a handful of econometric tests have been developed for testing positive assortative mating (Chiappori, Salanié, and Weiss, 2011; Anderson and Teng, 2013; Siow, 2015; Gualdani and Sinha, 2023).

A strand of literature on marriage matching market is initiated by Choo and Siow (2006b). They propose a marriage matching function that is based on the transferable utility matching framework with random preferences.[1,2] The critical assumptions of Choo and Siow (2006b) are (a) additive separability of joint surplus, (b) large marriage market, and (c) extreme value distribution of the shock (Galichon and Salanié, 2021). Choo and Siow (2006a) applied the

[1] See Chiappori and Salanié (2016) for the survey of the empirical marriage matching models under different assumptions: transferable utility, non-transferable utility, and imperfectly transferable utility models.

[2] See Fox (2010), Fox (2018), and Fox, Yang, and Hsu (2018) for an alternative method employing a maximum score approach based on the rank-order property associated with sorting. Suen and Liu (1999) develop an empirical model of spouse selection to examine complementarity in the marital production function.

marriage matching function to quantify the effect of the legalization of abortion on gains to marriage.

Based on the transferable utility framework, papers in the literature extend Choo and Siow (2006b) in different directions. Extending the structural model of Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanié, and Weiss (2017) build a marriage matching model with parental investment and embed it into the Choo and Siow (2006b) marriage matching function. They measure the changes in assortative mating by estimating the returns to education in the US marriage market. Mourifié and Siow (2021) incorporate peer effects into the marriage matching function of Choo and Siow (2006b). Dupuy and Galichon (2014) suggest how to amend the Choo and Siow (2006b) model for continuous attributes based on the continuous choice framework developed by Dagsvik (1994) and Dagsvik (2000). Galichon and Salanié (2021) show that the extreme value assumption of Choo and Siow (2006b) can be relaxed. Fernández, Guner, and Knowles (2005) construct a search model with positive assortative mating as an outcome. Instead of testing for positive assortative mating, they want to estimate the relationship between the degree of positive assortative mating and other variables of interest (e.g. wage gaps between skilled and unskilled workers).

The literature on worker-firm matching deals with a different setting but shares a similar interest in that it investigates the strength and the direction of assortative mating between two agents (workers and firms). Using employer-employee matched datasets, Abowd, Kramarz, and Margolis (1999) estimate the correlation between worker fixed effect and the firm fixed effect from wage equations. Pointing out that wage does not represent the underlying types such as job amenities, papers in this literature explore the correlation between types in a different way. One set of papers builds and estimates equilibrium search models and investigate the sign of sorting (Lise, Meghir, and Robin, 2016; Bagger and Lentz, 2019). This approach provides a consistent behavioral framework but comes with the cost of specifying a potentially complicated model. On the other hand, another set of papers presents a simpler method of testing for sorting at the expense of more restrictive assumptions. For example, Mendes, van den Berg, and Lindeboom (2010) assume that the observed skill level can identify the worker's type. Then they estimate the correlation coeficients of the fixed effects based on their production function equations. Taking into account for the noise in the worker wage rankings, Bartolucci, Devicienti, and Monzón (2018) propose a test for the sign of sorting between workers and firms.

Few papers develop econometric tests for positive assortative mating. Siow (2015) proposes a test for positive assortative mating based on maximum likelihood estimation. The test statistics are based on the theoretical framework of Choo and Siow (2006b), and the test procedure requires numerical optimization with different restrictions. Also, their test statistics have non-standard asymptotic distributions. Our method does not require numerical optimization and can easily be calculated.

Suppose there are I types of men, $i = 1, 2, .., I$, and J types of women, $j = 1, 2, .., J$, where types are vertically ordered. The likelihood ratio test of

Siow (2015) mainly uses two kinds of restrictions to define the null hypothesis: one imposes the positive assortativeness restriction for all (i, j)-type pairs, and the other restriction solely focuses on the preference for own types (TP2[3] and DP2[4]). It is not clear which of the restrictions in Siow (2015) are more suitable to assess positive assortative mating.[5] For example, the log-likelihood with the TP2 restriction is rejected against the unrestricted likelihood, but the log-likelihood with the DP2 restriction is not rejected. Siow (2015) states that the rejection of the TP2 likelihood is driven by the couples whose completed education is distant. As the extreme matches account only for less than 0.2% of marriages, TP2 restriction is arguably sensitive to outliers.[6] Our method requires only the unrestricted estimates of the log odds ratios. Note that this also means that the researcher does not have to choose among the appropriate restrictions to impose. Also, it exploits all types but is not as restrictive as imposing TP2.

We describe our method in Section 2. In Section 3, we provide empirical examples using Current Population Survey samples between 1962 and 2019. We conclude in Section 4.

2 Alternative Tests

Let n_{ij} be the number of observations where a type-i male, $i = 1, 2, .., I$ is matched with a type-j female, $j = 1, 2, .., J$, and define $N = \sum_{ij} n_{ij}$ as the total number of sampled marriages. Let p_{ij} be the probability that a randomly sampled marriage is of type ij. Siow (2015) states that the unrestricted log likelihood function for this problem is

$$\begin{aligned} L &= \sum_{i,j} n_{ij} \log p_{ij} \\ st\ 1 &= \sum_{ij} p_{ij}. \end{aligned}$$

[3] Define the (i, j) local log odds ratio as

$$\log \mu(i, j) + \log \mu(i + 1, j + 1) - \log \mu(i, j + 1) - \log \mu(i + 1, j),$$

where $\mu(\cdot)$ is the proportion of the (i, j) pairs in the population. TP2 is when all local log odds ratios are positive of order 2 usually denoted as "$\prec_2$." For two random variables A, B, we say that $A \prec_2 B$ iff

$$\int_{-\infty}^{x} [\Pr(B > t) - \Pr(A > t)]\, dt \geq 0 \quad \forall x$$

and $\exists x:$ the inequality in the equation is positive. See, for example, Shaked and Shanthikumar (2007).

[4] DP2 is when all log odds ratios of the same types $(i = j)$ are positive.

[5] Chiappori, Salanié and Weiss (2017) also propose a test based on their structural model that incorporates returns to parental investment. Their method requires using minimum distance of the moment conditions that come from their structural model. The main purpose of this test is to examine the changes in the intensities of educational homogamy.

[6] Anderson and Teng (2013) develop a test statistic very similar to the TP2 test in Choo and Siow (2006b).

The unrestricted MLE of p is $\widehat{p}^u = \{\widehat{p}^u_{ij}\}$ with $\widehat{p}^u_{ij} = n_{ij}/N$. Siow (2015) and others cited by Siow (2015) argue that the way to test for positive assortative mating is to also estimate $\widehat{p}$ under the restrictions imposed by positive assortative mating and then perform a likelihood ratio test. The null hypothesis in Siow (2015) is a region with positive measure, and the alternative hypothesis is a region with positive measure. To see this, consider a simple case where there are just two types of men and two types of women. The parameter space is the unit square, and the condition for positive assortative mating is

$$p_{11} + p_{22} - p_{12} - p_{21} \geq 0$$

which simplifies to

$$p_{11} + p_{22} \geq 1. \tag{1}$$

The region consistent with the null hypothesis is all values of (p_{11}, p_{22}) that satisfy equation (1), and the region consistent is the part of the unit square that does not satisfy equation (1). This is a quite uncommon problem in the literature. Meanwhile, our testing problem is standard: the null hypothesis is a point, and the alternative is a region surrounding the point.

A similar example of interest in the literature is testing a unit root hypothesis for a time series. Almost all of the literature on testing for a unit root assumes that the null hypothesis is nonstationarity. A strong reason for making this assumption instead of stationarity being the null hypothesis is that, when nonstationarity is the null hypothesis, it can be represented by a single $AR(1)$ parameter equalling 1. Thus, the situation is turned into a standard case where the null hypothesis corresponds to a point. However, from an economic point of view, the null hypothesis should be stationarity in that the model behaves better when there is stationarity (we would like to not reject the null hypothesis and then rely on the nice properties of stationary processes). Alternatively, Kwiatkowski et al. (1992), Leybourne and McCabe (1994), and Xiao (2001) all use a null hypothesis of stationarity by constructing a time series model with a random walk where the random walk is the only source of nonstationarity. The null hypothesis is then that the variance of the random walk error is equal to zero.

A second example, much closer to our problem, is to test whether the Slutsky substitution matrix is negative semidefinite. Gill and Lewbel (1992) choose a null hypothesis of semidefiniteness (which has positive measure) and show how to construct a consistent test statistic. Gill and Lewbel (1992) state that the asymptotics of this problem are very similar to that in Kodde and Palm (1986) where the null and alternative hypotheses have positive measure.[7] An easier approach is to construct a Wald test, requiring only the unrestricted estimates.[8] This avoids the cost associated with imposing the positive assortative mating restrictions, and it allows for more general tests.

[7] Unfortunately, Cragg and Donald (1996, 1997) show that the asymptotics for a related rank condition are incorrect, and neither paper addresses the correctness of the argument of Gill and Lewbel (1992) about testing for semidefiniteness.

[8] It's not clear which should be considered the restricted model and which the alternative.

Researchers have developed different methods to test hypotheses related to assortative mating, although not all directly focus on assessing the positivity of assortative mating. For instance, Chiappori, Salanie, and Weiss (2017) focuses on the change of the degree of educational assortative mating over time. They reject the null hypothesis that the degree of educational assortative mating has remained constant over time. Mourifié and Siow (2021) propose a Cobb-Douglas marriage matching function and emphasize the significance of peer and scale effects in marital matching. Gualdani and Sinha (2023) employ a set identification approach with nonparametric distributional assumptions on unobserved heterogeneity. Their findings indicate the presence of positive educational sorting among less educated couples.

2.1 Wald Test

In the next two subsections, we develop a Wald test, a likelihood ratio test, and a Lagrange multiplier test. Starting with the Wald test, the condition for positive assortative mating is[9]

$$R_{ij} = (\log p_{ij} + \log p_{i+1,j+1}) - (\log p_{i+1,j} + \log p_{i,j+1}) > 0 \ \forall ij, \tag{2}$$

and the proposed test statistic for testing for positive assortative mating is

$$T = \sum_{i,j} \left(\log \widehat{p}^u_{ij} + \log \widehat{p}^u_{i+1,j+1}\right) - \left(\log \widehat{p}^u_{i+1,j} + \log \widehat{p}^u_{i,j+1}\right)$$

where a superscript u indicates "unrestricted."

One might worry that the distribution of T under either the null or alternative would be difficult to evaluate. However, the same is true of Siow (2015) because the restrictions are inequalities instead of equality restrictions (Kudo, 1962; Gourieroux, Holly, and Monfort, 1982; Kodde and Palm, 1986; Wolak, 1989). Siow (2015) uses parametric bootstrapping to simulate critical values. The same can be done for T. Under positive assortative mating, $T > 0$, and, with total positive assortative mating (TP2 in Siow, 2015), $T \to \infty$ because $p_{i+1,j}p_{i,j+1} = 0$. Under an alternative specification, random sorting (Agresti, 2002), where $\log p_{ij} = \log x_i + \log y_j$, using the definition of R_{ij} in equation (2),

$$R_{ij} = (\log x_i + \log y_j + \log x_{i+1} + \log y_{j+1})$$

$$- (\log x_{i+1} + \log y_j + \log x_i + \log y_{j+1}) \equiv 0.$$

One can simulate a critical value $\widehat{c}$ such that

$$\Pr\left[T > \widehat{c} \mid R_{ij} = 0 \ \forall ij\right] = \delta.$$

[9] Note that $R_{ij} > 0 \ \forall ij \Rightarrow$

$$\left(\log p_{ij} + \log p_{i+k,j+k}\right) > \left(\log p_{i+k,j} + \log p_{i,k+2}\right) \ \forall k.$$

Note that one could test the hypothesis that $R_{ij} = 0 \; \forall ij$ with power against the alternative of (positive or negative) assortative mating as a Wald test with

$$\sum_{i,j} \frac{\widehat{R}_{ij}^2}{s_{ij}^2} \sim \chi_K^2$$

where

$$\widehat{R}_{ij}^2 = \left(\log \widehat{p}_{ij}^u + \log \widehat{p}_{i+1,j+1}^u\right) - \left(\log \widehat{p}_{i+1,j}^u + \log \widehat{p}_{i,j+1}^u\right),$$

$$\begin{aligned} s_{ij}^2 &= Var\left[\left(\log \widehat{p}_{ij}^u + \log \widehat{p}_{i+1,j+1}^u\right) - \left(\log \widehat{p}_{i+1,j}^u + \log \widehat{p}_{i,j+1}^u\right)\right] \\ &\approx \frac{(1-p_{ij})}{Np_{ij}} + \frac{(1-p_{i+1j+1})}{Np_{i+1j+1}} + \frac{(1-p_{i+1,j})}{Np_{i+1,j}} + \frac{(1-p_{i,j+1})}{Np_{i,j+1}}, \end{aligned}$$

using a first-order Taylor series approximation, and

$$K = (I-1)(J-1).$$

2.2 Likelihood Ratio Test and Lagrange Multiplier Test

Also note that the MLE of p under the restriction of random mating is $\widehat{p}^r = \left\{\widehat{p}_{ij}^r\right\}$ where

$$\widehat{p}_{ij}^r = \widehat{x}_i \widehat{y}_j, \quad \widehat{x}_i = \frac{\sum_j n_{ij}}{N}, \quad \widehat{y}_j = \frac{\sum_i n_{ij}}{N}.$$

Thus, one could test the null hypothesis of random mating against a general alternative with a likelihood ratio (LR) test,

$$\begin{aligned} T_{LR} &= 2\left[\sum_{i,j} n_{ij} \log \widehat{p}_{ij}^u - \sum_{i,j} n_{ij} \log \widehat{p}_{ij}^r\right] \\ &= 2\left[\sum_{i,j} n_{ij} \log \frac{\widehat{p}_{ij}^u}{\widehat{p}_{ij}^r}\right] \\ &= 2\left[\sum_{i,j} n_{ij} \log \frac{\widehat{p}_{ij}^u}{\left(\sum_k \widehat{p}_{ik}^u\right)\left(\sum_k \widehat{p}_{kj}^u\right)}\right] \sim \chi_K^2 \end{aligned}$$

(which requires only estimation of $\widehat{p}^u$) with

$$K = MF - 1$$

which is larger than the degrees of freedom in for the Wald test. This difference exists because, for the Wald test, one loses degrees of freedom at $m = M$ and $f = F$ which does not happen for the LR test.

Alternatively, one could construct a Lagrange Multiplier (LM) test. Let

$$L = \sum_{i,j} n_{ij} \log p_{ij}.$$

Then,

$$\begin{aligned}
\frac{\partial L}{\partial p_{ij}} \quad | \quad {}_{p_{ij}=\widehat{p}^{r}_{ij}} &= \frac{n_{ij}}{\widehat{p}^{r}_{ij}} \\
&= \frac{n_{ij}}{\frac{\sum_k n_{ik}}{N}\frac{\sum_k n_{kj}}{N}} \\
&= \frac{N\widehat{p}^{u}_{ij}}{\left(\sum_k \widehat{p}^{u}_{ik}\right)\left(\sum_k \widehat{p}^{u}_{kj}\right)}
\end{aligned}$$

implying that the LM has the same form and the same distribution as the LR test.

3 Distribution of Test Statistics

In order to simulate the distribution of our test statistics, we start with a model of assortative mating. Define a man as having an integer value of x_m between 1 and 10, and define a woman as having an integer value of x_w between 1 and 10. Assume that the density of matches in an economy is[10,11]

$$f\left(x_m, x_w\right) = \frac{\exp\left\{-\alpha\left|x_m - x_w\right|\right\}}{\sum_{x'_m=1}^{10}\sum_{x'_w=1}^{10}\exp\left\{-\alpha\left|x'_m - x'_w\right|\right\}}.$$

Figure 1 shows the contour of $f\left(x_m, x_w\right)$ as $\left(x_m, x_w\right)$ moves from $(1, 10)$ to $(10, 1)$ for $\alpha = 0.1$, 0.4, and 0.7. The larger α, the more assortative mating there is.

Figure 2 shows the distributions of the Wald test statistics for samples of 500, 1,000, and 10,000 when $\alpha = 0$ along with the χ^2_{81} distribution.[12] The χ^2_{81} distribution exhibits less variance than the Wald test statistics for samples of at least 1,000, and it stochastically dominates the Wald test statistic distribution for a sample of 500. The 5% critical value for the χ^2_{81} distribution is off by a meaningful amount relative to the simulated Wald test statistic distributions. The 5% critical value for a test statistic with a χ^2_{81} distribution is 103.0, while the Wald test statistics with sample sizes 500, 1,000, and 10,000 are respectively 99.0, 113.9, and 114.0. The difference between the two occurs because the simulated Wald test statistic provides the small-sample distribution of the test statistic, while the simulated LR test relies significantly on asymptotic approximations.

Figure 3 shows the distributions of the LR test statistic for samples of 500, 1,000, and 10,000 when $\alpha = 0$ along with the χ^2_{99} distribution. The χ^2_{99} distribution stochastically dominates the LR test statistic distributions for all three sample sizes. The 5% critical value for the χ^2_{99} distribution is off by

[10] Note that $\sum_{x_m=1}^{10}\sum_{x_w=1}^{10} f\left(x_m, x_w\right) = 1$.

[11] Also, note that this somewhat like a triangle kernel function with $\alpha = b^{-1}$ where b is the bandwidth (Kokonendji and Zocchi, 2010).

[12] The degrees of freedom are $(M-1)(F-1)$.

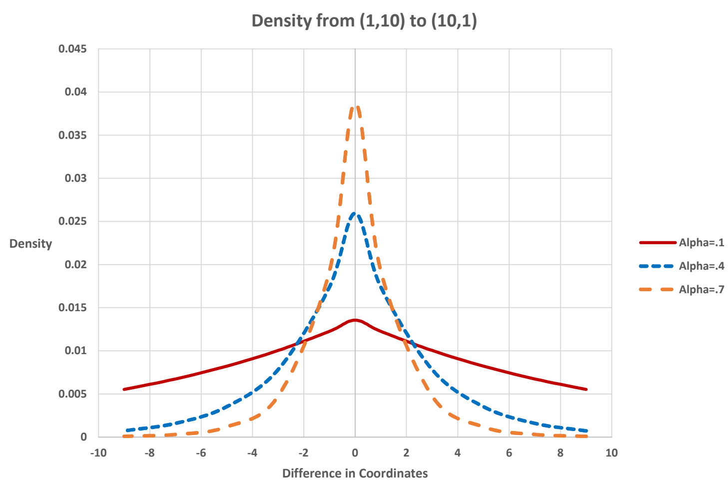

Figure 1: Density from (1,10) to (10,1)

Figure 2: Distribution of Pseudo-Wald Test Statistic for $\alpha = 0$

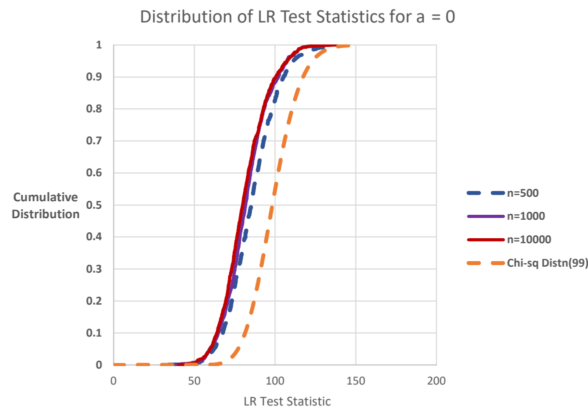

Figure 3: Distribution of LR Test Statistic for $\alpha = 0$

a meaningful amount relative to the simulated LR test statistic distributions. The 5% critical value for a test statistic with a χ^2_{99} distribution is 123.2, while, for the LR test statistic distributions with sample sizes 500, 1,000, and 10,000, they are respectively 111.3, 107.3, and 107.4.

Figure 4 shows the power functions for both test statistics for different values of α using the 5% critical values under the null hypothesis of $\alpha = 0$. The Wald test statistic has weak power for sample sizes less than 10,000, and the LR test statistic has very strong power. Part of the good performance of the LR test statistic relative to the Wald test statistic is due to it having more degrees of freedom. Also, the LR test statistic distribution is dominated by the χ^2_{99} distribution while the Wald test statistic is not dominated by the χ^2_{81} distribution; i.e., the LR test statistic has more power than implied by its χ^2_{99} asymptotic distribution while the Wald test statistic does not have more power than its χ^2_{81} asymptotic distribution.

4 Empirical Example

4.1 Wage Equations

Carroll, Kang, and Stern (2021) collect data from 18 Current Population Survey (CPS) samples between 1962 and 2019. For each sample, they estimate log wage rate equations using Heckman (1979), then compute log predicted wages using the inverse Mills' ratio $\phi(\cdot) / [1 - \Phi(\cdot)]$ for observed log wages and $-\phi(\cdot) / \Phi(\cdot)$ for unobserved log wages, and then measure the amount of assortative mating by log predicted wage rates among married couples. They also measure the amount of assortative mating by level of education. Among other analyses, they use the Wald test statistic and the LR test statistic presented in this paper to test for the existence of positive assortative mating. Test results are reported

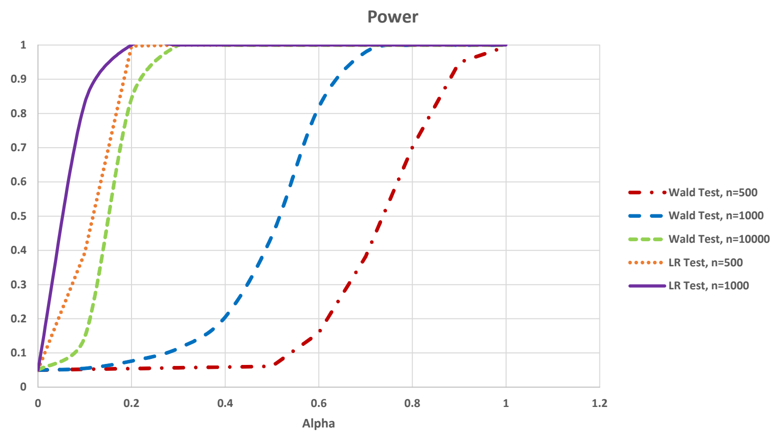

Figure 4: Power

in Table 1. As can be seen in the table, the LR test statistics are an order of magnitude larger than the Wald test statistics, both for log predicted wages and years of education. However, for all four test statistics, the null hypothesis of no assortative mating is rejected at any reasonable significance level for every year of CPS data in favor of the existence of positive assortative mating.

Table 1: Assortative Mating Test Statistics

	log Predicted Wage		Years of Education	
Year	Wald Test	LR test	Wald Test	LR test
1962	525.5	7314.0	2443.6	5642.5
1967	773.3	6298.7	2981.7	5968.1
1970	604.1	5317.9	4453.5	8447.1
1972	497.9	7318.3	3952.1	8404.5
1975	269.1	4329.7	3456.1	7756.3
1978	318.5	2382.0	3857.4	8507.5
1980	672.9	3119.2	5022.2	10139.1
1985	871.8	3101.7	5121.0	9977.7
1990	564.9	5818.5	6061.8	10134.3
1995	451.1	5098.8	4344.7	8250.6
2000	544.9	5764.1	3799.6	7642.9
2005	631.3	8087.4	5372.7	11930.9
2007	654.3	7502.8	5108.6	11536.9
2010	807.4	7046.4	4525.0	10983.2
2014	1004.0	8168.8	3654.2	10317.1
2015	935.5	7930.8	3946.9	10395.8
2018	1043.6	5283.8	2964.6	8854.5
2019	1183.9	5208.0	2897.4	8604.3
# Cells	10		5	
DF	81	99	16	24

4.2 Education

As an additional exercise, we use the Census 1970 and 2000 datasets, which were employed in Siow's (2015) analysis. The results of the tests are presented in Table 2. It is worth noting that the LR test statistics yield smaller values compared to the Wald test statistics. Across all four test statistics, the null hypothesis of no assortative mating is rejected at a significant level for each year of the Census data, providing evidence in support of the presence of positive assortative mating.

Table 2: Test Results for Siow (2015)

Year	Wald Test	LR test
1970	1,664,919	90,758
2000	5,906,027	186,646
#cell	5	
df	16	

5 Conclusion

We develop empirical tests on positive assortative mating. Our tests are straightforward to use and do not require numerical optimization, which offer advantages over existing tests on positive assortative mating. The empirical applications using CPS and Census data suggest that the null hypothesis of no assortative mating is rejected for selected years.

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