$\boxed{2019/10/16}$

$\boxed{\text{- Determinant}}$

↳ $\det(A)$, $\det A$, $|A|$

- $\det([a]) = a$

  ↳ "If it is a scalar"

- If A is a matrix, we use "Cofactor expansion" to calculate the determinant.

① $$\det(A) = \sum_{i=1}^{N} \{(-1)^{i+j_0} M_{i,j_0}\} a_{i,j_0}$$

or

② $$\det(A) = \sum_{j=1}^{N} \{(-1)^{i_0+j} M_{i_0,j}\} a_{i_0,j}$$

$$\underbrace{C_{i,j}}_{\text{"Cofactor"}} = (-1)^{i+j} M_{i,j}$$

$$\det(A) = \sum_{i=1}^{N} C_{i,j_0} a_{i,j_0} = \sum_{j=1}^{N} C_{i_0,j} a_{i_0,j}$$

(For example) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$\det(A) = \{(-1)^{1+1} M_{1,1}\} a_{1,1} + \{(-1)^{2+1} M_{2,1}\} a_{2,1} + \{(-1)^{3+1} M_{3,1}\} a_{3,1}$$

$$= M_{1,1} a_{1,1} + (-M_{2,1} a_{2,1}) + M_{3,1} a_{3,1}$$

$$= M_{1,1} \cdot 1 - M_{2,1} \cdot 4 + M_{3,1} \cdot 7$$

i) $M_{1,1}$ $\begin{bmatrix} \cancel{1} & \cancel{2} & \cancel{3} \\ \cancel{4} & 5 & 6 \\ \cancel{7} & 8 & 9 \end{bmatrix} = \det\left(\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}\right) \Rightarrow M_{1,1} = \{(-1)^{1+1} M'_{1,1}\} a'_{1,1} + \{(-1)^{2+1} M'_{1,2}\} a'_{1,2}$

$\begin{bmatrix} \cancel{5} & \cancel{6} \\ \cancel{8} & 9 \end{bmatrix} \rightarrow M'_{1,1} = \det([9]) = 9$

$\begin{bmatrix} \cancel{5} & \cancel{6} \\ 8 & \cancel{9} \end{bmatrix} \rightarrow M'_{1,2} = \det([8]) = 8$

$M_{1,1} = 9\times5 - 8\times6 = \underline{-3}$

2) $M_{2,1}$ $\begin{bmatrix} \cancel{1} & 2 & 3 \\ \cancel{4} & \cancel{5} & \cancel{6} \\ \cancel{7} & 8 & 9 \end{bmatrix} = \det\left(\begin{bmatrix} 2 & 3 \\ 8 & 9 \end{bmatrix}\right) = \underline{-6}$

3) $M_{3,1}$ $\begin{bmatrix} \cancel{1} & 2 & 3 \\ \cancel{4} & 5 & 6 \\ \cancel{7} & \cancel{8} & \cancel{9} \end{bmatrix} = \det\left(\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}\right) = \underline{-3}$

↳ $\det(A) = -3 - (-6)\times4 + (-3)\times7 = \boxed{0}$