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- As we have previously ~~seen~~, the A^TA's return will be always square matrix.
  - Let's review again with an example.

$$A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}_{3\times1}, \quad A^T = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}_{1\times3}$$

$$A^TA = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}_{1\times3} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}_{3\times1} = [4+9+1] = [14] \Rightarrow \underbrace{1\times1}_{\text{"Square"}}$$

- Let's say "$Ax \approx b$"

$$\Downarrow$$

$$A^TAx \approx A^Tb$$

↳ If an Inverse Matrix exists for A^TA,

↳ $(A^TA)^{-1}(A^TA)x \approx (A^TA)^{-1}A^Tb$

↳ $\boxed{x \approx \left((A^TA)^{-1}A^T\right)b}$

↳ $(A^TA)^{-1})A^T$ ⇒ we call it as "pseudo inverse" and symbolized as "A^+"

$$\boxed{A^+ = \left((A^TA)^{-1}A^T\right)}$$

$$\boxed{x \approx A^+b}$$