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## - Norm

$$\|A\|_p = \left(\sum_{i=1}^{N}\sum_{j=1}^{M}|a_{ij}|^p\right)^{1/p}$$

Generally, $p$ is 1 or 2 or $\infty$, but mostly $p=2$ used.
we call it "Frobenius norm" ($p=2$). "$\|A\|_F$"

$$\|A\| = \|A\|_2 = \|A\|_F = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{M} a_{ij}^2}$$

↳ "Always, the norm is equal or greater than 0."
"Norm can be defined any shape of matrix".
- $(\text{Vector's norm})^2 = (\text{Vector's RSS})$

$$\boxed{\|x\|^2 = \sum_{i=1}^{N} x_i^2 = x^T x}$$

↳ (Therefore) $\boxed{\text{Minimizing Norm} == \text{Minimizing Vector's RSS}}$

## - Norm's features

① $\|A\| \geq 0$
② $\|\alpha A\| = |\alpha|\,\|A\|$
③ $\|A+B\| \leq \|A\| + \|B\|$
④ $\|AB\| \leq \|A\|\,\|B\|$ (square Matrix)

## - Trace

↳ Only apply for the "Square matrix".

$$tr(A) = a_{11} + a_{22} + \cdots + a_{NN} = \sum_{i=1}^{N} a_{ii}$$

- $tr(I_N) = N$
- trace could be a "negative number".
- $tr(cA) = c \cdot tr(A)$
- $tr(A^T) = tr(A)$
- $tr(A+B) = tr(A) + tr(B)$
- $tr(AB) = tr(BA)$