(3) If $A=[1,2,3]$, $B=\begin{bmatrix}4&7\\5&8\\6&9\end{bmatrix}$,
try to calculate both AB and BA if it is possible,
and see the result.

$$AB=[1,2,3]\begin{bmatrix}4&7\\5&8\\6&9\end{bmatrix}=[(4+10+18),(7+16+27)]=[32,50]$$

$1\times3 \quad 3\times2 \rightarrow 1\times2$.

$$BA=\begin{bmatrix}4&7\\5&8\\6&9\end{bmatrix}[1,2,3] \Rightarrow$$

Can't calculate since the shapes (3,2) and (1,3) not aligned

(4) If $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$, $B=\begin{bmatrix}5&6\\7&8\end{bmatrix}$,
try to calculate both AB and BA if possible,
and see the result.

$$AB=\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}5&6\\7&8\end{bmatrix}=\begin{bmatrix}5+14&6+16\\15+28&18+32\end{bmatrix}=\begin{bmatrix}19&22\\43&50\end{bmatrix}$$

$2\times2 \quad 2\times2 \rightarrow 2\times2$

$$BA=\begin{bmatrix}5&6\\7&8\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}5+18&10+24\\7+24&14+32\end{bmatrix}=\begin{bmatrix}23&34\\31&46\end{bmatrix}$$

$AB \neq BA$

(5) If $A=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}$, Calculate AA^T and A^TA.
See the shape of AA^T and A^TA. Is it square Matrix?

$$AA^T=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}\begin{bmatrix}(1+4)&(3+8)&(5+12)\\(3+8)&(9+16)&(15+24)\\(5+12)&(15+24)&(25+36)\end{bmatrix}=\begin{bmatrix}5&11&17\\11&25&39\\17&39&51\end{bmatrix}$$

$3\times2 \quad 2\times3 \Rightarrow 3\times3$

$$A^TA=\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}=\begin{bmatrix}(1+9+25)&(2+12+30)\\(2+12+30)&(4+16+36)\end{bmatrix}=\begin{bmatrix}35&44\\44&56\end{bmatrix}$$

$2\times3 \quad 3\times2 \Rightarrow 2\times2$

$AA^T \neq A^TA$ But, it became all square Matrix