exercise   (unit vector => length of vector is equal to 1)

U) Find a unit vector that orthogonal with x.

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$a = \begin{bmatrix} 0, & 1 \end{bmatrix}$$

$$\Rightarrow \quad ax = \begin{bmatrix} 0, & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 + 0 = 0$$

2) Find a unit vector that orthogonal with x.

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$a = \begin{bmatrix} -\frac{1}{\sqrt{2}}, & +\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Rightarrow \quad ax = \begin{bmatrix} -\frac{1}{\sqrt{2}}, & +\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

$$\|a\| = \sqrt{a^T a} = \sqrt{a_1^2 + \cdots + a_N^2} = \sqrt{\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(+\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$$

3) Find a unit vector that satisfy $a \perp x$ and $a \perp y$.

$$x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad a^T x = \begin{bmatrix} 0, 0, 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$a^T y = \begin{bmatrix} 0, 0, 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0$$

$$a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$