$\boxed{2019/10/17}$

1) $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

i) $x^T A x = [x_1, x_2, x_3] \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$1\times3 \quad 3\times3 \quad 3\times1$

$= \left[(2x_1 - x_2), (-x_1 + 2x_2 - x_3), (-x_2 + 2x_3)\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$= x_1(2x_1 - x_2) + x_2(-x_1 + 2x_2 - x_3) + x_3(-x_2 + 2x_3)$

$= 2x_1^2 - x_1x_2 + (-x_1x_2) + 2x_2^2 - x_2x_3 - x_2x_3 + 2x_3^2$

$= 2x_1^2 - 2x_1x_2 + 2x_2^2 - 2x_2x_3 + 2x_3^2$

$= x_1^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + x_3^2 > 0$

Answer: "positive-definite"

ii) Find trace.

$tr(A) = a_{11} + a_{22} + \dots + a_{NN} = \sum_{i=1}^{N} a_{ii}$

$tr(A) = 2 + 2 + 2 = 6$

Answer: $tr(A) = 6$

iii) Find determinant of A. (I am not going to use the simple equation)

$\det(A) = \{(-1)^{1+1} M_{1,1}\} a_{1,1} + \{(-1)^{2+1} M_{2,1}\} a_{2,1} + \{(-1)^{3+1} M_{3,1}\} a_{3,1}$

$= M_{1,1} a_{1,1} - M_{2,1} a_{2,1} + M_{3,1} a_{3,1}$

$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$\det(A) = 8 + 0 + 0 - 0 - (-1\times-1\times2) - (2\times-1\times-1) = 8 - 2 - 2 = 4$ ($\cancel{8}$)

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \rightarrow M_{1,1} = \det\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\right) \rightarrow \{(-1)^{1+1} M'_{1,1}\} a'_{1,1} + \{(-1)^{2+1} M'_{2,1}\} a'_{2,1}$

$= M'_{1,1} a'_{1,1} - M'_{2,1} a'_{2,1}$

$= 2\times2 - (-1\times-1) = 3$

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \rightarrow M_{2,1} = \det\left(\begin{bmatrix} -1 & 0 \\ -1 & 2 \end{bmatrix}\right) \Rightarrow \{(-1)^{1+1} M^x_{1,1}\} a^x_{1,1} + \{(-1)^{2+1} M^x_{2,1}\} a^x_{2,1}$

$= M^x_{1,1} a^x_{1,1} - M^x_{2,1} a^x_{2,1}$

$= (2\times-1) - (0) = -2$

re-confirm

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \rightarrow M_{3,1} = \det\begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix}$

$= M^{\checkmark}_{1,1} a^{\checkmark}_{1,1} - M^{\checkmark}_{2,1} a^{\checkmark}_{2,1}$

$= (-1)(-1) - (0) = +1$

Answer: $(3\times2) + (2\times-1) + (1\times0) = \boxed{4}$