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- Quadratic Form

from the previous equation, there was $w^T X^T X w$.

$X^T X$ is a square matrix, and make it as A. Then,

$w^T A w = w^T X^T X w$ ⇒ we call it as "quadratic Form"

"row vector" × [square matrix] × [column vector]

$$x^T A x = [x_1, x_2, \cdots, x_N] \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_{ij}$$

Practice Calculate it!

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$x^T A x = [x_1 \; x_2 \; x_3] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$1 \times 3 \quad 3 \times 3 \Rightarrow 1 \times 3 \quad 3 \times 1 = 1 \times 1$

$$= [a_{11}x_1 + a_{21}x_2 + a_{31}x_3, \; a_{12}x_1 + a_{22}x_2 + a_{32}x_3, \; a_{13}x_1 + a_{23}x_2 + a_{33}x_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= (a_{11}x_1 + a_{21}x_2 + a_{31}x_3)x_1 + (a_{12}x_1 + a_{22}x_2 + a_{32}x_3)x_2 + (a_{13}x_1 + a_{23}x_2 + a_{33}x_3)x_3$$