2019-10-07

2.2 Vector and Matrix Operations

- Vector/Matrix addition and subtraction

$$x = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$x + y = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 14 \end{bmatrix}, \quad x - y = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 24 \\ 34 & 44 \end{bmatrix}$$

- scalar multiply vector/Matrix

$$c \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix}, \quad c \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{bmatrix}$$

- broad casting

$$\begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} - 10 = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} - 10 \cdot 1 = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} - \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \rightarrow x - m = \begin{bmatrix} x_1 - m \\ x_2 - m \\ \vdots \\ x_N - m \end{bmatrix}, \quad m = \frac{1}{N} \sum_{i=1}^{N} x_i$$

→ In a data analysis, we often use the mean removed vector or zero-mean vector

- Linear Combination

$$c_1 x_1 + c_2 x_2 + c_3 x_3 + \cdots + c_L x_L = x$$

$$c_1 A_1 + c_2 A_2 + c_3 A_3 + \cdots + c_L A_L = A$$

$$c_1, c_2, \ldots, c_L \in \mathbf{R}$$

$$x_1, x_2, \ldots, x_L, \; x \in \mathbf{R}^M$$

$$A_1, A_2, \ldots, A_L, \; A \in \mathbf{R}^{M \times N}$$

$$c_1 \boxed{x_1} + c_2 \boxed{x_2} + \cdots + c_L \boxed{x_L}, \quad c_1 \boxed{A_1} + c_2 \boxed{A_2} + \cdots + c_L \boxed{A_L}$$