2019-10-19

problem Calculate the Inverse matrix

(1) $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}^{-1}$

$A^{-1} = \frac{1}{\det(A)} C^T$

i) find $\det(A)$, $\det(A) = \frac{1}{\frac{1}{2}} = ad - bc = 2$

$A^{-1} = \frac{1}{2}\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{4}{2} \end{bmatrix}$

$C_{11} = (-1)^{1+1} M_{1,1} \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 1$

$C_{12} = (-1)^{1+2} M_{1,2} \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$C_{21} = (-1)^{2+1} M_{2,1} \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$C_{22} = (-1)^{2+2} M_{2,2} \Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 2$

(2) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^{-1}$

$\det(A) = ad - bc = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}\right) = 1$

$A^{-1} = 1 \times \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$C_{11} = (-1)^{1+1} M_{1,1} \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}$

$C_{12} = (-1)^{1+2} M_{1,2} \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = -\frac{1}{\sqrt{2}}$

$C_{21} = (-1)^{2+1} M_{2,1} \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = +\frac{1}{\sqrt{2}}$

$C_{22} = (-1)^{2+2} M_{2,2} \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}$

$$A^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$