

20/9/14/05

$$\textcircled{1} \sum_{i=1}^N x_i = \sum_{j=1}^N x_j$$

$$\textcircled{2} \sum_{i=1}^N c x_i = c \sum_{i=1}^N x_i$$

$$\textcircled{3} \sum_{i=1}^N (x_i + y_i) = \sum_{i=1}^N x_i + \sum_{i=1}^N y_i$$

$$\textcircled{4} \sum_{i=1}^N \sum_{j=1}^M x_{ij} = \sum_{j=1}^M \sum_{i=1}^N x_{ij}$$

$$\prod_{i=1}^N \prod_{j=1}^M = \prod_{j=1}^M \prod_{i=1}^N$$

Example)  $\sum_{i=1}^2 \sum_{j=1}^3 x_{ij} = (x_{11} + x_{12} + x_{13}) + (x_{21} + x_{22} + x_{23})$

$\sum_{j=1}^3 \sum_{i=1}^2 x_{ij} = (x_{11} + x_{21}) + (x_{12} + x_{22}) + (x_{13} + x_{23})$

### Practice

(1) prove both are same.

$$\left( \sum_{i=1}^3 x_i \right)^2 = \sum_{i=1}^3 \sum_{j=1}^3 x_i x_j$$

$$\textcircled{a} \left( \sum_{i=1}^3 x_i \right)^2 = \left( \sum_{i=1}^3 x_i \right) \left( \sum_{i=1}^3 x_i \right)$$

$$= (x_1 + x_2 + x_3)(x_1 + x_2 + x_3)$$

$$= (x_1^2 + x_1 x_2 + x_1 x_3) + (x_1 x_2 + x_2^2 + x_2 x_3) + (x_1 x_3 + x_2 x_3 + x_3^2)$$

$$= [x_1^2 + x_2^2 + x_3^2 + 2x_1 x_2 + 2x_1 x_3 + 2x_2 x_3]$$

$$\textcircled{b} \sum_{i=1}^3 \sum_{j=1}^3 x_i x_j = \sum_{i=1}^3 \left( \sum_{j=1}^3 x_i x_j \right) = \sum_{i=1}^3 (x_i x_1 + x_i x_2 + x_i x_3)$$

$$= (x_1 x_1 + x_1 x_2 + x_1 x_3) + (x_2 x_1 + x_2 x_2 + x_2 x_3) + (x_3 x_1 + x_3 x_2 + x_3 x_3)$$

$$= [x_1^2 + x_2^2 + x_3^2 + 2x_1 x_2 + 2x_1 x_3 + 2x_2 x_3]$$