9/14: Up to Speed on Stirling & Spitters

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1 Reminders of some notions from my first chapter

2 Two Yoneda embeddings and normalization by evaluation

2.1 Reification, reflection

2.2 Just work from wikipedia for this part

2.3 The readback used in StirSpit

Note that I haven't yet seen exactly the readback operation (in terms of presheaves) planned yet. Their presentation is abysmally bad on this point actually... they delay explaining/defining an object of deep conceptual and technical import to the result until like 10 pages in. Maybe they suppose

that the reader already has experience with normalization by evaluation, which I guess is kind of fair, but this was personally my first exposure to it in earnest.

3 A pickle: too much quotienting

The definition of the syntactic category (category of contexts and substitutions) given in the chapter I wrote a few weeks ago has for its equations governing equality of morphisms those given by the "substitution lemma." It turns out that this identifies far too many terms for our uses. In particular, terms which are related by the various beta rules are identified, meaning that a normal form (a term for which no further beta reduction can be performed) is identified with its (manifestly not normal) beta-predecessor. The upshot is that we can't isolate the normal forms as a class of terms, which totally bungles our whole project of investigating which terms (all of them) of the simply typed lambda calculus have normal forms.

4 Unpickling ourselves: the category of renamings

5 The relative hom functor

Stirling & Spitters follow Fiore in defining the "relative hom functor", which they suggestively call $\mathfrak{Tm} : \mathrm{Cl}_\Sigma \to \mathrm{Ren}_\Sigma{}^{\mathrm{Set}}$. The suggestion hinted at by the name, that this functor defines a presheaf of open terms, turns out to be a (small?) lie. Let's look at what it actually does. $\mathfrak{Tm}$ is defined by adjusting the hom functor (i.e, the Yoneda embedding) by precomposition with the inclusion of the category of renamings into the category of (contexts and) substitutions. In particular, StirSpit define $\mathfrak{Tm}(\Delta) = \mathrm{Cl}_\Sigma[i(-), \Delta]$. In plain terms, $\mathfrak{Tm}(\Delta)$ takes a context in the category of renamings to the *substitutions on terms* **out of** Δ (recalling that the action of the category of contexts on its clones is contravariant). This can be (very loosely) construed as a presheaf of open terms. For a (renaming) context Γ, we have $\mathfrak{Tm}(\Delta)(\Gamma) = \mathrm{Cl}_\Sigma[i(\Gamma), \Delta]$, the context Γ just falls through and we get the substitutions $\gamma^* : \Delta \vdash \tau \to \Gamma \vdash \tau$ for arbitrary τ. In particular, any (possibly) open term $\Delta \vdash t : \tau$ is included in $\mathfrak{Tm}(\Delta, \tau)(\Delta)$ as the single substitution $[t/x]$. The reason I regard as misleading the suggestion that the relative hom functor defines a presheaf of open terms is that the morphisms in the syntactic category aren't just single substitutions, but also single omissions $\hat{x}$ and all the compositions of these two classes of maps.