Exercise Walkthrough: Probability of At Least One 'One'

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1 Overview

This document provides a step-by-step solution to the problem of calculating the probability of obtaining at least one [illegible]

2 Step 1: Defining the Probability Space

[illegible] $(\Omega', \mathcal{A}', P)$ [illegible] Ω' [illegible] $\mathcal{A}'$ [illegible] P

The Sample Space Ω' [illegible] $\Omega = \{0, 1, 2, 3, 4, 5\}$ [illegible] $(0, 4, 1, 0, 5, 2)$

… Ω' …

$$\Omega' = \Omega^6 = \{(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6) \mid \omega_j \in \Omega \ \ldots\ j = 1, \ldots, 6\}$$

… $|\Omega'|$ … $|\Omega|^6 = 6^6 = 46,656$ …

The Event Space (σ-algebra) $\mathcal{A}'$ … Ω' … Ω' … $\mathcal{P}(\Omega')$ … Ω' …

$$\mathcal{A}' = \mathcal{P}(\Omega')$$

The Probability Measure P … $\omega \in \Omega'$ … $E \in \mathcal{A}'$ … E … Ω' …

$$P(E) = \frac{|E|}{|\Omega'|} = \frac{|E|}{6^6}$$

… $(\Omega^6, \mathcal{P}(\Omega^6), P)$ …

3 Step 2: Defining the Event of Interest

Ω'

$$A = \{\omega \in \Omega' \mid \exists j \in \{1, \ldots, 6\} \; \omega_j = 0\}$$

$|A|$

4 Step 3: Using the Complement Rule

A A^c A A^c

$$A^c = \{\omega \in \Omega' \mid \forall j \in \{1, \ldots, 6\}, \omega_j \neq 0\}$$

A

$$P(A) = 1 - P(A^c)$$

A^c

5 Step 4: Calculating the Probability of the Complement

$P(A^c)$ $|A^c|$ A^c $\{1, 2, 3, 4, 5\}$

$$|A^c| = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6 = 15,625$$

$$P(A^c) = \frac{|A^c|}{|\Omega'|} = \frac{5^6}{6^6} = \left(\frac{5}{6}\right)^6$$

6 Step 5: Final Calculation

$$p = P(A) = 1 - P(A^c) = 1 - \left(\frac{5}{6}\right)^6$$

$$p = 1 - \frac{15,625}{46,656} = \frac{46,656 - 15,625}{46,656} = \frac{31,031}{46,656} \approx 0.6651$$

Check Your Understanding

7 Summary and Takeaways

$(\Omega, \mathcal{A}, P)$

$A \subseteq \Omega$

In-Depth Explanations

[1] Power Set (Definition A.3) [illegible] $\mathcal{P}(\Omega)$ [illegible] 2^{Ω} [illegible] $\emptyset$ [illegible] Ω [illegible] n [illegible] 2^n [illegible] $\Omega = \{a, b\}$ [illegible] $\mathcal{P}(\Omega) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ [illegible] Ω [illegible] σ [illegible]

[2] Laplace Probability / Uniform Distribution (Example 1.36 (i), 1.37) [illegible] Ω [illegible] $\omega \in \Omega$ [illegible] $p(\omega) = 1/|\Omega|$ [illegible] $\omega \in \Omega$ [illegible] $A \subseteq \Omega$ [illegible]

$$P(A) = \sum_{\omega \in A} p(\omega) = \sum_{\omega \in A} \frac{1}{|\Omega|} = \frac{|A|}{|\Omega|}$$

[illegible] A [illegible] Ω.

[3] Complement Rule (Derived from Definition 1.1 & 1.18) A Ω A^c Ω A ‹$A^c = \Omega \setminus A$› $\ast$ A A^c ‹$A \cap A^c = \emptyset$› ‹$A \cup A^c = \Omega$› ∇

- $P(\Omega) = 1$
- ∇ $P(A \cup A^c) = P(A) + P(A^c)$ σ

$P(A) + P(A^c) = P(A \cup A^c) = P(\Omega) = 1$

$$P(A) = 1 - P(A^c)$$