From Election to Interpolation: Ballot Completion in Ranked Choice Voting

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Acknowledgements

I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class (for LaTeX) and the R bookdown package, in general.

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Abstract

One of the arguments against implementing ranked-choice voting (RCV) is that RCV is harder for voters to participate in. Two of the reasons for this are the more complicated ballot design and the extra effort that goes into forming a ordered preference of candidates. To evaluate this claim, we examine rates of ballot errors and undervoting (ranking fewer than the allowed number of candidates) in some American elections conducted with RCV. Results show that idk yet.

Second paragraph of abstract starts here.

Dedication

You can have a dedication here if you wish.

Introduction

Welcome to the *R Markdown* thesis template. This template is based on (and in many places copied directly from) the Reed College LaTeX template, but hopefully it will provide a nicer interface for those that have never used TeX or LaTeX before. Using *R Markdown* will also allow you to easily keep track of your analyses in **R** chunks of code, with the resulting plots and output included as well. The hope is this *R Markdown* template gets you in the habit of doing reproducible research, which benefits you long-term as a researcher, but also will greatly help anyone that is trying to reproduce or build onto your results down the road.

Hopefully, you won't have much of a learning period to go through and you will reap the benefits of a nicely formatted thesis. The use of LaTeX in combination with *Markdown* is more consistent than the output of a word processor, much less prone to corruption or crashing, and the resulting file is smaller than a Word file. While you may have never had problems using Word in the past, your thesis is likely going to be about twice as large and complex as anything you've written before, taxing Word's capabilities. After working with *Markdown* and **R** together for a few weeks, we are confident this will be your reporting style of choice going forward.

Why use it?

R Markdown creates a simple and straightforward way to interface with the beauty of LaTeX. Packages have been written in **R** to work directly with LaTeX to produce nicely formatting tables and paragraphs. In addition to creating a user friendly interface to LaTeX, *R Markdown* also allows you to read in your data, to analyze it and to visualize it using **R** functions, and also to provide the documentation and commentary on the results of your project. Further, it allows for **R** results to be passed inline to the commentary of your results. You'll see more on this later.

Who should use it?

Anyone who needs to use data analysis, math, tables, a lot of figures, complex cross-references, or who just cares about the final appearance of their document should use *R Markdown*. Of particular use should be anyone in the sciences, but the user-friendly nature of *Markdown* and its ability to keep track of and easily include figures, automatically generate a table of contents, index, references, table of figures, etc. should make it of great benefit to nearly anyone writing a thesis project.

Chapter 1

What is ranked choice voting?

Ranked choice voting (RCV), also known as the alternative vote (AV) or instant-runoff voting (IRV) is an alternative voting method to the first-past-the-post (FPTP) or "plurality" election system more familiar to American voters, where the candidate with the most votes wins. Each voter, instead of choosing their highest preference among a set of candidates for an office, ranks some subset of the candidates in order of preference. This system (or a close variant) is used in Australia, Maine, and some American municipalities: San Francisco, CA; Minneapolis, MN; and Cambridge, MA; among others.

The single-winner RCV tabulation algorithm generally proceeds as follows:

1. For each voter, identify their most preferred candidate that has not yet been eliminated. Count up these preferences by candidate.

2. If one candidate has a majority (50% + 1) of the unexhausted votes, they are declared the winner and counting stops.

3. The candidate with the lowest number of votes is eliminated.

4. The ballots counted for that candidate are each transferred to the voter's next choice if one exists, or if one does not exist the ballot is "exhausted" and removed from counting for further rounds.

5. Return to 1.

Most jurisdictions that use RCV have slightly different rules for edge cases and ballot errors, but this algorithm is what distinguishes RCV from other ranked voting systems (e.g. Borda, Condorcet, Contingent, etc.). A close variant of RCV is the single transferrable vote (STV) method[1], which can be used to elect multiple candidates, i.e. for a school board, instead of just one. In the US, this is used in Cambridge, MA and Minneapolis, MN to elect multi-member offices.

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[1]More accurately, RCV is the single-winner implementation of the STV algorithm.

1.1　Frequently Used Terms

Below are some definitions for frequently used terms later on. These are not all ubiquitous (for example, "undervote" has another meaning in most voting research), but we define them here for clarity later on.

- *Overvote*: when a voter ranks multiple candidates in the same slot. This slot is typically thrown out entirely in counting, because it's often not possible to determine which candidate was preferred.

- *Undervote*: when a voter does not rank candidates in all of the slots available to them. This is different than other definitions of "undervote", which refer to a voter participating in one election on a ballot but not another one. This is not a problem in counting, and is explicitly allowed in the laws of most jurisdictions. A plurality election analog would be voting in high-profile races (e.g. presidential), but not down-ticket decisions (e.g. local water board).

- *Skipped vote*: when a voter ranks no candidate at slot x, but ranks a candidate at slot $y > x$. This is typically not a problem in counting, but different jurisdictions have different rules about whether a voter's ballot is exhausted at this point or continues on to their next ranked choice. Plurality voting has no analog to this, because each race only has one "ranking" (first!).

- *Duplicated vote*: when a voter ranks the same candidate for distinct slots x and y. This is typically not a problem for counting, and the first ranking for the candidate is used. Similar to a skipped vote, plurality voting has no analog to this.

- *Ballot exhaustion*: as ballot counting progresses, some ballots will become "exhausted" when all the candidates selected are eliminated. Suppose the final count in an election is between candidates B and D, and a voter ranked candidates C-A-E. Their ballot would not be counted in this final round, as they expressed no preference for either candidate B or D. An analogous situation in a plurality election might be voting in the general election but not a runoff, that is only having a say in part of the election.

Over-, skipped, and duplicated votes are really only interpretable as "ballot mistakes": for example, even if a voter truly prefers two candidates equally, the ballot instructions (should) make it clear that ranking them at the same slot is not allowed.

1.2　Claims about RCV

There are plenty of arguments both for and against implementing RCV in place of plurality in different jurisdictions (see the literature review), but here we'll focus on evaluating one major argument against it - RCV is harder for voters to participate in than a plurality system. There are two major reasons cited for this:

- The physical design of an RCV ballot is usually more complicated than a plurality ballot, because there has to be a system to encode a more full preference among the candidates than just selecting one candidate

- The process of forming a multi-candidate preference inherently takes more mental energy than just choosing a favorite candidate

The first facet of this argument should be reflected in ballot errors made by voters. Compared to plurality voting, we expect more errors in an RCV ballot just because the ballot is more complicated. There are also more potentials for error in the RCV system generally. The only "errors" in a plurality ballot are incompletely marking a candidate (think incorrect Scantron bubbling, or hanging chads) or overvoting, both of which are potential pitfalls for a ranked choice ballot as well. On top of these, there are the potential errors of duplicated and skipped votes unique to ranked ballots[2].

The second facet should be reflected in incomplete ballots filled out by voters. Given that they understand how to encode their preferences on the ballot, there is still the non-trivial task of forming such a preference. Structurally, some of the factors that should affect this incompleteness are:

- The number of candidates running for a position

- The number of candidates voters can rank

- The number of seats elected in a given race

This first variable is at the election level (different for every election), the second is at the jurisdiction level, and the third is a mix of both. For a clear example of these differences, consider a 2016 San Francisco Board of Supervisors race (District 3) versus a 2017 Cambridge City Council race.

Factor	San Francisco 2016	Cambridge 2017
Candidates running	2	27
Candidates rankable	2 (Generally, up to 3[3])	27 (Generally, all)
Seats elected	1	9

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[2]These types of errors are not uniform, and some jurisdictions are more forgiving than others about rules for counting these errors. While it may be apparent that a voter who listed the same candidate 3 times (A-A-A) prefers that candidate, a candidate ranking of A-B-A is harder to extract a clear preference from. Skipped votes are where we see the most variance in jurisdiction counting rules: if a voter marks the ballot A-___-B, skipping the second slot, some jurisdictions will ignore the skip and treat B as the voter's second choice, while others will stop counting after A is eliminated (ignoring their vote for B), and others yet will throw out the ballot entirely.

[3]This is changing in 2019 to be up to 10.

1.3   History of RCV in the US (SF in particular)

In the United States, there have been two major periods of RCV implementation in various jurisdictions. Between 1915 and 1950, 24 American cities chose to institute RCV as a form of local election. By 1965, however, all of these except for Cambridge, MA had eliminated the policy change. Then, in the 2000s, there was a resurgence of uptake in a different set of American cities[4], including Minneapolis and a handful in the San Francisco Bay Area. While Cambridge has consistently used the multi-winner (STV) method to elect City Council and School Board seats, the modern resurgence of RCV almost universally deals with single-winner elections. Research argues that RCV appears in jurisdictions where there is strong multi-party support for the reform - the RCV method itself gives individual parties less power in the election process, so powerful single parties usually don't have reason to support it.

1.4   Why, or why not, implement RCV?

There are plenty of arguments on both sides of implementing RCV in jurisdictions that consider it.

1.4.1   Pros

No secondary elections

There are two major types of "secondary elections" used in American voting: primary elections and runoff elections. Primaries are used by political parties to select their nominee for a general election, so the voters of any one party aren't split between different candidates. Runoffs are most often used when no candidate in the general election surpasses 50% of the vote total. Typically the top two candidates from the general election[5] advance to a later runoff. These secondary elections face two main challenges: low turnout and high cost.

Secondary elections as a whole face low turnout (Wright, 1989; Ranney, 1972). Reasons: Research shows that people don't actually like voting that much - the more frequently elections are held, the lower turnout will be for all of them generally (Boyd, 1986). Secondary elections increase the number of elections in a period, so this is one possible reason why they generally have low turnout. Further research indicates that holding elections concurrently with a presidential election "increase[s] the likelihood that citizens will vote" (Boyd, 1986). This is seen in off-year Congressinoal elections, where turnout drops from presidential years. Typically general elections are held concurrently with presidential elections (second Tuesday in November, super high media coverage, lots of voter outreach, yadda yadda), so secondary elections cannot be held at the same time as a presidential election and they should thus suffer in

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[4]Mostly in the American West: there are 9 cities west of the Mississippi River currently using RCV and only 4 east of it

[5]Or primary election - Seven Southern states require primary winners to obtain 50% of the vote to get on the general election ballot, and some other states have a requirement of 40%. (WaPo article)

turnout. This low turnout has consequences for representation in the system. The same research (Ranney, 1972) finds that while primary voters are not ideologically unrepresentative of general election voters, they are both demographically unrepresentative and unrepresentative on some major issues. Traditional knowledge holds that primary voters are more committed partisans than general election voters, leading the eventual candidates in a general election to be polarized away from the "center" of political ideas (double check this but I'm pretty sure the cite is Hill's *Instant Runoff Voting*).

The higher costs associated with secondary elections are a little more intuitive than turnout issues - it takes money to hold elections. Pollworkers have to be paid, facilities have to be reserved, and candidates have to do more campaigning. A 2011 City Council runoff in Plano, TX cost the city an extra $73,000 (Plano Star Courier, 2011). A 2012 Alabama runoff for multiple seats cost the state about $3 million.

Since RCV eliminates the need for primary and runoff elections while still ensuring majority rule (which is the main reason for these elections), it should avoid the problems of lower turnout and higher costs associated with secondary elections[6]. Summed up, RCV decreases costs and "boosts turnout via elimination of low-turnout elections" (Morales, 2018).

Ensures majority rule

In jurisdictions without rules for 50% minimums, a common phenomenon is a candidate winning an election with less than 50% of the vote (a plurality, rather than a majority). The major conceptual issue with this is that more people preferred a candidate other than the one who was elected[7]. RCV requires that a winning candidate receive at least 50% of the votes remaining[8], ensuring that a majority of voters prefer the elected candidate to other candidates.

This is particularly important in jurisdictions (like Maine) with strong third-party support and more than two viable candidates. Former Maine Governor Paul LePage, a Republican, won his first election in 2010 with 38.1% of the vote, compared to Independent Eliot Cutler's 36.7%[9].

Reduces strategic voting - spoiler candidates and third parties

Strategic voting[10] is a scenario where a voter does not reflect their true preferences on their cast ballot in order to affect the outcome of the election. For example, a third party supporter may cast their vote for one of the two major party candidates because otherwise they feel like "it won't count".

The "spoiler effect" is when a third party candidate draws votes away from the ideologically closest major party candidate, thus contributing to the election of the

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[6]Or at least some of it - costs overall are indeterminate (Rhode comference paper)

[7]The 'ideal' for electoral systems is the Condorcet condition: the candidate elected should beat all other candidates in one-on-one contests.

[8]See below for issues with this 'remaining' concept.

[9]a margin of about 7,500 votes. Democrat Libby Mitchell received 19%.

[10]Also known as "tactical" or "insincere" voting.

other major party candidate. The most recent large-scale accusation of this was in the 2000 election. Green Party candidate Ralph Nader drew about 3% of the national vote, more than the margin of victory for George W. Bush over Al Gore[11]. In the especially consequential state of Florida, Nader took 1.6% of the vote: almost 200 times greater than the margin between the two major party candidates of less than .01 percentage points[12] (source from FEC). Many believed that Nader, generally seen as more liberal than the Democrat Gore, drew votes from the Democratic base that would have helped Gore win the election otherwise. While research into third-party voters casts some doubt on this theory's applicability in 2000 (Herron, Lewis)[13], public opinion still rests on the idea that Nader cost Gore the presidency[14]. In fact, a pro-Republican PAC aired campaign ads promoting Nader in Democratic states in an attempt to pull votes from Gore (Meckler, 2000).

One of the reasons for major party voters to support RCV is that is avoids this spoiler issue. Under a plurality system, voters are discouraged from voting for third-party candidates because it could help elect their least-preferred of the two major candidates - "the greater of two evils", so to speak. Under RCV, however, since voters can be heard throughout multiple rounds across separate candidates, third-party voters can vote for their most preferred candidate, then still have their vote count for a better major-party candidate if their first option is eliminated.

Conversely, one of the reasons for third-party voters to support RCV is that it helps third party candidates get elected. Voters can ignore this aforementioned facet of strategic voting[15] and select their truly preferred candidate. Third-party supporters who were worried about the spoiler effect, then, can vote for their true preference of a third party and not inadvertently help a less-preferred major candidate get elected. As people abandon this strategy, third parties will receive more votes from people no longer worried about the spoiler effect, and this could get third party candidates elected.

Disincentivizes negative campaigning

Ranked choice voting should incentivize candidates to avoid negative campaigning. In a plurality election, since candidates don't care about voters who are committed to their competitors, a well-thought out negative campaign will only ostracize voters who were never going to support another candidate in the first place, and perhaps bring

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[11]Admittedly, the margins are less clear-cut than this at the state level, where the margins actually matter for the Electoral College.

[12]Not to point fingers at Nader alone in this case - while he was the most popular third party candidate by far, all 8 official third-party candidates received more votes than the major-candidate margin of only 537 votes

[13]In short - while Nader's Florida voters potentially would have broken enough for Gore to put him over the top, this was more a factor of the unusually close margin between the two major candidates than anything that Nader aided in particular.

[14]One of the sections of Nader's Wikipedia page is entitled "Spoiler controversy" in regards to this election.

[15]Or any strategic voting - while not impossible, it's infeasible to vote strategically under RCV (Bartholdi and Orlin, 1990).

more swing voters to their side. Under RCV, however, alienating another candidate's voters could backfire in the event that candidate is eliminated and these voters decide to support your opponent in the next round, causing your defeat . Research supports this - a 2016 study showed that voters in cities using RCV are more satisfied with campaigns than in cities who use plurality methods, and consider the campaign to have a less negative tone overall (Donovan et al.). In San Francisco's first RCV election, there were joint fundraisers between candidates, and one district even saw regular "Candidates Collaborative" public meetings between many candidates to discuss issues affecting the district, where "the setting [was] decidedly congenial" (Murphy, 2004).

An interesting case study of this phenomena is in the 2018 San Francisco mayoral election . There were three frontrunners heading into election day, all incumbent members of the city's Board of Supervisors: London Breed, Jane Kim, and Mark Leno. As polls showed Breed ahead about a month before the election, Kim and Leno held a joint press conference to endorse the other as voters' second choices. By drawing second-choice votes from the other candidate, the remaining candidate hoped to overcome the gap between them and Breed. In the actual election, the standing when it came time to proceed to the final round of counting was 102,767 for Breed, 68,707 for Leno, and 66,043 for Kim. While a significant proportion of Kim's voters transferred to Leno after her elimination, in the final round Breed surpassed Leno by about 2,000 votes.

Though it's outside the scope of this research to tell if this cross-endorsement was effective[16], there is some evidence in favor of this theory. Leno received almost 70% of the votes previously counted for Kim compared to Breed's 20%, bringing Breed's final margin of victory down to only 1 percentage point. In previous rounds of the election, no single candidate ever received more than 35% of the transferred votes from an eliminated candidate[17], so this is at least an unusual observation.

Minority representation

While much of the American literature on minority representation under voting systems has focused on gerrymandering and single- versus multi-member districts, there is some study of representation under RCV independent of the number of seats up for election.

John, Smith, and Zack (2018) find an increase under RCV in the number of racial/ethnic minority candidates who run in the Bay Area, and an increase in the chance that women and minority women win their election. Their theory for this result is that RCV lowers the barrier to entry in an election, making it more feasible for minority candidates to campaign, and that women (particularly women of color) are better suited in general to the less negative campaigning and coalition-building that RCV promotes in candidates.

Because minority voter turnout in secondary elections is significantly worse than

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[16]Other confounding factors counld exist: maybe Kim and Leno had similar enough positions that this scenario would have happened without the endorsement, maybe this number is only significant because in the final rounds there were only 2 candidates for second choice votes to flow to, etc.

[17]Except for round 2, where all 3 votes for the same write-in candidate transferred to Breed.

white turnout (moreso than in a general election), the elimination of these sec-
ondary elections should increase the relative say of minority voters in elections overall
(Callaghan, 2017).

Turnout improvements

Voting system reform advocates claim that these improvements to the voting process
will boost turnout by generally increasing public trust in the effectiveness of elections.
While there are many different strategies employed by campaigns and advocacy groups
to boost turnout, these methods don't have as much effect as changing methods to
ranked voting. Phone and direct mail get-out-the-vote campaigns "typically [yield]
less than 1 percentage point boosts in voter turnout", and while in-person efforts are
better they are less efficient at contacting voters.

Another argument is a little more complicated to reason through - "[IRV] boosts
turnout via elimination of low-turnout elections". While we typically think about
turnout in terms of the number of people voting across similar elections (e.g. change
in turnout from 2010 to 2014 in Congressional races), RCV improves turnout by
(allegedly) increasing the number of people who get to participate in an election overall
compared to the number of people who participate in typically low-turnout primary
and general elections. (all Morales, 2018)

1.4.2   Cons

What is a majority, anyway?

Under RCV, the problem of ballot elimination can result in "majorities" that actually
aren't. Since it's not always possible to voters to rank every candidate[18], there will
almost certainly be some number of voters who did not list a ranking for any of the
candidates still remaining in the final round of the election. Thus the majority that
the winner has collected is only a majority of the unexhausted ballots, which may
make it less than 50% of the total counted ballots (Petrangelo, 2013). This does
introduce a less expansive form of the spoiler problem - while voters aren't "punished"
for putting a less electable candidate for their first choice, they are punished for filling
all three available slots with minor candidates. In a 2015 study of four RCV elections,
Burnett and Kogan found that all four had enough eliminated ballots to only give
the winner an overall plurality, not the majority sought after. This is a problem that
requires a substantial fix, as "even individuals who mark three distinct choices often
face the prospect of exhaustion, so education alone will not fix the problem" (Burnett
and Kogan 2015, p. 49).

A research question that might address this issue of majorities is whether the
pluralities generated by RCV are generally "bigger" (closer to the ideal of a true
majority) in some way than under FPTP, and how this is affected by technical rules

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[18]This may be disallowed, as SF only allows you 3 rankings (due to technical limitations), or it
may just be infeasible - Cambridge often has 20+ candidates on the ballot.

like the number of candidates that voters are allowed to rank. Some research[19] shows that RCV does perform better than a two-round runoff in this case - as a percentage of the voters in the first round, RCV consistently has more voters counted in the final round compared to top-two runoffs (Richie, Brown, 2017).

Legal challenges

(All sourced from Maine Legislature) In addition to the political hurdle of passing RCV legislation[20], there are legal challenges as well. In the aftermath of Maine's recent ballot initiative instituting RCV, legal questions were brought to the state Supreme Court by the State Senate, due to conflict between the initiated bill's language about a "majority" versus the state constitution's language requiring a "plurality". There was a ruling from the court that resulted in an amended law instituting RCV for only primary elections, but a "People's Veto" later occured through initiative that removed the law, followed by a later suit by the state Republican Party to block RV. Maine has so far used RCV in 2018 for both the primary and general elections to elect certain offices.

This anecdote examplifies the issue - while public support may be behind RCV in jurisdictions that enact it, legal challenges abound. A (failed) legal challenge was brought against RCV in San Francisco by a defeated candidate in 2011 (Hawkins, 2011). Another challenge (also failed) was brought by Bruce Poliquin against Maine after his loss to Jared Golden in their 2018 Senate race (Mistler, 2018). In Pierce County, WA (home of Tacoma), RCV was repealed by voters after the US Supreme Court court sided with a challenge to reinstate Washington's top two primary-runoff system. With the re-implementation of the top two primary, there was no need to have an RCV election (between two candidates) and the measure was repealed (Eberhard, 2017).

More complicated tabulation

By its nature, the tabulation of RCV elections is more complicated than counting results in plurality races. There are multiple rounds, so counting takes longer. Typically ballot counts are done at the precinct or county level, and then numbers are sent to a state elections office for full tabulation and verification of the results. Under RCV, however, the process typically requires all ballots to be sent (physically or electronically) to the state office for the multiple rounds of counting, further increasing the time needed to count. Additionally, if the jurisdiction is unable to obtain software to count ballots, hand counting of the ballots is necessary. This increased time in counting is an issue because "The elapsed time between Election Day and providing some results is one of the critical factors to maintaining voter engagement and trust in the process" (League of Women Voters of Maine, Comments on Proposed Rules for RCV).

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[19]from the pro-RCV advocacy group FairVote, admittedly

[20]Neither major party is particularly interested in pushing the issue, and Republicans in particular are often opposed to it (Woodard, 2018).

In jurisdictions with some form of electronic voting, the cost of transitioning to RCV can be a disincentive as well. Not all jurisdictions have hardware capable of conducting an RCV election, and of those that do the vendor (typically jurisdictions contract to election system vendors) may not currently have software that can tabulate the results of the election. Getting around these problems is significant, as it requires either a technology upgrade[21] or a switch to some combination of paper ballots and hand-counting. (Morales and Eberhard, 2017)

Less intuitive rules

The plularity idea, while clearly not the only voting rule out there, is one of the most simple. One of the problems with RCV is that it doesn't always satisfy this idea simply - RCV does not always agree with the plurality method on choosing a winner. In the Senate race in Maine between Bruce Poliquin and Jared Golden, while Poliquin was on top at the end of the first round of counting, neither had 50% and Golden took the lead (and the election) after other candidates were eliminated and their votes transferred to voters' second choices (Portland Press-Herald). The more complicated process and this seemingly unfair (at first glance) rejection of plurality means that voters in general have a harder time understanding and trusting the results of an RCV election. This complication leads to an increase in overvotes (Kimball), which causes more ballots to be fully or partially rejected in the counting process. RCV also sees increased costs (particularly for the first election conducted) in voter education - In Minneapolis' first RCV election, 30% of the estimated $365,000 additional cost[22] of RCV was spent on voter education (Kimball, 2010).

Three types of errors - fatigue (intentionally opting out, or accidentally skipping a portion of the ballot), confusion, and lack of information to form a preference (Neely and Cook, 2008).

Lack of true adoption by voters, only listing first choice

While RCV provides the opportunity for voters to rank multiple candidates, not all voters will do so[23]. This is entirely reasonable - forming a complex preference between multiple candidates is naturally more difficult than selecting a favorite from the same set. This phenomenon of undervoting, however, means that some ballots become exhausted in the course of tabulating the election and some voters don't have a say in the final round of the election. If a voter only lists their first choice and skips the second and third choices, they lose out on two extra rounds of tabulating where their vote still counts towards the decision.

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[21]This is not a problem unique to RCV - election technology across the country is aging and in need of replacement (NCSL, Funding Elections Technology).

[22]About a third of this total was one-time costs for implementing the new system.

[23]Which is fully legal - nobody is forced to vote.

1.5   Research into SF?

The San Francisco Bay Area in particular has been an RCV hotspot in the modern American resurgence, so some good literature exists specifically studying the impact of RCV there.

Dennis (2004) finds higher rates of overvotes in SF's first RCV election (November 2004) than were expected, somewhat positively correlated with the number of candidates on the ballot in each district. Almost a third of the ballots contained an undervote. Looking at racial and ethnic groups, while Hispanic voters were more likely to find the RCV method easy to use than Asian voters, the former were also more likely to have completed their ballot incorrectly. One possible explanation of this phenomenon is that advocacy groups "succeeded in heightening awareness about the new voting system amongst the Asian community" prior to the election, increasing their wariness of it as well.

McNeely and Cook (2008) extend this basic analysis of ballot errors to explore some theories related to racial and ethnic divisions (and others). There was a significant decrease in undervoting among racial and ethnic minority groups across 4 years of elections, and a significant increase in undervotes among women as well. Districts with more candidates[24] and more campaign spending were less likely to have undervotes. The likelihood of ranking all 3 available candidates was affected by racial and ethnic categories, but not as much as factors such as prior exposure to RCV and available information (campaign spending, number of candidates).

SF Examiner article (from Elections employees, fairly) about how good it's going - lower costs, more minority candidates/officeholders, high turnout, low over/undervoting rate, low number of exhausted ballots

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[24]These also had more overvotes, in keeping with Dennis' findings.

Chapter 2

Methods and Structure

Inevitably, the data format that would be ideal to conduct this research doesn't exist publicly (as it shouldn't - voter anonymity and such). The ideal model might be a logistic or other classification model that predicts whether a given voter has a ballot error or fills out the ballot incompletely, given their demographic qualities.

2.1 Data Structure and Source

2.1.1 Election-specific data

The data used comes from two main sources. From the San Francisco Elections Office , we have a cast ballot record of the election in question (source?). This is presented by the city as two text files: - The ballot image, a 45-character fixed-width file with fields corresponding to election, candidate, unique voter number (anonymized and disconnected from any voter registration ID), ranking, precinct, and other information. Each of these is encoded numerically, so each line of the file appears as a 45-digit number. - The lookup table, an 83-character fixed-width file that defines the encodings used to create the numerical values in the ballot image.

In the ballot file, each voter is spread across three rows of data (one for each ranking). We use the `clean_ballot()` function from our `rcv` R package to read in the data for easier manipulation. From this data we have the full ranked preference set of candidates (up to 3) from each voter.

Since the voter-level data is fully anonymized, we have no demographic information at the individual level. For any given voter, since the most identifying piece of information we have about them is their precinct, it is impossible to know any one voter's gender or ethnicity. To gain insight into these demographic trends, we instead aggregate up to the precinct level.

At the precinct level, we can now only study the rates of these ballot phenomena (as opposed to an individual's response). For example, in Precinct 1703, we may see 14% of voters undervote. Rather than building a classification model (error / no error), we can now instead build regression models with a numerical dependent variable - the rate of ballot error. Moving up this level of abstraction does remove some granularity from the model (inference and prediction at the precinct level is less specific than

at the individual level), but this is the least we can do and still be able to access demographic information.

2.1.2 Demographic data

Edit this paragraph because it's not right - ACS, no MOE, etc. Now that we have precinct-level rates, we need to obtain precinct-level demographic information in order to build a model. Our source of demographic data is the 2010 U.S. Census as part of the 2018 Census Planning Database (source? Also map source). While this data is slightly out of date (about 8 years), there is more certainty about the accuracy of the measurements collected. While a more temporally accurate data set like the yearly American Community Survey (ACS) could have been used, the error margins of this data set proved too large / complicated to be useful for this study.

One consideration with this is that the voting population is not always representative of the general population. It might be more informative to obtain demographics about the voting population specifically, rather than the entire population of each precinct. However, the only information available about voters is their age (from the county voter registration file), and the discontinuity introduced by using two different data sets for demographic information is more of an "error" to me than using a less accurate (but universal) census data set.

We now encounter a problem. Since census regions (block groups, tracts) are set by the federal government, and election precincts are set by San Francisco County[1], the regions don't line up nicely[2]. Given this mismatch, how do we obtain demographic estimates for our precincts?

Stated more generally: if we have a division of a geographic region and some set of properties on the divisions, how do we estimate measures of these properties for other possible divisions?

Areal Interpolation

One field of GIS (geographic information system) research that looks into this is called *areal interpolation.* The goal of areal interpolation is to take a variable distributed over a set of "source zones" and estimate its distribution over a set of "target zones" (Schroeder, 2007). These two zoning systems must overlap at least partially to get any estimate for the target zones (e.g. information about Oregon doesn't directly tell us anything about information in Washington, under any zoning system), but are incompatible in some way[3]. When this is not possible, however, areal interpolation can help get parameter estimates for these incompatible areas.

Soem common types of areal interpolation are:

---

[1]The city and county government are unified in this case, because the county comprises entirely of the city of San Francisco.

[2]There's no inherent reason that they should, it's just unfortunate for this study.

[3]A set of "compatible" zoning systems would be something like the US Census' hierarchical systems: multiple blocks are combined to make a block group, multiple block groups are combined into a census tract, etc. Since the areas overlap neatly, you can directly add together certain numbers (like population) from block groups to get a very accurate estimate for the census tract.

- Areal weighting, using the assumption that populations are distributed uniformly on a region.

- Modified areal weighting, which creates a continuous map from region to region to better reflect changes in population density.

- Target density weighting, which uses extra information about the target zones, typically population density, to increase accuracy (Schroeder, 2007).

- Dasymetric mapping, which uses alternative data, also called *ancillary data* to produce a continuous estimate of population density (Sleeter and Gould, 2008). Examples of ancillary data include land cover information, parcel classifications, and street locations.

In this work we will use areal weighting for our interpolation. The other methods above, while typically more accurate are infeasible under the constraints of the research[4]. The assumption of uniformity is not necessarily correct, but in an urban area such as San Francisco it is more accurate than an area like the entire state of California, with a large urban/rural spread.

Areal weighting

Areal weighting first makes the assumption that populations are distributed uniformly over a space. If Region X has 100 people and we split X in half spatially, then we assume that 50 people are in each half. This also applies to sub-populations: if Region X has 20 non-White Hispanic people, then we assume each half has 10 non-white Hispanic people. This type of data, counts that can be divided into sub-regions, is called *spatially extensive* data. Extensive data is data that applies to an entire region, but not any given sub-region.

Conversely, data that applies to any given sub-region of a region is called *spatially intensive*. Properties like population density are spatially intensive under the uniformity assumption, because the ratio of population to area does not change upon examining a sub-region. Percentages are also spatially intensive - considering the sub-population as above, the percentage of non-White Hispanic people in Region X (20 out of 100, 20%) does not change when we look at one of the sub-regions (10 out of 50, 20%).

We will use the following example case to illustrate the areal interpolation process. Suppose regions A, B, C, and D are the source regions (each taking up a quadrant of the square), and regions X, Y, and Z are the target regions (each taking up a third of the square vertically). Further suppose that the boundaries of the source regions and target regions are fully coincident, that is $A \cup B \cup C \cup D = X \cup Y \cup Z$[5].

```
Reading layer `source_zones' from data source `/Users/jaylee/Desktop/thesis/data/sou
Simple feature collection with 4 features and 1 field
geometry type:  POLYGON
```

---

[4]Mostly time limitations; see Conclusion section for further research ideas.

[5]This example ignores the case where the covered regions are not coincident, which is a possibility in general. In this research we enforce coincidence in our data, however.

```
dimension:      XY
bbox:           xmin: -3 ymin: -3 xmax: 3 ymax: 3
epsg (SRID):    4267
proj4string:    +proj=longlat +datum=NAD27 +no_defs

Reading layer `target_zones' from data source `/Users/jaylee/Desktop/thesis/data/target_zo
Simple feature collection with 3 features and 1 field
geometry type:  POLYGON
dimension:      XY
bbox:           xmin: -3 ymin: -3 xmax: 3 ymax: 3
epsg (SRID):    4267
proj4string:    +proj=longlat +datum=NAD27 +no_defs
```
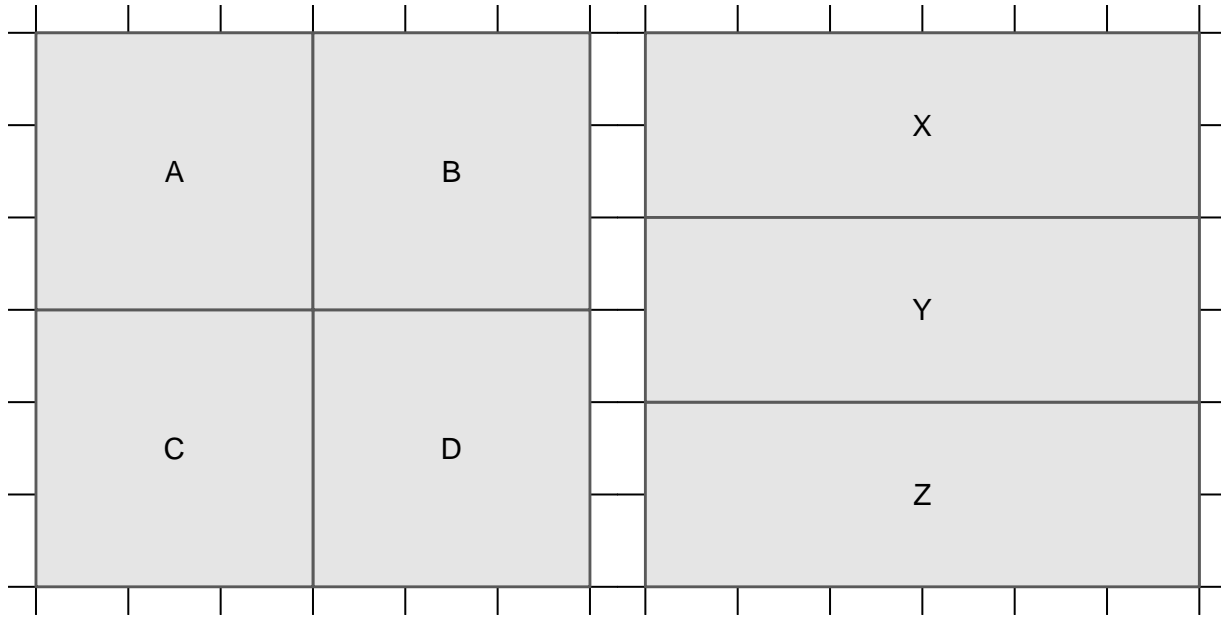
Source regions   Target regions

In general, denote a source region by S_i (over index set I) and a target region by T_j (over index set J). For any region R, denote the area of R by $Area(R)$, the measure of a given extensive property of R by x_R, and the measure of a given intensive property of R by y_R. These measures are known in the source regions, but unknown in the target regions (hence, the interpolation). Denote an estimate of a quantity with a carat, e.g. $\widehat{y_R}$.

Say we want to estimate x_{T_j}, the measure of the extensive property in region T_j. For all $j \in J$, we can estimate this property with

$$\widehat{x_{T_j}} = \sum_{i \in I} \frac{Area(S_i \cap T_j)}{Area(S_i)} \cdot x_{S_i}$$

For each source region, calculate the proportion of the region that lies inside the target region. These proportions are the weights to be multiplied by the source regions'

properties. In the example case, consider target region X:

$$\widehat{x_X} = \frac{Area(A \cap X)}{Area(A)} \cdot x_A + \frac{Area(B \cap X)}{Area(B)} \cdot x_B + 0 + 0 = \frac{2}{3}(x_A + x_B).$$

Since A and B each have $2/3$ of their area inside target region X, we estimate that $2/3$ of the extensive quantity x_A is inside region X (similarly for x_B). Adding these together gives us an estimate $\widehat{x_X}$.

For intensive properties, the process is slightly different. For all $j \in J$, we can estimate an intensive property y_{T_j} with

$$\widehat{y_{T_j}} = \sum_{i \in I} \frac{Area(S_i \cap T_j)}{Area(T_j)} \cdot x_{S_i}$$

Since the intensive property isn't "divisible" within a region, we instead take a weighted average of the component source regions of the target region, where each component is weighted by the amount of the target region it takes up. Again considering target region X:

$$\widehat{y_X} = \frac{Area(A \cap X)}{Area(X)} \cdot y_A + \frac{Area(B \cap X)}{Area(X)} \cdot y_B + 0 + 0 = \frac{1}{2}(y_A + y_B).$$

Since A and B each take up half of target region X, each of them gets weighted by half before being added to get the estimate of y_X.

To validate the application of this method to the data at hand, we can visually compare the spatial distribution of a variable before the interpolation to its distribution after the interpolation.

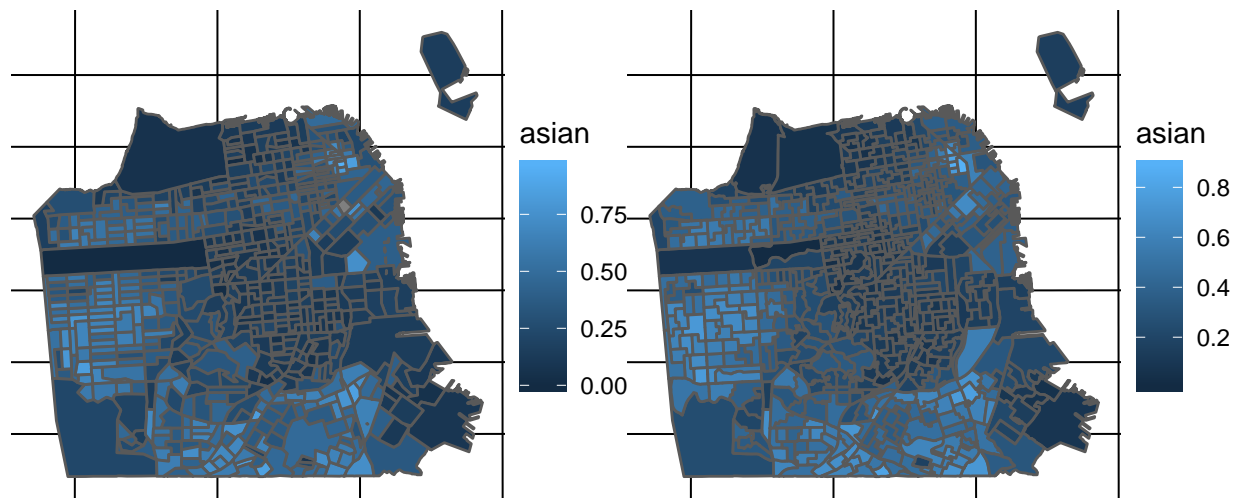Pre–interpolation   Post–interpolation

Figure 2.1: Proportion of non-Hispanic Asian residents

Interpolation - counts vs. percentages

One consideration in the data preparation stage was whether to use counts or percentages as input for the regression. The census data contains total population in a region, as well as a count and percentage for a given variable (say people between the ages of 25 and 44). The percentage can equivalently be calculated by dividing the count by the population. After performing the areal interpolation on the data, I ran this percentage calculation again to double check that it lined up with the reported percentages (post-interpolation). My intuition was that these steps should be commutative - calculating a percentage and then interpolating should have the same result as interpolating and then calculating the percentage. This was not the case, however - in the variable for population between 25 and 44, the error between these two methods ranged from -12 to 26 percentage points.

```
Warning: Removed 4 rows containing missing values (geom_point).
```
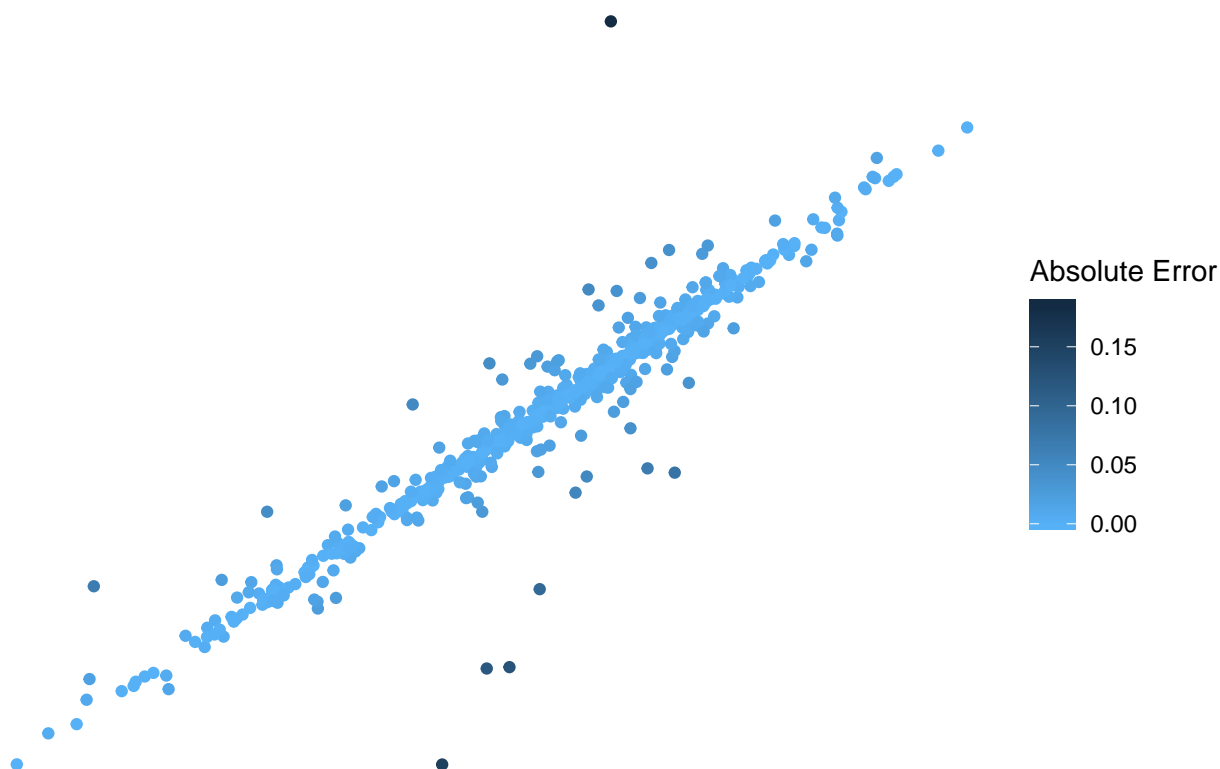

Figure 2.2: Example error in intensive interpolation

As it turns out, these steps are not commutative in this way, and the observed error is mostly a function of the weighting between different steps in the process. For example, consider a simple case: suppose source regions A and B are fully contained in target region X, and split X in half. Let the number of people between 25 and 44 in A be 4 (out of 10 total) and in B be 3 (out of 5 total). Taking the percentages first gives

us a proportion of 0.4 in A and 0.6 in B. Using the intensive interpolation method on these proportions, both are weighted by the amount of X that the region takes up (half, in each case) and added, so the average weighted by area is 0.5. Conversely, using the extensive interpolation method on the count and popuation, we see that X has 7 people between 25 and 44 (out of 15 total). Taking the percentage, we see that the proportion of this variable in X is ~0.47.

In short, the "calculating proportion" and "areal interpolation" steps are NOT commutative, because of the differences in weights when using intensive vs. extensive interpolation methods. Both are calculating a weighted mean of sorts, but the former is weighting by AREA, while the latter is weighting by POPULATION. In this case we see that the latter is more accurate - source regions with more people should have greater impact on the estimated measures in target regions, because the variables we are dealing with are human-centered rather than space-centered. As such, for this study we will interpolate only the count data (population, numbers of people for each measured variable) and then calculate proportions in the target regions after the interpolation. These proportions are better suited to the regression to ensure that variables are of the same scale and we can compare coefficient estimates.

Boundary mismatch

A further issue appears - just like the precinct and census boundaries don't line up because they come from different sources, the outside boundaries of the precinct and census files don't line up. The census boundaries have a lower resolution on the whole than then precinct boundaries. This leads to issues when performing the areal interpolation, because parts of a census boundary that are outside of a precinct will be dropped and cause an underestimate of the true measure of the variables. We address this by bounding both files to only consider the space that is contained in both[6]. Since we are dealing with a fundamental unit of people instead of space, this ensures that every person is counted, and changes some of the spatial weights calculated. The assumption here is that any region which is only contained in one of the files (precincts without census, or census without precincts) has zero population, because every person should be contained in both a precinct and a census region.

---

[6]No census tracts were fully removed in this process, but this causes one precinct, Precinct 9900, to be cut off. However, this precinct is a semi-exclave of the county on Alameda Island, across the San Francisco Bay. Since this land, an undeveloped former naval air base, is uninhabited (Levi, 2018) its removal does not impact our results.

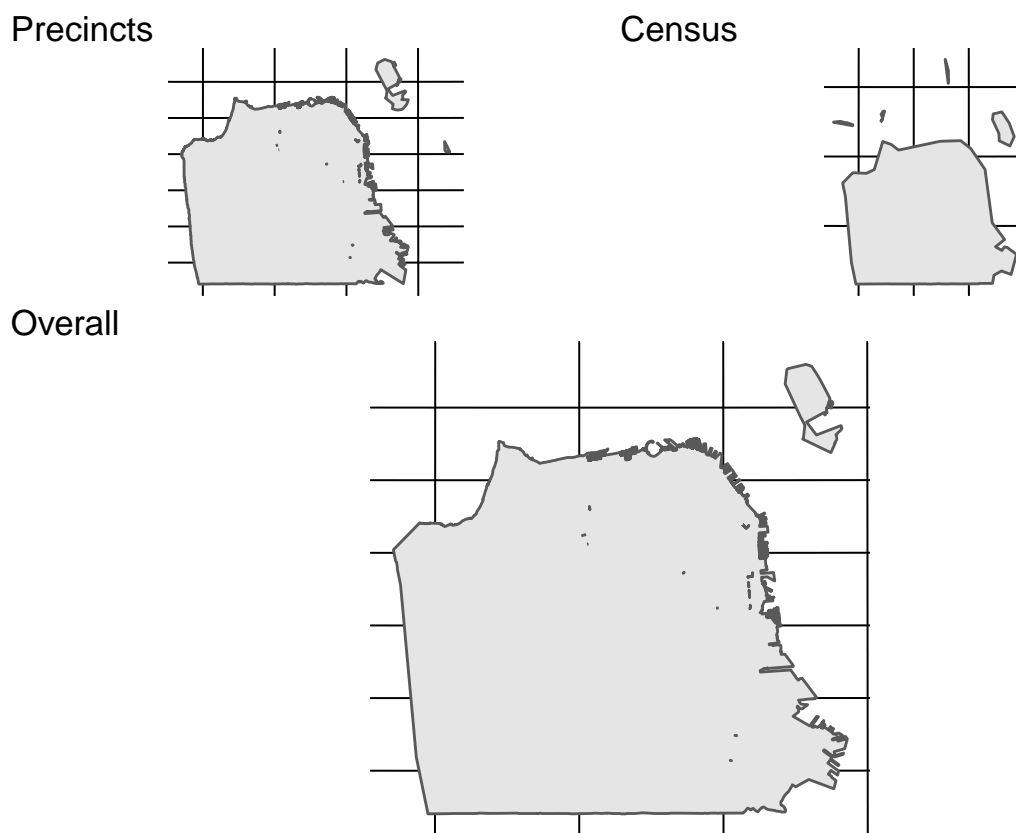Precincts   Census

Overall

Figure 2.3: Boundary intersection between shapefiles

    This plot displays the result of this intersection between the two regions. The impact on the census tracts is quite visible - notably the eastern coastline is much more defined, and the gap in Treasure Island (the large island off the northeastern shore) appears. The impact on the precinct boundaries is less apparent, but still there - the exclave on Alameda Island (small sliver to the East) has disappeared, and in the southeastern corner it is visible how the angles in the precinct boundary have softened to the more rounded final shape.

    Below is a comparison of the original shapefiles[7] (including all divisions) to the bounded versions.

---

[7]The original census regions contained the Farallon Islands, which have been removed from this plot because they are uninhabited, 30 miles into the Pacific Ocean, and messed up the scale of the graph.
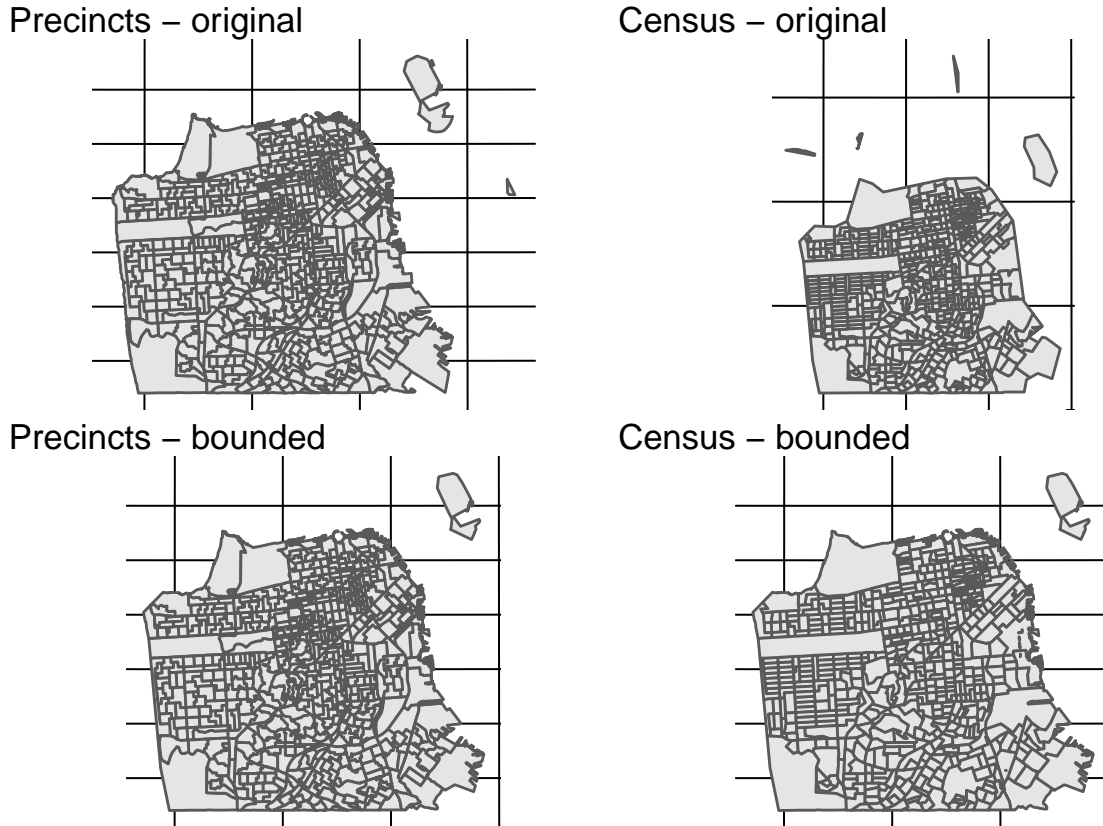
Figure 2.4: Original versus bounded shapefiles

Precinct Consolidation

One peculiarity in San Francisco is the combination of certain precincts during elections. California state law allows for counties to "consolidate" precincts with low numbers of registered voters during an election. This eases administration by not requiring counties to set up and staff a full polling place in a precinct with few voters, while still giving voters a physical polling location in their approximate area[8]. While San Francisco does this less often than other counties, there are still some precincts that are consolidated each election.

The areal interpolation process produces demographic estimates for areas in the map, however, which are non-consolidated. This causes a mismatch: we have demographic data for individual precincts, but election data for the consolidated precincts. To address this issue, we have split the election data (a count of overvotes and undervotes) in the consolidated precincts into their two component precincts. This split is weighted by the population of each precinct in 2010[9]. For example, suppose

---

[8]This method of precinct consolidation may change with the 2018 California Voter's Choice Act, which lets counties (except for Los Angeles County) move fully to vote-by-mail in combination with voting centers (California Senate Bill 450).

[9]The most simple weight would be 50-50 for each split, but this is inaccurate for many of these precincts: see the gap in the weights for Pct 7527/7528. The flaw in this method was discovered after calculating turnout of over 1000% for Precinct 7527. . .

Precinct X/Y had 100 undervotes, the population of Precinct X in 2010 was 325, and the population of Precinct Y in 2010 was 175. Then the "population split" between X and Y is 65%-35%, and we adjust the undervotes accordingly: Precinct X should have 65 of the 100 undervotes, and Precinct Y should have the remaining 35.

Examples of this in the data are below. The first table is the initial data, with full consolidated counts doubled in the precincts (and thus doubled across rows), and the second table is the data with proper weights applied.

Table 2.1: Consolidated Precincts - Original Data

Election Pct.	Pct. Number	Overvotes	Not over	Undervotes	Not under
Pct 1104/1105	1104	1	459	101	359
Pct 1104/1105	1105	1	459	101	359
Pct 7509/7511	7511	1	491	132	360
Pct 7509/7511	7509	1	491	132	360
Pct 7527/7528	7527	5	396	98	303
Pct 7527/7528	7528	5	396	98	303

Table 2.2: Consolidated Precincts - Weighted Data

Election Pct.	Pct. Number	Weight	Overvotes	Not over	Undervotes	Not under
Pct 1104/1105	1104	0.4060452	0.4060452	186.374741	41.010564	145.770223
Pct 1104/1105	1105	0.5939548	0.5939548	272.625259	59.989436	213.229777
Pct 7509/7511	7511	0.7438036	0.7438036	365.207545	98.182069	267.769279
Pct 7509/7511	7509	0.2561964	0.2561964	125.792455	33.817931	92.230721
Pct 7527/7528	7527	0.0129859	0.0649297	5.142434	1.272622	3.934741
Pct 7527/7528	7528	0.9870141	4.9350703	390.857566	96.727378	299.065259

2.2   Calculating over/undervote info

RCV package (with Matthew) Data from the city elections board Data is missing some ballots that had to be hand-counted because it spits out straight from the machine, so this won't necessarily match the official reports.

Data comes in in weird double file form, then gets cleaned up and looks like this.

Table 2.3: Processed Ballot Image

Contest	Voter ID	1	2	3
Mayor	000012886	JANE KIM	ELLEN LEE ZHOU	MARK LENO
Mayor	000012887	JANE KIM	ANGELA ALIOTO	RICHIE GREENBERG
Mayor	000012888	JANE KIM	LONDON BREED	ANGELA ALIOTO
Mayor	000012889	JANE KIM	LONDON BREED	ANGELA ALIOTO

Mayor	000012890	LONDON BREED	JANE KIM	MARK LENO
Mayor	000012891	MARK LENO	LONDON BREED	ANGELA ALIOTO
Mayor	000012892	MARK LENO	LONDON BREED	JANE KIM
Mayor	000012893	MARK LENO	ANGELA ALIOTO	MICHELLE BRAVO
Mayor	000012894	LONDON BREED	ANGELA ALIOTO	MARK LENO
Mayor	000012895	ELLEN LEE ZHOU	MICHELLE BRAVO	JANE KIM

To summarize the data

```
[1] 0.004074546
```

```
[1] 0.2524841
```

2.3 Regressions

Methods for regression:

- Linear ?
    - Pretty simple honestly
- Logistic with the "binomial-style" input data
    - This is more useful because our output (rate) is 0-1 limited.
    - It also overweights precincts with more people - is this good? (Ask Heather)
- For whatever goes in, I need to pick a better model selection method.
- Also test / train data set
- Cross-validation?
- ex post facto model validation! The residual plots and all that. This could also go in the results section.

Chapter 3

Results

Variables considered for this are LIST/TABLE/CALCULATION

I have some results and they are here. First model is the overvoting linear model, second is undervoting linear.

```
Call:
lm(formula = overvote_rate ~ hispanic + black + no_english, data = sf_precincts)

Residuals:
      Min        1Q    Median        3Q       Max
-0.008424 -0.002606 -0.000584  0.001633  0.037469

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0018971  0.0002937   6.459 2.18e-10 ***
hispanic    0.0037156  0.0013724   2.707  0.00698 **
black       0.0153094  0.0018998   8.059 4.20e-15 ***
no_english  0.0092404  0.0015438   5.985 3.72e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003841 on 599 degrees of freedom
Multiple R-squared:  0.1728,    Adjusted R-squared:  0.1687
F-statistic: 41.72 on 3 and 599 DF,  p-value: < 2.2e-16


Call:
lm(formula = overvote_rate ~ pop_18_24 + pop_25_44 + pop_45_64 +
    white + black + no_hs + college, data = sf_precincts)

Residuals:
      Min        1Q    Median        3Q       Max
-0.009142 -0.002600 -0.000599  0.001656  0.038353
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0043119  0.0022074   1.953   0.0512 .
pop_18_24   -0.0018829  0.0029419  -0.640   0.5224
pop_25_44    0.0009263  0.0020995   0.441   0.6592
pop_45_64   -0.0002122  0.0036628  -0.058   0.9538
white        0.0033238  0.0017292   1.922   0.0551 .
black        0.0130410  0.0021664   6.020 3.06e-09 ***
no_hs        0.0067164  0.0032870   2.043   0.0415 *
college     -0.0055218  0.0023552  -2.345   0.0194 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003872 on 595 degrees of freedom
Multiple R-squared:  0.1652,    Adjusted R-squared:  0.1554
F-statistic: 16.82 on 7 and 595 DF,  p-value: < 2.2e-16
```

Writeup of said results is forthcoming because it's way past my bedtime right now. Not sure why the undervote model is the full model - nothing in it is significant, but apparently that had the best MSE. I'll check it more seriously tomorrow.

And these are logistic for over and under - still need to do training/test or cross validation, plus better model selection.

```
Call:
glm(formula = cbind(over_count, no_over_count) ~ +hispanic +
    black + no_english, family = binomial, data = sf_precincts)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.4763  -0.8224  -0.2002   0.4779   3.4130

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -5.99377    0.06082 -98.549  < 2e-16 ***
hispanic     0.94028    0.24326   3.865 0.000111 ***
black        2.49121    0.30587   8.145 3.81e-16 ***
no_english   1.86161    0.26677   6.978 2.99e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 869.52  on 602  degrees of freedom
Residual deviance: 746.04  on 599  degrees of freedom
```

```
AIC: 1952.2

Number of Fisher Scoring iterations: 5



Call:
glm(formula = cbind(under_count, no_under_count) ~ +pop_18_24 +
    pop_25_44 + pop_45_64 + hispanic + white + black + asian +
    no_hs + college, family = binomial, data = sf_precincts)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-6.0425  -0.9543  -0.0741   0.8984   5.1072

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.79707    0.17597 -10.212  < 2e-16 ***
pop_18_24   -0.82711    0.10140  -8.157 3.43e-16 ***
pop_25_44   -1.06927    0.06324 -16.908  < 2e-16 ***
pop_45_64   -0.61586    0.10886  -5.657 1.54e-08 ***
hispanic     0.58464    0.17750   3.294 0.000988 ***
white        0.97672    0.17320   5.639 1.71e-08 ***
black        1.18790    0.19900   5.969 2.38e-09 ***
asian        0.62484    0.17213   3.630 0.000283 ***
no_hs        0.64791    0.10176   6.367 1.93e-10 ***
college      0.85892    0.07075  12.140  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2460.6  on 602  degrees of freedom
Residual deviance: 1291.2  on 593  degrees of freedom
AIC: 5006.1

Number of Fisher Scoring iterations: 3
```

Conclusion

If we don't want Conclusion to have a chapter number next to it, we can add the `{-}` attribute.

More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

Appendix A

The First Appendix

This first appendix includes all of the R chunks of code that were hidden throughout the document (using the `include = FALSE` chunk tag) to help with readibility and/or setup.

In the main Rmd file

```r
# This chunk ensures that the thesisdown package is
# installed and loaded. This thesisdown package includes
# the template files for the thesis.
if(!require(devtools))
  install.packages("devtools", repos = "http://cran.rstudio.com")
if(!require(thesisdown))
  devtools::install_github("ismayc/thesisdown")
library(thesisdown)
knitr::opts_chunk$set(echo = FALSE, eval = TRUE)
# generally I don't want code to show up in the doc
# I'll just link to the (eventually public) thesis repository to find source code
# or put them as appendices? who knows
```

In Chapter ??:

Appendix B

The Second Appendix, for Fun

References

Angel, E. (2000). *Interactive computer graphics : A top-down approach with opengl.* Boston, MA: Addison Wesley Longman.

Angel, E. (2001a). *Batch-file computer graphics : A bottom-up approach with quicktime.* Boston, MA: Wesley Addison Longman.

Angel, E. (2001b). *Test second book by angel.* Boston, MA: Wesley Addison Longman.