Due: 04/07/2024 23:59

Foundations of Artificial Intelligence: Homework 2

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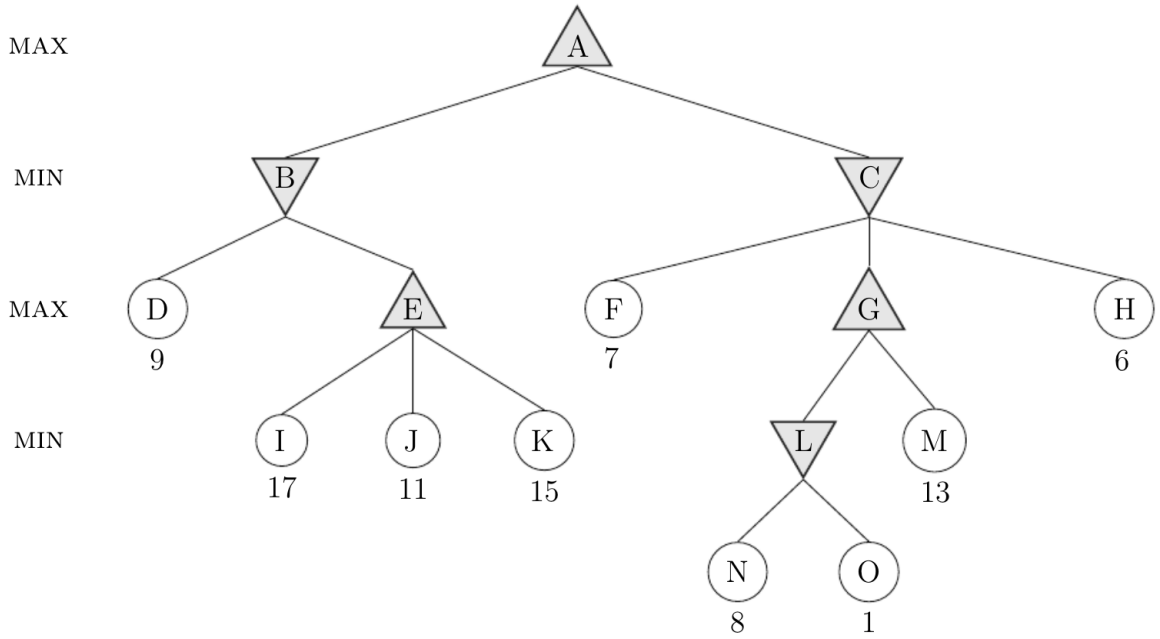
Problem 1 (10 points)

Consider the `MAX-MIN` game tree shown below where the numbers underneath the leaves of the tree are utility values from the first player's point of view (`MAX`).



a) Draw a copy of the tree on paper and perform the **minimax** algorithm algorithm on it by hand. Write the resulting minimax values next to every node

b) Do the same, but with **left-to-right alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

c) Do the same, but with **right-to-left alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

α : MAX's best option on path to root (其它 min 給出來的)

β : MIN's best option on path to root. (其它 max 給出來的)

求 min 的時候，在乎 min 的是 Max !! ⟶ 所以途中要和 α 比

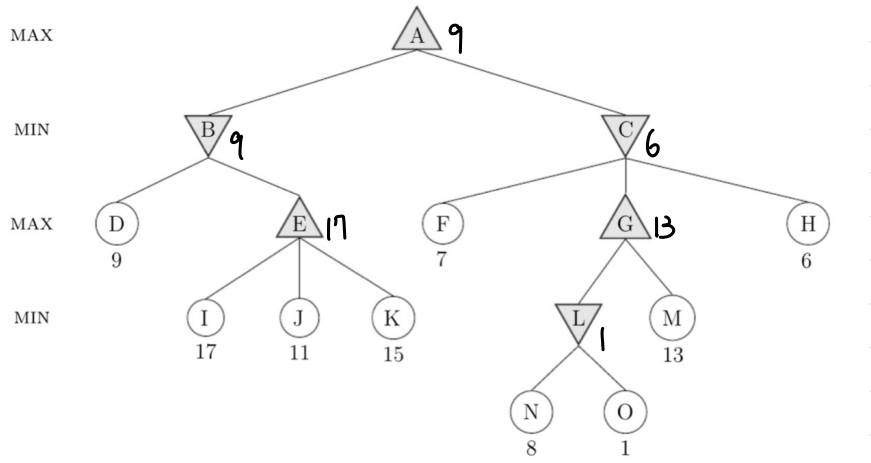
⟶ 同時記得更新 β

做 min : 先比 α (if $v \le \alpha$, ret v)，更新 β ($\beta = \min(\beta, v)$)

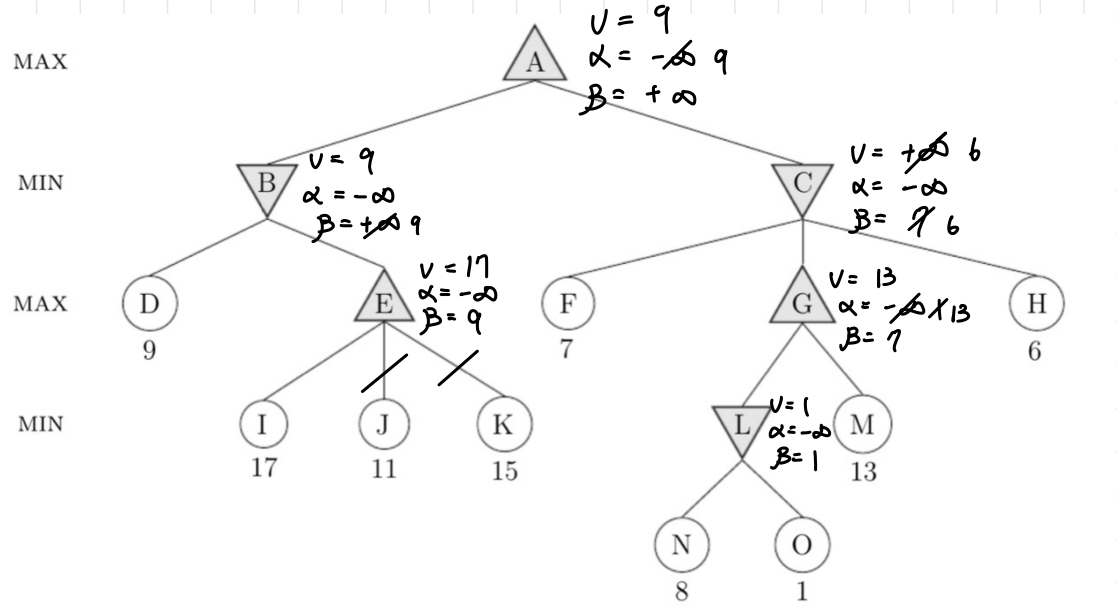
做 max，比 β (if $v \ge \beta$, ret v)，更新 α ($\alpha = \max(\alpha, v)$)

Problem 1

(a)

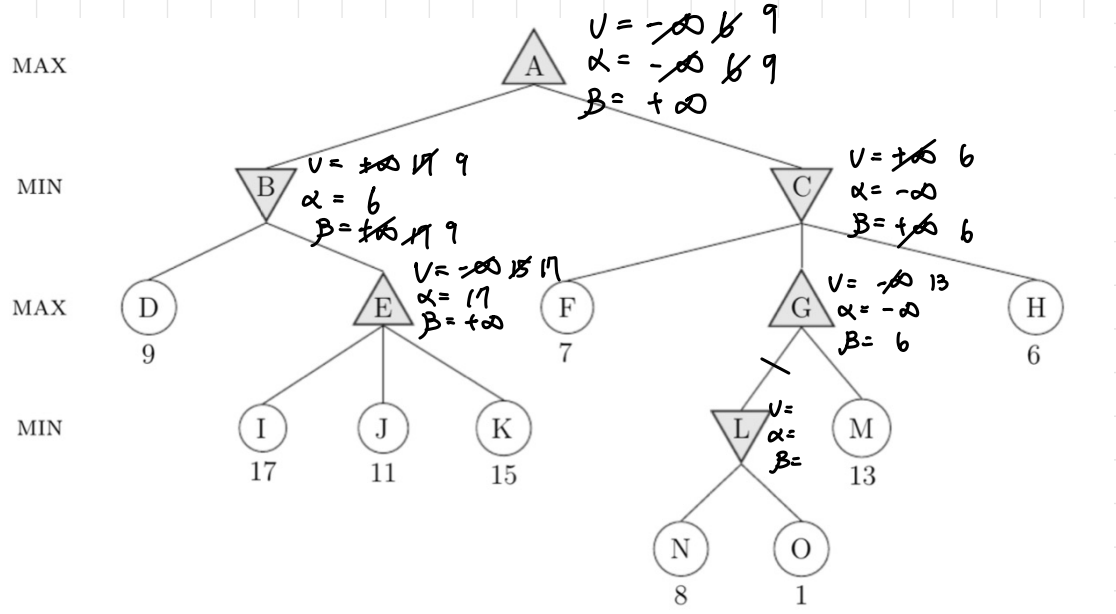


(b)



Node J and node K will not be examined because we apply prunning.

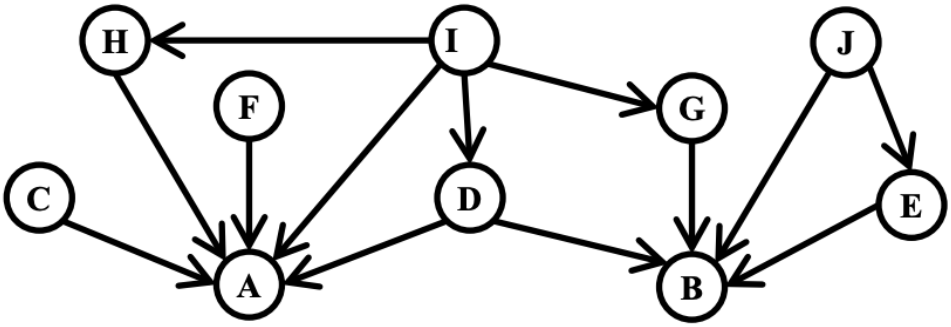
(c)



Node L, node N and node O will not be examined because of prunning.

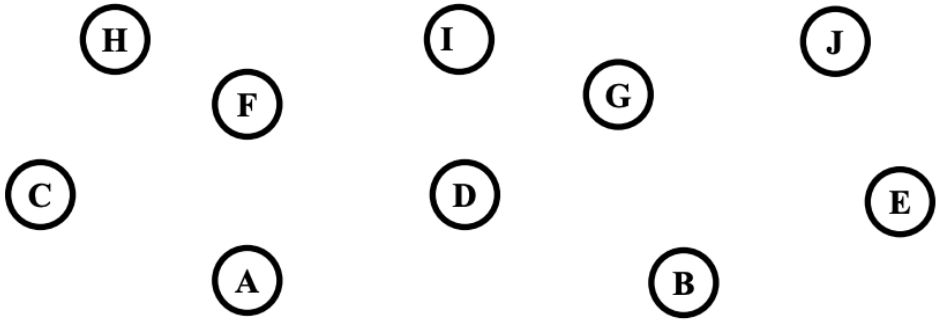
Problem 2 (10 points)

(a) Write down the factored joint probability distribution according to the following Bayesian Network.

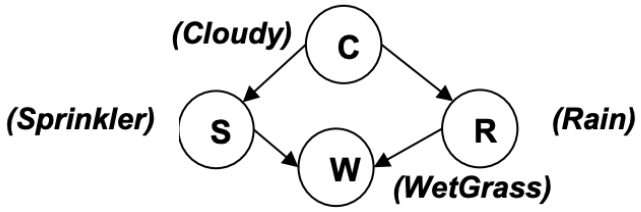


(b) Draw the Bayesian Network that corresponds to this conditional probability:

$$P(A|C,D,F,H)P(B|D,E,J)P(C|H)P(D|G,J)P(E)P(F|G,I)P(G|I,J)P(H)P(I)P(J)$$



(c) Below is the Bayesian network for the WetGrass problem.



P(C)
.5

C	P(S)
t	.1
f	.5

C	P(R)
t	.8
f	.2

S	R	P(W)
t	*t*	.99
t	*f*	.90
f	*t*	.90
f	*f*	.00

Write down an expression that will evaluate to

$$P(C = f \wedge R = f \wedge S = t \wedge W = t).$$

You do not need to carry out the multiplication to produce a single number (probability).

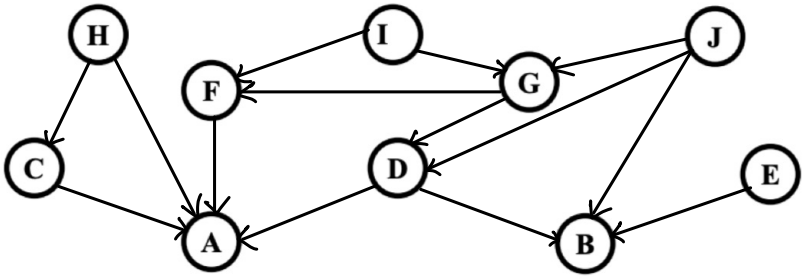
Problem 2

(a)

Joint probability distribution :

$P(A|C,H,F,I,D) \cdot P(B|D,G,J,E) \cdot P(C) \cdot P(D|I) \cdot P(E|J) \cdot P(F) \cdot P(G|I) \cdot P(H|I) \cdot P(I) \cdot P(J)$ #

(b)



(c)

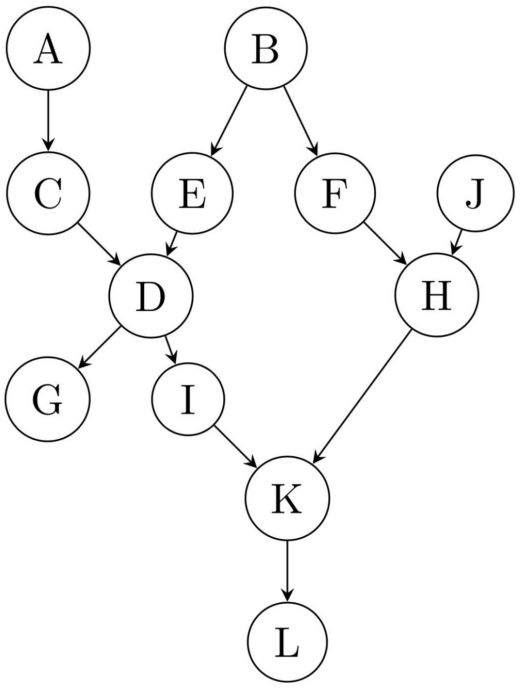
$P(C=f \wedge R=f \wedge S=t \wedge W=t)$

$= P(C=f) \cdot P(R=f \mid C=f) \cdot P(S=t \mid C=f) \cdot P(W=t \mid S=t \wedge R=f)$

$= 0.5 \times 0.8 \times 0.5 \times 0.90$

Problem 3 (10 points)

According to the following Bayesian Network,



(a) List all the variables that are d-separated from F given E.

(b) List all the variables that are d-separated from F given E and K.

Problem 4 (10 points)

Draw a Bayes net with four states $\{A, B, C, D\}$, that follows all of the independence constraints below.

(a) $A \perp\!\!\!\perp B$

(b) $A \not\perp\!\!\!\perp D \mid B$

(c) $A \perp\!\!\!\perp D \mid C$

(d) $A \not\perp\!\!\!\perp C$

(e) $B \not\perp\!\!\!\perp C$

(f) $A \not\perp\!\!\!\perp B \mid D$

(g) $B \perp\!\!\!\perp D \mid A, C$

Problem 3.

(a)

d-seperated:

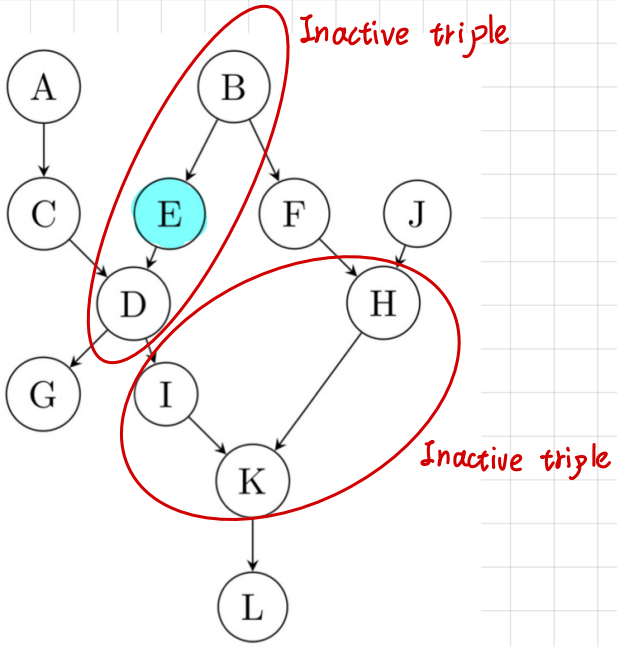
J, D, I, G, C, A

d-connected:

B, H, K, L

↳ F, H, K, J is an active path

↳ F, H, K is an active path



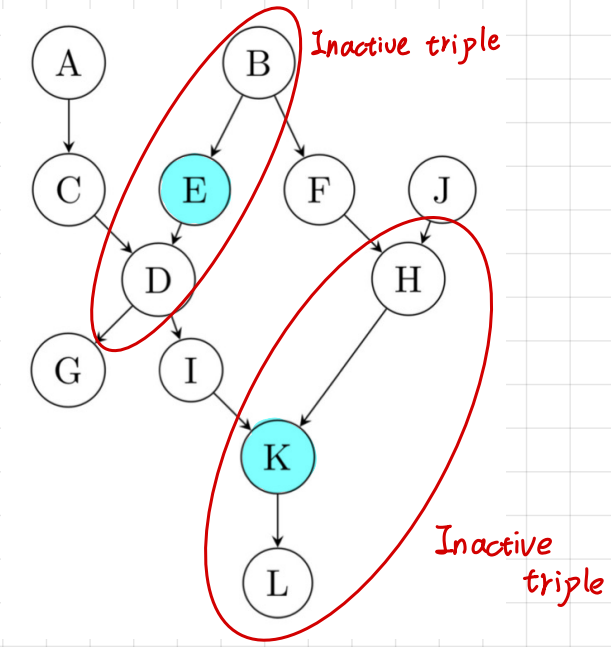
(b)

d-seperated:

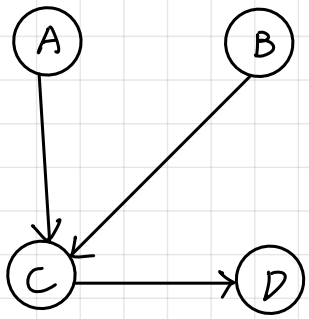
L

d-connected:

B, H, J, I, D, G, A, C



Problem 4.



(a) $A \perp\!\!\!\perp B$ ✓

(b) $A \not\perp\!\!\!\perp D \mid B$ ✓

(c) $A \perp\!\!\!\perp D \mid C$ ✓

(d) $A \not\perp\!\!\!\perp C$ ✓

(e) $B \not\perp\!\!\!\perp C$ ✓

(f) $A \not\perp\!\!\!\perp B \mid D$ ✓

(g) $B \perp\!\!\!\perp D \mid A, C$ ✓