

2. (a)

$$H(z) = \frac{(z+2)(z^2-z+1)}{(z^2+z+1)}$$

$$= \frac{z(1+2z^{-1})(1-\frac{1+j\sqrt{3}}{4}z^{-1})(1-\frac{1-j\sqrt{3}}{4}z^{-1})}{(1+\frac{1-j\sqrt{3}}{4}z^{-1})(1+\frac{1+j\sqrt{3}}{4}z^{-1})}$$

$$A=1, a_1=-2, a_2=\frac{1+j\sqrt{3}}{4}, a_3=\frac{1-j\sqrt{3}}{4}$$

$$c_1=\frac{-1+j\sqrt{3}}{4}, c_2=\frac{-1-j\sqrt{3}}{4}$$

$$\hat{h}[n] = \begin{cases} -\frac{(-2)^n}{n}, & n > 0 \\ 0, & n \leq 0 \end{cases}$$

(b)

$$|a_1|=2, |a_2|=|a_3|=|c_1|=|c_2|=\frac{1}{\sqrt{2}}$$

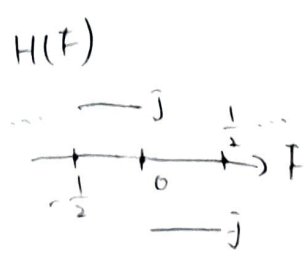
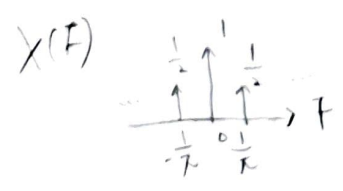
$$H_1(z) = H(z) \cdot a_1 \frac{z-\bar{a}_1}{z-a_1} = -2H(z) \frac{1+\frac{1}{2}z^{-1}}{1+2z^{-1}}$$

(c)

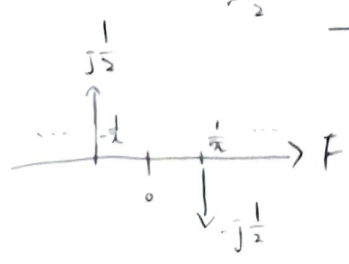
The minimum phase filter is stable and causal.

3. (a)

$$1 \xrightarrow{DTFT} \sum_{k=-\infty}^{\infty} \delta(F-k), \quad \cos(\pi n) \xrightarrow{DTFT} \frac{1}{2} \sum_{k=-\infty}^{\infty} [\delta(F-\frac{1}{2}-k) + \delta(F+\frac{1}{2}-k)]$$

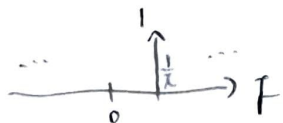


$$X_H(F) = X(F) \cdot H(F)$$



(b),

$$X_a(F) = \begin{cases} 2X(F), & 0 < F < 0.5 \\ X(F), & F=0 \\ 0, & -0.5 < F < 0 \end{cases}$$



4,

(a), Difference filter and Hilbert filter

(b), Kalman filter and particle filter

5,

$$l[n] = \begin{cases} 0, & n = 40, 60, 80 \\ 1, & \text{other} \end{cases}$$

$$x[n] = y[n] \cdot l[n]$$

6,

(a), 1. It's easier to obtain the channel

2. echos can be filtered out by removing higher frequencies

(b), 1. no phase ambiguity

2. much less probability for $\gamma[m] \rightarrow -\infty$

3. $B_m[k]$ fits human perception

4. using DCT instead of IDFT to reduce the complexity

7,

(a), $\sin(3600\pi t)$ sounds the loudest because human hearing is most sensitive around 3000 Hz.

(b), $\sin(400\pi t)$ can propagate longest because it's of lowest frequency.

Extra:

1. What is the absolute value of $0.4 - 0.3j$?

$$|0.4 - 0.3j| = \sqrt{(0.4)^2 + (0.3)^2} = 0.5$$

2. What problem might occur when taking log in obtaining cepstrum?

1. $X(f) = 0 \rightarrow \log(X(f)) = -\infty$

2. $\arg(X(f))$ has inf number of solutions.