Assignment 4

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Euler Forward and Backward Method

Overview

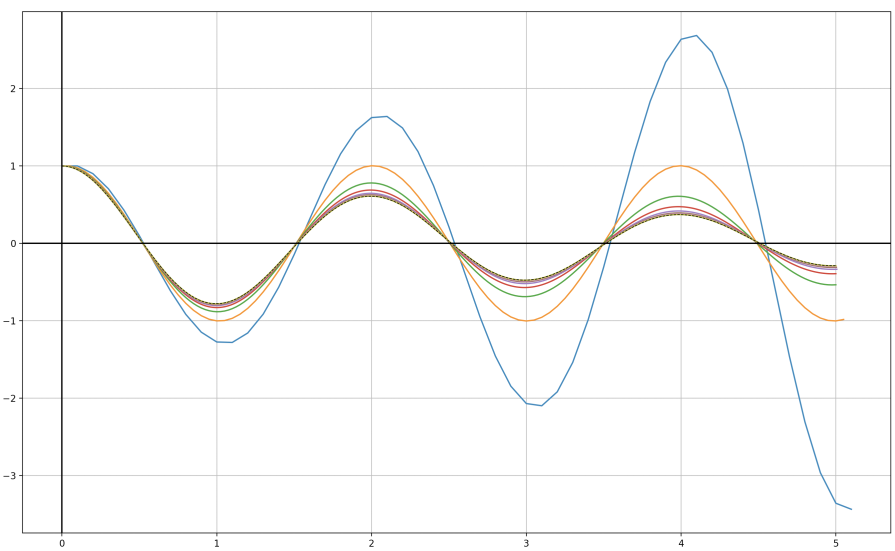
In this assignment, we were asked to use ***Euler Methods*** for approximating the characteristics of *Damped Harmonic Oscillator*. With the help ideas discussed in the class, such as **Backward Method** and **Forward Method**, I was able to plot and see the relation between error and the time gap implemented.
I used `Python` programming language for coding the assignment and the `matplotlib` module for plotting.

Forward Method

Forward Method works on the direct approximation of the instant next occurrence from the present one. The Method uses following equations.

$$x(t + \delta t) = x(t) + \delta t * v(t)$$
$$v(t + \delta t) = v(t) + \delta t * a(t)$$
$$a(t) = -k * x(t) - c * v(t)$$

Now, using these equations in iterative manner, I was able to get the following graph.



Here, the dashed black line represents the actual (correct) function. I have used 10 different values of δt of the form $\frac{1}{10*2^i}$, i ranging from (0-9)

Some observations are...

- The error is of diverging nature. (Approaches ∞)
- The energy is increasing using this method.
- The more is the value of δt the more is the curve diverging.

Backward Method

Backward Method works by estimating the next occurrence by using the old occurrence for the same entity and next occurrences of all other entities.

$$x(t+\delta t) = x(t) + \delta t * v(t+\delta t)$$
$$v(t+\delta t) = v(t) + \delta t * a(t+\delta t)$$
$$a(t) = -k * x(t) - c * v(t)$$

These equations combined lead to these two simultaneous linear equations.

$$x(t+\delta t) - \delta t * v(t+\delta t) = x(t)$$
$$k * \delta t * x(t+\delta t) + (1 + c * \delta t) * v(t+\delta t) = v(t)$$

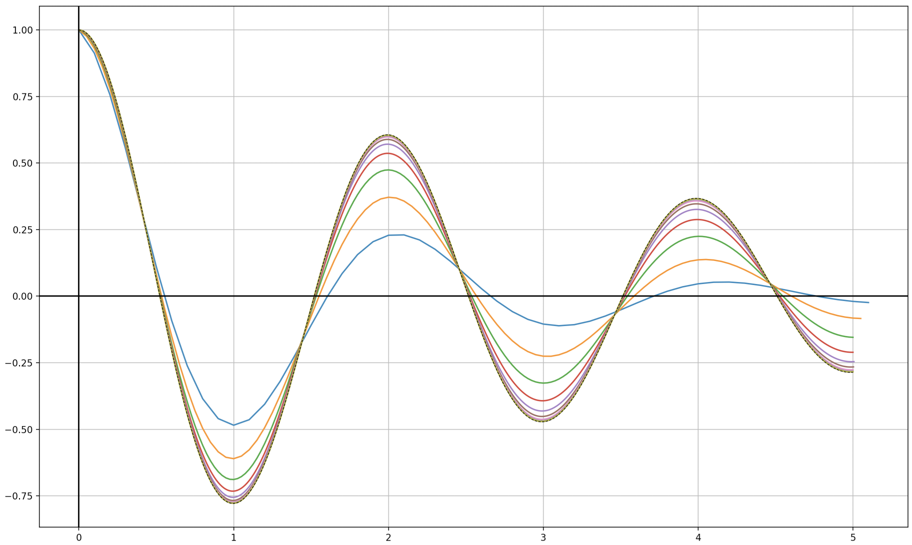
Solving these we get the follwing.

$$v(t+\delta t) = \frac{v(t) - k * \delta t * x(t)}{k * \delta t^2 + c * \delta t + 1}$$
$$x(t+\delta t) = \frac{v(t) * \delta t + (c * \delta t + 1) * x(t)}{k * \delta t^2 + c * \delta t + 1}$$

We may also define $div_t = k * \delta t^2 + c * \delta t + 1$ since it only depends on δt and is constant for one complete iteration. Thus, final answer becomes...

$$v(t+\delta t) = \frac{v(t) - k * \delta t * x(t)}{div_t}$$
$$x(t+\delta t) = \frac{v(t) * \delta t + (c * \delta t + 1) * x(t)}{div_t}$$

Now, using these equations in iterative manner, I was able to get the following graph.



Here, the dashed black line represents the actual (correct) function. I have used 10 different values of δt of the form $\frac{1}{10*2^i}$, i ranging from (0-9)

Some observations are...

- The energy is decreasing using this method.

- The more is the value of δt the faster (w.r.t time) is the convergence. (to 1).

Summary

Using these methods, I understood the need of better Method methods to map real life physical simulations. Overall, this assignment was an amazing experience. Regards. Thanks a lot!