

Characteristic polynomials, eigenvalues and eigenvectors

TOTAL POINTS 10

1.Question 1

Given a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, recall that one can calculate its eigenvalues by solving the characteristic polynomial $\lambda^2 - (a+d)\lambda + (ad-bc) = 0$. In this quiz, you will practice calculating and solving the characteristic polynomial to find the eigenvalues of simple matrices.

For the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

☒ $\lambda^2 - 3\lambda + 2 = 0$

$\lambda_1 = 1, \lambda_2 = 2$

☐ $\lambda^2 - 3\lambda - 2 = 0$

$\lambda_1 = 1, \lambda_2 = -2$

☐ $\lambda^2 + 3\lambda + 2 = 0$

$\lambda_1 = -1, \lambda_2 = -2$

☐ $\lambda^2 + 3\lambda - 2 = 0$

$\lambda_1 = -1, \lambda_2 = 2$

2.Question 2

Recall that for a matrix A , the eigenvectors of the matrix are vectors for which applying the matrix transformation is the same as scaling by some constant.

For $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☒ $\begin{bmatrix} 1 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & -1 \end{bmatrix}$

☒ $\begin{bmatrix} 0 & 2 \end{bmatrix}$

☒ $\begin{bmatrix} 0 & 3 \end{bmatrix}$

3.Question 3

For the matrix $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

☐ $\lambda^2 - 8\lambda - 15 = 0$
 $\lambda_1 = -3, \lambda_2 = 5$

☐ $\lambda^2 + 8\lambda - 15 = 0$
 $\lambda_1 = 3, \lambda_2 = -5$

☐ $\lambda^2 + 8\lambda + 15 = 0$
 $\lambda_1 = -3, \lambda_2 = -5$

☒ $\lambda^2 - 8\lambda + 15 = 0$
 $\lambda_1 = 3, \lambda_2 = 5$

4.Question 4

For the matrix $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☒ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

☒ $\begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$

☒ $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

☐ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

5.Question 5

For the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

☒ $\lambda^2 - 5\lambda + 4 = 0$
 $\lambda_1 = 1, \lambda_2 = 4$

☐ $\lambda^2 + 5\lambda + 4 = 0$
 $\lambda_1 = -1, \lambda_2 = -4$

☐ $\lambda^2 + 5\lambda - 4 = 0$

$$\lambda_1=1, \lambda_2=-4$$

☐ $\lambda^2-5\lambda-4=0$

$$\lambda_1=-1, \lambda_2=4$$

6.Question 6

For the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☐ $\begin{bmatrix} 3 & 2 \end{bmatrix}$

☐ $\begin{bmatrix} 3 & -1 \end{bmatrix}$

☒ $\begin{bmatrix} 3 & 1 \end{bmatrix}$

☒ $\begin{bmatrix} 0 & 1 \end{bmatrix}$

7.Question 7

For the matrix $A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

☐ $\lambda^2-25=0$

$$\lambda_1=\lambda_2=5$$

☐ $\lambda^2+25=0$

$$\lambda_1=\lambda_2=-5$$

☒ $\lambda^2-25=0$

$$\lambda_1=-5, \lambda_2=5$$

☐ $\lambda^2+25=0$

$$\lambda_1=-5, \lambda_2=5$$

8.Question 8

For the matrix $A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☒ $\begin{bmatrix} 4 & -1 \end{bmatrix}$

☒ $\begin{bmatrix} -1 & -1 \end{bmatrix}$

☒ $\begin{bmatrix} 1 & 1 \end{bmatrix}$

☐ [0 2]

9.Question 9

For the matrix $A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

☒ $\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = \lambda_2 = 1$

☐ $\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = \lambda_2 = -1$

☐ $\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = -1, \lambda_2 = 1$

☐ $\lambda^2 - 2\lambda + 1 = 0$

No real solutions.

10.Question 10

For the matrix $A = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

☐ $\lambda^2 - \lambda + 1 = 0$

No real solutions.

☒ $\lambda^2 + \lambda + 1 = 0$

No real solutions.

☐ $\lambda^2 - \lambda - 1 = 0$

$\lambda_1 = (1 - \sqrt{5})/2, \lambda_2 = (1 + \sqrt{5})/2$

☐ $\lambda^2 + \lambda - 1 = 0$

$\lambda_1 = (-\sqrt{5} - 1)/2, \lambda_2 = (\sqrt{5} - 1)/2$