Eigenvalues and eigenvectors

LATEST SUBMISSION GRADE

100%

1.Question 1
This assessment will test your ability to apply your knowledge of eigenvalues and eigenvectors to some special cases.

Use the following code blocks to assist you in this quiz. They calculate eigenvectors and eigenvalues respectively:

```
# Eigenvalues

M = np.array([[4, -5, 6],

          [7, -8, 6],

          [3/2, -1/2, -2]])

vals, vecs = np.linalg.eig(M)

vals

[ 1. -4. -3.]

# Eigenvectors - Note, the eigenvectors are the columns of the output.

M = np.array([[4, -5, 6],

          [7, -8, 6],

          [3/2, -1/2, -2]])

vals, vecs = np.linalg.eig(M)

vecs

 [[ 3. -2.  1.]

 [ 3. -2. -1.]

 [ 1.  1. -2.]]
```

To practice, select all eigenvectors of the matrix, *A* = [[4, -5, 6], [7, -8, 6], [3/2, -1/2, -2]]

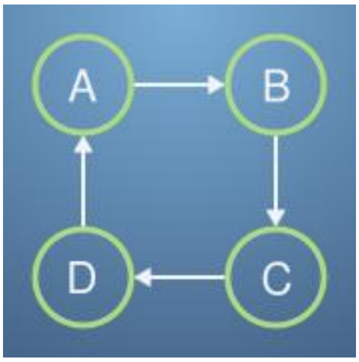
1 / 1 point

- [x] [−3 −3 −1]
- [ ] None of the other options.
- [ ] [−3 −2 1]
- [ ] [−1 1 −2]
- [x] [−2/sqrt(9) −2/sqrt(9) 1/sqrt(9)]
- [x] [1/2 −1/2 −1]

2.Question 2

Recall from the *PageRank* notebook, that in PageRank, we care about the eigenvector of the link matrix, L*L*, that has eigenvalue 1, and that we can find this using *power iteration method* as this will be the largest eigenvalue.

PageRank can sometimes get into trouble if closed-loop structures appear. A simplified example might look like this,



With link matrix, *L* = [[0 0 0 1], [1 0 0 0], [0 1 0 0], [0 0 1 0]]

Use the calculator in Q1 to check the eigenvalues and vectors for this system.

What might be going wrong? Select all that apply.

1 / 1 point

- [ ] Some of the eigenvectors are complex.
- [ ] None of the other options.
- [ ] The system is too small.
- [x] Because of the loop, *Procrastinating Pat*s that are browsing will go around in a cycle rather than settling on a webpage.

☑ Other eigenvalues are not small compared to 1, and so do not decay away with each power iteration.

3.Question 3

The loop in the previous question is a situation that can be remedied by damping.

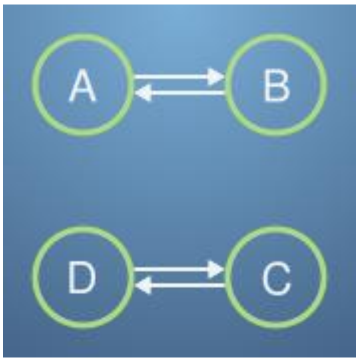
If we replace the link matrix with the damped, *L* ′= [[0.1 0.1 0.1 0.7], [0.7 0.1 0.1 0.1], [0.1 0.7 0.1 0.1], [0.1 0.1 0.7 0.1]], how does this help?

1 / 1 point

- ☑ There is now a probability to move to any website.
- ☐ None of the other options.
- ☐ The other eigenvalues get smaller.
- ☐ The complex number disappear.
- ☐ It makes the eigenvalue we want bigger.

4.Question 4

Another issue that may come up, is if there are disconnected parts to the internet. Take this example,



with link matrix, *L* = [[0 1 0 0], [1 0 0 0], [0 0 0 1], [0 0 1 0]].

This form is known as block diagonal, as it can be split into square blocks along the main diagonal, i.e., *L* = [[*A* 0], [0 *B*]], with *A*=*B*=[[0 1], [1 0]] in this case.

What is happening in this system?

1 / 1 point

- ☐ The system has zero determinant.
- ☑ There isn't a unique PageRank.

- [ ] None of the other options.
- [x] There are two eigenvalues of 1.
- [x] There are loops in the system.

5.Question 5
By similarly applying damping to the link matrix from the previous question. What happens now?

1 / 1 point

- [ ] Damping does not help this system.
- [x] None of the other options.
- [ ] There becomes two eigenvalues of 1.
- [ ] The negative eigenvalues disappear.
- [ ] The system settles into a single loop.

6.Question 6
Given the matrix *A*=[[3/2 -1], [−1/2 1/2]], calculate its characteristic polynomial.

1 / 1 point

- ○ λ^2+2λ−1/4
- ● λ^2−2λ+1/4
- ○ λ^2+2λ+1/4
- ○ λ^2−2λ−1/4

7.Question 7
By solving the characteristic polynomial above or otherwise, calculate the eigenvalues of the matrix *A*=[[3/2 -1], [−1/2 1/2]].

1 / 1 point

- ○ λ1=1−sqrt(5)/2,λ2=1+sqrt(5)/2
- ○ λ1=−1−sqrt(3)/2,λ2=−1+sqrt(3)/2
- ○ λ1=−1−sqrt(5)/2,λ2=−1+sqrt(5)/2
- ● λ1=1−sqrt(3)/2,λ2=1+sqrt(3)/2

8.Question 8
Select the two eigenvectors of the matrix *A*=[[3/2 -1], [−1/2 1/2]].

1 / 1 point

- ○ **v1**=[−1−sqrt(5) 1],**v2**=[−1+sqrt(5) 1]
- ○ **v1**=[1−sqrt(5) 1],**v2**=[1+sqrt(5) 1]
- ○ **v1**=[1−sqrt(3) 1],**v2**=[1+sqrt(3) 1]
- ◉ **v1**=[−1−sqrt(3) 1],**v2**=[−1+sqrt(3) 1]

9.Question 9
Form the matrix *C* whose left column is the vector v1 and whose right column is v2 from immediately above.

By calculating *D*=*C*^(−1)*AC* or by using another method, find the diagonal matrix *D*.

1 / 1 point

- ○ [[−1−sqrt(3)/2 0], [0 −1+sqrt(3)/2]]
- ○ [[1−sqrt(5)/2 0], [0 1+sqrt(5)/2]]
- ○ [[−1−sqrt(5)/2 0], [0 −1+sqrt(5)/2]]
- ◉ [[1+sqrt(3)/2 0], [0 1-sqrt(3)/2]]

10.Question 10
By using the diagonalisation above or otherwise, calculate A^2.

1 / 1 point

- ○ [[−11/4 2], [1 −3/4]]
- ◉ [[11/4 −2], [-1 3/4]]
- ○ [[11/4 -1], [−2 3/4]]
- ○ [[−11/4 1], [2 −3/4]]