

Non-square matrix multiplication

TOTAL POINTS 8

1.Question 1

In the previous lecture we saw the Einstein summation convention, in which we sum over any indices which are repeated. In traditional notation we might write, for example, $A_{ij}v_j = A_{i1}v_1 + A_{i2}v_2 + A_{i3}v_3$. With the Einstein summation convention we can avoid the big sigma and write this as $A_{ij}v_j$. We know that we sum over j because it appears twice.

We saw that thinking about this type of notation helps us to multiply non-square matrices together. For example, consider the matrices

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, and remember that in the A_{ij} notation the first index i represents the row number and the second index j represents the column number. For example, $A_{12} = 2$.

Let's define the matrix $C = AB$. Then in Einstein summation convention notation $C_{mn} = A_{mj}B_{jn}$.

Using the Einstein summation convention, calculate $C_{21} = A_{2j}B_{j1}$.

- ☐ $C_{21}=3$
- ☐ $C_{21}=4$
- ☒ $C_{21}=5$
- ☐ $C_{21}=6$

1 / 1 point

2.Question 2

We can use the same method to calculate every element of $C=AB$. Doing so we see that we are multiplying A 's rows with B 's columns in exactly the same way as we would for square matrices.

In fact, we can multiply any matrices together as long as the terms which we sum over have the same number of elements. For example, there are the same number of values for j in $C_{mn}=A_{mj}B_{jn}$. The resulting matrix C will have as many rows as A and as many columns as B .

Using the same matrices as before, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, what is $C = ABC = AB$?

- ☒ $C = \begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 1 \end{bmatrix}$

1 point

3.Question 3

Let's practice multiplying together a few more matrices which are not square.

Calculate the product:

$$\begin{bmatrix} 2 & 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

- ☐ 29
☒ 30
☐ 31
☐ 32

1 / 1 point

4.Question 4

Calculate the product:

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & 5 & 6 \end{bmatrix}$$

- ☒ $\begin{bmatrix} 2 & 4 & 5 & 6 \\ 6 & 12 & 15 & 18 \\ 4 & 8 & 10 & 12 \\ 2 & 4 & 5 & 6 \end{bmatrix}$

1 / 1 point

5.Question 5

Calculate the product:

$$\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 4 & -1 \\ -2 & 0 & 0 & 2 \end{bmatrix}$$

- ☒ $\begin{bmatrix} 2 & 2 & 8 & -4 \\ -6 & 0 & 0 & 6 \\ 0 & 1 & 4 & -1 \end{bmatrix}$

1 / 1 point

6.Question 6

We have seen that we can multiply an $m \times n$ matrix with an $n \times k$ matrix, and the resultant matrix will be an $m \times k$ matrix. You can check this is consistent with your previous answers.

Let $D = ABC$ where A is a 5×3 matrix, B is a 3×7 matrix and C is a 7×4 matrix.

What are the dimensions of the matrix DD ?

- ☒ D is a 5×4 matrix
- ☐ D is a 4×5 matrix
- ☐ D is a 3×7 matrix
- ☐ A , B and C cannot be multiplied together because they have the wrong dimensions.
- ☐ D is a 5×7 matrix

1 / 1 point

7.Question 7

Calculate the product:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- ☒ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

1 / 1 point

8.Question 8

Let u and v be vectors with n elements. Which of the following are equal to the dot product of these two vectors?

- ☒ $\sum_{i=1}^n u_i v_i$
- ☒ $[u_1 u_2 \dots u_n][v_1, v_2, \dots, v_n]$
- ☒ $u_i v_i$
- ☒ $u \cdot v$