

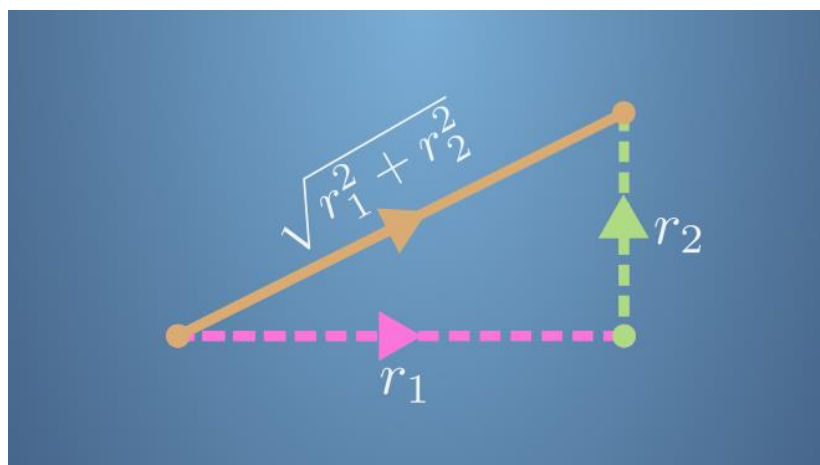
Dot product of vectors

TOTAL POINTS 6

1.Question 1

As we have seen in the lecture videos, the dot product of vectors has a lot of applications. Here, you will complete some exercises involving the dot product.

We have seen that the size of a vector with two components is calculated using Pythagoras' theorem, for example the following diagram shows how we calculate the size of the orange vector $r = [r_1 \ r_2]$:



In fact, this definition can be extended to any number of dimensions; the size of a vector is the square root of the sum of the squares of its components. Using this information, what is the size of the vector

$$s = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 2 \end{bmatrix}?$$

- ☐ $|s| = \sqrt{10}$
- ☐ $|s| = 30$
- ☐ $|s| = 10$
- ☒ $|s| = \sqrt{30}$

1 / 1 point

2.Question 2

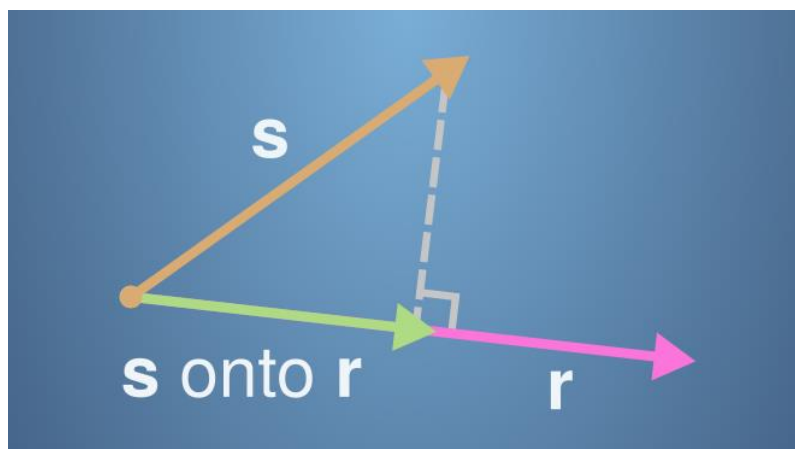
Remember the definition of the dot product from the videos. For two n component vectors, $a \cdot b = a_1b_1 + a_2b_2 + \dots + a_nb_n$. What is the dot product of the vectors $r = \begin{bmatrix} -5 \\ 3 \\ 2 \\ 8 \end{bmatrix}$ and $s = \begin{bmatrix} 12 \\ -10 \end{bmatrix}$?

- ☐ $\mathbf{r} \cdot \mathbf{s} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -56 & -20 \end{bmatrix}$
- ☐ $\mathbf{r} \cdot \mathbf{s} = 1$
- ☒ $\mathbf{r} \cdot \mathbf{s} = -1$
- ☐ $\mathbf{r} \cdot \mathbf{s} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -45 & 19 \end{bmatrix}$

1 / 1 point

3.Question 3

The lectures introduced the idea of projecting one vector onto another. The following diagram shows the projection of \mathbf{s} onto \mathbf{r} when the vectors are in two dimensions:



Remember that the scalar projection is the *size* of the green vector. If the angle between \mathbf{s} and \mathbf{r} is greater than $\pi/2$, the projection will also have a minus sign.

We can do projection in any number of dimensions. Consider two vectors with three components, $\mathbf{r} = \begin{bmatrix} 1 & 3 & -40 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 1 & 105 & -6 \end{bmatrix}$.

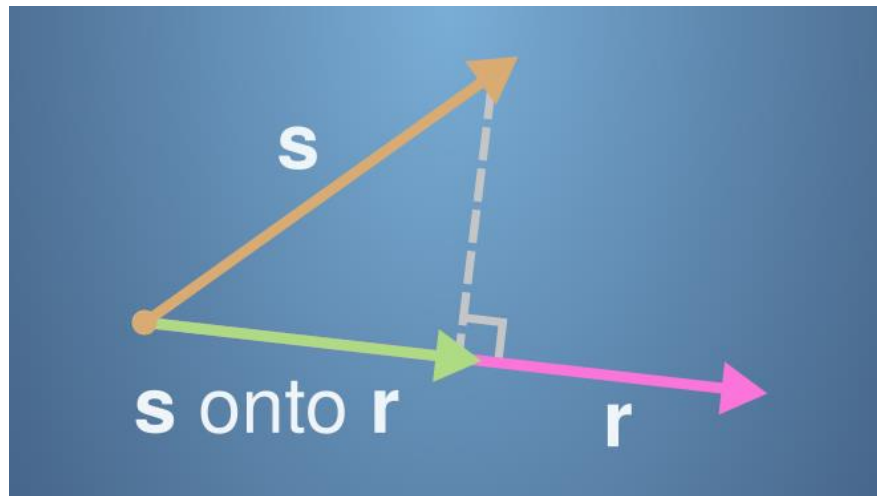
What is the scalar projection of \mathbf{s} onto \mathbf{r} ?

- ☒ 2
- ☐ $-1/2$
- ☐ -2
- ☐ $1/2$

1 / 1 point

4.Question 4

Remember that in the projection diagram, the vector projection *is* the green vector:



Let $r = \begin{bmatrix} 3 \\ -40 \end{bmatrix}$ and let $s = \begin{bmatrix} 105 \\ -6 \end{bmatrix}$.

What is the vector projection of s onto r ?

- ☒ $\begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 30 \\ -20 \\ 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 6 \\ -8 \\ 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$

1 / 1 point

5.Question 5

Let $a = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 5 \\ 12 \end{bmatrix}$.

Which is larger, $|a+b|$ or $|a|+|b|$?

- ☒ $|a+b| < |a|+|b|$
- ☐ $|a+b| = |a|+|b|$
- ☐ $|a+b| > |a|+|b|$

1 / 1 point

6.Question 6

Which of the following statements about dot products are correct?

- ☒ We can find the angle between two vectors using the dot product.
- ☒ The size of a vector is equal to the square root of the dot product of the vector with itself.
- ☒ The vector projection of \mathbf{s} onto \mathbf{r} is equal to the scalar projection of \mathbf{s} onto \mathbf{r} multiplied by a vector of unit length that points in the same direction as \mathbf{r} .
- ☐ The scalar projection of \mathbf{s} onto \mathbf{r} is always the same as the scalar projection of \mathbf{r} onto \mathbf{s} .
- ☐ The order of vectors in the dot product is important, so that $\mathbf{s} \cdot \mathbf{r} \neq \mathbf{r} \cdot \mathbf{s}$