# How To Prove It: A Structured Approach
# Proof Solutions

By jayms

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**Theorem 1.** *Suppose $a$ and $b$ are real numbers. If $0 < a < b$ then $a^2 < b^2$.*

*Proof.* Suppose $0 < a < b$. Then $b - a < 0$.
Then $(b - a)(b + a) < 0$.
Then $b^2 - a^2 < 0$.
Since $b^2 - a^2 < 0$, it follows that $a^2 < b^2$.
Therefore if $0 < a < b$ then $a^2 < b^2$. ■

**Theorem 2.** *Suppose $a$ and $b$ are real numbers. If $a < b < 0$ then $a^2 > b^2$.*

*Proof.* Suppose $a < b < 0$. Then $a - b < 0$.
Then $(a - b)(a + b) < 0$.
Then $a^2 - b^2 < 0$.
Since $a^2 - b^2 < 0$, it follows that $a^2 > b^2$.
Therefore if $a < b < 0$ then $a^2 > b^2$. ■

**Theorem 3.** *Suppose $a$ and $b$ are real numbers. If $0 < a < b$ then $\frac{1}{b} < \frac{1}{a}$.*

*Proof.* Suppose $0 < a < b$. Then $\frac{a}{ab} < \frac{b}{ab}$.
Then $\frac{1}{b} < \frac{1}{a}$.
Therefore if $0 < a < b$ then $\frac{1}{b} < \frac{1}{a}$. ■

**Theorem 4.** *Suppose $a$ is a real number. If $a^3 > a$ then $a^5 > a$.*

*Proof.* Suppose $a^3 > a$. Then $a^3 - a > 0$.
Then $(a^3 - a)(a^2 + 1) > 0$.
Then $a^5 - a > 0$.
Since $a^5 - a > 0$, it follows that $a^5 > a$.
Therefore if $a^3 > a$ then $a^5 > a$. ■

**Theorem 5.** *Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. If $x \notin D$ then $x \in B$.*

*Proof.* Suppose $x \notin D$. Since $x \notin D$, it follows that $x \notin C \cap D$.
Then $x \notin A \setminus B$.
Then either $x \notin A$ or $x \in B$.
Since $x \in A$, $x$ must be a member of $B$.
Therefore if $x \notin D$ then $x \in B$. ■

**Theorem 6.** *Suppose $a$ and $b$ are real numbers. If $a < b$ then $\dfrac{a+b}{2} > b$.*

*Proof.* Suppose $a < b$. Then $a + b < 2b$. Then $\dfrac{a+b}{2} < b$.
Therefore if $a < b$ then $\dfrac{a+b}{2} < b$. ■

**Theorem 7.** *Suppose $x$ is a real number and $x \neq 0$. If $\dfrac{\sqrt[3]{x}+5}{x^2+6} = \dfrac{1}{x}$ then $x \neq 8$.*

*Proof.* Suppose $x = 8$. Then $\dfrac{\sqrt[3]{8}+5}{8^2+6} \neq \dfrac{1}{8}$.
Then $\dfrac{1}{10} \neq \dfrac{1}{8}$.
Since $\dfrac{1}{10} \neq \dfrac{1}{8}$, it follows that $\dfrac{\sqrt[3]{x}+5}{x^2+6} \neq \dfrac{1}{x}$ for $x = 8$.
Therefore, by the law of contraposition, if $\dfrac{\sqrt[3]{x}+5}{x^2+6} = \dfrac{1}{x}$ then $x \neq 8$. ■