Discrete Structures - Functions

Tutorial 6

PG-43 Jaynam Modi. G3. August 28th.

Q7. Determine whether injective, surjective or bijective

(a) $f(1)=2,\ f(2)=1,\ f(3)=3,\ f(4)=4.$

$F=\{(1,2),(2,1),(3,3),(4,4)\}$

since the function is both one-one & onto, it is bijective.

(b) $F=\{(1,2),(2,2),(3,4),(4,3)\}$

None → Neither surjective, nor injective.

(c) $f=\{(1,4),(2,2),(3,4),(4,3)\}$

nether surjective nor injective

(d) $F=\{(1,2),(2,3),(3,4),(4,1)\}$

Bijective.

Q2. $f_1(x) = x^3$ $f_2(x) = x^2 + 5x + 6$.

(a) sum → $f_1 + f_2$ @ $x = 2$.

$y = f_1 + f_2 = x^3 + x^2 + 5x + 6$.

at $x = 2$,

thus, $y(2) = 8 + 4 + 10 + 6 = \underline{28}$

hence, $f_1 + f_2$ @ $x = 2$ is $\underline{28}$.

(b) product → $f_1 . f_2$ for $x = 2$.

$y = f_1 . f_2 = (x^3) . (x^2 + 5x + 6)$

$= x^5 + 5x^4 + 6x^3$

at $x = 2$,

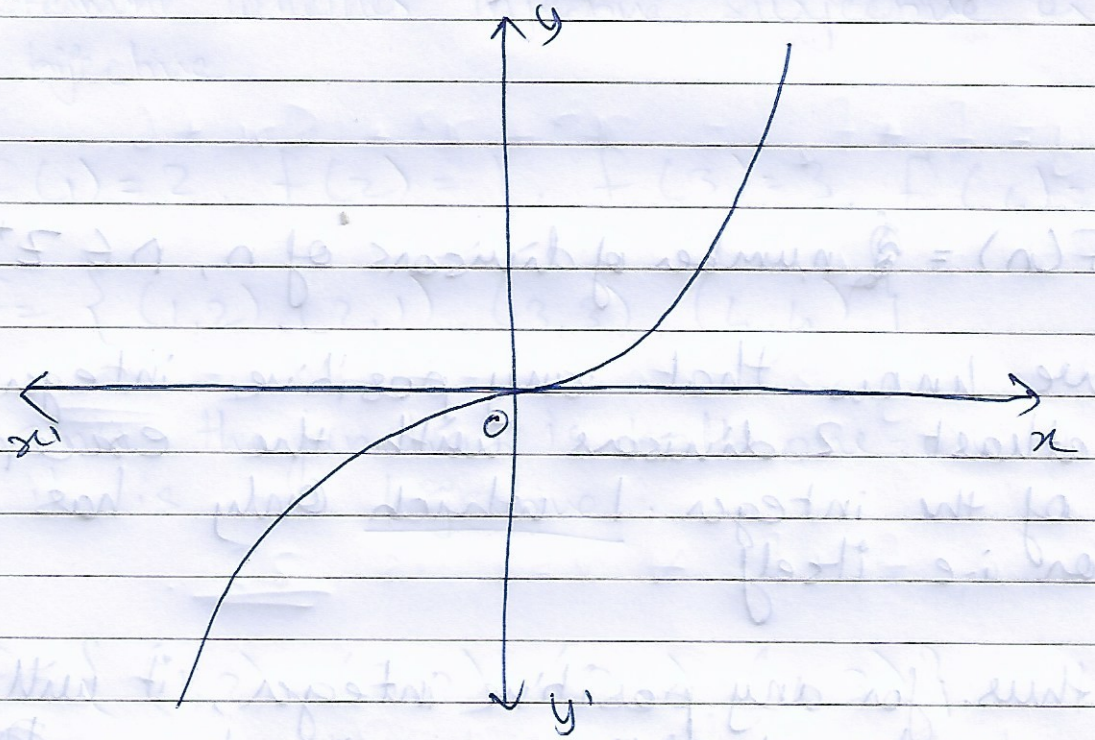
thus, $y(2) = 32 + 80 + 48$.

$= \underline{160}$.

hence, $f_1 . f_2$ @ $x = 2$ is $\underline{160}$.

Q3 $f(x) = x^3$

since we can easily plot the graph of this function.



from the graph it is clear that the function $f(x) = x^3$ is both one-one i.e. injective & onto i.e. surjective, since this is a bijective function for the given domain & range, it is invertible.

to find the inverse,

let $y = x^3$, taking cube root on both sides

we get, $y^{1/3} = (x^3)^{1/3}$

i.e $\sqrt[3]{y} = x$.

hence, $F^{-1}(y) = \sqrt[3]{y}$ or $F^{-1}(x) = \sqrt[3]{x}$

hence, the inverse of the given function

$$f(x) = x^3$$

is $\boxed{f^{-1}(x) = x^{1/3}}$.

Qn.

(a) $F(n) =$ number of divisors of n, $n \in Z^+$

we know that any positive integer has atleast 2 divisors with the exception of the integer 1 which only has one divisor i.e. itself.

thus, for any positive integer, it will have a number of divisors ranging from $[1, \infty)$.

hence, the function is onto.

But, since two numbers can share the number of divisors, such as prime numbers which only have 2 divisors, the function is not one-one.

hence, the function is onto i.e surjective only.

(b) $f(n)$ = Maximum of $n \leq 50$ $\forall n \in Z^+$

for any positive integer less than 50, $f(n) = 50$, hence the function cannot be one-one.

and, similarly, the function cannot be onto either.

Hence, the function is neither onto i.e. surjective nor one-one i.e. injective.

Q5. Determine whether f is a function from Z to R.

(a) $f(n) = \pm n$.

this is not a function since there are 2 values of $f(n)$ corresponding to one value of n.

(b) $f(n) = 1/(n^2-4)$

clearly, for $n = \pm 2$, the value of $f(n)$ will be $1/(4-4) = 1/0$. which is not defined

hence, there exists no value in the range of $f(n)$ which is an image of ± 2 in the domain.

So, $f(n)$ is Not a function as all values in the domain i.e. $\mathbb{Z}$ do not have an image in the Range, $\mathbb{R}$.