Page No.:

Discrete Structures Tutorial - 5.

PG-43. G3.
Jaynam Modi.

Q1. (a) $P(P(S)) = P(S)$

~~since~~ considering S has 'n' elements,

$P(S)$ has n^2 elements

& $P(P(S))$ has $(n^2)^2$ elements

since $n^2 \neq (n^2)^2$ for all $n \in N$, it is ~~false~~ not always true.

(b) $P(S) \cap P(P(S)) = \{\phi\}$

TRUE

(c) $P(S) \cap S = P(S)$

as we know, the intersection of a set & it's power set is empty set, this is false.

(d) $S \notin P(S)$

False, since a power set is the set of all subsets including the empty set & the set itself. thus $S \in P(S)$.

Page No.:

2. $x_1 + x_2 + x_3 \leq 11$

let there be a non negative value x_4,

such that $x_1 + x_2 + x_3 + x_4 = 11$

then clearly, $x_1 + x_2 + x_3 = 11 - x_4$

since x_4 is non negative, $11 - x_4 \leq 11$.

thus, x_4 also represents the number of non negative solutions to the given inequality,

hence, number of solutions $= {}^{4+11-1}C_{11} = {}^{14}C_{11}$

$$= {}^{14}C_3 = \frac{14!}{3! \times 11!} = \frac{14 \times 13 \times \cancel{12}^{2}}{\cancel{3} \times \cancel{2} \times 1}$$

$$= \underline{\underline{364}}$$

3. $A = \{a, b, c\}$

largest equivalence relation $= A \times A$

$= \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$

since $A \times A$ is the largest set possible with the elements of A, and because it is equivalence,

the number of elements in the largest equivalence set = number of elements in $A \times A = \underline{\underline{n^2}}$

smallest equivalence relation = $\{(a,a),(b,b),(c,c)\}$

hence, clearly, the number of elements in the smallest equivalence relation is the number of elements in the given set itself. i.e. n.

4. $A = \{0, 1, 2, 3\}$

(a) $\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0)\}$

since (2,2) is not in the relation, it is not equivalence as it is not reflexive

(b) $\{(0,0), (0,2), (2,3), (1,1), (2,2)\}$

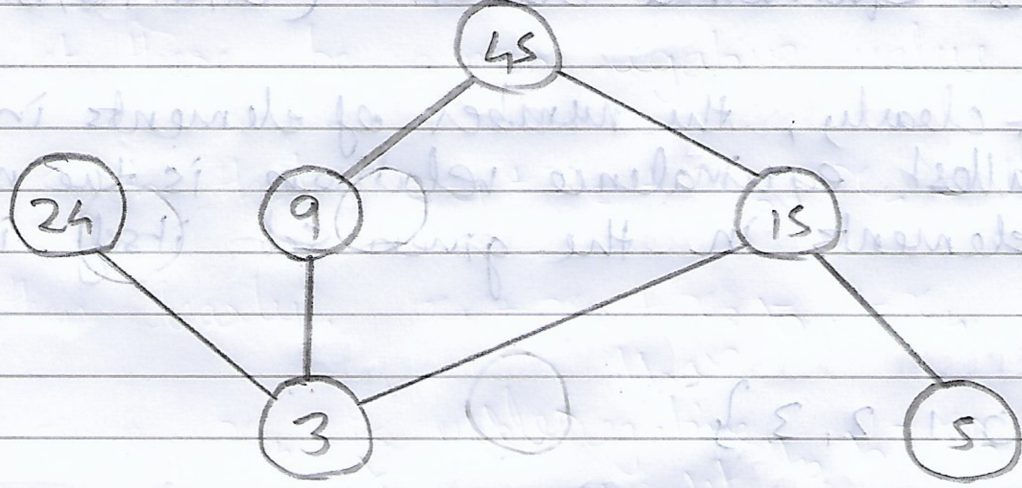
since (3,3) is not in the relation, it is not equivalence as it is not reflexive.

5. Poset = $(\{3, 5, 9, 15, 24, 45\}, |)$

$S = \{3, 5, 9, 15, 24, 45\}$

$R = \{(a,b) \mid a \text{ divides } b\}$

Hasse Diagram :



(a) Maximal elements : 24, 45

(b) Minimal elements : 3, 5

(c) upper bounds of (3,5) : 15, 45

(d) lower bounds of (15,45) : 3, 5, 15

6. Is (S,R) a poset if S is the set of all people in the world....

(a) a is taller than b.

since a is taller than b is our condition, (a,a) cannot be included in the Relation R. thus, R is not reflexive.
thus, (S,R) is not a poset.

Page No.:

(b) a is not taller than b.

i.e. a is equal in height to b or shorter than b.

since a person is always equal in height to his/her-self, $(a,a) \in R$, thus, R is reflexive.

if a is not taller than b, then b is taller than or equal in height to b

but since, they can have equal heights

for two people having equal heights,

$(a,b) \in R$ & $(b,a) \in R$

thus, R is not Antisymmetric.

hence, $(R, \leq)$ is not poset.