

## Discrete Structures Theory Assignment

Jaynam Medi. PG-43 G-3 Aug 15, 2020.

$$1. \quad x \oplus y = x^2 + y^2$$

let us consider two integers  $a, b$  then,

$$a \oplus b = a^2 + b^2 = b^2 + a^2 \quad [\text{since addition is commutative}]$$

$$\text{but, } b \oplus a = b^2 + a^2 = a^2 + b^2$$

$$\text{thus, clearly, } a \oplus b = b \oplus a$$

hence  $\oplus$  is commutative,

considering another integer,  $c$ ,

$$(a \oplus b) \oplus c = (a^2 + b^2) \oplus c$$

$$= (a^2 + b^2)^2 + c^2$$

$$= a^4 + b^4 + 2a^2b^2 + c^2$$

$$\text{and } a \oplus (b \oplus c) = a^2 + (b \oplus c)^2$$

$$= a^2 + (b^2 + c^2)^2$$

$$= a^2 + b^4 + c^4 + 2b^2c^2$$



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$$\text{and } a \oplus (b \oplus c) = a^2 + (b \oplus c)^2$$

$$= a^2 + (b^2 + c^2)^2$$

$$= a^2 + b^4 + c^4 + 2b^2c^2$$



hence,  $(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$

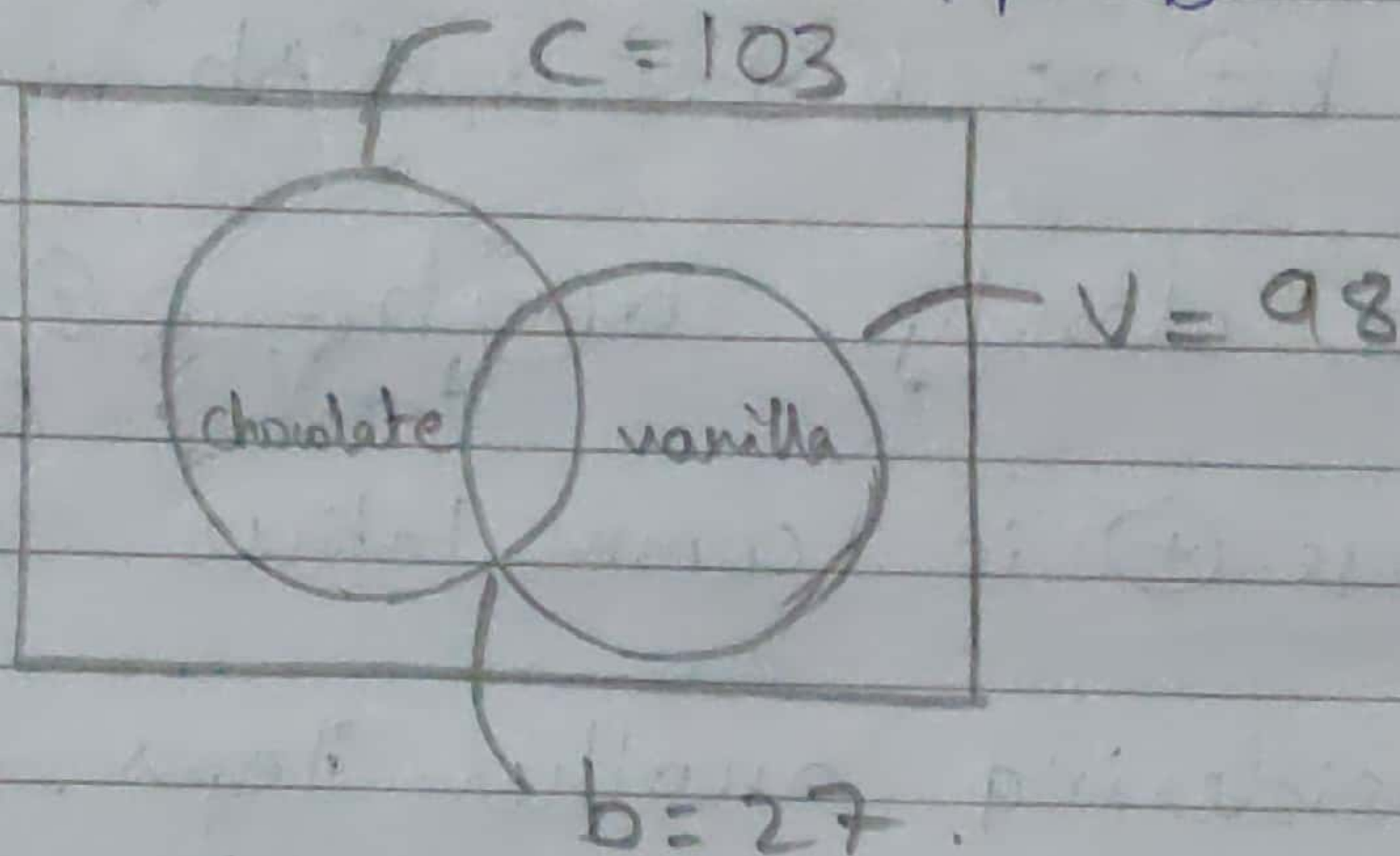
hence,  $\oplus$  is not associative.

thus,  $\oplus$  is (A) Commutative, but not Associative

2. customers who like chocolate,  $C = 103$ .

customers who like vanilla,  $V = 98$ .

customers who like both,  $b = 27$ .



Number of people who like only chocolate

$$= C - b = 103 - 27 = \underline{\underline{76}}.$$



3. We know that there are 20 people & 20 slices of pizza.

10 of the slices of pizza have olives & 10 of them don't.

We also know that each person can either like olives or they can dislike olives.

Thus, considering the slices of pizza which have olives, in accordance with the possibility that each person has only two choices, ~~we get that~~ by the pigeonhole principle,

$$\underline{\underline{\lceil 10/2 \rceil = 5}}$$

i.e. at least 5 people will be happy with the slices of pizza containing olives.

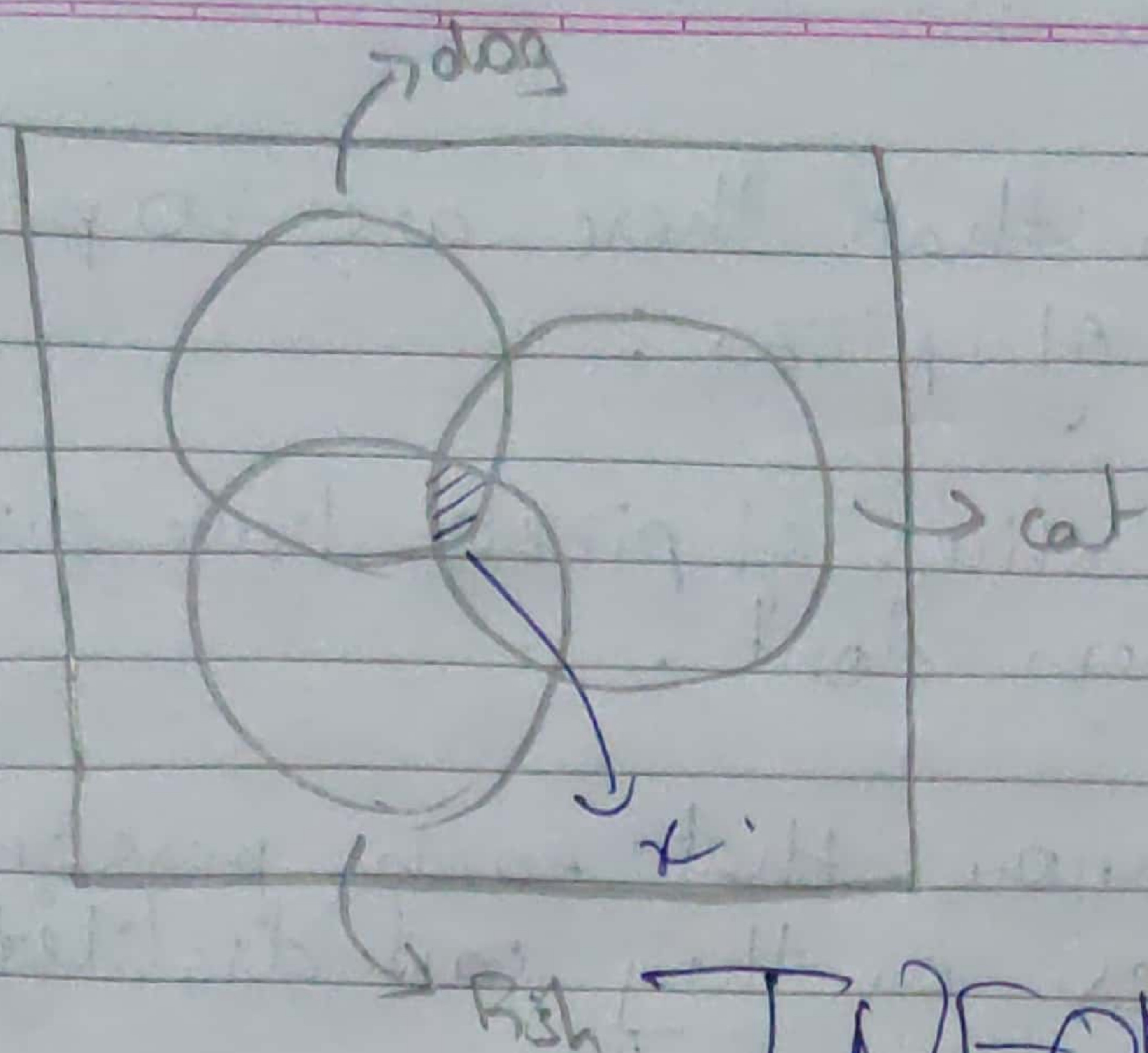
Similarly, for the slices without olives,

$\lceil 10/2 \rceil = 5$ , at least 5 people will be happy with the slices of pizza ~~cont~~ without olives.

i.e. in all at least 10 people will be happy with the slice they receive.



4.



# INFORMATION.

## INSUFFICIENT

Total number of students = 400  
 number of students with pets =  $400 - 228 = 172$

dogs,  $d = 100$  ; cat,  $c = 124$ .

fish,  $f = 26$  ; dog & cat only,  $dc = 20$ .

dog & fish only,  $df = 10$ .

hence, using the venn diagram & given information,

number of people having all pets =  ~~$x$~~   $x$ .

then,

$$d \cap c - x = 20$$

$$\Rightarrow d \cap c = 20 + x$$

$$d \cap f = x = 10 \Rightarrow d \cap f = 10 + x$$



5. Since 10 friends are to be chosen out of 20 friends,

there are,  ${}^{10}C_{20} = \frac{20!}{10! 10!}$  ways

to do so.

6. In most cases, the value of set theory is not in any particular theorem but in the language & direction it provides us. Nowadays, the heart of computer science, the Turing machine is described in set theory.

The main advantage <sup>of</sup> ~~given by~~ using it over other forms is that it enables us to unambiguously answer questions.

In day to day life, social networking, the current COVID tracking apps in some way or other rely on the concepts of set theory in order to distinguish & trace ~~at~~ users as entities. for instance, a simple mutual friends finder for facebook uses sets to define it.

USER A

xyz

abc

123

456

567

USER B

abc

xyz

890

567

pqr

→ mutual friends  
analysed by sets