Sample Solution

Project Objective:

What is multicollinearity and how it affects the regression model? Multicollinearity occurs when the independent variables of a regression model are correlated and if the degree of collinearity between the independent variables are high, it becomes difficult to estimate the relationship between each independent variable and the dependent variable and the overall precision of the estimate coefficients. Even though the regression models with high multicollinearity can give you a high R squared but hardly any significant variables.

The objective of the project is to use the dataset *Factor-Hair-Revised.csv* to build a regression model to predict *satisfaction*.

Project Approach:

- Data Exploration
- Collinearity of the variables
- Initial Regression analysis
- Factor Analysis
- Labelling and interpreting of the factors
- Regression analysis using the factors as independent variable
- Model performance measures

Data Exploration:

Let's import the data and check the basic descriptive statistics.

```
1. data <- read.csv("Factor-Hair-Revised.csv", header = TRUE, sep = ",")
2. head(data)</pre>
```

```
ID ProdQual Ecom TechSup CompRes Advertising ProdLine SalesFImage
1 2 3
            8.5
                  3.9
                            2.5
                                      5.9
                                                     4.8
                                                                4.9
   2
                  2.7
            8.2
                            5.1
                                      7.2
                                                     3.4
                                                                7.9
                                                                               3.1
   3
            9.2
                  3.4
                            5.6
                                      5.6
                                                     5.4
                                                                7.4
                                                                               5.8
4
   4
            6.4
                            7.0
                                                                               4.5
                                                     4.7
                                                                4.7
                  3.3
                                      3.7
5
                                                                               4.5
3.7
   5
            9.0
                            5.2
                                                     2.2
                  3.4
                                      4.6
                                                                6.0
   6
                  2.8
            6.5
                            3.1
                                      4.1
                                                     4.0
                                                                4.3
  ComPricing WartyClaim OrdBilling DelSpeed Satisfaction
1
2
3
                                      5.0
3.9
                                                                 8.2
5.7
           6.8
                        4.7
                                                 3.7
                        5.5
           5.3
                                                 4.9
                                                                 8.9
           4.5
                                      5.4
                                                 4.5
4
           8.8
                        7.0
                                      4.3
                                                 3.0
                                                                 4.8
5
6
           6.8
8.5
                                                                 7.1
4.7
                                      4.5
                        6.1
                                                 3.5
                        5.1
                                                 3.3
                                      3.6
> dim(data)
[1] 100 13
```

Now let's check the structure and summary statistics of the data set.

```
> str(data)
'data.frame':
                         s. of 13 variables:
1 2 3 4 5 6 7 8 9 1
8.5 8.2 9.2 6.4 9 6
                  100 obs. of
                                       7 8 9 10
                  : int
   ID
                                        6.4 9 6.5 6.9 6.2 5.8 6.4 ...
   ProdQual
                  : num
 $ Ecom
                                                           3.3 3.6 4.5 ...
                          3.9
                                   3.4
                                        3.3 3.4 2.8 3.7
                    num
                              2.7
                                       7 5.2 3.1
3.7 4.6 4
                              5.1
7.2
                                   5.6
5.6
                                            2 3.1 5 4.6 4.1
                    num
                          2.5
                                                      3.9
                                                           5.1
   TechSup
                                                      2.6 4.8
                          5.9
                                                               6.7
   CompRes
                    num
   Advertising
                    num
                          4.8
                              3.4
                                   5.4 4.7
                                            2.2 4 2.1 4.6 3.7 4.7
                              7.9
                                       4.7
   ProdLine
                   num
                          4.9
                                   7.4
                                            6 4.3 2.3
                                                        3.6
                                                             5.9
                                               3.7
                          6 3.1 5.8 4.5 4.5
                                                        5.1
   SalesFImage:
                   num
                                                    5.4
                          6.8
                              5.3 4.5
                                       8.8 6.8 8.5 8.9 6.9 9.3 8.4 ...
   ComPricing
                    num
                              5.5
                                   6.2
                                               5.1 4.8
3.6 2.1
                                                        5.4
                                                             5.9
                          4.7
   WartyClaim
                    num
                                          6.1
                          5 3.9 5.4 4.3 4.5
                                                        4.3 4.4 4.1 ...
   OrdBilling
                    num
                              5.7 8.9 4.8 7.1 4.7
                                                      3.7
   DelSpeed
                    num
                          3.7
                                                          4.6
                          8.2
                                                      5.7 6.3 7
   Satisfaction: num
```

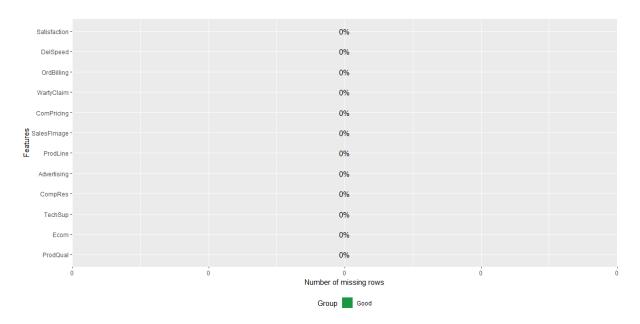
```
> describe(data)
                                    sd median trimmed
                                                                                   skew kurtosis
               vars
                          mean
                                                          mad min
                                                                      max range
                    100 50.50 29.01
                                                        37.06 1.0
                                        50.50
                                                 50.50
                                                                    100.0
                                                                            99.0
                                                                                   0.00
                                                                                            -1.24
ProdQual
                  2
                                         8.00
                                                   7.85
                                                               5.0
                                                                             5.0
                                                                                            -1.17
                    100
                          7.81
                                 1.40
                                                         1.78
                                                                     10.0
                                                                                  -0.24
                          3.67
                                                                                             0.57
                  3
                    100
                                 0.70
                                                         0.52 2.2
ECOM
                                         3.60
                                                   3.63
                                                                      5.7
                                                                             3.5
                                                                                   0.64
                                                         1.85 1.3
                                                                      8.5
                                                                             7.2
                                 1.53
                  4
                    100
                          5.37
                                         5.40
                                                   5.40
                                                                                  -0.20
TechSup
                                                                                            -0.63
                          5.44
                                         5.45
                                                                      7.8
6.5
                  5
                    100
                                 1.21
                                                  5.46
                                                         1.26
                                                               2.6
                                                                             5.2
                                                                                  -0.13
                                                                                            -0.66
CompRes
                  6
                    100
                                                         1.19
                                                               1.9
                                                                             4.6
                                                                                            -0.94
Advertising
                          4.01
                                         4.00
                                                   4.00
                                                                                   0.04
                                 1.32
                                                         1.56
                                                               2.3
                                                                                            -0.60
ProdLine
                    100
                          5.80
                                         5.75
                                                   5.81
                                                                      8.4
                                                                             6.1
                                                                                  -0.09
SalesFImage
                  8
                    100
                          5.12
                                 1.07
                                         4.90
                                                   5.09
                                                         0.89 2.9
                                                                      8.2
                                                                             5.3
                                                                                   0.37
                                                                                             0.26
                  9
                          6.97
                                 1.55
ComPricing
                    100
                                         7.10
                                                  7.01
                                                         1.93 3.7
                                                                      9.9
                                                                             6.2 - 0.23
                                                                                            -0.96
WartyClaim
OrdBilling
                 10
                    100
                          6.04
                                 0.82
                                                  6.04
                                                         0.89 4.1
                                                                      8.1
                                                                             4.0
                                                                                            -0.53
                                         6.10
                                                                                   0.01
                    100
                          4.28
                                                  4.31
                                                         0.74 2.0
                                                                      6.7
                                                                             4.7
                                                                                             0.11
                 11
                                 0.93
                                         4.40
                                                                                  -0.32
                          3.89
                                         3.90
                                                         0.74
DelSpeed
                 12
                    100
                                 0.73
                                                  3.92
                                                               1.6
                                                                      5.5
                                                                             3.9
                                                                                  -0.45
                                                                                             0.09
                                         7.05
                                                                      9.9
Satisfaction
                 13 100
                          6.92
                                 1.19
                                                  6.90
                                                         1.33 4.7
                                                                                   0.08
                                                                                            -0.86
```

The variable ID is unique number given to individuals and as such should not have any explanatory power for explaining *Satisfaction* in the regression equation. So we can safely drop ID from the dataset.

```
1. data1 <- subset(data, select = -c(1))</pre>
```

Now let's check for the missing values in the dataset.

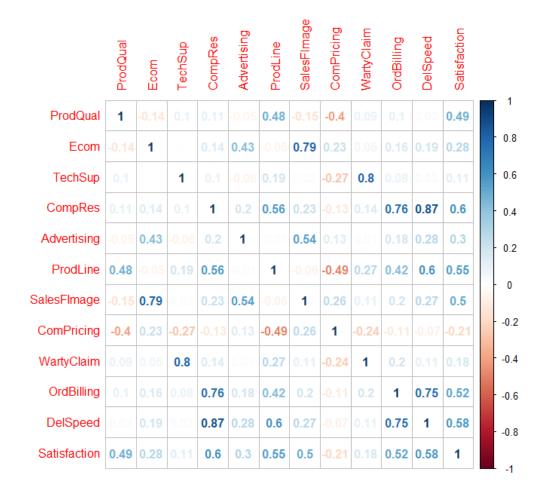
```
    library(DataExplorer)
    plot_missing(data)
```



There are no missing values in the datasets.

Inter-item Correlation analysis:

Now let's plot the correlation matrix plot of the data.



Observations:

- 1. CompRes and DelSpeed are highly correlated
- 2. OrdBilling and CompRes are highly correlated
- 3. WartyClaim and TechSupport are highly correlated
- 4. CompRes and OrdBilling are highly correlated
- 5. OrdBilling and DelSpeed are highly correlated
- 6. Ecom and SalesFImage are highly correlated

For examining the patterns of multicollinearity, it is required to conduct t-test for correlation coefficient. Let's use the *ppcor* package to compute the partial correlation coefficients along with the t-statistics and corresponding *p* values for the independent variables.

```
1. library(ppcor)
2. pcor(data1, method = "pearson")
```

```
> pcor(data1, method = "pearson")
$estimate
                  ProdQual
                                      ECOM
                                                TechSup
                                                               CompRes
                                                                         Advertising
                                                         -0.056354142
-0.033513777
                                                                       0.1123767461
-0.0002972504
               1.000000000
                             0.1549597037
                                            0.002424222
ProdQual
                                            0.082359000
               0.154959704
                             1.0000000000
ECOM
                             0.0823590001
                                            1.00000000
TechSup
               0.002424222
                                                          0.143603415
                                                                       -0.0593002540
                                                          1.000000000
CompRes
              -0.056354142
                            -0.0335137768
                                            0.143603415
                                                                       -0.0648060931
                            -0.0002972504
Advertising
               0.112376746
                                           -0.059300254
                                                         -0.064806093
                                                                        1.000000000
                                                          0.020305650
ProdLine
               0.281144724
                             0.1538660545
                                           -0.125349050
                                                                        0.1319231866
                                                                        0.2628793429
              -0.376228551
                             0.7321011890
                                           -0.093310796
SalesFImage
                                                         -0.022150933
                                                                       -0.0632980381
ComPricing
              -0.014021386
                             0.0149131857
                                           -0.132972485
                                                         -0.004487151
                                                                        0.0275909587
                            -0.1232553822
                                                         -0.109686211
WartyClaim
              -0.042752416
                                            0.787729606
OrdBilling
               0.054523215
                             0.1546990521
                                           -0.165914724
                                                          0.288344382
                                                                        -0.0326580430
                                           -0.021914916
              -0.335084210
                                                          0.528932328
                                                                        0.2046544064
                            -0.0083917930
DelSpeed
               0.607438787
                                            0.055111867
                                                          0.172416347
                                                                       -0.0449735411
Satisfaction
                            -0.3308440834
                            SalesFImage
                                           ComPricing
                                                       WartyClaim
-0.04275242
                 ProdLine
                                                                     OrdBilling |
                           -0.376228551
                                                                     0.05452322
               0.28114472
                                         -0.014021386
ProdQual
ECOM
               0.15386605
                            0.732101189
                                          0.014913186
                                                       -0.12325538
                                                                     0.15469905
                                                        0.78772961
                                                                    -0.16591472
              -0.12534905
                           -0.093310796
                                         -0.132972485
TechSup
               0.02030565
                           -0.022150933
                                         -0.004487151
                                                       -0.10968621
                                                                     0.28834438
CompRes
                                         -0.063298038
Advertising
              -0.13192319
                            0.262879343
                                                        0.02759096
                                                                    -0.03265804
ProdLine
               1.00000000
                           -0.230170570
                                         -0.361757904
                                                        0.25718360
                                                                    -0.28098068
                                                        0.18930027
SalesFImage
              -0.23017057
                            1.000000000
                                          0.126612489
                                                                    -0.18235930
              -0.36175790
                                          1.000000000
                                                                     0.08650087
ComPricing
                            0.126612489
                                                        0.02035155
WartyClaim
               0.25718360
                            0.189300271
                                          0.020351554
                                                        1.00000000
                                                                     0.25984451
                                                                     1.0000000
OrdBilling
              -0.28098068
                           -0.182359301
                                         -0.086500869
                                                        0.25984451
               0.50189505
                            0.005889925
                                          0.190437993
                                                       -0.09164900
DelSpeed
                                                                     0.35053021
               0.18325156
                            0.660251850 -0.087467711 -0.08868071
                                                                     0.14880055
Satisfaction
                  DelSpeed
                           Satisfaction
ProdQual
              -0.335084210
                              0.60743879
ECOM
              -0.008391793
                             -0.33084408
                              0.05511187
TechSup
              -0.021914916
               0.528932328
                              0.17241635
CompRes
Advertising
               0.204654406
                             -0.04497354
               0.501895051
                              0.18325156
ProdLine
               0.005889925
                              0.66025185
SalesFimage
ComPricing
               0.190437993
                             -0.08746771
WartyClaim
              -0.091649002
                             -0.08868071
OrdBilling
               0.350530212
                              0.14880055
                              0.08955614
               1.000000000
DelSpeed
Satisfaction
               0.089556143
                              1.00000000
```

```
$p.value
                  ProdOual
                                               TechSup
                                                             Compres Advertising
                                     ECOM
              0.000000e+00 1.447435e-01 9.819081e-01 5.977987e-01
                                                                       0.2916350\bar{1}
ProdQual
                                                        7.538375e-01
              1.447435e-01 0.000000e+00 4.402823e-01
                                                                       0.99778145
ECOM
TechSup
              9.819081e-01 4.402823e-01 0.000000e+00 1.769171e-01
                                                                       0.57876015
              5.977987e-01 7.538375e-01 1.769171e-01 0.000000e+00
                                                                       0.54395113
CompRes
              2.916350e-01 9.977814e-01 5.787601e-01 5.439511e-01
                                                                       0.0000000
Advertising
ProdLine
              7.269030e-03 1.476353e-01 2.391187e-01 8.493380e-01
                                                                       0.21517368
                                                                       0.01230672
              2.576212e-04 2.442012e-16 3.817042e-01 8.358301e-01
SalesFImage
              8.956443e-01 8.890480e-01 2.115149e-01 9.665194e-01
ComPricing
                                                                       0.55338281
WartyClaim
              6.890839e-01 2.471182e-01 3.262868e-20
                                                        3.034147e-01
                                                                       0.79629803
OrdBilling
              6.097697e-01 1.454288e-01 1.180864e-01 5.850710e-03
                                                                       0.75993047
DelSpeed 1.245192e-03 9.374303e-01 8.375553e-01 8.359069e-08 Satisfaction 2.182238e-10 1.447603e-03 6.059096e-01 1.041593e-01
                                                                       0.05300070
                                                                       0.67382405
                                            ComPricing
                                                        WartyClaim OrdBilling 6.890839e-01 0.6097697424
                  ProdLine
                             SalesFImage
              7.269030e-03 2.576212e-04 0.8956443272
ProdQual
              1.476353e-01 2.442012e-16 0.8890479591
                                                        2.471182e-01 0.1454287628
ECOM
                            3.817042e-01 0.2115148546
TechSup
              2.391187e-01
                                                        3.262868e-20 0.1180864174
                                                        3.034147e-01 0.0058507099
              8.493380e-01 8.358301e-01 0.9665194173
CompRes
Advertising
              2.151737e-01 1.230672e-02 0.5533828069
                                                        7.962980e-01 0.7599304715
                                                        1.440249e-02 0.0073046065
ProdLine
              0.000000e+00 2.907563e-02 0.0004593443
                                                        7.394541e-02 0.0853804743
8.490015e-01 0.4175555780
              2.907563e-02
                            0.000000e+00 0.2343791374
SalesFImage
              4.593443e-04 2.343791e-01 0.0000000000
ComPricing
WartyClaim
              1.440249e-02 7.394541e-02 0.8490014635
                                                        0.000000e+00 0.0133877361
OrdBilling |
              7.304606e-03 8.538047e-02 0.4175555780
                                                        1.338774e-02 0.0000000000
              4.664278e-07
                            9.560619e-01 0.0721942454 3.902770e-01 0.0007064114
DelSpeed
                            1.449719e-12 0.4123500152 4.058729e-01 0.1615974575
Satisfaction 8.383625e-02
                  DelSpeed Satisfaction
              1.245192e-03 2.182238e-10
ProdQual
ECOM
              9.374303e-01 1.447603e-03
              8.375553e-01 6.059096e-01
TechSup
              8.359069e-08 1.041593e-01
CompRes
Advertising
              5.300070e-02 6.738241e-01
ProdLine
              4.664278e-07 8.383625e-02
              9.560619e-01 1.449719e-12
SalesFImage
ComPricing
              7.219425e-02 4.123500e-01
WartyClaim
              3.902770e-01 4.058729e-01
OrdBilling
              7.064114e-04 1.615975e-01
              0.000000e+00 4.012356e-01
DelSpeed
Satisfaction 4.012356e-01 0.000000e+00
$statistic
                 ProdQual
                                             TechSup
                                                                    Advertising
                                   ECOM
                                                          CompRes
                                                                    1.060907458
               0.00000000
                            1.471424516
                                          0.02274128
                                                      -0.52949015
ProdQual
                                                      -0.31456380
1.36122814
               1.47142452
ECOM
                            0.00000000
                                          0.77522957
                                                                   -0.002788456
               0.02274128
                                          0.00000000
TechSup
                            0.775229571
                                                                   -0.557266374
              -0.52949015
                           -0.314563798
                                                       0.00000000
                                                                   -0.609215688
CompRes
                                          1.36122814
Advertising
               1.06090746
                           -0.002788456
                                         -0.55726637
                                                      -0.60921569
                                                                    0.00000000
              2.74821968
-3.80921125
                                                       0.19052316
                            1.460787002
                                         -1.18522655
                                                                   -1.248460807
ProdLine
                                                                    2.555921881
                           10.081854496 -0.87916864
                                                      -0.20784517
SalesFImage
              -0.13154519
Compricing
                            0.139913642 -1.25856891
                                                      -0.04209363 -0.594981366
WartyClaim
              -0.40142023
                           -1.165122048 11.99562484
                                                      -1.03519395
                                                                    0.258924708
OrdBilling
               0.51223505
                            1.468888754
                                         -1.57829305
                                                       2.82489237
                                                                   -0.306523103
              -3.33624278
                           -0.078724768
                                         -0.20562952
                                                       5.84663073
                                                                    1.961341689
DelSpeed
                                          0.51778207
                                                       1.64199901 -0.422316520
               7.17336519
                           -3.288799958
Satisfaction
                ProdLine SalesFImage
                                        Compricing WartyClaim OrdBilling
                                                                               DelSpeed
ProdQual
               2.7482197
                          -3.80921125
                                       -0.13154519
                                                    -0.4014202
                                                                 0.5122350
                                                                            3.33624278
                                        0.13991364
                                                                 1.4688888
                                                   -1.1651220
11.9956248
               1.4607870
                         10.08185450
                                                                           -0.07872477
ECOM
TechSup
              -1.1852266
                          -0.87916864
                                       -1.25856891
                                                               -1.5782930
                                                                           -0.20562952
               0.1905232
                          -0.20784517
                                       -0.04209363
                                                    -1.0351939
                                                                 2.8248924
                                                                             5.84663073
CompRes
                           2.55592188 -0.59498137
                                                                -0.3065231
              -1.2484608
                                                     0.2589247
                                                                             1.96134169
Advertisina
                                                                             5.44344736
               0.0000000
                          -2.21876450 -3.64012829
                                                     2.4965744
                                                               -2.7464786
ProdLine
                                                     1.8084929
                                                               -1.7398559
                           0.00000000
                                        1.19736653
                                                                             0.05525335
SalesFImage
              -2.2187645
                                                     0.1909540
                                                               -0.8145030
                                                                             1.81976992
                                        0.00000000
Compricing
              -3.6401283
                           1.19736653
WartyClaim
               2.4965744
                           1.80849286
                                        0.19095404
                                                     0.000000
                                                                 2.5242649
                                                                            -0.86337748
OrdBilling |
              -2.7464786
                          -1.73985585
                                       -0.81450302
                                                     2.5242649
                                                                 0.000000
                                                                             3.51103503
               5.4434474
DelSpeed
                           0.05525335
                                        1.81976992
                                                    -0.8633775
                                                                 3.5110350
                                                                             0.0000000
Satisfaction
               1.7486638
                           8.24679932 -0.82367672 -0.8351894
                                                                 1.4115877
                                                                             0.84350046
              Satisfaction
```

```
ProdQual
                 7.1733652
ECOM
                -3.2888000
TechSup
                 0.5177821
CompRes
                 1.6419990
                -0.4223165
Advertising
ProdLine
                 1.7486638
                 8.2467993
SalesFImage
ComPricing
                -0.8236767
WartyClaim
                -0.8351894
OrdBilling
                 1.4115877
DelSpeed
                 0.8435005
Satisfaction
                 0.0000000
[1] 100
$gp
[1] 10
$method
[1] "pearson"
```

As expected the correlation between *sales force image* and *ecommerce* is highly significant; so is the correlation between *delivery speed* and *order billing* with *complaint resolution*. Also, the correlation between *order* & *billing* and delivery speed. We can safely assume that there is a high degree of collinearity between the independent variables.

Initial Regression Model using the data as it is.

Let's build a model using all the independent variables and check its performance.

```
1. model0 = lm(Satisfaction~., data1)
  2. summary(model0)
  3. vif(model0)
lm(formula = Satisfaction ~ ., data = data1)
Residuals:
                            3Q
0.37190
                1Q
                     Median
     Min
                                           Max
-1.43005 -0.31165
                    0.07621
                                      0.90120
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
                                  -0.824
                         0.81233
(Intercept) -0.66961
                                           0.41199
ProdQual
              0.37137
                         0.05177
                                    7.173
                                          2.18e-10
             -0.44056
                         0.13396
                                   -3.289
                                           0.00145 **
ECOM
              0.03299
TechSup
                         0.06372
                                    0.518
                                           0.60591
              0.16703
                         0.10173
                                           0.10416
CompRes
                                    1.642
Advertising -0.02602
                         0.06161
                                    -0.422
                                           0.67382
                                    1.749
ProdLine
              0.14034
                         0.08025
                                           0.08384
                                    8.247
SalesFImage
                         0.09775
                         0.04677
                                   -0.824
                                           0.41235
ComPricing
             -0.03853
             -0.10298
                         0.12330
WartyClaim
                                   -0.835
                                           0.40587
OrdBilling |
              0.14635
                         0.10367
                                    1.412
                                           0.16160
                         0.19644
                                    0.844
                                           0.40124
DelSpeed
              0.16570
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.5623 on 88 degrees of freedom
Multiple R-squared: 0.8021, Adjusted R-squared: 0.7774
```

```
F-statistic: 32.43 on 11 and 88 DF, p-value: < 2.2e-16
> vif(model0)
   ProdQual
                             TechSup
                                         CompRes Advertising
                   ECOM
               2.756694
                            2.976796
                                        4.730448
   1.635797
                                                     1.508933
                                      WartyClaim
   ProdLine SalesFImage
                                                  OrdBilling
                         ComPricing
               3.439420
                                        3.198337
                                                     2.902999
   3.488185
                           1.635000
   DelSpeed
   6.516014
```

Here, even though independent variables explain 78% of variance of the dependent variable, only 3 variables are significant out of 11 independent variables.

Now let's study the VIF values.

High Variable Inflation Factor (VIF) is a sign of multicollinearity. There is no formal VIF value for determining presence of multicollinearity; however in weaker models VIF value greater than 2.5 may be a cause of concern.

From the VIF values we can infer that variables DelSpeed and CompRes are a cause of concern.

Remedial Measures:

Two of the most commonly used methods to deal with multicollinearity in the model is the following.

- Remove some of the highly correlated variables using VIF or stepwise algorithms.
- Perform an analysis design like principal component analysis (PCA)/ Factor Analysis on the correlated variables.

Factor Analysis:

Now let's check the factorability of the variables in the dataset.

First create a new dataset by taking a subset of all the independent variables in the data and perform Kaiser-Meyer-Olkin (KMO) Test.

```
    data2 <- subset(data1, select = -c(12))</li>
    datamatrix<-cor(data1)</li>
    KMO(r=datamatrix)
```

```
Kaiser-Meyer-Olkin factor adequacy
Call: KMO(r = datamatrix)
Overall MSA = 0.66
MSA for each item =
    ProdQual
                                                                           ProdLine
                       Ecom
                                  TechSup
                                                CompRes
                                                          Advertising
        0.49
                       0.59
                                     0.52
                                                   0.83
                                                                 0.83
                                                                                0.70
                ComPricing
0.77
                                             OrdBilling
                                                             DelSpeed Satisfaction
 SalesFImage 0.52
                              WartyClaim
                                                   0.79
                                     0.52
                                                                 0.72
                                                                                0.66
```

Since MSA > 0.5, we can run Factor Analysis on this data.

Bartlett's test of sphericity should be significant.

```
1. cortest.bartlett(datamatrix, n = 50)
```

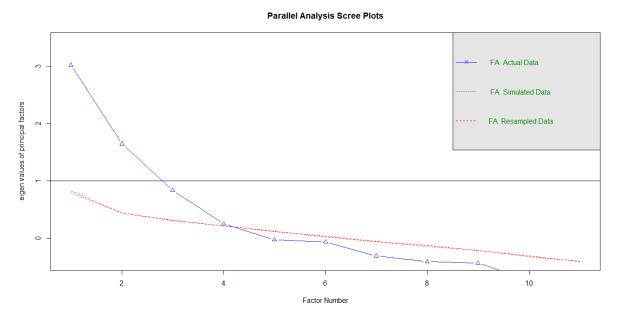
```
$chisq
[1] 360.9826

$p.value
[1] 3.049209e-42

$df
[1] 66
```

Now let's use *Psych* package's *fa.parallel* function to execute parallel analysis to find acceptable number of factors and generate the scree plot.

```
1. parallel <- fa.parallel(data2, fm = 'minres', fa = 'fa')</pre>
```



The blue line shows eigenvalues of actual data and the two red lines (placed on top of each other) show simulated and resampled data. Here we look at the large drops in the actual data and spot the point where it levels off to the right.

Looking at the plot 3 or 4 factors would be a good choice.

Let's use 4 factors to perform the factor analysis. Also, let's use orthogonal rotation (varimax) because in orthogonal rotation the rotated factors will remain uncorrelated whereas in oblique rotation the resulting factors will be correlated.

1. fa1<- fa(r=data2, nfactors = 4, rotate="varimax",fm="pa")</pre>

print(fa1)

```
fa.diagram(fa1)
Factor Analysis using method = pa
Call: fa(r = data2, nfactors = 4, rotate = "varimax", fm = "pa")
Standardized loadings (pattern matrix) based upon correlation matrix
             PA1
                   PA2
                         PA3
                               PA4
                                     h2
                                           u2 com
            0.02 -0.07 0.02 0.65 0.42 0.576 1.0
ProdQual
ECOM
            0.07 0.79
                        0.03 -0.11 0.64 0.362 1.1
            0.02 -0.03 0.88 0.12 0.79 0.205 1.0
TechSup
CompRes
            0.90 0.13 0.05 0.13 0.84 0.157 1.1
Advertising 0.17 0.53 -0.04 -0.06 0.31 0.686 1.2
            0.53 -0.04 0.13 0.71 0.80 0.200 1.9
ProdLine
SalesFImage 0.12 0.97 0.06 -0.13 0.98 0.021 1.1
Compricing -0.08 0.21 -0.21 -0.59 0.44 0.557 1.6
WartyClaim
            0.10 0.06 0.89 0.13 0.81 0.186 1.1
OrdBilling
            0.77  0.13  0.09  0.09  0.62  0.378  1.1
            0.95 0.19 0.00 0.09 0.94 0.058 1.1
DelSpeed
                      PA1 PA2 PA3 PA4
                     2.63 1.97 1.64 1.37
SS loadings
Proportion Var
                     0.24 0.18 0.15 0.12
Cumulative Var
                     0.24 0.42 0.57 0.69
Proportion Explained 0.35 0.26 0.22 0.18
Cumulative Proportion 0.35 0.60 0.82 1.00
Mean item complexity = 1.2
Test of the hypothesis that 4 factors are sufficient.
The degrees of freedom for the null model are 55 and the objective funct
ion was 6.55 with Chi Square of 619.27
The degrees of freedom for the model are 17 and the objective function wa
s 0.33
The root mean square of the residuals (RMSR) is 0.02
The df corrected root mean square of the residuals is 0.03
The harmonic number of observations is 100 with the empirical chi square
3.19 with prob < 1
The total number of observations was 100 with Likelihood Chi Square = 3
0.27 with prob < 0.024
```

Tucker Lewis Index of factoring reliability = 0.921

RMSEA index = 0.096 and the 90 % confidence intervals are 0.032 0.139

BIC = -48.01

Fit based upon off diagonal values = 1

Measures of factor score adequacy

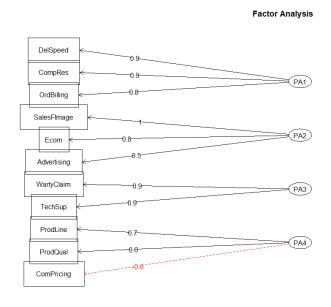
PA1 PA2 PA3 PA4

Correlation of (regression) scores with factors 0.98 0.99 0.94 0.88

Multiple R square of scores with factors 0.96 0.97 0.88 0.78

Minimum correlation of possible factor scores 0.93 0.94 0.77 0.55

Let's visualize the factors.



Labelling and interpretation of the factors.

#	Factors	Variables	Label	Short Interpretation
1	PA1	DelSpeed, CompRes, OrdBilling	Purchase	All the items are related to purchasing the product; from placing order to billing and getting it delivered.
2	PA2	SalesFImage, Ecom, Advertising	Marketing	In this factors the items are related to marketing processes like the image of sales force and spending on advertising.
3	PA3	WartyClaim, TechSup	Post Purchase	Post purchase activities are included in this factor; like warranty claims and technical support.
4	PA4	ProdLine, ProdQual, CompPricing	Product Position	Product positioning related items are grouped in this factor.

Regression analysis using the factors scores as the independent variable.

Let's combine the dependent variable and the factor scores into a dataset and label them.

```
    regdata <- cbind(data1[12], fa1$scores)</li>
    head(regdata)
    names(regdata) <- c("Satisfaction", "Purchanse", "Marketing", "Post_purchase", "Prod_positioning")</li>
```

```
> head(regdata)
 Satisfaction Purchanse Marketing Post_purchase Prod_positioning
1
          8.2 -0.1338871 0.9175166 -1.719604873
                                                      0.09135411
2
          5.7 1.6297604 -2.0090053 -0.596361722
                                                      0.65808192
3
          8.9 0.3637658 0.8361736
                                      0.002979966
                                                      1.37548765
          4.8 -1.2225230 -0.5491336
4
                                      1.245473305
                                                     -0.64421384
5
          7.1 -0.4854209 -0.4276223 -0.026980304
                                                      0.47360747
          4.7 -0.5950924 -1.3035333 -1.183019401 -0.95913571
```

Let's the dataset into training and testing dataset (70:30)

```
    set.seed(100)
    indices= sample(1:nrow(regdata), 0.7*nrow(regdata))
    train=regdata[indices,]
    test = regdata[-indices,]
```

Let's train the regression model.

```
    model1 = lm(Satisfaction~., train)
    summary(model1)
```

```
call:
lm(formula = Satisfaction ~ ., data = train)
Residuals:
            1Q Median
   Min
                            3Q
                                  Max
-1.6857 -0.4018 0.1051 0.4027 1.2036
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                            0.08263 83.827 < 2e-16 ***
(Intercept)
                 6.92625
                                     7.377 3.73e-10 ***
Purchanse
                 0.62022
                            0.08408
Marketing
                 0.57735
                            0.08047
                                     7.175 8.50e-10 ***
                                     1.104 0.274
Post_purchase
                 0.09567
                            0.08667
                           0.09374 7.101 1.15e-09 ***
Prod_positioning 0.66562
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.6814 on 65 degrees of freedom
Multiple R-squared: 0.7079, Adjusted R-squared:
F-statistic: 39.39 on 4 and 65 DF, p-value: < 2.2e-16
```

The factors Purchasing, Marketing and Prod_positioning are significant in the model.

Let's check the VIF scores.

All the VIF values are reasonably alright; we don't have multicollinearity in the model.

Root mean square error of the model:

```
> rmse1 <- sqrt((mean(model1$residuals^2)))
> rmse1
[1] 0.6566331
```

Now let's check prediction of the model in the test dataset.

```
    pred=predict(model1, newdata = test, type = "response")
    test$Satisfaction.Predict <- pred</li>
```

```
> head(test[c(1,6)],10)
    Satisfaction Satisfaction.Predict
                 8.2
5.7
                                         7.269232
                                         7.158146
8.550469
2
3
4
5
7
                 8.9
                                         5.541333
                  4.8
                  7.1
                                         6.690958
                  5.7
                                         4.661277
14
21
23
27
                  7.6
                                         7.963941
                  5.4
                                         5.570249
                  7.0
                                         7.704405
                                         7.361437
```

Now let's check the R-squared in the test data set.

```
> cor(test$Satisfaction, test$Satisfaction.Predict)^2
[1] 0.6396429
```

Root mean square error of the model in the test dataset:

```
> rmse2 <- sqrt(mean((test$Satisfaction - pred)^2))
> rmse2
[1] 0.6618739
```

#	R-Squared	RMSE
Train	0.69	0.656633
Test	0.6396429	0.661874

The R squared is not varying much for the model both in test and train dataset and so is the RMSE; so we can infer that the model is valid and also not overfit.