A lottery preference-based explanation of realized kurtosis puzzle in Chinese stock market

ABSTRACT

High kurtosis corresponds to fat tails on both sides and under risk-aversion assumption investors' dislike of left-tail loss outweighs their preference for right-tail gain. Therefore, high kurtosis characteristic of stock should predict high expected return. However, the high-frequency data based empirical results on Chinese stock market are just the opposite, which we refer to as the "realized kurtosis puzzle". Using the double sorts and firm-level cross-sectional regression methods, we further demonstrate investors' preference for lottery-like stocks or lottery preference is key to solve the puzzle. Our further empirical research verifies stocks with higher retail investors' shareholding proportion and unavailable for short show stronger "realized kurtosis puzzle". In addition, the puzzle is particularly significant in high lottery preference periods while less apparent in low lottery preference times.

KEYWORDS

Realized kurtosis puzzle; lottery preference; stock pricing; Chinese stock market

JEL CLASSIFICATION

G12; C32

I. Introduction and literature review

The capital asset pricing theory (CAPM) proposed by Sharpe (1964), Lintner (1965) and Black (1972) asserts exists a linear relationship between the expected return and market beta, while other factors fail to explain the expected return. Though challenged by numerous following empirical studies (for example, Fama and French, 1992), this model lays the foundation for subsequent researches on the asset pricing factors and its core idea still holds - the pricing factor behind the asset reflect the risk the asset face, and the risk premium exists to compensate for the risk.

Since the market beta is constructed on the second-order moment, along this line following scholars have conducted more in-depth researches. Literatures on the relationship between second-order moment (variance and corresponding volatility) and the expected return of stock are numerous. It is widely accepted that high volatility is an embodiment of stock risk, and to compensate for this risk, high volatility stocks should have high expected returns. However, empirical study conducted by Ang et al. (2006, 2009) shows that the stock's aggregate or idiosyncratic volatility and the expected return are negatively related. This anomaly has raised a great deal of academic interest and has led to a series of further studies (Bali and Cakici, 2008; Adrian and Rosenberg, 2008; Stambaugh et al., 2015; Hou and Loh, 2016). Some literatures further focus on the relationship between stock's third-order moment (skewness) and expected return. Since investors prefer stocks with positive skewed returns, holding desirable positive-skewed stocks must be at the expense of lower expected return. The skewness factor is incorporated into expected return models by Kraus and Litzenberger (1976, 1983). At the same time, a series of empirical studies also support this view (Harvey and Siddique, 2000; Chen et al., 2001; Boyer et al., 2010).

Unlike the above, the kurtosis (fourth-order moment) and its relationship with expected return have received relatively less attention mainly for two reasons. First, high kurtosis corresponds to heavy tails on both sides of the distribution. Investors prefer right-tail gains and dislike left-tail losses. Since two forces are working in the opposing directions, the overall result can be quite obscure and surely not as obvious as the skewness. Second, how to accurately estimate the kurtosis turns out to be a practical problem when data is limited. There are two main ways to deal with this issue. First, following the studies of Bakshi and Madan (2000) and Bakshi et al. (2003), implied kurtosis information can be derived from the trading data of derivatives. Second, when it is unlikely to use low-frequency data to accurately estimate kurtosis, we can instead take advantage of high-frequency data to solve this problem. Based on these two ideas, researchers have conducted several studies on this issue. Using option prices data, Conrad et al. (2013) extract risk-neutral high-order moment information and finds a positive link between kurtosis and expected return. Similarly, Chang et al. (2013) derive market fourth-order moment information (i.e., kurtosis) from S&P 500 options. Empirical research based on this indicator shows that stocks more sensitive to market kurtosis enjoy higher expected returns. Similarly, Bali et al. (2017) also make use of options to extract high-order moment information, and confirm that the high kurtosis stocks possess high expected returns. Furthermore, they define the difference between the risk-neutral high-order moment and the physical high-order moment as the risk premium, and find that the risk premium is positively correlated with the expected return. Unlike scholars mentioned above who derive kurtosis information from option prices, Amaya et al. (2016) resort to high-frequency data. With intraday high-frequency data, their empirical research demonstrates that in most specifications the realized kurtosis is positively correlated with stock expected return in U.S. stock market.

Overall, most studies support the view that high kurtosis stocks have high expected returns, and this result is not surprising [1]. Given a variance, kurtosis depicts the extent to which the distribution is concentrated in the tail (Darlington, 1970). The higher the kurtosis, the fatter the tail is. The fat tail on the left means great possibility of poor performance disgusted by investors. In contrast, the fat tail on the right means that the large likelihood of excellent performance preferred by investors. If investors are risk averse, then their aversion to losses will outweigh their preference for returns. Therefore, the result of two contradictory effects should be that investors dislike high kurtosis stocks. In consequence, stocks with higher kurtosis should have higher expected returns so as to make investors willing to hold them.

Note that the above empirical conclusions about stock's kurtosis and expected return are based on U.S. stock market. Our focus is, however, on the Chinese capital market. Due to the short history of the Chinese option market, few option varieties and relatively low market trading volume, we are temporarily unable to effectively derive high-quality kurtosis information from the option-based method over a long time period. Thus, we instead turn to obtain kurtosis from high-frequency trading data of the stock market. Based on the five-minute high-frequency trading data of individual stocks, we construct the measure of the realized kurtosis, with which we further study the relationship between kurtosis and stock expected return of Chinese stock market. To our surprise, our empirical result based on Chinese stock market is exactly the opposite of the one from U.S. stock market. The expected returns of high realized kurtosis stocks are significantly lower than that of stocks with low realized kurtosis. We call this phenomenon the "realized kurtosis puzzle". Compared to the two-hundred-year-old U.S. stock market, Chinese stock market is relatively young, and it will not be too astonishing to discover a great deal of differences between the two. The purpose of this paper is therefore to explain the "realized kurtosis puzzle" occurred in Chinese stock market. On the basis of discovering the "realized kurtosis puzzle", we further attempt to prove it is the strong lottery preference in Chinese stock market that is the key to solve the puzzle. As Statman (2002) emphasizes, lottery revenue and stock investment returns share a great many common points, and investors' speculation behavior will be reflected in stock investment. Kumar (2009) and Bali et al. (2011) provide empirical evidence for investors' lottery preference in stock investment. Barberis and Huang (2008) point out that lottery preference can be used to explain a series of seemingly unrelated financial phenomena. Lottery-like stocks are referred to such stocks may experience extraordinarily excellent return performance in the future just like holding lottery tickets. There is no doubt that lottery characteristic is desirable and investors are willing to accept lower expected return as a price. This means that lottery-like stocks should bear lower expected returns while non-lottery-like stocks should enjoy higher expected returns. If high kurtosis stocks are suffering much heavier lottery preference pressure than low kurtosis stocks, then investors' strong preference for lottery (no matter on rational or irrational basis) in Chinese stock market can reasonably explain the "realized kurtosis puzzle".

A natural question to ask is why the "realized kurtosis puzzle" exists in Chinese stock market but not in its U.S. counterpart. A direct explanation is that the lottery preference in Chinese stock market is much stronger than that in U.S. market. This might be related to the two major characteristics of Chinese stock market. First, the proportion of retail investors in Chinese stock market is remarkably high. Relative to institutional investors, retail investors with limited expertise and limited stock

[1] One exception is Blau and Whitby (2017). This paper shows high idiosyncratic kurtosis stocks suffer low return. However, the kurtosis measure of individual stocks in their paper are calculated based on daily data in a month. Since there're only about 21 trading days in one month, the fourth moment can hardly be well estimated, if not impossible.

trading experience apt to show stronger speculation tendency. Second, Chinese stock market has its restrictions on short selling. Despite the fact that Chinese stock market started its short selling mechanism in 2010, the stocks available for short selling so far account for only one-third of all stocks, and the annual interest rate for margin trading is up to 10%. The sky-high cost greatly restrains investors from short selling and thus block the major channel which rational market force can take advantage of to alleviate the lottery preference effect. No wonder, Jinglian Wu, a well-known Chinese economist, once put forward the view that "Chinese stock market is a casino".

We validate the above viewpoints through a series of empirical studies. (1) According to our single-sorted results, we find that the "realized kurtosis puzzle" is significant in Chinese stock market. Further, based on the low-minus-high investment portfolio sorted by the realized kurtosis, we find that the "realized kurtosis puzzle" remains relatively stable through time. (2) Similarly, through the single sorts method, we confirm that there exists a strong lottery preference effect in Chinese stock market. When we make use of the double sorts method and control the lottery preference variables, we can to a large extent explain the "realized kurtosis puzzle". In stark contrast to this, non-lottery preference factors appear to be ineffective in solving the puzzle. (3) Based on double sorts method, our empirical study confirms that stocks with higher retail investors' shareholding proportion show stronger "realized kurtosis puzzle". By splitting the stocks into two parts – unavailable for short and available for short, our single sort results demonstrate that the former displays stronger "realized kurtosis puzzle" than the latter. (4) We then conduct further studies by firm-level cross-sectional regressions. When controlling three lottery preference variables, the "realized kurtosis puzzle" is no longer statistically significant. In comparison, the non-lottery preference factors are almost incapable of explaining the puzzle. (5) We further examine the time-varying characteristics of the puzzle and their relations to the lottery preference. For this reason, we split the sample into the high and low lottery preference periods by the lottery preference variable. We find that the "realized kurtosis puzzle" during the high lottery preference period is obviously stronger than that of low lottery preference. Generally speaking, the period of stock market crash is right the time when terrified investors are busy searching for safe harbor and do not care about the lottery opportunities. Thus we find that the puzzle in stock market crash years is less evident.

The remainder of the paper proceeds as follows. The first part is the introduction and literature review. The second part proposes the variable construction method and provides the source of data. The third part verifies that there exists a "realized kurtosis puzzle" in Chinese stock market. We perform a double sorts analysis in the fourth part to see whether lottery preference theory can explain the "realized kurtosis puzzle". The fifth part tests whether lottery preference can explain the "realized kurtosis puzzle" from the perspective of two important characteristics of the Chinese stock market - high retail investors' shareholding proportion and short selling restrictions. The sixth part applies the conventional firm-level cross-sectional regressions method and Hou and Lou's new method to further study the above issues. The seventh part provides more supportive evidence by investigating the time-varying nature of the "realized kurtosis puzzle". Robustness checks are provided in the eighth part. Finally, the eighth part concludes.

II. Variables and data

Variable construction

In this section, we illustrate how to construct the kurtosis variable and lottery preference variables. Firstly, we clarify how to construct the kurtosis variable. As mentioned above, since the Chinese option market is established recently, we cannot extract the implied kurtosis information from option data for long time period. Following Barndorff-Nielsen and Shephard (2004) we construct the kurtosis index from high-frequency data. The daily realized kurtosis index is calculated as follows:

$$RKT_d \equiv \frac{N\sum_{n=1}^{N} r_{n,d}^4}{RVOL_d^4} \tag{1}$$

where RKT_d denotes the realized kurtosis index of the day d. $r_{n,d}$ indicates the n-th five-minute high-frequency return of the day d, and N indicates the number of five-minute high-frequency returns in one day (for Chinese stock market, N is 48). $RVOL_d$ denotes the realized volatility of the day d (the definition of realized volatility is shown below). Then, we transform the daily kurtosis index into monthly index. The corresponding transformation formula is as follows:

$$RKT_m \equiv \frac{1}{D}\left(\sum_{d=1}^{D} RKT_d\right) \tag{2}$$

The RKT_m denotes the realized kurtosis index of month m, D indicates the number of trading days in month m, and the RKT_d is the daily realized kurtosis index calculated above.

Secondly, we illuminate how to define lottery preference variables. Bali et al. (2011) take the average of several maximum daily returns in a month as an indicator of lottery preference (the number of days can range from 1 day to 5 days). Kumar (2009) documents lottery-like stocks are characterized by high volatility and high skewness. Therefore, combining the above literatures, we take the average of the three maximum daily returns in the month (MAX), the realized volatility and the realized skewness as proxy variables for the lottery preference. The construction method of MAX is as follows: for any stock, we search for the three trading days with the highest daily returns of that stock in the month, and then average these three returns to get the index. The construction of the daily index of realized volatility and realized skewness is as follows:

$$RVOL_d \equiv \sqrt{\sum_{n=1}^{N} r_{n,d}^2} \tag{3}$$

$$RSK_d \equiv \frac{\sqrt{N}\sum_{n=1}^{N} r_{n,d}^3}{RVOL_d^3} \tag{4}$$

$RVOL_d$ denotes the realized volatility index of the day d, while RSK_d denotes the realized skewness index of the day d. $r_{n,d}$ and N are defined as above. Similarly, we transform the daily indices into monthly indices:

$$RVOL_m \equiv \frac{1}{D}\left(\sum_{d=1}^{D} RVOL_d\right) \tag{5}$$

$$RSK_m \equiv \frac{1}{D}\left(\sum_{d=1}^{D} RSK_d\right) \tag{6}$$

Data sources

The high-frequency data in this paper is from the CSMAR high-frequency database. Since the start year of the data provided by this database is 2007, the sample covers period from 2007-1-4 (the first trading day of year 2007) to 2017-12-29 (the last trading day of year 2017). The sampling frequency of the high-frequency data in this paper is five-minute. On the one hand, sampling frequency needs to be high enough to meet the convergence requirement. On the other hand, sampling frequency have to be lowered to avoid market micro-noise. The choice of the sampling frequency of five minutes in this paper is the tradeoff between the two restrictions above. Furthermore, this sampling frequency is also the most common choice in literatures in this field (Hansen and Lunde, 2006). Daily trading time in Chinese stock market is from 9:30 to 11:30 and from 13:00 to 15:00. Altogether, we obtain approximately 270 million observations in 11 years, or 290,000 company-month combinations for Chinese A-shares. In addition, the non-high-frequency stock data (including daily price data and company characteristics data) in this paper come from the RESSET database.

III. Realized kurtosis puzzle

Single sorts based on realized kurtosis

We sort all stocks into 5 portfolios in the ascending order of the realized kurtosis, and then observe these portfolios' returns in the next month. We consider equal-weighted returns and value-weighted returns, respectively. The corresponding results are shown in Table 1.

Table 1. Single sorts based on realized kurtosis.

	Equal-Weighted			Value-Weighted		
	Level	Return	FF3F	Level	Return	FF3F
1	4.12	75.58	-30.78	3.90	14.56	-17.35
2	4.74	72.23	-39.03	4.73	3.83	-47.04
3	5.21	53.64	-59.79	5.20	-14.20	-73.24
4	5.77	43.31	-73.13	5.76	-11.73	-79.19
5	7.10	-16.29	-130.03	7.00	-60.41	-133.70
5-1		-91.87	-99.25		-74.97	-116.35
		(-4.40)	(-5.46)		(-1.86)	(-3.60)

The left half part of Table 1 corresponds to the equal-weighted case, and the right half part corresponds to the value-weighted case. The first column is the level of realized kurtosis of each portfolio. The realized kurtosis level of portfolio 1 is the lowest, and its mean value is 4.12. In contrast, the realized kurtosis level of portfolio 5 is the highest, with its mean value 7.10. The first five figures in the second column correspond to the time series average of monthly returns for each portfolio. We find that the returns of portfolio 1 to portfolio 5 show a monotonously decreasing trend. The monthly average return falls from 75.58 basis points (bps) or 0.76% for portfolio 1 all the way to -16.29 basis points (bps) or -0.16% for portfolio 5. The sixth figure is the average return resulting from longing the portfolio 5 and shorting the portfolio 1 at the same time (the high-minus-low portfolio). This value reaches -91.87 bps and its corresponding Newey-West t statistic is -4.40. Portfolio 5 has the highest realized kurtosis, meaning this portfolio has the fattest tails. If risk averse investors' dislike of left-tail loss exceeds their preference for right-tail gain, stocks in this portfolio should offer the highest expected returns in order to make risk averse investors willing to hold them. However, the single-sorted results based on Chinese stock market reach the opposite conclusion: the returns of high realized kurtosis stocks are significantly lower than that of low realized kurtosis stocks. We call this phenomenon the "realized kurtosis puzzle".

We further study whether the puzzle is caused by common systemic risks. Therefore, we use the Fama-French 3 Factors (FF3F) to adjust the returns and obtain the corresponding alpha. Alpha refers to the return that cannot be explained by these three systemic risk factors, and is shown in the second column. It can be seen that after the FF3F adjustment, the differences between the portfolios do not disappear. The average return of the high-minus-low portfolio is -99.25 bps, corresponding to a Newey-West t statistic of -5.46. This result is similar to the situation unadjusted by FF3F.

Analogously, we consider the value-weighted case and its FF3F adjusted counterpart. The average value-weighted return of the high-minus-low portfolio without FF3F adjustment is -74.97 bps with the corresponding Newey-West t statistic -1.91, while the average value-weighted return of the FF3F-adjusted high-minus-low portfolio is -116.35 bps with the Newey-West t statistic -3.60. In value-weighted case, the decreasing trend of the average return from portfolio 1 to portfolio 5 is retained, and the return of high-minus-low portfolios is still economically and statistically significant. In addition, in the value-weighted case, the absolute value of the return of the high-minus-low portfolio adjusted by the FF3F (which includes the market value factor) is significantly larger than that without the FF3F adjustment. Further empirical investigation in Section IV show that although "realized kurtosis puzzle" exist for stocks of different market capitalizations, the puzzle is more prominent among small-cap stocks than that in big-cap stocks. Therefore, the "realized kurtosis puzzle" can be better highlighted under equal weights construction method. For this reason, in the following sections we will pay special attention to the equal-weighted case.

Portfolio Performance

In this section we construct the low-minus-high investment portfolio based on the realized kurtosis (reverse operation of the high-minus-low portfolio, i.e., shorting portfolio 5 and longing portfolio 1). To evaluate portfolio performance, we introduce the market excess return portfolio as the reference to see whether our portfolio beat the market or not.

Figure 1 shows the cumulative returns of our low-minus-high portfolio and the market excess return portfolio. As can be seen, there is a clear discrepancy between the performance of the two. The cumulative return of low-minus-high investment portfolio goes up steadily, with its curve sloping upward to the right. Over the 11-year sample period, only one year's return is negative (year 2010). Correspondingly, the trajectory of the market portfolio over the same period is like experiencing a roller coaster ride. Its cumulative return is soaring from time to time, and then followed by cliff-like fall. In contrast, the trajectory of the market portfolio return over the same period however experiences a roller coaster ride with its cumulative return soaring from time to time, and then followed by cliff-like falls.

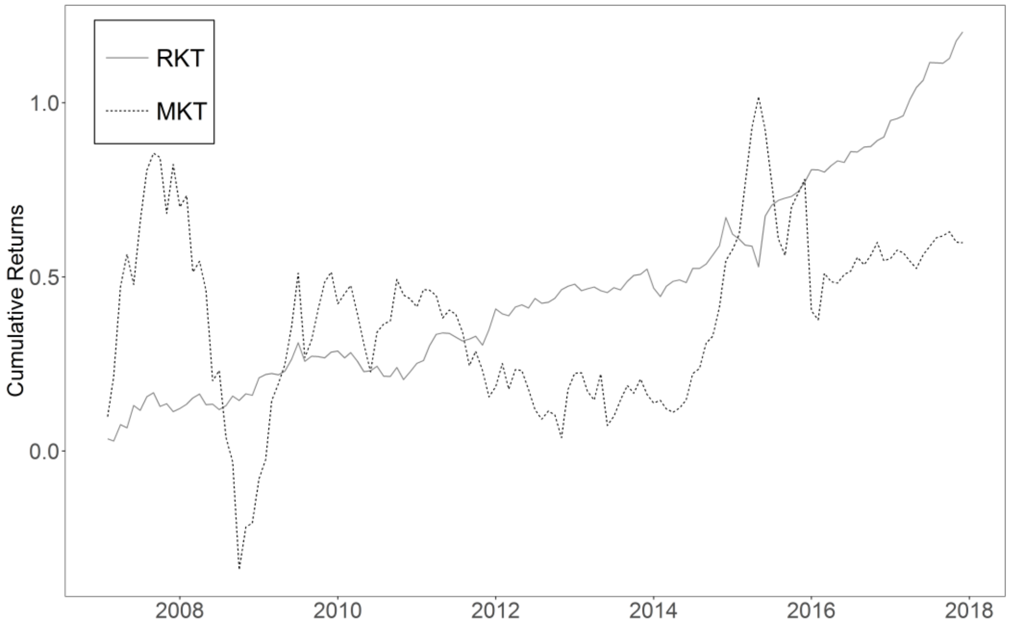

Figure 1. Cumulative return of portfolio.

We further compare the performance of these two portfolios from two dimensions - return and risk. The corresponding results are shown in Table 2. We first examine the performance of both portfolios from the perspective of return. Assume that the value of the portfolio at the beginning of the period is 1 yuan. It accumulates for 11 years on the return of the low-minus-high portfolio sorted by realized kurtosis to 3.33 yuan, and its corresponding annualized return is 11.02%. The value of the market portfolio is only 1.81 yuan at the end of the period, corresponding to an annualized return of mere 5.48%. We then turn our attention to the risk and employ the methods of Value at Risk (VaR) and Expected Shortfall (ES). Given a certain confidence level, VaR refers to the maximum possible loss of the portfolio in a certain period. ES at the p% level refers to the expected return of the portfolio in the worst p% scenario. The results of both indicators are shown from columns 2 to 5. Columns 2 and 3 correspond to VaR at the 1% level and 5% level respectively, while columns 4 and 5 correspond to the ES at the 1% level and 5% level respectively. It can be seen, from the viewpoint of the VaR and ES, the risk of our low-minus-high investment portfolio is significantly lower than that of market portfolio. To combine information on both return and risk, we further compute the Sharpe ratios of these two portfolios. The Sharpe ratio of low-minus-high portfolio is as high as 1.19 while the value of market portfolio is only 0.17. The steady return performance of low-minus-high portfolio demonstrates stability of the “realized kurtosis puzzle” over time.

Table 2. The return and risk performance.

	Return	VaR (1%)	VaR (5%)	ES (1%)	ES (5%)	Sharpe Ratio
RKT	11.02%	-5.42%	-3.03%	-5.69%	-4.56%	1.19
MKT	5.48%	-29.47%	-16.02%	-34.33%	-25.16%	0.17

IV. “Realized kurtosis puzzle” and lottery preference explanation: double sorts

Lottery preference effect

Before embracing lottery preference explanation, let us first verify whether lottery preference effect exists in Chinese stock market. We proxy the lottery preference effect using three variables, namely MAX (the average of three maximum daily returns in the month), the realized volatility and the realized skewness [2]. The single-sorted results based on the lottery preference variables are shown in Table 3.

Table 3. Single-sorted results based on lottery preference variables.

	MAX		RVOL		RSK	
	Equal-Weighted	Value-Weighted	Equal-Weighted	Value-Weighted	Equal-Weighted	Value-Weighted
1	89.61	41.51	79.36	42.47	110.57	54.22
2	117.35	33.83	103.26	9.63	89.93	44.64
3	77.44	4.28	78.53	2.94	62.17	2.06
4	23.65	-0.73	40.85	-22.39	23.59	-46.68
5	-79.35	-106.48	-73.31	-126.60	-57.70	-91.08
5-1	-168.96	-147.98	-152.67	-169.07	-168.27	-145.30
	(-6.03)	(-3.60)	(-4.94)	(-3.56)	(-5.84)	(-3.60)

The three parts of Table 3 correspond to the single-sorted results by MAX, the realized volatility and the realized skewness, respectively. Since the lottery characteristics of stocks are desirable, investors are willing to accept lower expected return for lottery-like stocks as the cost. If lottery preference effect exists, the expected return for portfolio 1 (the portfolio with the lowest lottery preference) to portfolio 5 (the portfolio with the highest lottery preference) should show a downward trend. This is indeed the case. Table 3 shows there exists strong lottery preference effect in Chinese stock market. In the equal-weighted case, moving from portfolio 1 to portfolio 5 we get a decreasing trend of returns for all three lottery preference proxies. The returns of the high-minus-low portfolios are -168.96, -152.67, and -168.27 bps respectively, and are all statistically significant. The conclusions hold when we turn to the value-weighted case as well. Note that in our unreported double sorts analysis, we still observe strong lottery preference effect in Chinese stock market even controlling for other common stock pricing factors.

Double sorts controlling lottery preference

In this section we use double sorts approach to investigate whether the lottery preference can explain the "realized kurtosis puzzle". We first divide the stocks into five groups according to the lottery preference variable (control variable), and then divide each group of stocks into five groups

[2] Firstly, we consider using the maximum daily return in the month and the average of five maximum daily returns in the month instead of the average of three maximum daily returns in the month. In these two cases we get similar results. Secondly, we also consider using the aggregate volatility and idiosyncratic volatility obtained from the daily return data (instead of intraday high frequency data) to replace the realized volatility. In these two cases similar results are obtained. Thirdly, we consider using the aggregate skewness and idiosyncratic skewness calculated based on the daily data within the month to replace the realized volatility as well. In this case, the first two results are slightly worse than the last one. This may be due to the fact that the daily return data within one month of each stock is limited, and the the high-order moment measures (degrees of skewness) based on this are naturally not accurate. When we possess high-frequency data, we naturally choose the most accurate measurement method.

based on the realized kurtosis (variable of interest). In doing so we get a 5×5 matrix of portfolios. The equal-weighted results are shown in Table 4.

Table 4. Double sorts controlling lottery preference variables.

	1	2	3	4	5	Average
Panel A. MAX as the lottery preference variable						
1	103.56	124.97	80.15	34.90	-56.95	57.33
2	108.07	136.36	99.33	44.21	-61.18	65.36
3	87.47	109.40	73.92	36.14	-76.79	46.03
4	81.18	129.50	83.52	27.98	-99.35	44.57
5	67.80	86.60	50.65	-24.61	-102.44	15.60
5-1	-35.77	-38.37	-29.51	-59.51	-45.48	-41.73
	(-1.29)	(-1.46)	(-1.15)	(-2.58)	(-1.42)	(-2.19)
Panel B. Realized volatility as the lottery preference variable						
1	103.65	125.02	95.02	48.42	-73.46	59.73
2	96.79	131.34	93.89	49.61	-44.93	65.34
3	81.22	100.32	74.62	59.62	-64.72	50.21
4	76.20	101.65	85.26	40.39	-81.36	44.43
5	39.32	58.47	44.10	6.63	-101.76	9.35
5-1	-64.33	-66.55	-50.91	-41.78	-28.29	-50.37
	(-2.84)	(-2.44)	(-2.08)	(-1.67)	(-1.01)	(-2.76)
Panel C. Realized skewness as the lottery preference variable						
1	131.19	95.73	56.70	14.89	-59.61	47.78
2	126.68	104.49	77.26	31.36	-34.81	61.00
3	110.25	102.76	78.29	29.55	-53.80	53.41
4	121.21	91.56	74.23	27.05	-43.67	54.08
5	63.68	55.88	24.86	15.10	-96.26	12.65
5-1	-67.50	-39.85	-31.84	0.21	-36.65	-35.13
	(-2.20)	(-1.39)	(-1.27)	(0.01)	(-1.62)	(-1.76)

Table 4 Panel A shows the double-sorted results based on the MAX (control variables) and the realized kurtosis (variable of interest) (The correlation coefficient between the two variables is 25.24%). The first five figures in each column are corresponding to portfolio returns sorted by realized kurtosis, given a certain level of MAX. The sixth figure for each column gives the average return of the high-minus-low portfolio (portfolio 5 minus the portfolio 1) and the corresponding Newey-West t statistic. When controlling MAX, three out of the five high-minus-low portfolios are no longer statistically significant. For the remaining two, though still statistically significant, the absolute value of the returns of high-minus-low portfolios also decrease remarkably compared with the single-sorted results in Section III. To examine the overall effect of the variable of interest, we average the first five columns to get the sixth column. Note, since portfolios in this last column all contain control variables with all levels, their control variable levels are quite similar. Although we still observe a downward trend moving from portfolio 1 to portfolio 5, the return of the high-minus-low portfolio now falls to -41.73 bps. This means MAX alone can explain 54.58% $(= (91.87 -$

$41.73)/91.87)$ of the "realized kurtosis puzzle".

Table 4 Panel B is the double-sorted results based on the realized volatility (control variables) and the realized kurtosis (variable of interest) (The correlation coefficient between the two variables is 22.99%). When realized volatility is controlled, two out of the five high-minus-low portfolios are no longer statistically significant. Similarly, we average the first five columns to get the sixth column and find that the return of the high-minus-low portfolio falls to -50.37 bps. This means 45.17% $(= (91.87 - 50.37)/91.87)$ of the "realized kurtosis puzzle" can be interpreted by the realized volatility.

Table 4 Panel C is the double-sorted results based on the realized skewness (control variables) and the realized kurtosis (variable of interest) (The correlation coefficient between the two variables is 37.74%). In the case of controlling the realized skewness, the returns of four out of five high-minus-low portfolios are no longer statistically significant. As is shown in the sixth column, the return of the high-minus-low portfolio tumble to -35.13bps, meaning that 61.76% $(= (91.87 - 35.13)/91.87)$ of the puzzle is explained solely by the realized skewness.

The results above show that when lottery preference variables are controlled, the "realized kurtosis puzzle" can be largely explained. As we have stated earlier, the "realized kurtosis puzzle" under equal-weighted construction is more pronounced than that using value-weighted method. Thus, the puzzle naturally could be better interpreted by the lottery preference when we turn to value-weighted cases. Our unreported value-weighted results confirm this view. Returns of three out of five high-minus-low portfolios with MAX controlled are statistically insignificant, and the remaining two are barely significant. For the cases of controlling the realized volatility and the realized skewness respectively, all five high-minus-low portfolios are statistically insignificant. If we average out the three controlled variables respectively as above, the high-minus-low portfolios returns are all no longer statistically significant.

Double Sorts Controlling Non-Lottery Preferences

In this part we aim to investigate whether stock pricing factors other than the lottery preference variables can solve the "realized kurtosis puzzle". We consider the following stock pricing factors: market beta, firm size and book-to-market ratio (Fama and French, 1992; Fama and French, 1993), momentum factor (Jegadeesh and Titman, 1993; Carhart, 1997), reversal factor (Lehmann, 1990; Jegadeesh, 1990) and illiquidity factor (Amihud, 2002). See appendix for definitions. Table 5 gives the double-sorted results controlling non-lottery preference variables mentioned above.

Table 5. Double sorts controlling non-lottery preference variables.

	Beta	Size	B/M	Momentum	Reversal	Illiquidty
1	70.89	94.86	66.90	72.54	71.10	102.12
2	71.25	70.22	61.27	65.45	66.02	80.22
3	49.40	51.74	60.45	63.10	48.75	53.82
4	31.24	36.96	46.42	46.09	39.13	32.39
5	-20.00	-24.83	-6.12	-3.50	4.02	-39.55
5-1	-90.88	-119.69	-73.02	-76.04	-67.08	-141.67
	(-4.97)	(-7.36)	(-4.12)	(-4.18)	(-3.21)	(-7.34)

Before we apply double sorts to the listed non-lottery preference variables, a thorough theoretical analysis would be helpful and could serve as references. There is a weak positive correlation between the market beta and the realized kurtosis (4.52%), implying that the high market beta stocks are also those with high realized kurtosis (corresponding to low expected return). Due to the existence of the beta anomaly[3], the high market beta stocks bear low returns. Therefore, under the control of the market beta, the return (in absolute value) of the high-minus-low portfolio sorted by the realized kurtosis will somewhat decline. Firm size and the realized kurtosis are slightly correlated (-9.14%), that is, the small size stocks are the ones with high realized kurtosis. Because the small stocks enjoy higher returns, the return of the high-minus-low portfolio based on the realized kurtosis will rise with the control of the firm size. The correlation coefficient between book-to-market ratio and realized kurtosis is -23.14%, meaning that low book-to-market ratio stocks are characterized by high realized kurtosis. Since low book-to-market ratio stocks normally have lower return, the return of the high-minus-low portfolio might decline when book-to-market ratio factor is controlled. Despite a positive correlation between momentum factor and realized kurtosis (15.22%), as no significant pricing ability of momentum factor observed in Chinese stock market[4], the return of the high-minus-low portfolio when momentum factor controlled should not change too much. With the correlation coefficient being 9.00%, high reversal factor corresponds to high realized kurtosis. Because stocks with high reversal factor bear lower returns, the return of high-minus-low portfolio may reduce to some extent with the reversal factor controlled. Because the high reversal factor stocks bear lower returns, the return of high-minus-low portfolio may be reduced to some extent with the reversal factor controlled. There is a slight positive correlation between the illiquidity factor and the realized kurtosis (4.55%), meaning low liquidity stocks possess high realized kurtosis. Since low-liquidity stocks correspond to higher returns, the return of the high-minus-low portfolio will rise under the control of low liquidity factor.

The results in Table 5 fully support our theoretical analysis above. With market beta, book-to-market ratio, momentum factor and reversal factor controlled respectively, the corresponding returns of high-minus-low portfolios drop to -90.88, -73.02, -76.04 and -67.08 bps. In comparison, we notice that when firm size and illiquidity factor are controlled separately, the returns of high-minus-low portfolios rise to -119.69 and -141.67 bps instead. However, it's worth pointing out that, unlike lottery preference variables, non-lottery preference factors fail to explain the "realized kurtosis puzzle". For all the control variables considered above, a monotonically decreasing trend from portfolio 1 to 5 remains, and the returns of the high-minus-low portfolios are statistically significant, which are very much alike to the single-sorted results in Section III. In addition, in the unreported 5×5 complete results, all returns of high-minus-low portfolios are statistically significant at the 10% level. Except the lowest market beta and the highest momentum factor cases, in all situations returns of high-minus-low portfolios are at least statistically significant at the 5% level. The evidence above indicates that the puzzle remains unsolved by merely controlling non-lottery preference factors.

What merits our special attention is the case with firm size controlled. The corresponding results

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[3] Beta anomaly is first proposed by Black et al. (1972). Subsequent studies in this area include Fama and French (1992), Frazzini and Pedersen (2014), and Bali et al. (2017). Using past 60-month data, we construct the market beta factor, base on which we measure the beta anomaly of the Chinese stock market. In the sample ranging from 2000 to 2017, the annualized return of the high-minus-low portfolio based on the market beta is -6.54%, and is statistically significant at the 1% level. This evidence supports the existence of the beta anomaly in Chinese stock market.

[4] Literatures in this area mainly includes Zhou (2002), Zhu et al. (2003) and Wang et al. (2006). All of the above papers support the view that the momentum factor in Chinese stock market do not have a significant stock pricing capability. We further measure the momentum factor effect of the Chinese stock market. In the sample ranging from 2000 to 2017, the annualized return of the momentum factor based on the Jegadeesh and Titman (1993) method is merely 1.43%, while the annualized return of the momentum factor based on Carhart (1997) method is only - 2.36%. Both of them are not statistically significant.

are demonstrated in Table 6. For each column in Table 6, as we shift from portfolio 1 in the first row downward to portfolio 5 in the fifth row we observe a monotonically decreasing pattern, and the returns of high-minus-low portfolios are all statistically significant. Thus, the “realized kurtosis puzzle” exists for stocks with different firm sizes. However, with different firm capitalizations the “realized kurtosis puzzle” might reveal themselves differently. As is shown in Table 6, returns (in absolute value) of the high-minus-low portfolios in sixth row show a decreasing trend from left to right. High-minus-low return of small-cap stocks (the first column) triple its large size counterpart (the fifth column), implying the “realized kurtosis puzzle” is more noteworthy among small-cap stocks. Compared to blue chips, small-cap stocks are more likely to experience large fluctuations in stock prices and apt to be manipulated. Therefore, small-cap stocks are more prone to be the subject of lottery speculation. This further supports the lottery preference explanation for the “realized kurtosis puzzle”.

Table 6. Double sorts controlling firm size.

	1	2	3	4	5
1	194.04	138.59	75.59	51.09	14.99
2	142.20	108.60	77.99	34.94	-12.62
3	121.03	104.37	33.92	25.29	-25.90
4	107.18	77.63	14.85	10.72	-25.61
5	38.78	-17.24	-41.77	-62.06	-41.87
5-1	-155.26	-155.83	-117.36	-113.15	-56.86
	(-6.54)	(-9.15)	(-7.81)	(-4.64)	(-1.85)

V. High retail investors’ shareholding proportion, short selling restrictions and the “realized kurtosis puzzle”

The remarkably high shareholding proportion held by retail investors and short selling restrictions are two major characteristics of the Chinese stock market. Compared to professional institutions investors and skilled high net worth individual investors, retail investors show stronger lottery preference. Therefore, if the lottery preference can explain the “realized kurtosis puzzle”, then the larger proportion of stock shares retail investors hold, the stronger the puzzle is. In addition, due to the short selling restrictions, it is difficult for investors to arbitrage through short selling mechanisms to reduce the lottery effect of the market. Thus we can expect stocks unavailable for short should show stronger “realized kurtosis puzzle” than stocks available for short. In this section, from these two perspectives we demonstrate that lottery preference can explain the “realized kurtosis puzzle”.

Firstly, let's focus on the relationship between the retail investors’ shareholding proportion and the “realized kurtosis puzzle”. We use the top 10 stock holders’ holding percentage to reflect the shareholding proportion of retail investors. The higher the indicator is, the larger proportion of shares are held by professional institutions investors and skilled high net worth individual investors. In other word, the smaller proportion of stocks are in the hands of retail investors. The double sorts results controlling top 10 stock holders’ holding percentage are shown in Table 7. The first column corresponds to the stocks with the highest shareholding proportion possessed by retail investors, and the fifth column corresponds to the stocks with the lowest shareholding proportion held by retail investors. From the sixth row it can be seen that the absolute value of the high-minus-low portfolio returns show a monotonous decreasing trend from left to right. The high-minus-low portfolio return of stocks with the highest retail investors' shareholding proportion is about twice as high as the stocks with the lowest shareholding proportion possessed by retail investors. In the unreported results, we also take the total institutional investors holding percentage as the proxy. The disadvantage of this indicator over the top 10 stock holders' holding percentage is it does not include skilled high net worth individual investors. Fortunately, based on this indicator we also obtain similar results. Therefore, the evidence above supports the view of strong lottery preference of retail investors leading to significant "realized kurtosis puzzle" in Chinese stock market.

Table 7. Double sorts controlling top 10 stock holders' holding percentage.

	1	2	3	4	5
1	52.42	55.17	59.67	68.80	67.52
2	36.58	77.37	65.44	63.77	54.94
3	-5.21	48.08	53.74	58.26	43.56
4	-3.69	40.34	45.86	39.36	54.31
5	-58.16	-36.71	-23.53	-7.46	3.11
5-1	-110.58	-91.88	-83.20	-76.26	-64.41
	(-4.85)	(-3.99)	(-3.60)	(-2.84)	(-2.27)

Next, we turn our attention to the relationship between the short selling restrictions of stocks and the "realized kurtosis puzzle". Chinese stock market established its a short selling system in April 2010, and by the end of 2017, the number of stocks available for short was about 1/3 of the total number of equities. Thus we split the stock samples in the period with short selling system established into two parts - stocks unavailable for short and stocks available for short, and then study the significance of the "realized kurtosis puzzle" of these two types of stocks. The single sort results of these two types of stocks are shown in Table 8. The left side of Table 8 is corresponding to stocks cannot be shorted, and the right side is corresponding to stocks can be shorted. In the equal-weighted case, the high-minus-low portfolio return of stocks unavailable for short is -118.72 basis points, with its Newey-West t statistic highly significant. In comparison, the high-minus-low portfolio return of stocks available for short is only about half of ones unavailable for short, and only marginally significant at the 10% level. In the value-weighted case, the return gap between the two types of stocks narrows, but remarkable differences in statistical significance still persist. It should be pointed out in Chinese stock market the annual interest rate for margin trading is up to 10%, thus even stocks available for short still suffer short selling restrictions. Therefore, the real effect of short selling restrictions on the "realized kurtosis puzzle" is far more serious than that reflected above. But even so, the evidence above actively supports that owing to the short selling restrictions, smart investors can't take full advantage of the lottery effect to arbitrage. That's why significant "realized kurtosis puzzle" still exists in Chinese stock market.

Table 8. Single sorts results for stocks unavailable or available for short.

	Available for Short		Unavailable for Short	
	Equal-Weighted	Value-Weighted	Equal-Weighted	Value-Weighted
1	11.69	23.89	48.64	-1.20

2	0.86	-7.15	39.21	-3.30
3	2.83	-13.97	20.56	-7.28
4	-33.09	-51.34	1.33	-21.26
5	-57.33	-47.78	-70.07	-87.75
5-1	-69.02	-71.67	-118.72	-86.55
	(-1.81)	(-1.12)	(-5.50)	(-4.31)

VI. "Realized kurtosis puzzle" and lottery preference explanation: firm-level cross-sectional regressions

Conventional Approach

In Section IV we have discussed whether the lottery preference can account for the "realized kurtosis puzzle" in the context of double sorts approach. This method mainly has the following two defects. First, instead of controlling multiple variables at the same time, the double sorts can only control one variable at a time. Second, as a group-based approach, double sorts method ignores differences within each group. In response to these drawbacks, we introduce the firm-level cross-sectional regressions method. Essentially this method consists of a series of Fama-MacBeth cross-sectional regressions (Fama and MacBeth, 1973), each of which controls single or multiple variable(s). Specifically, for each month in our sample, we perform the following cross-sectional regression for all stocks traded during that month:

$$r_{i,t+1}^{e} = \alpha_t + \sum_{j=1}^{K} \beta_{j,t} Z_{j,i,t} + \varepsilon_{i,t+1} \tag{7}$$

where $r_{i,t+1}^{e}$ denotes the stock i's excess return in month $t+1$, and $Z_{j,i,t}$ denotes the observation of the j-th variable of stock i in month t. We use the above regression to obtain the estimated coefficients for each month. Then we average the time series of coefficient estimates and calculate the corresponding Newey-West t statistics. The results are presented in Table 9.

Table 9. Firm-level cross-sectional regressions.

Panel A. "Realized kurtosis puzzle" and lottery preference variables

	Intercept	RKT	MAX	RVOL	RSK	Adj. R^2
1	0.0317***	-0.0172***				0.0064
	(2.67)	(-4.95)				
2	0.0236**		-0.4322***			0.0189
	(2.52)		(-8.21)			
3	-0.0889***			-0.0265***		0.0222
	(-4.12)			(-4.82)		
4	0.0114				-0.0232***	0.0102
	(1.13)				(-6.05)	
5	0.0354***	-0.0085**	-0.3910***			0.0242
	(3.09)	(-2.13)	(-6.41)			
6	-0.0637**	-0.0091**		-0.0236***		0.0276
	(-2.08)	(-2.15)		(-3.51)		

7	0.0220*	-0.0072*			-0.0201***	0.0158
	(1.72)	(-1.83)			(-4.36)	
8	0.0017	-0.0033	-0.2955***	-0.0069	-0.0112**	0.0421
	(0.06)	(-0.76)	(-5.12)	(-0.94)	(-2.49)	

Panel B. “Realized kurtosis puzzle” and non-lottery preference variables

	Intercept	RKT	Beta	Size	B/M	Momentum	Reversal	Illiquidity	Adj. R^2
1	0.1525***	-0.0188***	-0.0087**	-0.0048***	0.0041**				0.0756
	(4.17)	(-6.50)	(-2.09)	(-3.12)	(2.20)				
2	0.1504***	-0.0182***	-0.0092**	-0.0047***	0.0031*	-0.0041			0.0833
	(4.11)	(-6.77)	(-2.40)	(-3.12)	(1.90)	(-1.37)			
3	-0.0694	-0.0170***	-0.0066	0.0045**	0.0034*	-0.0034	-0.0671***	0.0118***	0.1086
	(-1.57)	(-7.74)	(-1.60)	(2.27)	(1.89)	(-1.06)	(-6.18)	(8.07)	

Note: *, **, and *** indicate the significance at 10%, 5%, and 1% level, respectively.

We first use firm-level cross-sectional regressions to verify whether “realized kurtosis puzzle” exists in Chinese stock market. For this purpose, we run the regression with only one explanatory variable, i.e., realized kurtosis (except for the constant term). The corresponding result is demonstrated in the first row of panel A of Table 9. If the “realized kurtosis puzzle” exists, namely, high kurtosis stocks bearing low expected return, then the coefficient of the realized kurtosis ought to be negative. This is indeed the case. The estimated coefficient of the realized kurtosis is -0.0172, statistically significant with a Newey-West t statistic of -4.95. The standard deviation of the realized kurtosis after taking the natural logarithm is 0.2047. This means two standard deviations’ change of this variable corresponds to the stock’s monthly return change of -70.42 $(= 2 \times 0.2047 \times (-0.0172) \times 10000)$bps. In addition, in the realized kurtosis based single sorts part (the return of high-minus-low portfolio is -97.70 bps), the difference of the realized kurtosis (taking the natural logarithm) between Portfolio 1 and Portfolio 5 is $0.54(= 1.95 - 1.41)$. When we move from Portfolio 1 to Portfolio 5, the difference in returns caused by the realized kurtosis is -92.88$(= 0.54 \times (-0.0172) \times 10000)$bps, which is very close to -91.87 bps, the result we obtain via group-based method. Once again, the firm-level cross-sectional regressions confirm the existence of “realized kurtosis puzzle” in the Chinese stock market.

We then turn to the verification of lottery preference effect in Chinese stock market. To achieve this, we consider the three lottery preference variables respectively, namely the MAX, the realized volatility and the realized skewness. The second to fourth rows of panel A show that all three lottery preference variables are remarkably statistically significant. In addition, these three variables’ two standard deviations’ changes correspond to monthly stock return changes of 184.12, 208.72 and 130.89bps, respectively. Consistent with group-based approach, the regression analysis supports the strong lottery preference in Chinese stock market.

We continue our analysis by considering whether or not the lottery preference variables and the non-lottery preference factors can solve the “realized kurtosis puzzle”. Let’s first focus on the three lottery preference variables. The corresponding results are shown in the fifth to seventh rows of panel A. As the fifth row displays, with MAX controlled the coefficient estimate of the realized kurtosis decreases from the previous -0.0172 to -0.0085 with Newey-West t statistic dropping from -4.95 to -2.13, accounting for a 50.58%$(= (0.0172 - 0.0085)/0.0172)$ of the “realized kurtosis puzzle”. The sixth row corresponds to the case where realized volatility is controlled. In this case, the coefficient estimate of the realized kurtosis declines from the previous -0.0172 to -0.0091 with Newey-West t statistic falling from -4.95 to -2.15. This indicates that realized volatility alone can explain $47.09\% \ (= (0.0172 - 0.0091)/0.0172)$ of the puzzle. The seventh row shows the regression result with realized skewness included. When adding realized skewness, coefficient estimate of the realized kurtosis experiences a $58.14\%(= (0.0172 - 0.0072)/0.0172)$ decrease to -0.0072, with Newey-West t statistic going down to -1.88. Note that, though the estimated coefficients of realized kurtosis above remain statistically significant, remarkable drops in coefficients estimates and corresponding Newey-West t statistics are accompanied by the control of any one of lottery preference variables. The above results imply that each of the three lottery preference variables alone can explain the "realized kurtosis puzzle" to a large extent. Then the question is raised whether the puzzle could be fully solved if we include all three lottery preference variables at the same time? The answer is an absolute YES. As is presented in the eighth row, the coefficient estimate of the realized kurtosis drops drastically from the previous -0.0172 to -0.0033 and the Newey-West t statistic now becomes insignificant. This implies that the "realized kurtosis puzzle" has been successfully solved by the lottery preference.

Then we shift our focus to non-lottery preference stock pricing factors to investigate its role in explaining the "realized kurtosis puzzle". Panel B of Table 9 provides the results. We first consider the case of controlling the Fama-French three factors. See the first row for the result. We find that the coefficient estimate of realized kurtosis slightly increases from the original -0.0172 to -0.0194 with Newey-West t statistic rising from -4.95 to -6.48. It seems the Fama-French three factors not only fail to solve the puzzle, but also make things even slightly worse. Next, we move on to the case of Fama-French-Carhart four factors. The Fama-French-Carhart four factors contain not only the Fama-French three factors, but the momentum factor as well. See the second row for the corresponding result. Since the momentum factor has no significant pricing effect in Chinese stock market, the result based on the four factors are very similar to that of the three factors. Furthermore, we add the reversal factor and the illiquidity factors to the Fama-French-Carhart four factors. The corresponding result is listed in the third row. Surprisingly, under the control of the multiple stock pricing factors above, the coefficient estimate of the realized kurtosis is almost unchanged, and the corresponding Newey-West t statistic rises from the previous -4.95 to -6.97. Therefore, when it comes to the explanation of realized kurtosis puzzle, non-lottery preference factors seem powerless and are overshadowed by their lottery preference counterparts. The latter ones can not only interpret the "realized kurtosis puzzle" from the perspective of economics, but also provide positively supportive evidence from statistics. The poor performance of non-lottery preference pricing factors further backs up our lottery preference view of realized kurtosis puzzle.

Finally, one concern is that the "realized kurtosis puzzle" exists only in specific industry and the explanatory power of lottery preference is limited to specific industries. For this reason, we also consider the situations controlling the industry effect. We first conduct firm-level cross-sectional regressions of each industry's stocks on monthly basis and then average the estimated coefficients along industry and time dimension successively to obtain our final coefficient estimates and Newey-West t statistics. These results are shown in Table 10. We find that under the control of industry effect, the "realized kurtosis puzzle" still exists (see the first row), and the lottery preference can still explain the puzzle (see the second to fifth rows).

Table 10. Firm-level cross-sectional regressions (controlling industry effect).

	Intercept	RKT	MAX	RVOL	RSK	Adj. R^2
1	0.0324***	-0.0182***				0.0307
	(2.73)	(-5.86)				
2	0.0336***	-0.0068**	-0.4322***			0.0269
	(2.86)	(-2.40)	(-6.34)			
3	-0.0734**	-0.0091**		-0.0263***		0.0249
	(-2.08)	(-2.55)		(-3.25)		
4	0.0194	-0.0050			-0.0242***	0.0534
	(1.61)	(-1.44)			(-4.69)	
5	0.0030	-0.0005	-0.3448***	-0.0059	-0.0144***	0.0624
	(0.08)	(-0.10)	(-5.40)	(-0.72)	(-3.20)	

Note: *, **, and *** indicate the significance at 10%, 5%, and 1% level, respectively.

Hou and Loh Method

Hou and Loh (2016) proposes a brand-new approach to measure the capability of various possible explanatory variables to explain asset pricing anomaly. The idea behind this method is to split the coefficient of stock pricing factor with anomaly into two parts - the part that can be explained by a candidate explanatory variable and the part cannot be interpreted by that variable. Specifically, this method consists of three steps. The first step is same as the first specification of Table 9. The coefficient β_t of the realized kurtosis is estimated by the firm-level cross-sectional regression method (only realized kurtosis is included). The corresponding expression is as follows:

$$r_{i,t+1}^{e} = \alpha_t + \beta_t RKT_{i,t} + \varepsilon_{i,t+1} \tag{8}$$

The second step still makes use of the firm-level cross-sectional regression method. The difference is the explained variable becomes the realized kurtosis in this step, while the explanatory variable turns into a candidate variable that may explain the "realized kurtosis puzzle". The corresponding expression is as follows:

$$RKT_{i,t} = a_t + \delta_t X_{i,t} + \mu_{i,t} \tag{9}$$

where X represents a candidate explanatory variable of the "realized kurtosis puzzle". This step is implemented to obtain the coefficient estimator of the explanatory variable. The third step is the most critical one. This step completely splits β_t into two parts by the linearity of the covariance. The derivation process is as follows:

$$\begin{aligned}\beta_t &= \frac{Cov\left(r_{i,t+1}^{e}, RKT_{i,t}\right)}{Var\left(RKT_{i,t}\right)} \\ &= \frac{Cov\left(r_{i,t+1}^{e}, \delta_t X_{i,t} + a_t + \mu_{i,t}\right)}{Var\left(RKT_{i,t}\right)} \\ &= \frac{Cov\left(r_{i,t+1}^{e}, \delta_t X_{i,t}\right)}{Var\left(RKT_{i,t}\right)} + \frac{Cov\left(r_{i,t+1}^{e}, a_t + \mu_{i,t}\right)}{Var\left(RKT_{i,t}\right)} \\ &= \beta_t^C + \beta_t^R \end{aligned} \tag{10}$$

β_t^C indicates the portion of β_t that can be explained by X_t, while β_t^R denotes the unexplained residual part of β_t. Therefore, β_t^C/β_t represents the percentage of the "realized kurtosis puzzle" successfully explained by X_t in period t. By averaging the time series of β_t^C/β_t, we can learn to what extent the puzzle explained by X_t in the whole sample period. The explanatory power of each candidate explanatory variable for the "realized kurtosis puzzle" is shown in Table 11.

Table 11. Hou and Loh method results (%).

Lottery	MAX	RVOL	RSK
	57.28	56.43	71.57
Non-Lottery	Beta	Size	B/M
	0.50	-21.05	21.69
	Momentum	Reversal	Illiquidity
	13.67	24.83	-44.04

Table 11 again confirms the remarkable differences between lottery variables and non-lottery variables in explaining the "realized kurtosis puzzle". Based on the Hou and Loh (2016) method, each of the three lottery variables can explain at least half of the puzzle. The best performance comes from the realized skewness, which successfully explain about 3/4 of the puzzle. In comparison, non-lottery variables, though partially exhibiting certain explanatory potential under this method, are still defeated by lottery variables. It should be noted that, consistent with the result in Section IV, the size factor and the low liquidity factor further exacerbate the extent of the "realized kurtosis puzzle".

VII. Time-varying nature of "realized kurtosis puzzle" and lottery preference

In above we present evidence that the "realized kurtosis puzzle" is relatively stable to the sample period. As can be seen from the portfolio performance section of this paper, in 10 out of 11 years' sample period, the return of the low-minus-high portfolio is positive. Even in that special year, the annualized return of low-minus-high portfolio is not too bad (-5.71%). However, this does not mean that the influence of the realized kurtosis on stock returns is constant over time. If the lottery preference can effectively explain the "realized kurtosis puzzle", then the puzzle in high lottery preference period should be stronger, and less evident in low lottery preference period. Our verification process goes as follows. From the regression part we know the higher the lottery preference, the larger the corresponding coefficient is (in absolute value). Thus, we take the absolute value of lottery preference variable coefficient as our indicator of lottery preference strength and accordingly split the whole sample time into high and low lottery preference periods. Note the higher the return (in absolute value) of the realized kurtosis based high-minus-low portfolio, the stronger "realized kurtosis puzzle" is. To complete our verification, we then examine the return performance of high-minus-low portfolios in high and low lottery preference periods, respectively. The corresponding results are presented in Table 12.

Table 12. "Realized kurtosis puzzle" in high and low lottery preference periods.

	MAX	RVOL	RSK
High Lottery preference	-143.72	-160.01	-125.72
Period	(-4.09)	(-4.08)	(-4.72)
Low Lottery preference	-39.22	-22.68	-57.50
Period	(-1.33)	(-0.78)	(-1.59)

Column 1 to 3 in Table 12 correspond to the cases in which the MAX, the realized volatility and the realized skewness as the lottery preference strength indicator respectively. Regardless of the indicator used, the return performances in high and low lottery preference periods are remarkably different. Take the first column (treat MAX as the preference indicator) for instance. In high lottery preference period, the return of high-minus-low portfolio reaches -143.72 bps while this figure for low lottery preference period is only -39.22. The conclusion that the puzzle in high lottery preference period is much stronger is robust to the other two indicator choices (see the second and the third columns).

In addition, we have an interesting discovery. The three years with the worst return performances of the low-minus-high portfolio (reverse operation of high-minus-low portfolio) are 2008, 2010 and 2015. For these three years, the average of the annualized return of the low-minus-high portfolio is only 3.04%. In comparison, the average annualized return for the remaining years is 14.03%. The difference between the two is quite obvious. These three years were not tranquil for Chinese stock market. In fact, during these three years, Chinese stock market experienced severe plunges. Accompanied by the global financial crisis, the CSI 300 fell from 5731.76 points at the beginning of year 2008 all the way to 1677.83 points. In the stock market crash of year 2010, the CSI 300 dropped from 3353.23 points to 2512.65 points. In the year 2015's stock disaster, the CSI 300 decreased from 5,353.75 points all the way down to 3025.69 points. When the stock crisis befalls, investors' fear for risk and uncertainty overwhelmingly surpass their speculative desire. At that point, instead of seeking speculative opportunities investors try their best to find safe harbor for investment, say, the flight-to-quality. It can thus be seen that three years with the weakest "realized kurtosis puzzle" are precisely times of insufficient lottery preference. This evidence further supports the viewpoint that the "realized kurtosis puzzle" can be explained by the lottery preference.

VIII. Robustness test

Portfolio formation period

Note that the above results are based on one-month portfolio formation period. Therefore, one of our concerns is whether the "realized kurtosis puzzle" obtained above only holds with this particular portfolio formation period. In order to eliminate the doubt, we consider setting the portfolio formation period as three-month[5]. In this case, we make use of the past three-month information to calculate the realized kurtosis, based on which we form portfolios.

In the equal-weighted case, the returns of portfolio 1 (49.49 bps) to portfolio 5 (-20.63 bps) sorted by the realized kurtosis show a monotonously decreasing trend. The return of the high-minus-low portfolio is -70.11 bps, with the Newey-West t statistic of -3.10. We also reach similar conclusions in the value-weighted cases with and without FF3F adjustment. The empirical results above show the "realized kurtosis puzzle" still hold under three-month portfolio formation period.

Then, we further investigate whether the lottery preference can explain the puzzle when the realized kurtosis and lottery preference variables are constructed on a three-month basis. The firm-level cross-sectional regressions method is applied to answer this question. When we incorporate only one explanatory variable - realized kurtosis (except for the constant term), the estimated coefficient of realized kurtosis is -0.0156, similar to the one-month portfolio formation period case.

[5] We also consider the case where the portfolio construction periods are two months and half month. In both cases, we get the consistent conclusion.

The corresponding Newey-West t statistic is also statistically significant at the 1% level. When we separately control two lottery preference variables of the MAX and the realized volatility, the coefficient estimates of the realized kurtosis drop to -0.0041 and -0.0063, respectively, and their corresponding Newey-West t statistics are no longer significant. When the realized skewness is controlled, the estimated coefficient of the realized kurtosis changes from negative to positive and becomes 0.0017. Furthermore, when all three lottery preference variables are controlled at the same time, the estimated coefficient of the realized kurtosis once again turns to positive and becomes 0.0037. The robustness tests above reinforce the evidence of the lottery preference explanation for the "realized kurtosis puzzle".

Portfolio holding period

The results above are based on the one-month portfolio holding period. To check the robustness of this choice, we also consider the three-month case[6]. It should be pointed out that in this case, the length of the portfolio holding period is longer than the length of the portfolio formation period. Thus, for any month t, portfolio 1 to 5 contain three portfolios with equal weights, each of which is constructed based on the information of month $t-1$, month $t-2$ and month $t-3$, respectively.

As above, in the equal-weighted case, the return of portfolio 1 (46.83 bps) to portfolio 5 (-4.04 bps) also show a monotonously decreasing trend. The high-minus-low portfolio return is -50.87 bps, corresponding to a Newey-West t statistic of -2.84. Similar conclusions are obtained in the value-weighted cases with and without FF3F adjustment. The empirical results above indicate that the "realized kurtosis puzzle" still holds when the portfolio holding period is three months.

Our next step is checking whether the lottery preference can explain the puzzle when the stock holding period is three months. Similarly, the firm-level cross-sectional regressions method is employed to deal with this problem. When the explanatory variable is the only realized kurtosis, the coefficient estimate is -0.0265[7], statistically significant at the 1% level. When we separately control the three lottery preference variables, the estimated coefficient of the realized kurtosis decreases to -0.0105, -0.0099 and -0.0096 with insignificant Newey-West t statistic, respectively. What's more, when all three lottery preference variables are controlled at the same time, the estimated coefficient of the realized kurtosis becomes -0.0008, statistically insignificant as well. The evidence above once again supports the idea that "realized kurtosis puzzle" can be explained by the lottery preference.

Shanghai stock exchange (SSE) and Shenzhen stock exchange (SZSE)

The conclusions above are based on all Chinese A-shares. We consider separating the whole sample into A-shares listed in the SSE and in the SZSE to see whether the stock exchange affects our conclusions. For SSE listed stocks, when equal-weighted, the returns of the portfolio 1 (69.50 bps) to portfolio 5 (-36.74 bps) show a monotonously decreasing trend, resulting in the high-minus-low portfolio return of -106.24 bps, corresponding to a Newey-West t statistic of -5.20. For A-shares issued in SZSE, the returns of portfolio 1 (80.70 bps) to portfolio 5 (-1.34 bps) also show a

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[6] We also consider the case where the portfolio formation periods are two months and half month. In both cases, we get the consistent conclusion.

[7] Since we consider the return for the next three months, this coefficient estimate (in absolute value) is naturally greater than the one for the next one month.

monotonously decreasing trend. The return of the high-minus-low portfolio is -79.36 bps with its Newey-West t statistic -4.11. Similar conclusions are achieved in the value-weighted cases with and without FF3F adjustment. Therefore, the "realized kurtosis puzzle" exists both in the SSE and the SZSE.

We further examine whether the lottery preference can explain the puzzle both in the SSE case and the SZSE case. Let's first focus on the SSE. When the explanatory variable is only the realized kurtosis, the estimated coefficient is -0.0167 with Newey-West t statistic -4.65. When three lottery preference variables are separately controlled, the coefficient estimates of the realized kurtosis decline to -0.0084, -0.0085 and -0.0069, with their Newey-West t statistics dropping to -2.14, -2.17 and -1.62. The estimated coefficient of the realized kurtosis falls drastically to -0.0034 and is no longer statistically significant when all three lottery preference variables are controlled at the same time. We then turn our attention to the SZSE. When the explanatory variable is only the realized kurtosis, the estimated value is -0.0182 with Newey-West t statistic -4.52. When separately controlling three lottery preference variables, the coefficient estimates of the realized kurtosis decrease to -0.0094, -0.0102 and -0.0093 with Newey-West t statistics -2.02, -2.03, and -2.28 respectively. When all these three lottery preference variables are controlled at the same time, the estimated coefficient of the realized kurtosis drops drastically to -0.0053 and is statistically insignificant. Therefore, both the "realized kurtosis puzzle" of the SSE and SZSE can be explained by the lottery preference.

IX. Summary and conclusions

Based on the 5-minute high-frequency data of Chinese A-shares, we investigate the relationship between realized kurtosis and expected return in Chinese stock market and attempt to solve the puzzle it brings. The main conclusions are as follows:

(1) High kurtosis corresponds to thick tails on both sides and under risk-averse assumption investors' dislike of left-tail loss outweighs their preference for right-tail gain. High kurtosis thus should predict higher expected return. However, the single-sorted results by realized kurtosis based on high-frequency stock data show that high realized kurtosis stocks correspond to low returns, while low realized kurtosis stocks enjoy high returns. This result is both economically and statistically significant. We refer this anomaly as the "realized kurtosis puzzle".

(2) We construct a low-minus-high investment portfolio based on the realized kurtosis (reverse operation of high-minus-low portfolio) and compare it with market excess return portfolio. In our sample period, in terms of both the return and the risk, our low-minus-high portfolio outperforms the market portfolio. The key point is the steady performance of low-minus-high portfolio shows stability of the "realized kurtosis puzzle" over time.

(3) Based on the three lottery preference variables, namely the MAX, the realized volatility and the realized skewness, it can be seen there is a strong lottery preference in Chinese stock market. Using double sorts method controlling lottery preference variables, the "realized kurtosis puzzle" can be largely explained. In comparison, non-lottery preference pricing factors fail to solve the "realized kurtosis puzzle". This striking contrast highlights the superior interpretive ability of the lottery preference on the puzzle.

(4) In comparison to professional institutional investors and skilled high net worth individual investors, retail investors show stronger lottery preference. In addition, short selling restrictions

prevent investors from taking advantage of the lottery effect in the market. Our empirical research verifies stocks with higher retail investors' shareholding proportion and unavailable for short show stronger "realized kurtosis puzzle". The evidence above further supports the view of using lottery preference to explain "realized kurtosis puzzle".

(5) We use the firm-level cross-sectional regressions method to verify the above viewpoints again. We find when controlling three lottery preference variables at the same time, the estimated coefficient of the realized kurtosis is no longer statistically significant. In contrast, non-lottery preference factors are incapable of explaining the "realized kurtosis puzzle".

(6) Although the "realized kurtosis puzzle" is relatively stable through the sample period, we further study whether its time-varying nature is related to the lottery preference. We use the lottery preference variable to split the sample into high lottery preference period and low lottery preference period. It can be seen that the "realized kurtosis puzzle" in high lottery preference period is significantly stronger than that in the low lottery preference period. This view is once again reinforced during the period of stock market crash. It is generally believed that during the stock market crash period, instead of seeking speculative opportunities, fearful investors try their best to find safe harbor for the investments. It is in these years when the stock market experiences severe crisis that the "realized kurtosis puzzle" turns relatively weak.

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Appendix

In this paper we consider the following control variables: market beta, firm size, book-to-market ratio, momentum factor, reversal factor and illiquidity factor. Their definitions are as follows:

(1) Market beta: In month m, we use the past 60-month information for a certain stock i to perform the following regression:

$$Ret_{i,t}^{e} = \alpha_i + \beta_i MKT_t^{e} + \varepsilon_{i,t}$$
$$t = m-60, m-59, \ldots, m-1 \tag{8}$$

$Ret_{i,t}^{e}$ denotes the excess return of stock i in month t, and MKT_t^{e} represents the market excess return in month t. From the regression, the estimate of β_i is estimated and used as the market beta of the stock i for the month m.

(2) Firm size: The firm size of a specific stock on a certain trading day is defined as the product of number of shares outstanding and the closing price of that stock. Correspondingly, we average the firm size of that stock for all trading days in the month to obtain its monthly index.

(3) Book-to-market ratio: Book-to-market ratio = book value of the stock / firm size of the stock.

(4) Momentum Factor: We define the total return of a stock from the past 12 months to the past 2 months as its momentum factor.

(5) Reversal factor: We define the total return of a stock in the past one month as its reversal factor.

(6) Illiquidity factor: We define the illiquidity factor of a stock in a certain month as the average of the absolute return of the stock to the its trade volume in unit of currency (instead of share) for each trading day in the month. The formula is as follows:

$$ILLIQ_{i,t} = \frac{1}{N}\sum_d \left(\frac{|r_{i,d}|}{Trdsum_{i,d}}\right) \tag{9}$$

N represents the number of trading days of the stock i in the month, $r_{i,d}$ indicates the return of the stock i on the day d, and $Trdsum_{i,d}$ denotes the stock's trade volume in unit of currency on the day d.