

CS 440: Introduction to Artificial Intelligence

Homework #3 Part 1 Probabilistic Reasoning

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Professor Abdelsam Boularias

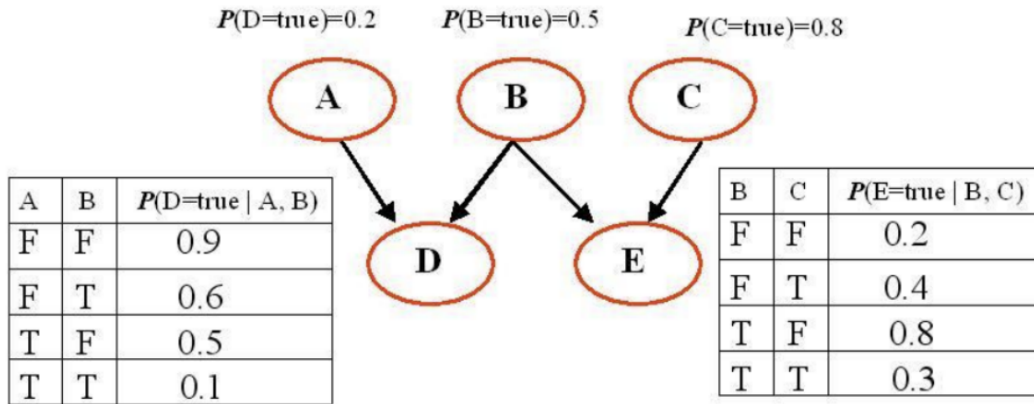
Jay Patwardhan 208001851

Alan Wu 208000574

Neel Shejwalkar 207004853

Problem 1

Consider the following Bayesian network, where variables A through E are all Boolean valued. Note: there is a typo in the image, it should be $P(A = \text{true}) = 0.2$ instead of $P(D = \text{true}) = 0.2$



Part A

What is the probability that all five of these Boolean variables are simultaneously true? [Hint: You have to compute the joint probability distribution. The structure of the Bayesian network suggests how the joint probability distribution is decomposed to the conditional probabilities available]

$$P(A, B, C, D, E) = P(A) \cdot P(B) \cdot P(C) \cdot P(D \mid A = T, B = T) \cdot P(E \mid B = T, C = T) \\ = 0.2 \cdot 0.5 \cdot 0.8 \cdot 0.1 \cdot 0.3 = 0.0024$$

Part B

What is the probability that all five of these Boolean variables are simultaneously false? [Hint: Answer similarly to above.]

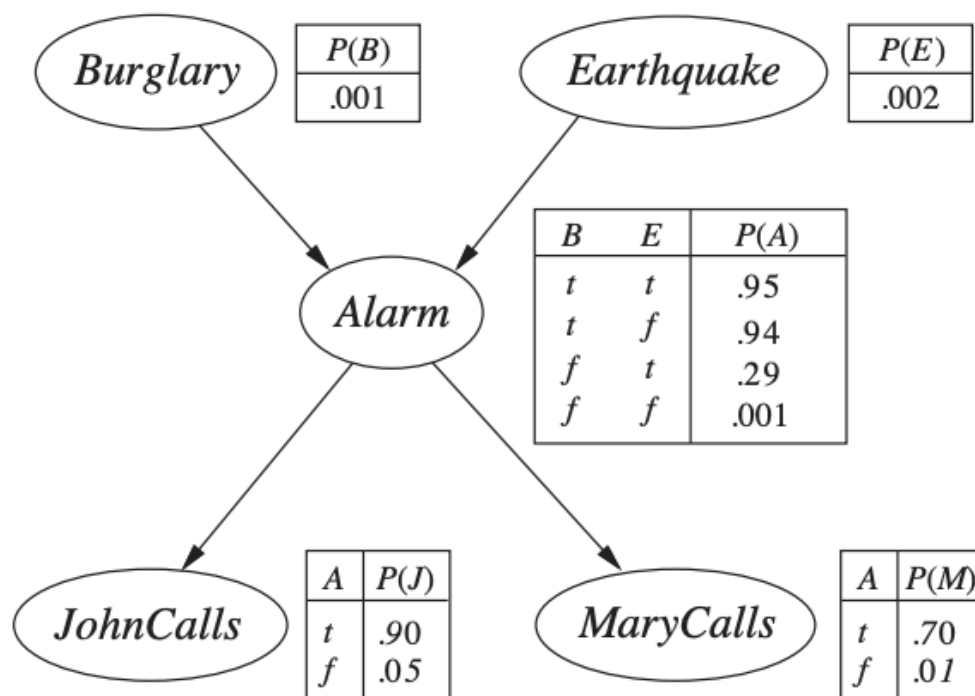
$$P(\neg A, \neg B, \neg C, \neg D, \neg E) = P(\neg A) \cdot P(\neg B) \cdot P(\neg C) \cdot P(\neg D \mid A = F, B = F) \cdot P(\neg E \mid B = F, C = F) \\ = (1 - 0.2) \cdot (1 - 0.5) \cdot (1 - 0.8) \cdot (1 - 0.9) \cdot (1 - 0.2) \\ = 0.8 \cdot 0.5 \cdot 0.2 \cdot 0.1 \cdot 0.8 = 0.0064$$

Part C

What is the probability that A is false given that the four other variables are all known to be true?

$$P(\neg A \mid B, C, D, E) = \frac{P(\neg A, B, C, D, E)}{P(B, C, D, E)} \\ P(\neg A, B, C, D, E) = P(\neg A) \cdot P(B) \cdot P(C) \cdot P(D \mid A = F, B = T) \cdot P(E \mid B = T, C = T) \\ = (1 - 0.2) \cdot 0.5 \cdot 0.8 \cdot 0.6 \cdot 0.3 = 0.0576 \\ P(B, C, D, E) = P(A, B, C, D, E) + P(\neg A, B, C, D, E) = 0.0024 + 0.0576 = 0.06 \\ P(\neg A \mid B, C, D, E) = \frac{0.0576}{0.06} = 0.96$$

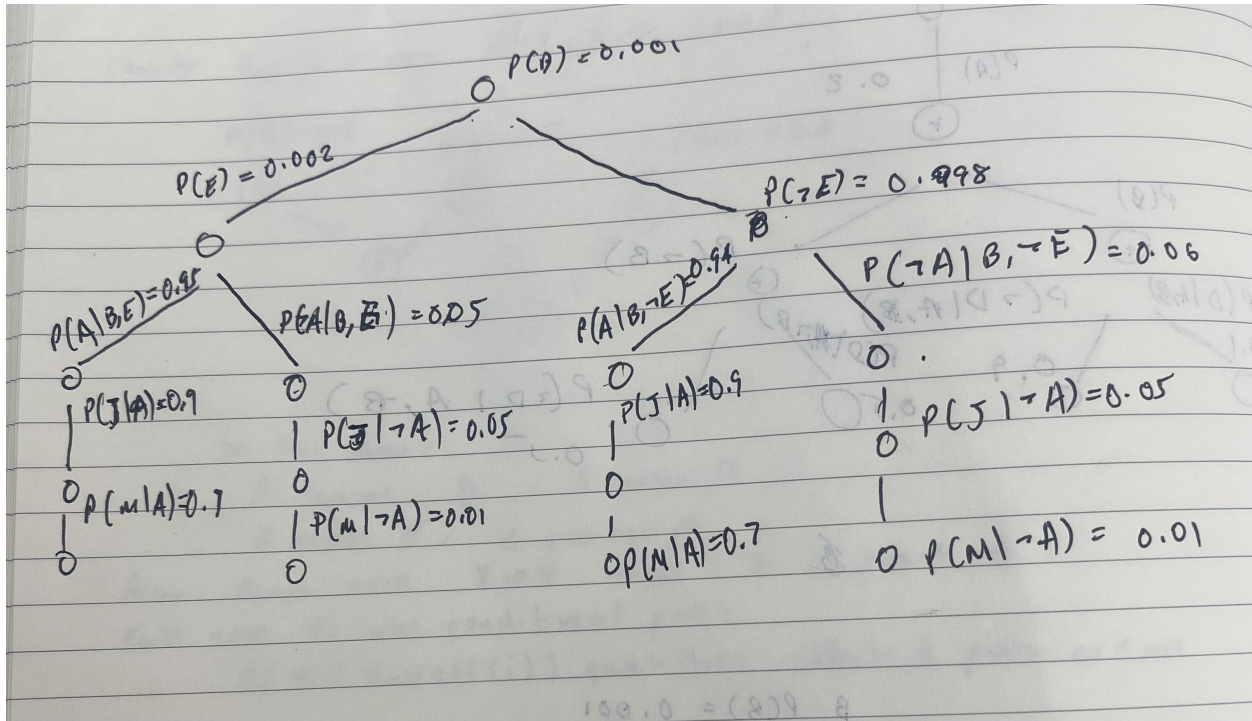
Problem 2



Part A

Calculate $P(\text{Burglary} | \text{JohnsCalls} = \text{true}, \text{MaryCalls} = \text{true})$ and show in detail the calculations that take place. Use your book to confirm that your answer is correct.

The enumeration tree is shown:



$$P(\text{Burglary} | \text{JohnsCalls} = \text{true}, \text{MaryCalls} = \text{true}) = P(B) \cdot \frac{P(J, M | B)}{P(J, M)}$$

$$= 0.001 \cdot \frac{(P(A|B) \cdot P(J, M | A)) + (P(\neg A|B) \cdot P(J, M | \neg A))}{(P(J, M | A) \cdot P(A)) + (P(J, M | \neg A) \cdot P(\neg A))}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(P(B) \cdot P(E) \cdot P(A|B, E)) + (P(B) \cdot P(\neg E) \cdot P(A|B, \neg E))}{P(B)}$$

$$= \frac{(0.001 \cdot 0.002 \cdot 0.95) + (0.001 \cdot 0.998 \cdot 0.94)}{0.001} = 0.94002$$

$$P(J, M | A) = P(J | A) \cdot P(M | A) = 0.9 \cdot 0.7 = 0.63$$

$$P(J, M | \neg A) = P(J | \neg A) \cdot P(M | \neg A) = 0.05 \cdot 0.01 = 0.0005$$

$$P(\neg A | B) = 1 - P(A | B) = 1 - 0.94002 = 0.05998$$

$$P(A) = (P(B) \cdot P(E) \cdot P(A|B, E)) + (P(B) \cdot P(\neg E) \cdot P(A|B, \neg E)) + (P(\neg B) \cdot P(E) \cdot P(A|\neg B, E)) + (P(\neg B) \cdot P(\neg E) \cdot P(A|\neg B, \neg E))$$

$$= (0.001 \cdot 0.002 \cdot 0.95) + (0.001 \cdot 0.998 \cdot 0.94) + (0.999 \cdot 0.002 \cdot 0.29) + (0.999 \cdot 0.998 \cdot 0.001)$$

$$= 0.0025$$

$$P(\neg A) = 1 - P(A) = 1 - 0.0025 = 0.99748$$

Therefore:

$$P(\text{Burglary} | \text{JohnCalls} = \text{true}, \text{MaryCalls} = \text{true}) = 0.001 \cdot \frac{(0.94002 \cdot 0.63) + (0.05998 \cdot 0.0005)}{(0.63 \cdot 0.0025) + (0.0005 \cdot 0.99748)}$$

$$= 0.2841718$$

Part B

Suppose a Bayesian network has the form of a *chain*: a sequence of Boolean variables X_1, \dots, X_n where $\text{Parents}(X_i) = X_{i-1}$ for $i = 2, \dots, n$. What is the complexity of computing $P(X_1 | X_n = \text{true})$ using enumeration. What is the complexity with variable elimination?

The complexity for Enumeration:

Above is the binary tree since we only care about the true values for X_1 and X_N . We are looking for $P(X_1|X_n = \text{true})$. Given the tree above, we know that the binary tree will have n levels that branch $n - 2$ times. With the graph shown with 5 levels, it only branches 3 times. The above graph is missing the top level node. In this tree design there is a total of 2^{n-1} nodes. The space complexity will be $O(n)$ for dfs to traverse the tree and it will take $O(2^n)$ time complexity, with 2^{n-1} nodes absorbed into 2^n .

Now we will calculate the complexity for variable elimination:

$$\begin{aligned}
 & P(X_1|X_n = \text{True}) \\
 &= \alpha P(X_1) \sum_{X_2} P(X_2|X_1) \sum_{X_3} P(X_3|X_2) \dots \sum_{X_{n-2}} X_{n-2}|X_{n-3} \sum_{X_{n-1}} X_{n-1}|X_{n-2} \cdot P(X_n|X_{n-1}) \\
 &= \alpha \sum_{X_2} \sum_{X_3} \dots \sum_{X_{n-1}} f_1(X_1) x f_2(X_2) x f_{n-1}(X_{n-1}) x f_n(X_n)
 \end{aligned}$$

Each $f_k(X_k)$ term is a 2×1 matrix that has $X_K|X_{K-1}$ on the top and $\neg X_K|X_{K-1}$ on the bottom. The total space complexity is $2n$ and therefore $O(n)$. The total time complexity is the time to multiply all the matrices, which is $2n$. This absorbs to runtime time complexity of $O(n)$

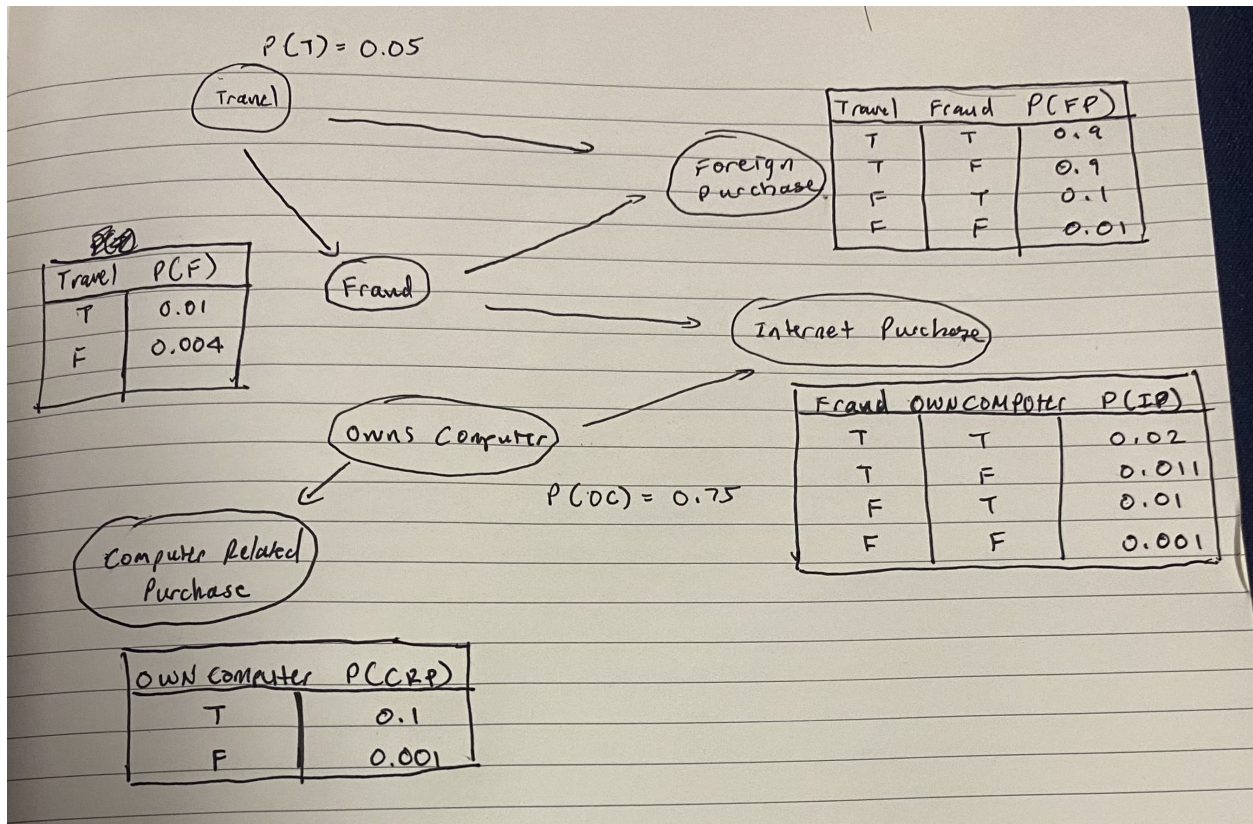
Problem 3

Suppose you are working for a financial institution and you are asked to implement a fraud detection system. You plan to use the following information:

- When the card holder is travelling abroad, fraudulent transactions are more likely since tourists are prime targets for thieves. More precisely, 1% of transactions are fraudulent when the card holder is travelling, where as only 0.4% of the transactions are fraudulent when she is not travelling. On average, 5% of all transactions happen while the card holder is travelling. If a transaction is fraudulent, then the likelihood of a foreign purchase increases, unless the card holder happens to be travelling. More precisely, when the card holder is not travelling, 10% of the fraudulent transactions are foreign purchases where as only 1% of the legitimate transactions are foreign purchases. On the other hand, when the card holder is travelling, then 90% of the transactions are foreign purchases regardless of the legitimacy of the transactions.
- Purchases made over the internet are more likely to be fraudulent. This is especially true for card holders who don't own any computer. Currently, 75% of the population owns a computer or smart phone and for those card holders, 1% of their legitimate transactions are done over the internet, however this percentage increases to 2% for fraudulent transactions. For those who don't own any computer or smart phone, a mere 0.1% of their legitimate transactions is done over the internet, but that number increases to 1.1% for fraudulent transactions. Unfortunately, the credit card company doesn't know whether a card holder owns a computer or smart phone, however it can usually guess by verifying whether any of the recent transactions involve the purchase of computer related accessories. In any given week, 10% of those who own a computer or smart phone purchase (with their credit card) at least one computer related item as opposed to just 0.1% of those who don't own any computer or smart phone.

Part A

Construct a Bayes Network to identify fraudulent transactions.



Part B

What is the prior probability (i.e., before we search for previous computer related purchases and before we verify whether it is a foreign and/or an internet purchase) that the current transaction is a fraud? What is the probability that the current transaction is a fraud once we have verified that it is a foreign transaction, but not an internet purchase and that the card holder purchased computer related accessories in the past week?

The probabilities we want to search for: $P(\text{Fraud})$ and $P(\text{Fraud} | FP = T, IP = F, CRP = T)$.

Part 1: $P(\text{Fraud})$

$$\begin{aligned}
 P(\text{Fraud}) &= (P(F|\text{Travel}) \cdot P(\text{Travel})) + (P(F|\neg\text{Travel}) \cdot P(\neg\text{Travel})) \\
 &= (0.01 \cdot 0.05) + (0.004 \cdot 0.95) = 0.0043
 \end{aligned}$$

Part 2:

$$\begin{aligned}
 \frac{P(\text{Fraud} | FP, \neg IP, CRP)}{\alpha} &= P(\text{Travel})P(OC)P(\text{Fraud}|\text{Travel})P(CRP|OC)P(\neg IP|OC, \text{Fraud})P(FP|\text{Travel}, \text{Fraud}) \\
 &\quad + P(\neg\text{Travel})P(OC)P(\text{Fraud}|\neg\text{Travel})P(CRP|OC)P(\neg IP|OC, \text{Fraud})P(FP|\neg\text{Travel}, \text{Fraud}) \\
 &\quad + P(\text{Travel})P(\neg OC)P(\text{Fraud}|\text{Travel})P(CRP|\neg OC)P(\neg IP|\neg OC, \text{Fraud})P(FP|\neg\text{Travel}, \text{Fraud}) \\
 &\quad + P(\neg\text{Travel})P(\neg OC)P(\text{Fraud}|\neg\text{Travel})P(CRP|\neg OC)P(\neg IP|\neg OC, \text{Fraud})P(FP|\neg\text{Travel}, \text{Fraud}) \\
 &= 0.05 \cdot 0.75 \cdot 0.01 \cdot 0.1 \cdot 0.98 \cdot 0.9 \\
 &= 0.95 \cdot 0.75 \cdot 0.004 \cdot 0.1 \cdot 0.98 \cdot 0.1 \\
 &= 0.05 \cdot 0.25 \cdot 0.01 \cdot 0.001 \cdot 0.989 \cdot 0.9 \\
 &= 0.95 \cdot 0.25 \cdot 0.004 \cdot 0.001 \cdot 0.989 \cdot 0.1 \\
 &\approx 0.0000612
 \end{aligned}$$

$$P(Fraud = True|FP, \neg IP, CRP) = \frac{1}{0.0000612+0.1026} \cdot 0.0000612 \approx 0.00592$$