CS 440: Introduction to Artificial Intelligence

Homework #3 Part 1 Probabilistic Reasoning

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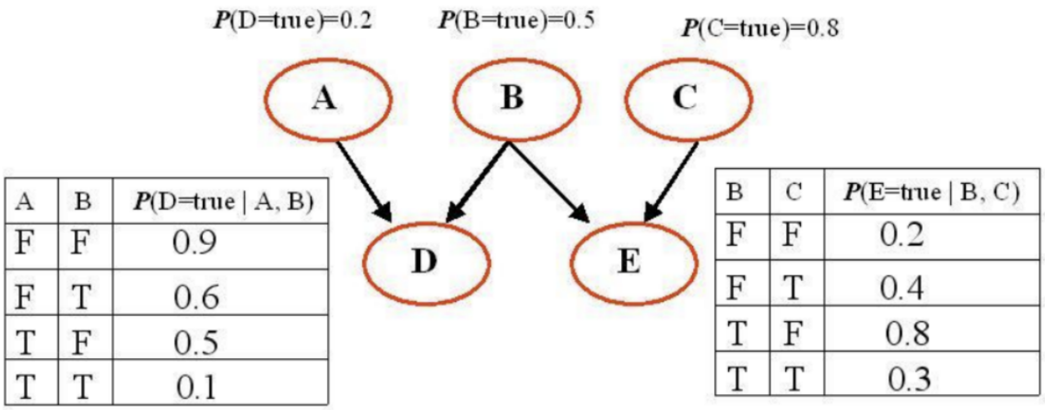
Problem 1

Consider the following Bayesian network, where variables A through E are all Boolean valued. Note: there is a typo in the image, it should be $P(A = true) = 0.2$ instead of $P(D = true) = 0.2$



Part A

What is the probability that all five of these Boolean variables are simultaneously true? [Hint: You have to compute the joint probability distribution. The structure of the Bayesian network suggests how the joint probability distribution is decomposed to the conditional probabilities available]

$P(A, B, C, D, E) = P(A) \cdot P(B) \cdot P(C) \cdot P(D|A = T, B = T) \cdot P(E|B = T, C = T)$
$= 0.2 \cdot 0.5 \cdot 0.8 \cdot 0.1 \cdot 0.3 = 0.0024$

Part B

What is the probability that all five of these Boolean variables are simultaneously false? [Hint: Answer similarly to above.]

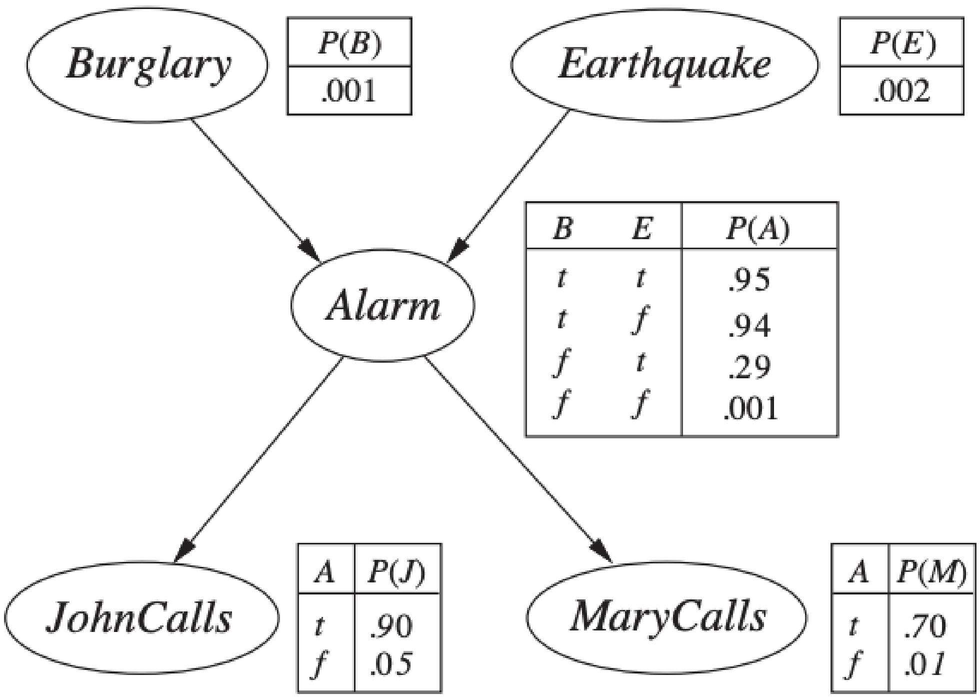
$P(\neg A, \neg B, \neg C, \neg D, \neg E) = P(\neg A) = P(\neg A) \cdot P(\neg B) \cdot P(\neg C) \cdot P(\neg D|A = F, B = F) \cdot P(\neg E|B = F, C = F)$
$= (1 - 0.2) \cdot (1 - 0.5) \cdot (1 - 0.8) \cdot (1 - 0.9) \cdot (1 - 0.2)$
$= 0.8 \cdot 0.5 \cdot 0.2 \cdot 0.1 \cdot 0.8 = 0.0064$

Part C

What is the probability that A is false given that the four other variables are all known to be true?

$P(\neg A|B, C, D, E) = \frac{P(\neg A, B, C, D, E)}{P(B, C, D, E)}$
$P(\neg A, B, C, D, E) = P(\neg A) \cdot P(B) \cdot P(C) \cdot P(D|A = F, B = T) \cdot P(E|B = T, C = T)$
$= (1 - 0.2) \cdot 0.5 \cdot 0.8 \cdot 0.6 \cdot 0.3 = 0.0576$
$P(B, C, D, E) = P(A, B, C, D, E) + P(\neg A, B, C, D, E) = 0.0024 + 0.0576 = 0.06$
$P(\neg A|B, C, D, E) = \frac{0.0576}{0.06} = 0.96$

Problem 2

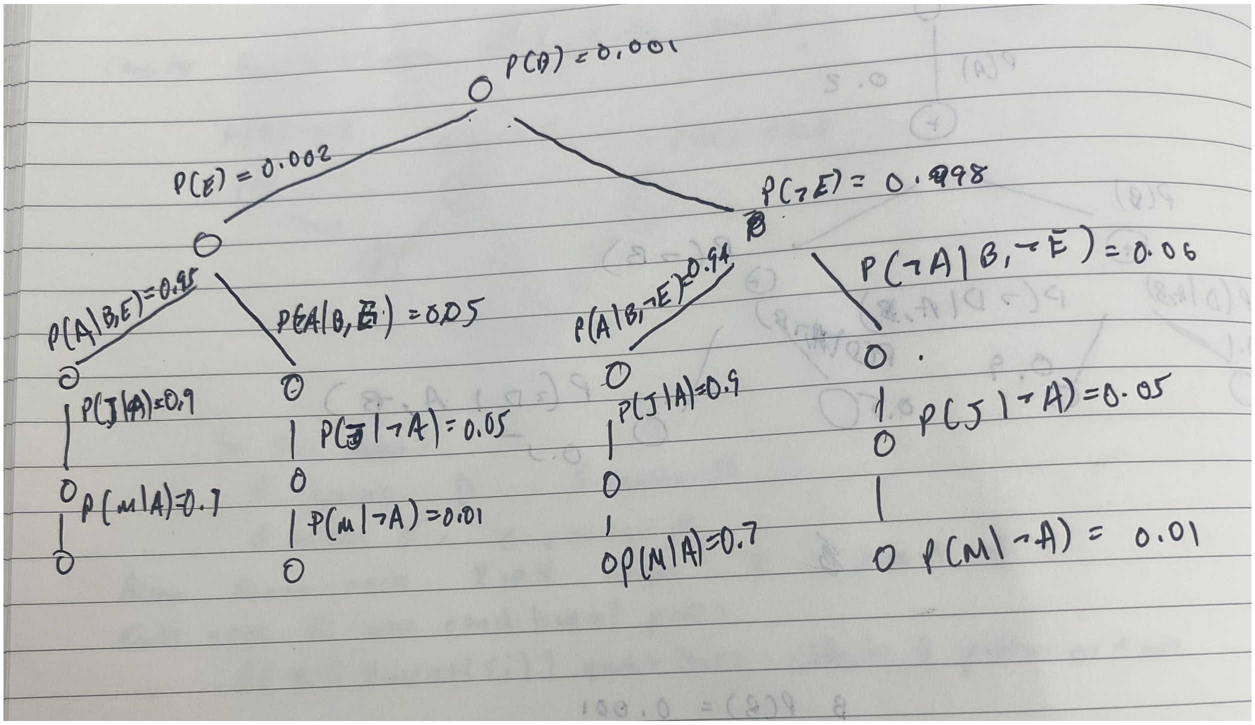


Part A

Calculate $P(Burglary|JohnsCalls = true, MaryCalls = true)$ and show in detail the calculations that take place. Use your book to confirm that your answer is correct.

The enumeration tree is shown:

Problem 2 continued on next page. . .



$P(Burglary|JohnsCalls = true, MaryCalls = true) = P(B) \cdot \frac{P(J,M|B)}{P(J,M)}$
$= 0.001 \cdot \frac{(P(A|B) \cdot P(J,M|A)) + (P(\neg A|B) \cdot P(J,M|\neg A))}{(P(J,M|A) \cdot P(A)) + (P(J,M|\neg A) \cdot P(\neg A))}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(P(B) \cdot P(E) \cdot P(A|B,E)) + (P(B) \cdot P(\neg E) \cdot P(A|B,\neg E))}{P(B)}$
$= \frac{(0.001 \cdot 0.002 \cdot 0.95) + (0.001 \cdot 0.998 \cdot 0.94)}{0.001} = 0.94002$

$P(J,M|A) = P(J|A) \cdot P(M|A) = 0.9 \cdot 0.7 = 0.63$

$P(J,M|\neg A) = P(J|\neg A) \cdot P(M|\neg A) = 0.05 \cdot 0.01 = 0.0005$

$P(\neg A|B) = 1 - P(A|B) = 1 - 0.94002 = 0.05998$

$P(A) = (P(B) \cdot P(E)P(A|B,E)) + (P(B) \cdot P(\neg E)P(A|B,\neg E)) + (P(\neg B) \cdot P(E) \cdot P(A|\neg B,E)) + (P(\neg B) \cdot P(\neg E) \cdot P(A|\neg B,\neg E))$
$= (0.001 \cdot 0.002 \cdot 0.95) + (0.001 \cdot 0.998 \cdot 0.94) + (0.999 \cdot 0.002 \cdot 0.29) + (0.999 \cdot 0.998 \cdot 0.001)$
$= 0.0025$

$P(\neg A) = 1 - P(A) = 1 - 0.0025 = 0.99748$

Therefore:
$P(Burglary|JohnCalls = true, MaryCalls = true) = 0.001 \cdot \frac{(0.94002 \cdot 0.63) + (0.05998 \cdot 0.0005)}{(0.63 \cdot 0.0025) + (0.0005 \cdot 0.99748)}$
$= 0.2841718$

Part B

Suppose a Bayesian network has the from of a *chain* : a sequence of Boolean variables $X_1, ...X_n$ where $Parents(X_i) = X_{i-1}$ for $i = 2, ..., n$. What is the complexity of computing $P(X_1|Xn = true)$ using enumeration. What is the complexity with variable elimination?

The complexity for Enumeration:

Above is the binary tree since we only care about the true values for X_1 and X_N. We are looking for $P(X_1|X_n = true)$. Given the tree above, we know that the binary tree will have n levels that branch $n-2$ times. With the graph shown with 5 levels, it only branches 3 times. The above graph is missing the top level node. In this tree design there is a total of 2^{n-1} nodes. The space complexity will be $O(n)$ for dfs to traverse the tree and it will take $O(2^n)$ time complexity, with 2^{n-1} nodes absorbed into 2^n.

Now we will calculate the complexity for variable elimination:

$P(X_1|X_n = True)$
$= \alpha P(X_1) \sum_{X_2} P(X_2|X_1) \sum_{X_3} P(X_3|X_2) ... \cdot \sum_{X_{n-2}} X_{n-2}|X_{n-3} \sum_{X_{n-1}} X_{n-1}|X_{n-2} \cdot P(X_n|X_{n-01})$
$= \alpha \sum_{X_2} \sum_{X_3} ... \sum_{X_{n-1}} f_1(X_1) x f_2(X_2) x f_{n-1}(X_{n-1}) x f_n(X_n)$

Each $f_k(X_k)$ term is a $2x1$ matrix that has $X_K|X_{K-1}$ on the top and $\neg X_K|X_{K-1}$ on the bottom. The total space complexity is $2n$ and therefore $O(n)$. The total time complexity is the time to multiply all the matrices, which is $2n$. This absorbs to runtime time complexity of $O(n)$

Problem 3

Suppose you are working for a financial institution and you are asked to implement a fraud detection system. You plan to use the following information:

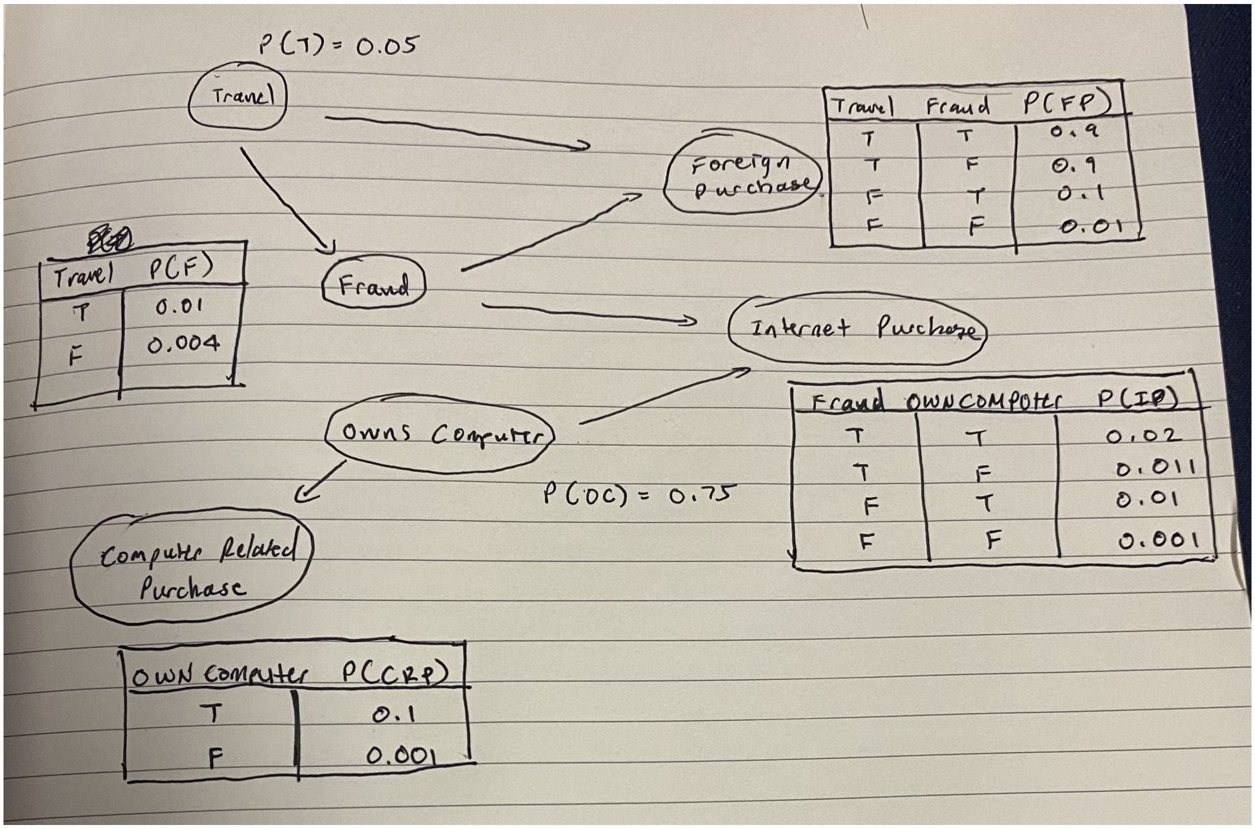
- When the card holder is travelling abroad, fraudulent transactions are more likely since tourists are prime targets for thieves. More precisely, 1% of transactions are fraudulent when the card holder is travelling, where as only 0.4% of the transactions are fraudulent when she is not travelling. On average, 5% of all transactions happen while the card holder is travelling. If a transaction is fraudulent, then the likelihood of a foreign purchase increases, unless the card holder happens to be travelling. More precisely, when the card holder is not travelling, 10% of the fraudulent transactions are foreign purchases where as only 1% of the legitimate transactions are foreign purchases. On the other hand, when the card holder is travelling, then 90% of the transactions are foreign purchases regardless of the legitimacy of the transactions.

- Purchases made over the internet are more likely to be fraudulent. This is especially true for card holders who don't own any computer. Currently, 75% of the population owns a computer or smart phone and for those card holders, 1% of their legitimate transactions are done over the internet, however this percentage increases to 2% for fraudulent transactions. For those who don't own any computer or smart phone, a mere 0.1% of their legitimate transactions is done over the internet, but that number increases to 1.1% for fraudulent transactions. Unfortunately, the credit card company doesn't know whether a card holder owns a computer or smart phone, however it can usually guess by verifying whether any of the recent transactions involve the purchase of computer related accessories. In any given week, 10% of those who own a computer or smart phone purchase (with their credit card) at least one computer related item as opposed to just 0.1% of those who don't own any computer or smart phone.

Part A

Construct a Bayes Network to identify fraudulent transactions.

Problem 3 continued on next page...



Part B

What is the prior probability (i.e., before we search for previous computer related purchases and before we verify whether it is a foreign and/or an internet purchase) that the current transaction is a fraud? What is the probability that the current transaction is a fraud once we have verified that it is a foreign transaction, but not an internet purchase and that the card holder purchased computer related accessories in the past week?

The probabilities we want to search for: $P(Fraud)$ and $P(Fraud|FP = T, IP = F, CRP = T)$.

Part 1: $P(Fraud)$

$P(Fraud)$

$= (P(F|Trav) \cdot P(Trav)) + (P(F|\neg Trav) \cdot P(\neg Trav))$

$= (0.01 \cdot 0.05) + (0.004 \cdot 0.95) = 0.0043$

Part 2:

$P(Fraud|FP, \neg IP, CRP)$

$= \frac{1}{\alpha} \cdot \sum_{Trav,OC} P(Fraud|FP, \neg IP, CRP)$

$= \frac{1}{\alpha} \cdot \sum_{Trav,OC} P(Trav) \cdot P(Fraud|Trav) \cdot P(FP|Fraud, Trav) \cdot P(OC) \cdot P(CRP, OC) \cdot P(\neg IP|OC, Fraud)$

All cases:

1. $Trav = True, OC = True$
   Total Probability $= 0.05 \cdot 0.01 \cdot 0.9 \cdot 0.75 \cdot 0.1 \cdot (1 - 0.02) = 0.000033075$

2. $Trav = True, OC = False$
   Total Probability $= 0.05 \cdot 0.01 \cdot 0.9 \cdot 0.25 \cdot 0.001 \cdot (1 - 0.011) = 0.0000001112625$

3. $Trav = False, OC = True$
   Total Probability $= 0.95 \cdot 0.004 \cdot 0.1 \cdot 0.75 \cdot 0.1 \cdot (1 - 0.02) = 0.00002793$

4. $Trav = False, OC = False$
   Total Probability $= 0.95 \cdot 0.004 \cdot 0.1 \cdot 0.25 \cdot 0.001 \cdot (1 - 0.011) = 0.000000093955$

$P(Fraud|FP, \neg IP, CRP)$
$= \frac{1}{\alpha} \cdot 0.000033075 + 0.0000001112625 + 0.00002793 + 0.000000093955$
$= \frac{1}{\alpha} \cdot 0.00006121021$
$= \frac{1}{0.00006121021 + 0.01026} \cdot 0.00006121021 = 0.00592$