Towards a new approach to reveal dynamical organization of the brain using topological data analysis

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April 4, 2022

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- They observe that the revealed individual differences in the dynamical organization of the subject were predictors of the task performance

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- The previous approaches have been unable to reveal the optimal temporal and spatial scales which best probe clinically and behaviorally relevant brain dynamics
- Additionally, they are unable to determine if the brain dynamics are best thought of as continuous or discrete or able to tell whether a particular brain activity is healthy or not
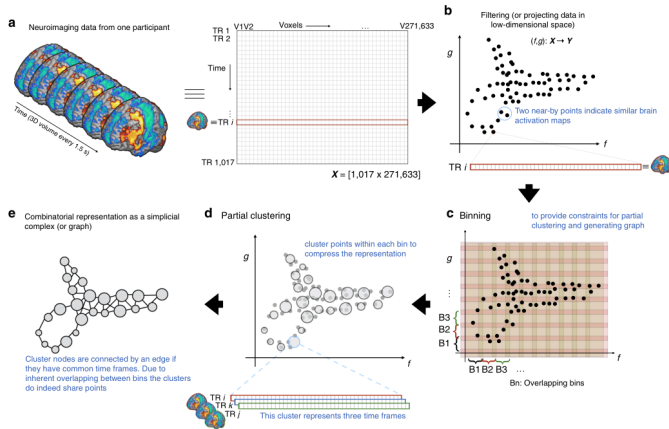
Figure: The method used to convert the 4-dimensional fMRI data into a simplicial complex. Steps b-e are a part of Mapper (the TDA-based algorithm/tool the authors used).

How Mapper works

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- Step 4 treats each cluster as a vertex of a graph and adds an edge between two vertices if they shared a point[4]

Filtering in Mapper

Filter Range : [0-4.2]
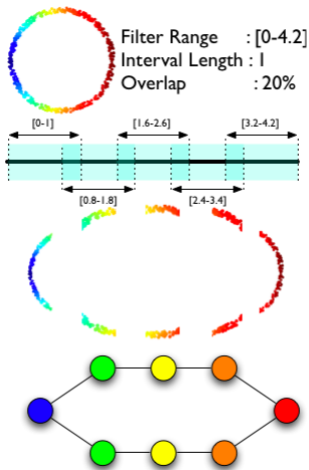Interval Length : 1
Overlap : 20%

Figure: Toy example of applying a filter to data[4]. The data is sampled from a noisy circle, and the filter used is $f(x) = ||x - p||^2$, where p is the left most point in the data. We divide the range of the filter into 5 intervals which have length 1 and a 20% overlap. For each interval we compute the clustering of the points lying within the domain of the filter restricted to the interval, and connect the clusters whenever they have non-empty intersection. At the bottom is the simplicial complex which we recover whose vertices are colored by the average filter value.
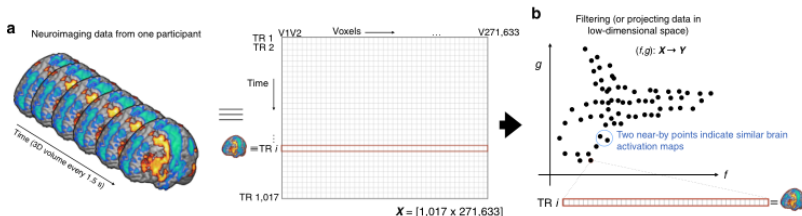
Figure: Our paper uses a very different filter function than the previous example. They use something called the Neighborhood Lens function to take their 271633-dimensional data into $\mathbb{R}^2$. This function is part of a patented software that appears to be standard.
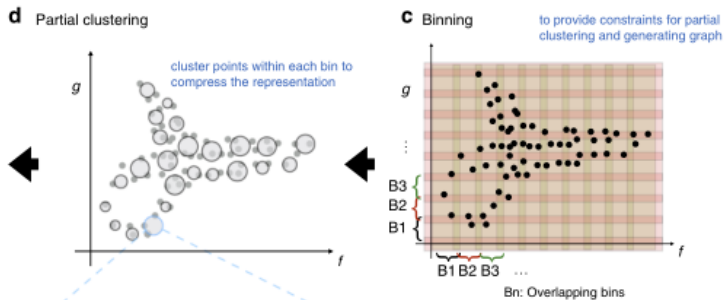
Figure: Mapper take two parameters when binning the data: Resolution and Gain. Resolution controls the number of bins and Gain controls the overlap between bins.

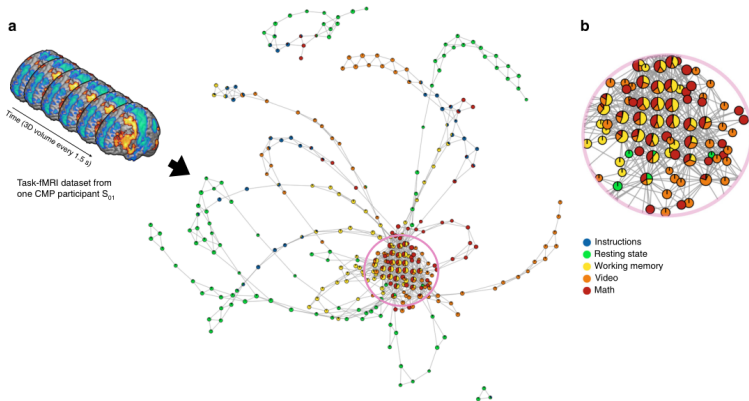Figure: After running Mapper on an individual's fMRI data, we are left with a graph like the above. The fMRI datasets (1017×271633) matrix) were compressed to 279 ± 60 points.

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- Two likely possibilities: the weight between two nodes is the number of timeframes they shared, or the weight is something that depends on the mean distance between the timeframes constituting the nodes

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- Q_{mod} (called the modularity) is the most commonly used metric to assess the community structure of a graph[1]

$$Q_{\mathrm{mod}} = \sum_{i,j} \left(A_{ij} - P_{ij} \right) \delta(g_i, g_j)$$

where A is the adjacency matrix, $P_{ij} = \frac{k_i k_j}{\sum_{ij} A_{ij}}$, k_i is degree of i, g_i is the community that i belongs to, and δ is the Kronecker delta.

Some intuition for Q_{mod}

- It is based on the idea that a random graph is not expected to have a cluster structure, so the possible existence of clusters is revealed by the comparison between the actual density of edges in a subgraph and the density one would expect to have in the subgraph if the vertices of the graph were attached regardless of community structure
- This expected edge density ($P_{ij} = \frac{k_i k_j}{2m}$ where $m = \sum_{ij} A_{ij}$) depends on the chosen null model, i.e. a copy of the original graph keeping some of its structural properties but without community structure[1]

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- In this case, the null model that produces the above expected edge density is called the configuration model. This a very standard type of model in network science because, as opposed to the Erdős-Renyi model, this model allows us to give the network arbitrary degree distributions. (The Erdős-Renyi model only allows for Poisson distribution for the degree sequence).
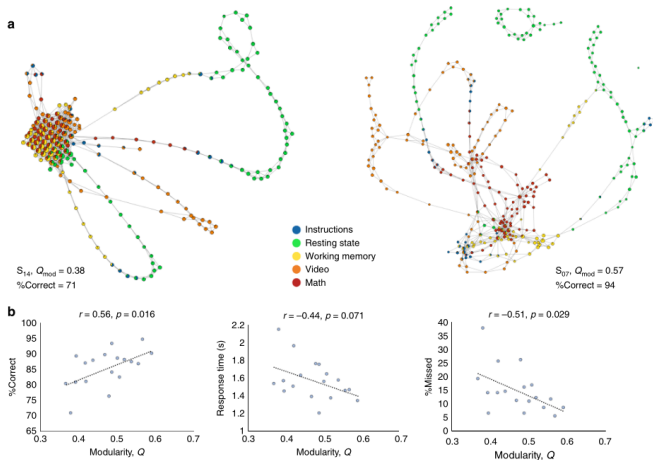
How Q_{mod} scales

Figure: There are two different participants' shape graphs and the relations between modularity and various metrics

Analyzing the core-periphery structure of the graph

- We assign each node a coreness score (CS) by giving higher scores to nodes which lie deeper in the network
- The Borgatti-Everett algorithm is an algorithm that assigns coreness scores to each vertex of the shape graph. If we call C_i is the coreness of vertex i, then we can define the core matrix C by $C_{ij} = C_i C_j$. We can define a quality function

$$R_{(\alpha,\beta)} = \sum_{i,j} A_{ij} C_{ij}$$

where (α, β) are the two parameters which determines the boundary between the core and periphery and size of the core respectively. A large α indicates a sharp transition

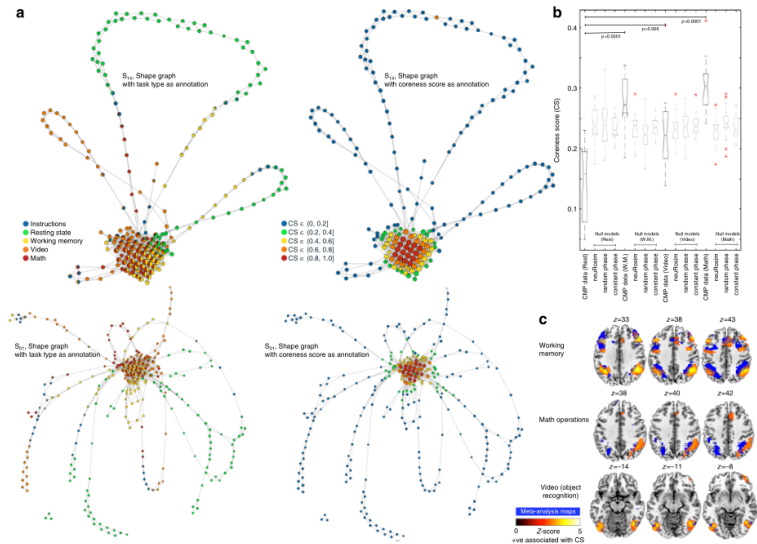# Definition of the aggregate of coreness scores

Recall that

$$R_{(\alpha,\beta)} = \sum_{i,j} A_{ij} C_{ij}$$

The aggregate of coreness of node i is

$$CS(i) = Z \sum_{(\alpha,\beta)} C_i(\alpha,\beta) \times R(\alpha,\beta)$$

where Z is a normalization factor to make the maximum CS be 1 and this sum is not actually over all α, β but instead over a uniform sampling of a discretization of the unit square.

How coreness score (CS) scales
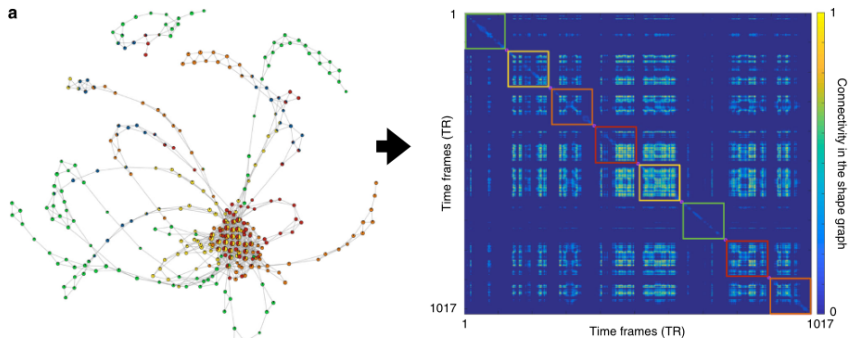
Figure: Shows which times during the scan were similar to other times by looking at the connections between times in the shape graph.
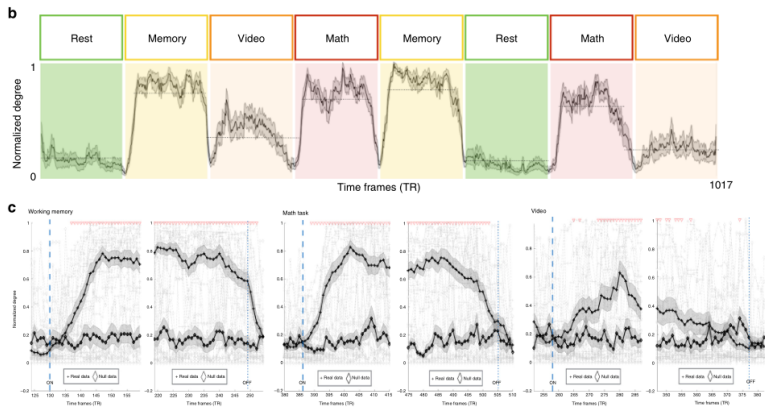
Figure: When there is a transition between tasks, this is captured by a dramatic change in the degree of the corresponding nodes. This phenomenon of nodes taken during strenuous activity having high degree is significant

[1] Santo Fortunato. "Community detection in graphs". In: *Physics Reports* 486.3-5 (2010), pp. 75–174. DOI: 10.1016/j.physrep.2009.11.002. URL: https://doi.org/10.1016%2Fj.physrep.2009.11.002.

[2] Saggar Manish et al. *Towards a new approach to reveal dynamical organization of the brain using topological data analysis*. 2018. URL: https://www.nature.com/articles/s41467-018-03664-4#citeas.

[3] Maria Giulia Preti, Thomas AW Bolton, and Dimitri Van De Ville. "The dynamic functional connectome: State-of-the-art and perspectives". In: *NeuroImage* 160 (2017). Functional Architecture of the Brain, pp. 41–54. ISSN: 1053-8119. DOI: https://doi.org/10.1016/j.neuroimage.2016.12.061. URL: https://www.sciencedirect.com/science/article/pii/S1053811916307881.

[4] Gurjeet Kaur Chatar Singh, Facundo Mémoli, and
     Gunnar E. Carlsson. "Topological Methods for the Analysis of High
     Dimensional Data Sets and 3D Object Recognition". In: (2007).

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