Stratocumulus Climate Transitions in a Simple Model

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Abstract
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Plain Language Summary

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1 Introduction

The purpose of this work is to create a minimalist, "almost analytical" model that describes how the coupled effects of tropical deep convection and subtropical stratocumulus convection influence the low-latitude climate system. Only equilibrium solutions are sought; the few prognostic equations of the model are merely tools used to find equilibria by time-marching.

The column relative humidity (CRH) plays a key role in our representation of the tropical atmosphere. It enters into the parameterizations of both tropical radiation and tropical convection.

(Schneider et al., 2019)

(Lilly, 1968)

(Wood, 2012)

(Pierrehumbert, 1995)

(Peters & Bretherton, 2005; Bretherton & Sobel, 2002)

(Wofsy & Kuang, 2012)

2 Summary of the basic approach

Like Pierrehumbert (1995) and Schneider et al. (2019), we define separate subtropical and tropical domains (Figure 1), denoted by subscripts *st* and *tr*, respectively and are expected to vary with height. The domain is assumed to be linear, with the subtropics on one side and the tropics on the other. The total width of the domain is denoted by L, with impermeable walls on the outer edges, as shown in the figure. The fractional areas covered by the tropical and subtropical domains are denoted by σ_{st} and σ_{tr}, respectively. They vary strongly with height; our motivation for this approach is explained below. We assume that

$$\sigma_{\mathrm{tr}} + \sigma_{\mathrm{st}} = 1 \; . \tag{1}$$

at every level.

The tropical troposphere is assumed to be represented by a convectively active layer topped by a "detrainment layer" that contains radiatively active high clouds. The subtropical troposphere is represented by a cloudy boundary layer and cloudless free troposphere above. The tropical troposphere is characterized by large-scale rising motion, and the subtropical troposphere by large-scale sinking motion. The upward and downward mass fluxes are defined by

$$M_{\mathrm{tr}} \equiv \rho_{\mathrm{tr}} w_{\mathrm{tr}} \sigma_{\mathrm{tr}} > 0 \; . \tag{2}$$

and

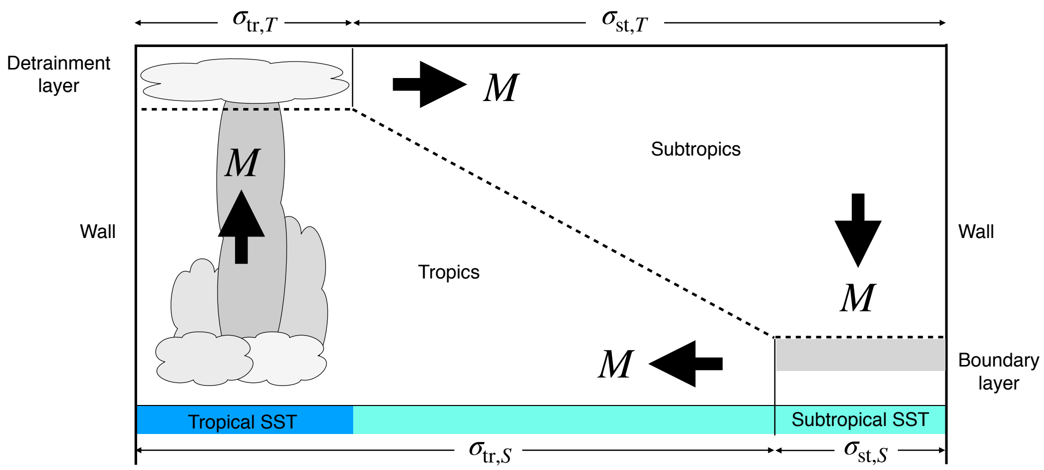

Figure 1. A sketch showing the tropical and subtropical domains, and the circulating mass flux M.

$$M_{\text{st}} \equiv \rho_{\text{st}} w_{\text{st}} \sigma_{\text{st}} < 0 \ , \tag{3}$$

respectively. Here ρ is the density of the air, and w is the vertical velocity. Mass exchanges between the subtropics and the tropics are assumed to occur only as a flow from the subtropical boundary layer into the tropics, and as a flow from the tropical detrainment layer into the subtropics. No mass crosses the diagonal dashed line in Fig. 1.

We assume that both M_{st} and M_{tr} are independent of height:

$$\frac{\partial M_{\text{tr}}}{\partial z} = \frac{\partial M_{\text{st}}}{\partial z} = 0 \ . \tag{4}$$

We also assume that horizontal temperature and density gradients are negligible in the free atmosphere, i.e., we make the "weak temperature gradient" (WTG) approximation (Charney, 1963; Sobel et al., 2001). In particular, we will use

$$\rho_{\text{tr}}(z) = \rho_{\text{st}}(z) \ , \tag{5}$$

and

$$s_{\text{tr}}(z) = s_{\text{st}}(z) \ , \tag{6}$$

where s is the dry static energy. Finally, we assume that the upward vertical mass flux in the tropics exactly compensates for the downward vertical mass flux in the subtropics, so that the total vertical mass flux is zero:

$$M_{\text{tr}} + M_{\text{st}} = 0 \ . \tag{7}$$

This allows us to write

$$M_{\rm tr} = -M_{\rm st} \equiv M \geq 0 \ . \tag{8}$$

As indicated in (8), we use the symbol M (no subscript) to denote the strength of the mass flux, which circulates around a "loop" that connects the tropics and the subtropics. See Figure 1. We assume that the mass flux from the subtropics to the tropics occurs within the subtropical boundary layer, and the mass flux from the topics to the subtropics occurs in a thin "detrainment" layer near the tropopause. When the subtropical boundary layer depth is in equilibrium, the downward subtropical mass flux balances the entrainment mass flux across the top of the subtropical boundary layer.

We include a slab model of the upper ocean, for both the tropics and the subtropics.

3 Water and energy budgets

In equilibrium, the global water budget is

$$\left(F_{q+l}\right)_{\rm tr,S} \sigma_{{\rm tr},S} + \left(F_{q+l}\right)_{{\rm st},S} \sigma_{{\rm st},S} - P_{\rm tr}\sigma_{{\rm tr},S} = 0 \ . \tag{9}$$

Here we have assumed that precipitation occurs only in the tropics. This implies a net flow of moisture from the subtropics into the tropics. We write the equilibrium tropical moisture budget as

$$\left(F_{q+l}\right)_{\rm tr,S} \sigma_{{\rm tr},S} - P_{\rm tr}\sigma_{{\rm tr},S} + M\left[q_M - q_{{\rm tr},T}\right] = 0 \ . \tag{10}$$

Similarly, in equilibrium the global energy budget is

$$\begin{aligned} &\left[\left(F_h\right)_{\rm tr,S} + R_{\rm tr,S} - S_{\rm tr,S}\right]\sigma_{{\rm tr},S} + \left[\left(F_h\right)_{\rm st,S} + R_{\rm st,S} - S_{{\rm st},S}\right]\sigma_{{\rm st},S} \\ &\quad = \left(R_{\rm tr,T} - S_{\rm tr,T}\right)\sigma_{{\rm tr},T} + \left(R_{\rm st,T} - S_{\rm st,T}\right)\sigma_{{\rm st},T} \ . \end{aligned} \tag{11}$$

We expect a net flow of energy from the tropics into the subtropics. We write the equilibrium tropical energy budget as

$$\left[\left(F_h\right)_{\rm tr,S} + R_{\rm tr,S} - S_{\rm tr,S}\right]\sigma_{{\rm tr},S} - \left(R_{\rm tr,T} - S_{\rm tr,T}\right)\sigma_{{\rm tr},T} + M\left(h_M - h_{{\rm tr},T}\right) = 0 \ . \tag{12}$$

Here we have assumed that all of the moist static energy flux into the subtropical boundary layer is transported into the tropics, and that in the detrainment layer the mass flux M carries moist static energy out of the tropics at the rate $Mh_{{\rm tr},T}$.

We will use these equations in Sections 4 and 5.

4 The subtropical stratocumulus regime

4.1 Prognostic equations

We use a simple cloud-topped mixed-layer model, following concepts introduced by Lilly (1968), with details similar to those described by Randall and Suarez (1984). The prognostic variables of the model are the mixed-layer moist static energy, h_M; the mixed-layer total (vapor plus liquid) water mixing ratio, q_M; and the pressure-thickness of the boundary layer,

$$\delta p_M \equiv p_S - p_B \ , \tag{13}$$

where p is pressure; and subscripts S, B, and $B+$ denote the surface, a level just below the boundary-layer top, and a level just above the boundary-layer top, respectively.

The pressure-thickness of the boundary layer is governed by

$$\frac{\partial}{\partial t}\left(\frac{\delta p_M}{g}\right) = E - \nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right)\right] \ . \tag{14}$$

Here g is the acceleration of gravity; E is the rate at which mass is entrained across the top of the boundary layer, $\mathbf{v}_M$ is the horizontal wind, and $\nabla\cdot[\mathbf{v}_M\,(\delta p_M)]$ is the rate at which mass diverges horizontally within the subtropical mixed-layer. In equilibrium, entrainment is balanced by horizontal mass flux divergence. Mass conservation implies that

$$\nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right)\right] = \frac{M}{\sigma_{\mathrm{st}}} \ . \tag{15}$$

Substituting (15) into (14), we obtain

$$\frac{\partial}{\partial t}\left(\frac{\delta p_M}{g}\right) = E - \frac{M}{\sigma_{\mathrm{st}}} \ . \tag{16}$$

In equilibrium,

$$E - \frac{M}{\sigma_{\mathrm{st}}} = 0 \ . \tag{17}$$

The governing equation for h_M is

$$\begin{aligned}\frac{\partial}{\partial t}\left(h_M \frac{\delta p_M}{g}\right) = &- \nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right) h_M\right] \\ &+ (F_h)_S + E h_{B+} + (R_{S,\mathrm{st}} - R_{B+,\mathrm{st}}) - (S_{S,\mathrm{st}} - S_{B+,\mathrm{st}}) \ ,\end{aligned} \tag{18}$$

Here $(F_h)_S$ is the surface flux of moist static energy; E is the mass flux due to entrainment across the boundary-layer top; R is the net upward flux of terrestrial radiation; and S is the net downward flux of solar radiation. Using h_M times (14) with (18), we find that

$$\begin{aligned}\frac{\delta p_M}{g}\frac{\partial h_M}{\partial t} = &- \nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right) h_M\right] + h_M \nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right)\right] \\ &+ (F_h)_S + E\,(h_{B+} - h_M) + (R_{S,\mathrm{st}} - R_{B+,\mathrm{st}}) - (S_{S,\mathrm{st}} - S_{B+,\mathrm{st}}) \ ,\end{aligned} \tag{19}$$

We use (15) and the approximation

$$\begin{aligned}\nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right) h_M\right] \cong & h_{\mathrm{dn}} \nabla \cdot \left[\mathbf{v}_M \left(\frac{\delta p_M}{g}\right)\right] \\ = & h_{\mathrm{dn}} \frac{M}{\sigma_{\mathrm{st}}}\end{aligned} \tag{20}$$

in (19) to obtain

$$\begin{aligned}\frac{\delta p_M}{g}\frac{\partial h_M}{\partial t} =& \frac{M}{\sigma_{\text{st}}}\left(-h_{\text{dn}}+h_M\right)\\ &+\left(F_h\right)_S+E\left(h_{B+}-h_M\right)+\left(R_{S,\text{st}}-R_{B+,\text{st}}\right)-\left(S_{S,\text{st}}-S_{B+,\text{st}}\right)\ ,\end{aligned} \tag{21}$$

Here h_{dn} is the value of the moist static energy that M carries from the subtropical domain into the tropical domain. We assume that

$$\begin{aligned}h_{\text{dn}} =& h_M+\left[\frac{\left(h_S\right)_{\text{tr}}-h_M}{L/2}\right]\sigma_{\text{st}}L/2\\ =& \left(h_S\right)_{\text{tr}}\sigma_{\text{st}}+h_M\left(1-\sigma_{\text{st}}\right)\ ,\end{aligned} \tag{22}$$

where

$$\left(h_S\right)_{\text{tr}} = c_p\text{SST}_{\text{tr}}+\text{CRH}Lq^*_{S,\text{tr}}\ . \tag{23}$$

Here $q^*_{S,\text{tr}} \equiv q^*\left(\text{SST},p_S\right)$ is the saturation mixing ratio evaluated using the tropical SST and the surface pressure. Using (22) in (21), we find that

$$\begin{aligned}\frac{\delta p_M}{g}\frac{\partial h_M}{\partial t} =& M\left[-\left(h_S\right)_{\text{tr}}+h_M\right]\\ &+\left(F_h\right)_S+E\left(h_{B+}-h_M\right)+\left(R_{S,\text{st}}-R_{B+,\text{st}}\right)-\left(S_{S,\text{st}}-S_{B+,\text{st}}\right)\ .\end{aligned} \tag{24}$$

Similarly, the governing equation for q_M is

$$\begin{aligned}\frac{\partial}{\partial t}\left[q_M\left(\frac{\delta p_M}{g}\right)\right] =& -\nabla\cdot\left[\mathbf{v}_M\left(\frac{\delta p_M}{g}\right)q_M\right]\\ &+\left(F_{q+l}\right)_S+E\left(q+l\right)_{B+}\ ,\end{aligned} \tag{25}$$

where $\left(F_{q+l}\right)_S$ is the surface moisture flux. The advective form of (33) is obtained by using q_M times (14). The result is

$$\begin{aligned}\left(\frac{\delta p_M}{g}\right)\frac{\partial}{\partial t}q_M =& -\nabla\cdot\left[\mathbf{v}_M\left(\frac{\delta p_M}{g}\right)q_M\right]+q_M\nabla\cdot\left[\mathbf{v}_M\left(\frac{\delta p_M}{g}\right)\right]\\ &+\left(F_{q+l}\right)_S+E\left[\left(q+l\right)_{B+}-q_M\right]\ .\end{aligned} \tag{26}$$

We use (15) and the approximation

$$\begin{aligned}\nabla\cdot\left[\mathbf{v}_M\left(\frac{\delta p_M}{g}\right)q_M\right] \cong& \left(q\right)_{\text{dn}}\nabla\cdot\left[\mathbf{v}_M\left(\frac{\delta p_M}{g}\right)\right]\\ =& \left(q\right)_{\text{dn}}\frac{M}{\sigma_{\text{st}}}\ .\end{aligned} \tag{27}$$

in (26) to obtain

$$\begin{aligned}\left(\frac{\delta p_M}{g}\right)\frac{\partial}{\partial t}q_M =&\frac{M}{\sigma_{\rm st}}\left[-\left(q\right)_{\rm dn}+q_M\right]\\ &+\left(F_{q+l}\right)_S+E\left[\left(q+l\right)_{B+}-q_M\right]\ .\end{aligned} \tag{28}$$

We assume that

$$\begin{aligned}\left(q\right)_{\rm dn} =&q_M+\left[\frac{\left(q_S\right)_{\rm tr}-q_M}{L/2}\right]\sigma_{\rm st}L/2\\ =&\left(q_S\right)_{\rm tr}\sigma_{\rm st}+q_M\left(1-\sigma_{\rm st}\right)\ ,\end{aligned} \tag{29}$$

and

$$\left(q_S\right)_{\rm tr}={\rm CRH}q^*\left({\rm SST}_{\rm tr},p_S\right)\ . \tag{30}$$

Using (16) and (27) in (26), we find that

$$\begin{aligned}\left(\frac{\delta p_M}{g}\right)\frac{\partial}{\partial t}q_M =&M\left[-\left(q_S\right)_{\rm tr}+q_M\right]\\ &+\left(F_{q+l}\right)_S+E\left[\left(q+l\right)_{B+}-q_M\right]\ .\end{aligned} \tag{31}$$

The surface flux of moist static energy is determined using

$$\left(F_h\right)_S=\rho_S|\mathbf{v}_S|c_T\left[h^*\left({\rm SST}_{\rm st},p_S\right)-h_M\right]\ . \tag{32}$$

In (32), ρ_S is the surface air density, $|\mathbf{v}_S|$ is the surface wind speed, c_T is a transfer coefficient, and h^* is the saturation moist static energy as a function of temperature and pressure. Similarly, the surface moisture flux is assumed to satisfy

$$\left(F_{q+l}\right)_S=\rho_S|\mathbf{v}_S|c_T\left[q^*\left({\rm SST}_{\rm st},p_S\right)-q_M\right]\ . \tag{33}$$

4.2 Subtropical water vapor

The total column water vapor in the subtropics, denoted by $W_{\rm st}$, is the sum of the water vapor in the boundary layer, denoted by W_{BL}, and the water vapor in the free troposphere, denoted by W_{FT}:

$$W_{\rm st}=W_{BL}+W_{FT}\ . \tag{34}$$

We obtain W_{BL} and W_{FT} using

$$W_{BL}=q_M\left(p_S-p_B\right)/g \tag{35}$$

and

$$W_{FT}=q_{B+}\left(p_B-p_T\right)/g\ , \tag{36}$$

respectively. Our method to determine q_{B+} is discussed later.

4.3 Finding the cloud-base level

We use an iterative method to determine height, temperature, and relative humidity of the boundary layer top. Using (A5), we can find the height of the boundary-layer top as follows:

$$z_B - z_S = \frac{T_S}{\Gamma}\left[1 - \left(\frac{p_B}{p_S}\right)^{\frac{\Gamma R}{g}}\right] . \tag{37}$$

In order to use (37), the value of Γ is needed. To start the iteration mentioned above, we use the "first guess" that $\Gamma = \Gamma_d$, where

$$\Gamma_d = g/c_p . \tag{38}$$

To test for the presence of cloud in the upper part of the mixed layer, we make a first guess at T_B using

$$T_B = \left[h_M - g z_B - L q_M\right]/c_p . \tag{39}$$

We then compute a first guess for the saturation mixing ratio at level B, based on our first guess for T_B and the known value of p_B. If this test gives $q^*\left(T_B, p_B\right)$ is larger than q_M, then we can be sure that there is no cloud in the boundary layer.

Otherwise a cloud is present, and we use a simple iterative method to find consistent values of T_B and $q^*\left(T_B, p_B\right)$, while holding z_B and Γ fixed. The value of T_B obtained through this iteration is used to update Γ, using

$$\Gamma = \frac{T_S - T_B}{z_B - z_S} . \tag{40}$$

We then return to (37) to obtain a new value of z_B. The iteration continues in this way until convergence is achieved.

Once the iteration has converged, the liquid water mixing ratio at level B can be computed from

$$l_B = \max\left[q_M - q^*\left(T_B, p_B\right), 0\right] , \tag{41}$$

the cloud thickness from

$$z_B - z_C = z_B \min\left\{\frac{l_B}{l_B - \left[q_M - q^*\left(T_S, p_S\right)\right]}, 1\right\} , \tag{42}$$

and the liquid-water path (LWP) from

$$\mathrm{LWP} = \rho_B \frac{1}{2}\left(z_B - z_C\right) l_B . \tag{43}$$

4.4 Entrainment parameterization

The vertically integrated turbulence kinetic energy (TKE) per unit mass has a source due to buoyancy, given by

$$B \equiv \frac{g}{c_p \rho_M} \int_{z_S}^{z_{B+}} \frac{F_{sv}}{T} \, dz \ . \tag{44}$$

Here F_{sv} is the vertical flux of virtual dry static energy. The TKE also has a source due to vertical momentum transport in the presence of shear, given by

$$S \equiv -\frac{1}{\rho_M} \int_{z_S}^{z_{B+}} \mathbf{F_V} \cdot \frac{\partial \mathbf{v}}{\partial z} \, dz \ , \tag{45}$$

where $\mathbf{F_V}$ is the vertical flux of horizontal momentum. We approximate B by

$$B \cong \frac{g}{2 c_p \rho_M} \left\{ \left[\frac{(F_{sv})_S}{T_S} + \frac{(F_{sv})_C}{T_C} \right] (z_C - z_S) + \left[\frac{(F_{sv})_{C+}}{T_C} + \frac{(F_{sv})_B}{T_B} \right] (z_B - z_C) \right\} \ . \tag{46}$$

Here $(F_{sv})_C$ is the buoyancy flux just below cloud base, and $(F_{sv})_{C+}$ is the buoyancy flux just above cloud base. We approximate S by

$$S \cong (\mathbf{F_V})_S \cdot \mathbf{v}_M / \rho_S > 0 \ . \tag{47}$$

Here we neglect the effects of shear across the boundary layer top.

We define the following thermodynamic parameters:

$$\varepsilon \equiv \frac{c_p T}{L} \cong 0.1 \ , \tag{48}$$

$$\gamma \equiv \frac{L}{c_p} \left(\frac{\partial q^*}{\partial T} \right)_p \ , \tag{49}$$

$$\beta \equiv \frac{1 + (1 + \delta) \gamma \varepsilon}{1 + \gamma} \ . \tag{50}$$

These are evaluated using the temperature and pressure of the mean state. The buoyancy flux in cloudy air is

$$F_{sv} = \beta F_h - \epsilon L F_{q+l} \text{ in cloud } \ , \tag{51}$$

and the buoyancy flux in clear air is

$$F_{sv} = (F_{sv})_{clr} \equiv F_h - (1 - \delta \varepsilon) L F_{q+l} \text{ in clear air} \ . \tag{52}$$

Inspection of (51) and (52) shows that, both in and out of cloud, the buoyancy flux depends on F_h and F_{q+l}, which vary linearly with pressure (and approximately linearly with height) through the depth of the mixed layer. The surface values of F_h and F_{q+l} are given by (32) and (33), respectively. At the boundary layer top, the fluxes are given by

$$(F_h)_B = -E \Delta h + \Delta R \ , \tag{53}$$

and

$$\left(F_{q+l}\right)_B = -E\Delta\left(q+l\right) \ . \tag{54}$$

In (53), $\Delta R \equiv R_{B+} - R_B$ is the "jump" in longwave radiation across the top of the boundary-layer cloud. Our radiation parameterizations are discussed in Appendix B. Using (51), (53), and (54), the buoyancy flux just below the boundary layer top can be written as

$$\left(F_{sv}\right)_B = -E\left[\Delta s_v - \left(\Delta s_v\right)_{\text{crit}}\right] + \beta\Delta R \text{ when cloud is present} \ , \tag{55}$$

and as

$$\left(F_{sv}\right)_B = -E\Delta s_v \text{ when the boundary layer is cloud-free} \ . \tag{56}$$

In (55),

$$\Delta s_v - \left(\Delta s_v\right)_{\text{crit}} = \beta\Delta h - \epsilon L\Delta\left(q+l\right) \ . \tag{57}$$

The cloud-base values of F_h and F_{q+l} are obtained by linear interpolation between the surface values and the boundary-layer-top values. Note that F_h and F_{q+l} are continuous across cloud base.

The buoyancy and shear integrals defined by (44) and (45) have the dimensions of velocity cubed. It turns out that the parametric dependence of B on the entrainment rate is linear, so we can write

$$B + S = G_0 + G_1 w_e \ , \tag{58}$$

where $w_e \equiv E/\rho_B$ is the entrainment velocity, and G_0 and G_1 are parametrically independent of w_e. Normally $G_0 > 0$, which means that there is positive production in the absence of entrainment; and $G_1 \le 0$, which means that entrainment tends to decrease the production rate.

Our entrainment parameterization is

$$w_e^3 = \epsilon_0 G_0 + \epsilon_1 G_1 w_e \ , \tag{59}$$

where ϵ_0 and ϵ_1 are positive nondimensional coefficients. For $G_0 > 0$ and $G_1 \le 0$, Eq. (59) has a single real root, always positive, and given by Cardano's formula:

$$w_e = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \ . \tag{60}$$

Here $p \equiv -\epsilon_1 G_1 > 0$ and $q \equiv -\epsilon_0 G_0 < 0$.

Consider two limiting cases: For sufficiently large (negative) values of G_1, we get

$$w_e \cong -\frac{\epsilon_0 G_0}{\epsilon_1 G_1} \ . \tag{61}$$

This is the usual case. On the other hand, for $G_1 = 0$ the entrainment rate is given by

$$w_e = \sqrt[3]{\epsilon_0 G_0} \ . \tag{62}$$

5 The tropical deep convection regime

5.1 Budgets of moisture and moist static energy for the tropical troposphere

As discussed in the introduction, we treat the convectively active tropical troposphere as a single layer, within which all levels are tightly coupled. In particular, we do not model the tropical boundary layer separately from the tropical free troposphere. The only time-stepped variable for the tropical sub-model is the tropical SST. The state of the tropical troposphere is diagnosed.

The tropical moisture balance is given by (10), which can be rearranged as

$$P_{\rm tr}\sigma_{{\rm tr},S} = \left(F_{q+l}\right)_{\rm tr,S}\sigma_{{\rm tr},S} - +M\left(q_M - q_{{\rm tr},T}\right) \ . \tag{63}$$

Here $\left(F_q\right)_{S,\rm tr}$ is a source of tropical moisture due to surface evaporation, and P is a sink due to precipitation. Eq. (63) also includes the effects of the circulating mass flux, M, which carries low-level moisture with mixing ratio $q_{\rm dn}$ in from from the subtropics, and upper-tropospheric moisture with mixing ratio $q_{\rm up}$ out to the subtropics. Since $q_{\rm dn} \gg q_{\rm up}$, the net effect of M is to add moisture to the tropical troposphere. A method to determine M is discussed in section 6.

Bretherton et al. (2004) found that over the tropical oceans the precipitation rate increases exponentially with the CRH. We use an updated version of this relationship presented by Rushley et al. (2018), and based on version 7 of the SSM/I data:

$$P = P_r e^{a_d \rm CRH} \ . \tag{64}$$

Here $P_r = 6.89 \times 10^{-5}$ mm day^{-1} and $a_d = 14.72$. These values imply that $P = 170$ mm day^{-1} for CRH $= 1$.

We assume that the surface evaporation rate over the tropical oceans is given by

$$\left(F_q\right)_{S,\rm tr} = \rho_S |\mathbf{v}_S| c_T q_S^* \left(1 - {\rm CRH}\right) \ . \tag{65}$$

In (65), the humidity of the atmosphere enters through the CRH, rather than the near-surface mixing ratio. This means that the “demand” for surface evaporation is exerted by the whole tropical troposphere, rather than a near-surface boundary layer.

Using (64) and (65) in (63), we find that the tropical moisture balance is expressed by

$$P_r e^{a_d \rm CRH} = \rho_S |\mathbf{v}_S| c_T q_S^* \left(1 - {\rm CRH}\right) + M\left(q_M - q_{{\rm tr},T}\right)/\sigma_{\rm tr} \ . \tag{66}$$

Eq. (66) will be used below.

Similarly, the moist static energy balance of the tropical troposphere is expressed by

$$\left[(F_h)_{\mathrm{tr,S}} + R_{\mathrm{tr,S}} - S_{\mathrm{tr,S}}\right]\sigma_{\mathrm{tr},S} - (R_{\mathrm{tr,T}} - S_{\mathrm{tr,T}})\,\sigma_{\mathrm{tr},T} + M\,(h_M - h_{\mathrm{tr},T}) = 0\ . \tag{67}$$

We assume that

$$(F_h)_{S,\mathrm{tr}} = L\,(F_q)_{S,\mathrm{tr}}\ , \tag{68}$$

which means that we neglect the tropical sensible heat flux.

5.2 A convective closure

To relate the tropical lapse rate to the CRH, we use a closure assumption for deep non-entraining clouds based loosely on Arakawa and Chen (1986):

$$\mathrm{CRH} = \frac{\varGamma_d - \varGamma}{\varGamma_d - \varGamma_m}\ . \tag{69}$$

See also Arakawa (1993) and Arakawa (2004). In Eq. (69), $\varGamma_d$ is the dry adiabatic lapse rate defined earlier;

$$\begin{aligned}\varGamma &\equiv -\,\partial T/\partial z \\ &= (\mathrm{SST} - T_T)\,/z_T\end{aligned} \tag{70}$$

is the actual lapse rate, which we assume to be constant with height in the tropical troposphere; and $\varGamma_m$ is the moist adiabatic lapse rate based on the sea surface temperature and the surface pressure. A similar closure was used by Kelly et al. (1999). The physical meaning of (69) is that deep convection couples the temperature and moisture soundings so that the lapse rate and CRH vary together. When the CRH approaches 1, the lapse rate approaches the moist adiabat. For smaller values of the CRH, the lapse rate is steeper than the moist adiabat. Using (69) to eliminate $\varGamma$ in (70), we find that

$$z_T = \frac{\mathrm{SST} - T_T}{\varGamma_d - \mathrm{CRH}\,(\varGamma_d - \varGamma_m)}\ . \tag{71}$$

Figure 2 shows a test of (69) using the ERA5 reanalysis data. The horizontal axis shows the CRH computed directly from the ERA5 reanalysis moisture and temperature data. The vertical axis shows the CRH estimated from (69), using $\varGamma$ and $\varGamma_m$ from the ERA5 temperature data. The lapse rate is computed using the surface temperature and the brightness temperature based on the outgoing longwave radiation. The data points are color code based on the precipitation rate reported by TRMM. For precipitation rates greater than about 4 mm day^{-1}, and for CRH $< 80\%$ along the horizontal axis, the data fall nearly along the diagonal line. There is also a set of strongly precipitating data points near the top center of the diagram. For these, the actual CRH is about 50%, and the CRH based on (69) is quite high, approaching 100% in some cases. The high CRH based on (69) means that $\varGamma$ is smaller (more stable) than would be expected when deep convection is active. We interpret these points as being associated with stratiform precipitation.

5.3 Nondilute ascent

We assume that the air at the cloud-top level is saturated and has the moist static energy obtained by combining the sea surface temperature and the surface saturation mixing ratio:

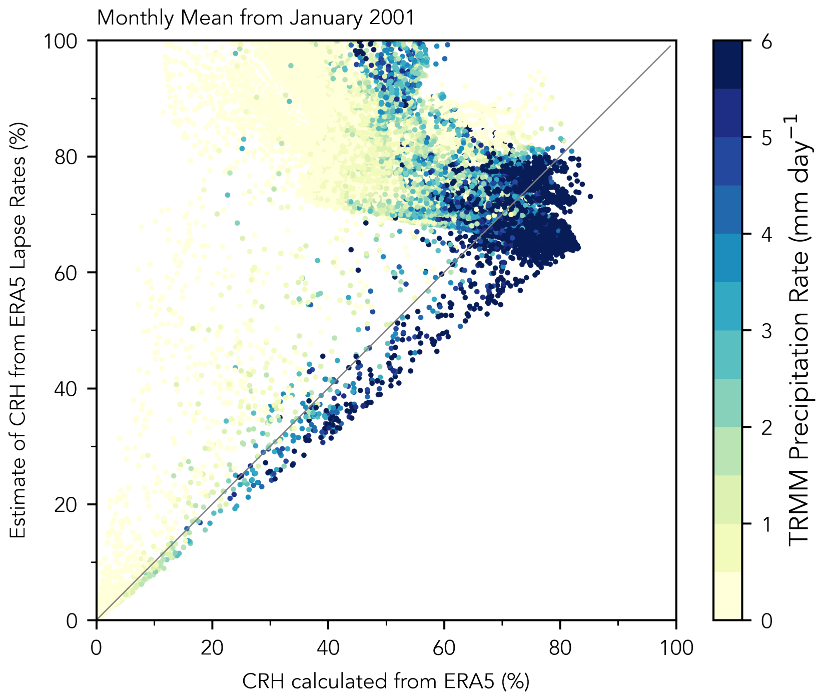

Figure 2. The horizontal axis shows the CRH computed directly from the ERA5 reanalysis moisture and temperature data. The vertical axis shows the CRH estimated from (69), using Γ and Γ_m from the ERA5 temperature data. The data points are color code based on the precipitation rate reported by TRMM. See the text for more details.

$$\begin{aligned} h_T =& c_p T_T + g z_T + L q_T^* \\ =& c_p \mathrm{SST} + L q_S^* \ . \end{aligned} \tag{72}$$

Here $q_T^* \equiv q^*\left(T_T, p_T\right)$. Eq. (72) is essentially a neutral-buoyancy condition for nondilute ascent. Use of (71) in (72) gives

$$T_T = \mathrm{SST} - \left[\frac{\mathit{\Gamma}_d - \mathrm{CRH}\left(\mathit{\Gamma}_d - \mathit{\Gamma}_m\right)}{\mathrm{CRH}\left(\mathit{\Gamma}_d - \mathit{\Gamma}_m\right)}\right]\left(L/c_p\right)\left(q_S^* - q_T^*\right) \ . \tag{73}$$

Note that T_T enters implicitly on the right-hand side of (73), through the q_T^* term. We also need p_T to evaluate q_T^*. It is related to the surface pressure by

$$p_T = p_S \left(\frac{T_T}{\mathrm{SST}}\right)^{\frac{g}{\mathit{\Gamma} R}} \ . \tag{74}$$

Use of (73) in (71) gives

$$z_T = \frac{\left(L/c_p\right)\left(q_S^* - q_T^*\right)}{\mathrm{CRH}\left(\mathit{\Gamma}_d - \mathit{\Gamma}_m\right)} \ . \tag{75}$$

5.4 Method of solution

The equations presented above are solved iteratively. We assume that M is known. The saturation mixing ratio of the detrainment layer appears in both (66) and (73). As a first guess, we set $q_T^* = 0$ in both equations, so that they simplify to

$$\rho_S |\mathbf{v}_S| c_T q_S^* \left(1 - \mathrm{CRH}\right) - P_r e^{a_d \mathrm{CRH}} + M q_M / \sigma_{\mathrm{tr}} = 0 \tag{76}$$

and

$$c_p T_T + g z_T = c_p \mathrm{SST} + L q_S^* \ , \tag{77}$$

respectively. Assuming that $M q_M$ is known, we can solve (76) for the CRH, using a simple iterative method. Once the CRH is known, the precipitation rate can be computed from (64).

Next, Eqs. (71) and (77) can be solved for the two unknowns z_T and T_T. The results are

$$z_T = \frac{\left(L/c_p\right) q_S^*}{\mathrm{CRH}\left(\mathit{\Gamma}_d - \mathit{\Gamma}_m\right)} \ , \tag{78}$$

and

$$T_T = \mathrm{SST} - \left(L/c_p\right) q_S^* \left[\frac{\mathit{\Gamma}_d - \mathrm{CRH}\left(\mathit{\Gamma}_\mathrm{d} - \mathit{\Gamma}_\mathrm{m}\right)}{\mathrm{CRH}\left(\mathit{\Gamma}_\mathrm{d} - \mathit{\Gamma}_\mathrm{m}\right)}\right] \ . \tag{79}$$

At this point, $\mathit{\Gamma}$ and p_T can be obtained from (70) and (74), respectively.

The iteration is continued by using T_T and p_T to obtain an updated estimate of the $L q_T^*$ term of (72) (replacing the first guess of zero). Eqs. (66) and (64) are then used to obtain updated values of the CRH and the precipitation rate. Finally, we solve (71) and (72) and for new values of z_T and T_T. The iteration proceeds in this way until convergence is achieved. A few passes suffice.

6 Coupling the tropics and subtropics

The discussion up to this point has covered the subtropics and the tropics, but not their interactions. The circulating mass flux, M, has not yet been determined. We also need to know the fractional areas covered by the subtropical and tropical portions of the domain.

The heating rates at levels between the subtropical boundary layer top and the tropical detrainment layer are denoted by Q. They have dimensions of energy per unit volume per unit time. Following Jenney et al. (2020), we assume that the area-averaged heating rate is zero at each level, so that

$$Q_{\mathrm{tr}}\sigma_{\mathrm{tr}} + Q_{\mathrm{st}}\sigma_{\mathrm{st}} = 0 \ . \tag{80}$$

We expect $Q_{\mathrm{tr}} > 0$ and $Q_{\mathrm{st}} < 0$. Invoking WTG, we assume that

$$\rho w_{\mathrm{tr}} \frac{\partial s}{\partial z} = Q_{\mathrm{tr}} \tag{81}$$

and

$$\rho w_{\mathrm{st}} \frac{\partial s}{\partial z} = Q_{\mathrm{st}} \tag{82}$$

where

$$\frac{\partial s}{\partial z} = -c_p \mathit{\Gamma} + g \tag{83}$$

takes the same value in the tropics and subtropics, and is independent of height.

Jenney et al. (2020) used (1) and (80) to show that

$$\sigma_{\mathrm{tr}} = \frac{-Q_{\mathrm{st}}}{Q_{\mathrm{tr}} - Q_{\mathrm{st}}} \ , \tag{84}$$

and

$$M = -\frac{1}{\partial s/\partial z}\left(\frac{Q_{\mathrm{st}}Q_{\mathrm{tr}}}{Q_{\mathrm{tr}} - Q_{\mathrm{st}}}\right) \ . \tag{85}$$

Using (84), can rewrite (85) as

$$M = \frac{\sigma_{\mathrm{tr}} Q_{\mathrm{tr}}}{\partial s/\partial z} \ . \tag{86}$$

The vertically integrated heating in the subtropical free troposphere is entirely due to radiation, and is given by

$$\int_{z_{B+}}^{z_T} Q_{\mathrm{st}} dz = R_{B+,\mathrm{st}} - (R)_{T,\mathrm{st}} + S_{T,\mathrm{st}} - S_{B+,\mathrm{st}} \ . \tag{87}$$

Similarly, the vertically integrated heating in the tropical troposphere is due to the combination of condensation and radiation:

$$\int_{z_{B+}}^{z_T} Q_{\mathrm{tr}} dz = LP_{\mathrm{tr}} + R_{S,\mathrm{tr}} - (R)_{T,\mathrm{tr}} + S_{T,\mathrm{tr}} - S_{S,\mathrm{tr}} \ . \tag{88}$$

Here we have assumed that the surface precipitation rate is equal to the vertically integrated condensation rate. Our parameterizations of radiation for the subtropics and tropics are discussed in the Appendix.

We now make the key assumption that

$$\frac{\partial w_{\mathrm{st}}}{\partial z} = 0 \ . \tag{89}$$

Using (89), we can integrate (82) to obtain

$$\int_{z_B}^{z_T} \rho dz \left(w_{\mathrm{st}} \frac{\partial s}{\partial z} \right) = \int_{z_B}^{z_T} Q_{\mathrm{st}} dz \ . \tag{90}$$

Using hydrostatics, this can be rearranged to

$$w_{\mathrm{st}} = \frac{g \int_{z_B}^{z_T} Q_{\mathrm{st}} dz}{\frac{\partial s}{\partial z} \left(p_B - p_T \right)} \ . \tag{91}$$

Since $\displaystyle\int_{z_B}^{z_T} Q_{\mathrm{st}} dz$ is known from (87), we can use (91) to calculate the height-independent value of w_{st}.

The height-independence of both M and w_{st} implies that

$$\frac{\partial}{\partial z} \left(\rho \sigma_{\mathrm{st}} \right) = 0 \ , \tag{92}$$

In the following discussion, we will denote the height-independent value of $\rho\sigma_{\mathrm{st}}$ by $(\rho\sigma_{\mathrm{st}})_T$, i.e., its value at the top of the model. From (92), we see that

$$\sigma_{\mathrm{st}} \left(z \right) = \frac{\left(\rho \sigma_{\mathrm{st}} \right)_T}{\rho \left(z \right)} \ . \tag{93}$$

This shows that $\sigma_{\mathrm{st}}(z)$ decreases strongly downward, but of course it remains positive. It follows from (1) that $\sigma_{\mathrm{tr}}(z)$ is given by

$$\sigma_{\mathrm{tr}} \left(z \right) = 1 - \frac{\left(\rho \sigma_{\mathrm{st}} \right)_T}{\rho \left(z \right)} \ . \tag{94}$$

which decreases strongly upward.

At this point, we can calculate M:

$$
\begin{aligned}
M =& - \rho\sigma_{\mathrm{st}}(z)\, w_{\mathrm{st}} \\
=& \frac{-(\rho\sigma_{\mathrm{st}})_T\, g \int_{z_B}^{z_T} Q_{\mathrm{st}} dz}{\frac{\partial s}{\partial z}(p_B - p_T)} \; .
\end{aligned} \tag{95}
$$

Using w_{st} from (91), we can use (82) to compute Q_{st} at each level. Then (80) can be used to compute Q_{tr} at each level.

Eq. (95) enables us to calculate the tropical vertical velocity:

$$
\begin{aligned}
w_{\mathrm{tr}}(z) =& \frac{M}{\rho(z)\,\sigma_{\mathrm{tr}}(z)} \\
=& -\frac{(\rho\sigma_{\mathrm{st}})_T}{\rho(z) - (\rho\sigma_{\mathrm{st}})_T} \frac{g \int_{z_B}^{z_T} Q_{\mathrm{st}} dz}{\frac{\partial s}{\partial z}(p_B - p_T)} \; .
\end{aligned} \tag{96}
$$

We can use the vertical velocity from (96) to calculate the tropical heating rate level-by-level, using (81):

$$
\begin{aligned}
Q_{\mathrm{tr}} =& \rho w_{\mathrm{tr}} \frac{\partial s}{\partial z} \\
=& -\frac{\rho(z)(\rho\sigma_{\mathrm{st}})_T}{\rho(z) - (\rho\sigma_{\mathrm{st}})_T} \frac{g \int_{z_B}^{z_T} Q_{\mathrm{st}} dz}{\frac{\partial s}{\partial z}(p_B - p_T)} \frac{\partial s}{\partial z} \\
=& -\left[\frac{1}{1 - (\rho\sigma_{\mathrm{st}})_T / \rho(z)}\right] \left[\frac{(\rho\sigma_{\mathrm{st}})_T\, g}{p_B - p_T} \int_{z_B}^{z_T} Q_{\mathrm{st}} dz\right] \; .
\end{aligned} \tag{97}
$$

The next step is to integrate (97) to obtain $\int_{z_{B+}}^{z_T} Q_{\mathrm{tr}} dz$. Note that the quantity in the second square brackets is independent of height, so the height-dependence of Q_{tr} comes entirely from the height-dependence of ρ, in the first square brackets.

From (A6) we have

$$
\rho_u = \rho_l \left[1 + (\mathit{\Gamma}/T_u)(z_u - z_l)\right]^{1 - \frac{g}{\mathit{\Gamma} R}} \; , \tag{98}
$$

where subscripts u and l denote upper and lower levels, respectively. This allows us to write

$$
\frac{\rho_T}{\rho(z)} = \left[1 + (\mathit{\Gamma}/T_T)(z_T - z)\right]^{1 - \frac{g}{\mathit{\Gamma} R}} \; . \tag{99}
$$

We thus need to evaluate this integral:

$$
\int_{z_{B+}}^{z_T} \frac{1}{1 - (\sigma_{\mathrm{st}})_T \left[1 + (\mathit{\Gamma}/T_T)(z_T - z)\right]^{1 - \frac{g}{\mathit{\Gamma} R}}} dz \; . \tag{100}
$$

7 Solution of the coupled system

The equations of our model are solved through nested iterations. The "outer" iteration determines M and the fractional areas covered by the subtropics and the tropics. The first guess is $M = 0$ kg m^{-2} s^{-1}, and $\sigma_{\mathrm{st}} = \sigma_{\mathrm{tr}} = 0.5$.

There are two "inner" iterations. The first one uses the current value of M to solve the tropical system, as summarized in section 5. The first guess is $M = 0$. The tropical SST is time-stepped to (near) equilibrium. The second inner iteration uses the current values of M and σ_{st} to time-step the subtropical model to (near) equilibrium.

We then return to the outer iteration is updated by solving Eqs. XXX and XXX for updated values of M and σ_{tr}.

8 Results

Randall and Suarez (1984); Pierrehumbert (1995); Schneider et al. (2019)

9 Conclusions

Appendix A Derivation of Eq. (74)

In this appendix we show the derivations of (37) and (74), both of which relate the pressure change across a layer to the corresponding height change, with the assumption of a constant lapse rate. The starting point is the hydrostatic equation in the form

$$\frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT} \; . \tag{A1}$$

Consider a upper level denoted by subscript u, and a lower level denoted by subscript l. We assume that

$$T_u = T_l - \Gamma \left(z_u - z_l \right) \tag{A2}$$

for $z_u \geq z_l$. Using (A2) to eliminate T in (A1), and integrating with respect to z from z_l to z_u, we find that

$$\ln\left(\frac{p_u}{p_l}\right) = \left(\frac{g}{\Gamma R}\right) \ln\left[1 - \Gamma\left(\frac{z_u - z_l}{T_l}\right)\right] \; , \tag{A3}$$

from which it follows that

$$\begin{aligned} p_u =& p_l \left[1 - \Gamma\left(\frac{z_u - z_l}{T_l}\right)\right]^{\frac{g}{\Gamma R}} \\ =& p_l \left(\frac{T_u}{T_l}\right)^{\frac{g}{\Gamma R}} \; . \end{aligned} \tag{A4}$$

Eq. (A4) can be rearranged to

$$z_u - z_l = \frac{T_l}{\Gamma}\left[1 - \left(\frac{p_u}{p_l}\right)^{\frac{\Gamma R}{g}}\right] \; . \tag{A5}$$

Finally, the density satisfies

$$
\begin{aligned}
\rho_u =& \frac{p_u}{RT_u} \\
=& \frac{p_l \left[1 - \frac{\Gamma(z_u - z_l)}{T_l}\right]^{\frac{g}{\Gamma R}}}{R\left[T_l - \Gamma\left(z_u - z_l\right)\right]} \\
=& \frac{p_l}{RT_l}\left[1 - \frac{\Gamma\left(z_u - z_l\right)}{T_l}\right]^{\frac{g}{\Gamma R}-1} \\
=& \rho_l\left[1 - \frac{\Gamma\left(z_u - z_l\right)}{T_l}\right]^{\frac{g}{\Gamma R}-1} \\
=& \rho_l\left[1 - \frac{\Gamma\left(z_u - z_l\right)}{T_u + \Gamma\left(z_u - z_l\right)}\right]^{\frac{g}{\Gamma R}-1} \\
=& \rho_l\left[\frac{T_u}{T_u + \Gamma\left(z_u - z_l\right)}\right]^{\frac{g}{\Gamma R}-1} \\
=& \rho_l\left[\frac{1}{1 + \left(\Gamma/T_u\right)\left(z_u - z_l\right)}\right]^{\frac{g}{\Gamma R}-1} \\
=& \rho_l\left[1 + \left(\Gamma/T_u\right)\left(z_u - z_l\right)\right]^{1-\frac{g}{\Gamma R}} \ .
\end{aligned}
\tag{A6}
$$

The exponent on the bottom line of (A6) is negative.

For the special case T =constant, (A1) integrates to

$$
p_u = p_l \exp\left[-\frac{g}{R}\left(\frac{z_u - z_l}{T_l}\right)\right] \ . \tag{A7}
$$

This simple result is well known. Comparing (A7) with (A4), we surmise that

$$
\lim_{\Gamma \to 0}\left\{\left(1 - \Gamma x\right)\right]^{A/\Gamma}\right\} = \exp\left(-Ax\right) \ . \tag{A8}
$$

Eq. (A8) can be proven by expanding both sides in a Taylor series about $x = 0$. For $\Gamma = 0$, both p and ρ are exponential functions of z.

Appendix B Radiation parameterizations

B1 Basic approach

We use the radiation parameterization of the Community Atmosphere Model version 6, which is called RRTMG (Clough et al., 2005), to compute three simple ratios of shortwave and longwave fluxes, as functions of water vapor path and CO_2 concentration from model output for the mid-20th century, with a CO_2 concentration of 367 ppmv, and also for a hypothetical future climate with a CO_2 concentration of 1830 ppmv.

The three ratios are as follows:

- The clear-sky transmissivity of a tropospheric layer (with no ozone) to solar radiation, defined by

$$
\begin{aligned}
J\left(W_{A,B},\ CO_2\right) \equiv& \left(S\downarrow\right)_{A,\mathrm{clr}} / \left(S\downarrow\right)_{B,\mathrm{clr}} \\
=& J_0 - \mu\left[1 - e^{-\nu W_{A,B}}\right] \ ,
\end{aligned}
\tag{B1}
$$

  Here the subscripts A and B denote an upper level and a lower level, respectively, and $W_{A,B}$ is the water vapor mass (per unit horizontal area) between those two levels. The data from RRTMG are shown in Figure B1. The second line of (B1) shows the function that we use to fit the RRTMG output. For a CO_2 concentration of 367 ppmv, we use $J_0 = 0.95$, $\mu = 0.2$ and $\nu = 0.03$. The values of J_0, μ and ν depend on the CO_2 concentration, as shown in Table B1.

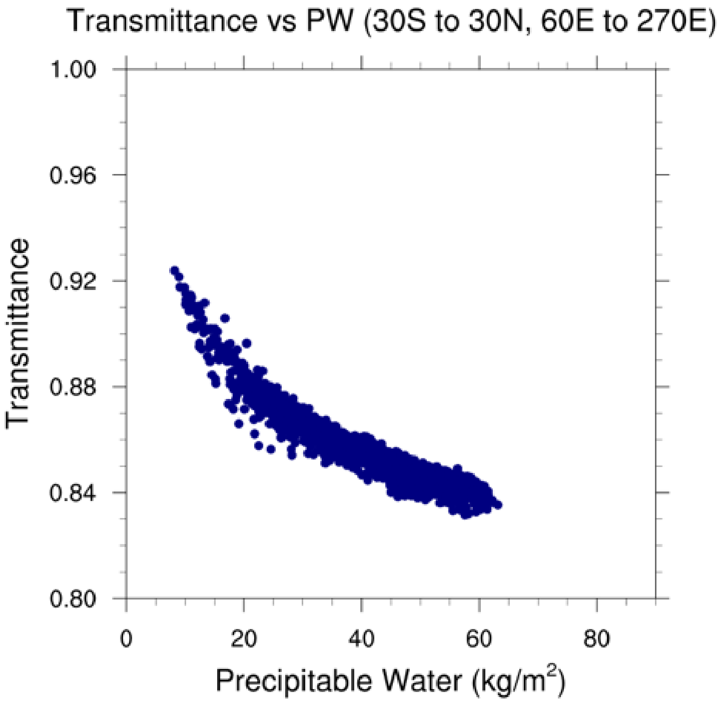

Figure B1. Clear-sky transmissivity of tropospheric air as a function of its water vapor content. These data were used to formulate the curve fit shown in Eq. (B1).

- We define the "bulk emissivity", ϵ_{bulk}, as the clear-sky outgoing longwave radiation (OLR) divided by the upwelling surface longwave radiation, as a function of water vapor path and CO_2 concentration:

$$\begin{aligned}\epsilon_{\text{bulk}}\left(W_{A,B},\ \mathrm{CO_2}\right) \equiv & \left(R\uparrow\right)_A / \left(R\uparrow\right)_B \\ = & c - dW_{A,B}\ .\end{aligned} \tag{B2}$$

The RRTMG data used to fit $\epsilon_{\text{bulk}}\left(W_{A,B},\ \mathrm{CO_2}\right)$ are shown in Figure B1. The values of c and d depend on the CO_2 concentration, as shown in Table B1.

- The clear-sky downwelling longwave radiation at a black-body surface such as the Earth's surface or the top of an optically thick cloud divided by the upwelling longwave radiation at the same level is called f_3:

$$\begin{aligned}f_3\left(W_{A,B}\right) \equiv & \left(R\downarrow\right)_{A,\text{clr,st}} / \left(R\uparrow\right)_{A,\text{st}} \\ = & a\left(1 - e^{-bW_{A,B}}\right)\ ,\end{aligned} \tag{B3}$$

where in this case $W_{A,B}$ is the total column water vapor above level A. The values of f_3 computed by RRTMG are shown in Figure B3
The values of a and b depend on the CO_2 concentration, as shown in Table B1.

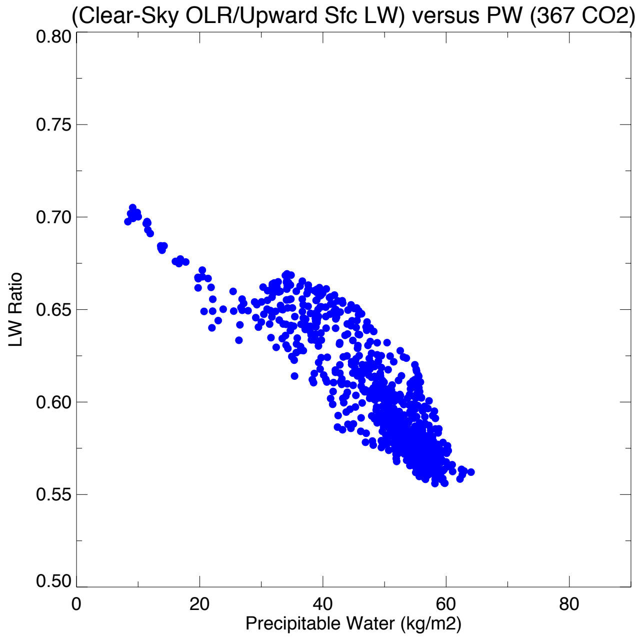

Figure B2. The function $\epsilon_{\mathrm{bulk}}(W)$ was found by fitting the data shown in this figure.

To help with the various curve fits, we ran RRTMG with each of the two CO_2 concentrations and very tiny water vapor concentrations to obtain the limiting values of J, ϵ_{bulk}, and f_3 as $W_{A,B} \to 0$.

B2 Subtropical radiation

B21 Subtropical longwave radiation

The upward surface longwave radiation in the subtropics is given by

$$(R\uparrow)_{S,\mathrm{st}} = \sigma_{SB} SST_{\mathrm{st}}^4 \; . \tag{B4}$$

The clear-sky downward flux at the surface is

$$(R\downarrow)_{S,\mathrm{clr,st}} = f_3\left(W_{A,B}\right)(R\downarrow)_{S,\mathrm{clr,st}} \,/\, (R\uparrow)_{S,\mathrm{st}} \;\; , \tag{B5}$$

The clear-sky OLR is assumed to be given by

$$(R)_{\infty,\mathrm{clr,st}} = (R\uparrow)_{S,\mathrm{st}}\, \epsilon_{\mathrm{bulk}}\left(W_{\mathrm{st}}\right) , \tag{B6}$$

Similarly, by analogy with (B6), we assume that the clear-sky upward longwave fluxes at levels B and B+ satisfy

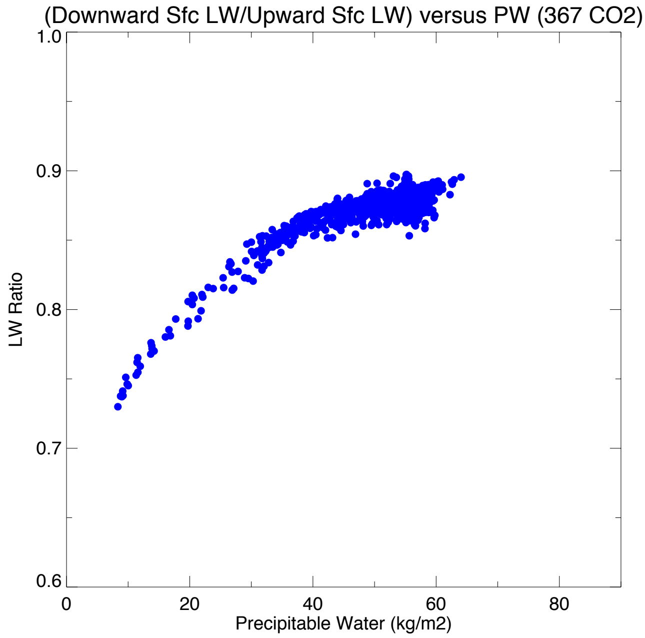

Figure B3. The function $f_3(W)$ was found by fitting the data shown in this figure.

CO_2 concentration	367 ppmv	1830 ppmv
a	0.90	?
b	0.11	?
c	0.73	?
d	0.003	?
J_0	0.95	?
μ	0.2	?
ν	0.03	?

Table B1. The numerical values of the radiation parameters as functions of the CO_2 concentration.

$$(R\uparrow)_{B,\mathrm{clr,st}} = (R\uparrow)_{B+,\mathrm{clr,st}} = (R\uparrow)_{S,\mathrm{st}}\,\epsilon_{\mathrm{bulk}}\,(W_{BL}) \ , \tag{B7}$$

When the upper portion of the boundary layer contains cloud, we use an exponential function of the liquid-water path to interpolate between the clear-sky and thick-cloud fluxes at level B+:

$$(R\uparrow)_{B+,\mathrm{st}} = \sigma_{SB} T_B^4 \left(1 - e^{-k_1 \cdot \mathrm{LWP}}\right) + (R\uparrow)_{B+,\mathrm{clr,st}}\, e^{-k_1 \cdot \mathrm{LWP}} \ . \tag{B8}$$

We assume that the downward longwave flux at level B+ is given by

$$(R\downarrow)_{B+,\mathrm{st}} = (R\downarrow)_{S,\mathrm{clr}} \frac{W_{FT}}{W_{\mathrm{st}}} \ . \tag{B9}$$

We also use

$$(R\downarrow)_{B,\mathrm{st}} = (R\downarrow)_{B+,\mathrm{st}}\, e^{-k_1\cdot \mathrm{LWP}} \ . \tag{B10}$$

The net longwave flux at level B+ is given by

$$R_{B+,\mathrm{st}} = (R\uparrow)_{B+,\mathrm{st}} - (R\downarrow)_{B+,\mathrm{st}} \ . \tag{B11}$$

We assume that the upward and downward longwave fluxes at level B both go to zero when the cloud is thick:

$$(R\uparrow)_{B,\mathrm{st}} = (R\uparrow)_{B,\mathrm{clr,st}}\, e^{-k_1\cdot \mathrm{LWP}} \ , \tag{B12}$$

$$(R\downarrow)_{B,\mathrm{st}} = (R\downarrow)_{B,\mathrm{clr,st}}\, e^{-k_1\cdot \mathrm{LWP}} \ . \tag{B13}$$

The net longwave flux at level B is thus given by

$$R_{B,\mathrm{st}} = \left[(R\uparrow)_{B,\mathrm{clr,st}} - (R\downarrow)_{B,\mathrm{clr,st}}\right] e^{-k_1\cdot \mathrm{LWP}} \ . \tag{B14}$$

The longwave flux "jump" across the cloud top is given by

$$\mathit{\Delta} R = R_{B+,\mathrm{st}} - R_{B,\mathrm{st}} \ . \tag{B15}$$

When cloud is present in the boundary layer, we compute the downward surface longwave radiation using

$$(R\downarrow)_{S,\mathrm{st}} = \sigma_{SB} T_C^4 \left(1 - e^{-k_1\cdot \mathrm{LWP}}\right) + (R\downarrow)_{B+,\mathrm{st}}\, e^{-k_1\cdot \mathrm{LWP}} \ . \tag{B16}$$

The net surface longwave flux is obtained by combining (B4) with (B16):

$$(R)_{S,\mathrm{st}} = (R\uparrow)_{S,\mathrm{st}} - (R\downarrow)_{S,\mathrm{st}} \ . \tag{B17}$$

As shown in Eq.(18), the boundary-layer is radiatively heated at the rate

$$(R)_{B,\mathrm{st}} = (R\uparrow)_{B,\mathrm{st}} - (R\downarrow)_{B,\mathrm{st}} \ . \tag{B18}$$

The subtropical free troposphere is radiatively heated at the (negative) rate

$$Q_{st,FT} = g\left[\frac{(R)_{B+,\mathrm{st}} - (R)_{\infty,\mathrm{st}}}{p_B - p_T}\right] \ . \tag{B19}$$

B22 Subtropical clear-sky solar radiatio

Following Kelly et al. (1999), the subtropical downward clear-sky shortwave radiation at the tropopause is

$$(S\downarrow)_{T,\mathrm{clr,st}} = (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\ , \tag{B20}$$

where $(S\downarrow)_{\infty,\mathrm{st}}$ is the incoming solar radiation at the top of the subtropical atmosphere, and A is the fractional absorption by stratospheric ozone. The subtropical downward clear-sky shortwave radiation at levels B+ and B is given by

$$(S\downarrow)_{B+,\mathrm{clr,st}} = (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\ , \tag{B21}$$

where $J_{\mathrm{st}}\,(W_{FT})$ is the clear-sky transmittance of the subtropical free troposphere as a function of its water vapor content. The clear-sky downward shortwave at the surface is

$$\begin{aligned}(S\downarrow)_{S,\mathrm{clr,st}} &= (S\downarrow)_{B+,\mathrm{clr,st}}\,J_{\mathrm{st}}\,(W_{BL}) \\ &= (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\,J_{\mathrm{st}}\,(W_{BL})\ ,\end{aligned} \tag{B22}$$

where $J_{\mathrm{st}}\,(W_{BL})$ is the clear-sky transmittance of the boundary layer as a function of its water vapor content.

The clear-sky upward solar radiation at the surface is

$$\begin{aligned}(S\uparrow)_{S,\mathrm{clr,st}} &= (S\downarrow)_{S,\mathrm{clr,st}}\,\alpha_S \\ &= (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\,J_{\mathrm{st}}\,(W_{BL})\,\alpha_S\ ,\end{aligned} \tag{B23}$$

where α_S is the surface albedo, and the clear-sky net shortwave at the surface is

$$S_{S,\mathrm{clr,st}} = (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\,J_{\mathrm{st}}\,(W_{BL})\,(1-\alpha_S)\ , \tag{B24}$$

The clear-sky upward shortwave radiation at levels B+ and B is

$$\begin{aligned}(S\uparrow)_{B+,\mathrm{clr,st}} &= (S\downarrow)_{B+,\mathrm{clr,st}}\,J^2_{\mathrm{st}}\,(W_{BL})\,\alpha_S \\ &= (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\,J^2_{\mathrm{st}}\,(W_{BL})\,\alpha_S\ .\end{aligned} \tag{B25}$$

The clear-sky net shortwave flux at levels B+ and B is then

$$\begin{aligned}S_{B+,\mathrm{clr,st}} &= (S\downarrow)_{B+,\mathrm{clr,st}} - (S\uparrow)_{B+,\mathrm{clr,st}} \\ &= (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT}) - (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\,J^2_{\mathrm{st}}\,(W_{BL})\,\alpha_S \\ &= (S\downarrow)_{\infty,\mathrm{st}}\,(1-A)\,J_{\mathrm{st}}\,(W_{FT})\left[1-J^2_{\mathrm{st}}\,(W_{BL})\,\alpha_S\right]\ .\end{aligned} \tag{B26}$$

The clear-sky upward shortwave flux at the tropopause (and the TOA) is

$$
\begin{aligned}
(S\uparrow)_{T,\mathrm{clr,st}} = (S\uparrow)_{\infty,\mathrm{clr,st}} &= (S\uparrow)_{B+,\mathrm{clr,st}}\, J_{\mathrm{st}}\left(W_{FT}\right) \\
&= (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right) J_{\mathrm{st}}^{2}\left(W_{FT}\right) J_{\mathrm{st}}^{2}\left(W_{BL}\right)\alpha_S \ .
\end{aligned} \tag{B27}
$$

Here we have assumed that the reflected solar radiation upwelling through the stratosphere does not experience any further absorption by ozone. The clear-sky net shortwave flux at the tropopause is

$$
\begin{aligned}
S_{T,\mathrm{clr,st}} &= (S\downarrow)_{T,\mathrm{clr,st}} - (S\uparrow)_{T,\mathrm{clr,st}} \\
&= (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right)\left[1 - J_{\mathrm{st}}^{2}\left(W_{FT}\right) J_{\mathrm{st}}^{2}\left(W_{BL}\right)\alpha_S\right] \ .
\end{aligned} \tag{B28}
$$

The clear-sky solar radiation absorbed by the free troposphere (above the boundary layer and below the stratosphere) is

$$
\begin{aligned}
S_{T,\mathrm{clr,st}} - S_{B+,\mathrm{clr,st}} &= (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right)\left[1 - J_{\mathrm{st}}^{2}\left(W_{FT}\right) J_{\mathrm{st}}^{2}\left(W_{BL}\right)\alpha_S\right] \\
&\quad - (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right) J_{\mathrm{st}}\left(W_{FT}\right)\left[1 - J_{\mathrm{st}}^{2}\left(W_{BL}\right)\alpha_S\right] \\
&= (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right)\left[1 - J_{\mathrm{st}}\left(W_{FT}\right)\right] \\
&\quad + (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right)\alpha_S J_{\mathrm{st}}^{2}\left(W_{BL}\right)\left[1 - J_{\mathrm{st}}\left(W_{FT}\right)\right] \ .
\end{aligned} \tag{B29}
$$

The first term shows the clear-sky absorption if $\alpha_S = 0$. The second term shows the additional clear-sky absorption if $\alpha_S > 0$. Finally, the clear-sky net shortwave flux at the TOA is

$$
\begin{aligned}
S_{\infty,\mathrm{clr,st}} &= (S\downarrow)_{\infty,\mathrm{st}} - (S\uparrow)_{\infty,\mathrm{clr,st}} \\
&= (S\downarrow)_{\infty,\mathrm{st}}\left[1 - \left(1-A\right) J_{\mathrm{st}}^{2}\left(W_{FT}\right) J_{\mathrm{st}}^{2}\left(W_{BL}\right)\alpha_S\right] \ .
\end{aligned} \tag{B30}
$$

B23 Cloud effects on subtropical solar radiation

We now consider the effects of boundary-layer clouds on the subtropical shortwave radiation. The downward solar radiation at levels B+ and B is not affected by the cloud, so that

$$
\begin{aligned}
(S\downarrow)_{B+,\mathrm{st}} &= (S\downarrow)_{B+,\mathrm{clr,st}} \\
&= (S\downarrow)_{\infty,\mathrm{st}}\left(1-A\right) J_{\mathrm{st}}\left(W_{FT}\right) \ .
\end{aligned} \tag{B31}
$$

Following Eq. (49) of Kelly et al. (1999), the downward shortwave radiation at the surface is assumed to be given by

$$
(S\downarrow)_{S,\mathrm{st}} = (S\downarrow)_{B+,\mathrm{st}}\, J_{\mathrm{st}}\left(W_{BL}\right)\left[1 + \alpha_C\alpha_S J_{\mathrm{st}}^{2}\left(W_{BL}\right)\right]\left(1-\alpha_C\right) \ , \tag{B32}
$$

where α_C is the cloud albedo, which is parameterized as a function of the liquid water path. The upward shortwave radiation at the surface is

$$
\begin{aligned}
(S\uparrow)_{S,\mathrm{st}} &= \alpha_S\left(S\downarrow\right)_{S,\mathrm{st}} \\
&= \alpha_S\left(S\downarrow\right)_{B+,\mathrm{st}}\, J_{\mathrm{st}}\left(W_{BL}\right)\left[1 + \alpha_C\alpha_S J_{\mathrm{st}}^{2}\left(W_{BL}\right)\right]\left(1-\alpha_C\right) \ ,
\end{aligned} \tag{B33}
$$

and the net shortwave radiation at the surface is

$$\begin{aligned} S_{S,\mathrm{st}} &= (S\downarrow)_{S,\mathrm{st}} - (S\uparrow)_{S,\mathrm{st}} \\ &= (S\downarrow)_{B+,\mathrm{st}}\, J_{\mathrm{st}}(W_{BL}) \left[1+\alpha_C\alpha_S J^2_{\mathrm{st}}(W_{BL})\right](1-\alpha_C)(1-\alpha_S)\ . \end{aligned} \tag{B34}$$

Following Eq. (48) of Kelly et al. (1999), the upward SW radiation at level B+ is

$$\begin{aligned} (S\uparrow)_{B+,\mathrm{st}} &= (S\downarrow)_{B+,\mathrm{st}} \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right] \\ &= (S\downarrow)_{\infty,\mathrm{st}}(1-A)\, J_{\mathrm{st}}(W_{FT}) \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\ . \end{aligned} \tag{B35}$$

The net solar radiation at level B+ is

$$\begin{aligned} S_{B+,\mathrm{st}} &= (S\downarrow)_{B+,\mathrm{st}} - (S\uparrow)_{B+,\mathrm{st}} \\ &= (S\downarrow)_{\infty,\mathrm{st}}(1-A)\, J_{\mathrm{st}}(W_{FT}) \left\{1-\left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\right\}\ . \end{aligned} \tag{B36}$$

The solar radiation absorbed by the boundary layer is $S_{B+,\mathrm{st}} - S_{S,\mathrm{st}}$.

The upward solar radiation at the tropopause and the top of the atmosphere is given by

$$\begin{aligned} (S\uparrow)_{T,\mathrm{st}} = (S\uparrow)_{\infty,\mathrm{st}} &= (S\uparrow)_{B+,\mathrm{st}}\, J_{\mathrm{st}}(W_{FT}) \\ &= (S\downarrow)_{\infty,\mathrm{st}}(1-A)\, J^2_{\mathrm{st}}(W_{FT}) \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\ . \end{aligned} \tag{B37}$$

The net SW radiation at the tropopause is thus

$$\begin{aligned} S_{T,\mathrm{st}} &= (S\downarrow)_{T,\mathrm{st}} - (S\uparrow)_{T,\mathrm{st}} \\ &= (S\downarrow)_{\infty,\mathrm{st}}(1-A) \left\{1 - J^2_{\mathrm{st}}(W_{FT}) \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\right\}\ . \end{aligned} \tag{B38}$$

The solar radiation absorbed by the subtropical free troposphere is

$$\begin{aligned} S_{T,\mathrm{st}} - S_{B+,\mathrm{st}} &= (S\downarrow)_{\infty,\mathrm{st}}(1-A) \left\{1 - J^2_{\mathrm{st}}(W_{FT}) \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\right\} \\ &\quad - (S\downarrow)_{\infty,\mathrm{st}}(1-A)\, J_{\mathrm{st}}(W_{FT}) \left\{1-\left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\right\} \\ &= (S\downarrow)_{\infty,\mathrm{st}}(1-A)\left[1-J_{\mathrm{st}}(W_{FT})\right] \\ &\quad + (S\downarrow)_{\infty,\mathrm{st}}(1-A) \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right] J_{\mathrm{st}}(W_{FT})\left[1-J_{\mathrm{st}}(W_{FT})\right] \\ &= (S\downarrow)_{\infty,\mathrm{st}}(1-A)\left[1-J_{\mathrm{st}}(W_{FT})\right] \left\{1 - J_{\mathrm{st}}(W_{FT}) \left[\alpha_C + (1-\alpha_C)^2\alpha_S J^2_{\mathrm{st}}(W_{BL})\right]\right\}\ . \end{aligned} \tag{B39}$$

B3 Tropical radiation

B31 Clear-sky longwave radiation in the tropics

The upward surface longwave radiation in the tropics is given by

$$(R\uparrow)_{S,\mathrm{tr}} = \sigma_{SB} SST_{\mathrm{tr}}^4 \ . \tag{B40}$$

The downward surface clear-sky longwave radiation is

$$(R\downarrow)_{S,\mathrm{clr,tr}} = \sigma_{SB} SST_{\mathrm{tr}}^4 f_3\left(W\right) \ . \tag{B41}$$

where $f_3\left(W\right) < 1$ is discussed in the Appendix. The clear-sky OLR is assumed to be given by

$$(R)_{\infty,\mathrm{clr,tr}} = \sigma_{SB} SST_{\mathrm{tr}}^4 \epsilon_{\mathrm{bulk}}\left(W\right) \ . \tag{B42}$$

Combining (B41) and (B42), the clear-sky longwave flux difference across the tropical atmosphere is

$$(R)_{S,\mathrm{clr,tr}} - (R)_{\infty,\mathrm{clr,tr}} = \sigma_{SB} SST_{\mathrm{tr}}^4 \left[1 - f_3\left(W\right) - \frac{1}{\epsilon_{\mathrm{bulk}}\left(W\right)}\right] \ . \tag{B43}$$

The all-sky longwave flux difference across the tropical atmosphere is given by

$$(R)_{S,\mathrm{tr}} - (R)_{\infty,\mathrm{tr}} = (R)_{S,\mathrm{clr,tr}} - (R)_{\infty,\mathrm{clr,tr}} + \mathrm{ACRE_{LW}}\left(\mathrm{CRH}\right) \ . \tag{B44}$$

Here $\mathrm{ACRE_{LW}}\left(\mathrm{CRH}\right)$ is the (positive) longwave atmospheric cloud radiative effect. As discussed in the Appendix, we parameterize $\mathrm{ACRE_{LW}}\left(\mathrm{CRH}\right)$ as a function of the CRH, following Needham and Randall (2021). Eq. (B44) can be rearranged as

$$\mathrm{ACRE_{LW}}\left(\mathrm{CRH}\right) = \mathrm{LWCRE_S} + \mathrm{LWCRE_\infty} \ , \tag{B45}$$

where the surface longwave cloud radiative effect is given by

$$\mathrm{LWCRE}_S \equiv (R)_{S,\mathrm{tr}} - (R)_{S,\mathrm{clr,tr}} < 0 \ , \tag{B46}$$

and the top-of-atmosphere longwave cloud radiative effect is given by

$$\mathrm{LWCRE}_\infty \equiv (R)_{\infty,\mathrm{clr,tr}} - (R)_{\infty,\mathrm{tr}} > 0 \ . \tag{B47}$$

Note with the definition given by (B46), the longwave cloud radiative effect tends to warm the surface, even though $\mathrm{LWCRE}_S < 0$.

We assume that the all-sky OLR satisfies

$$(R)_{\infty,\mathrm{tr}} = (R)_{\infty,\mathrm{clr,tr}}\, e^{-k_2\cdot\mathrm{IWP}} + \sigma_{SB} T_T^4 \left(1 - e^{-k_2\cdot\mathrm{IWP}}\right) \ . \tag{B48}$$

This implies that

$$\mathrm{LWCRE}_\infty = \left[(R)_{\infty,\mathrm{clr,tr}} - \sigma_{SB} T_T^4\right]\left(1 - e^{-k_2\cdot\mathrm{IWP}}\right) \ . \tag{B49}$$

Substituting (B48) into (B44), and solving for the all-sky surface net longwave radiation, we obtain

$$(R)_{S,\mathrm{tr}} = (R)_{S,\mathrm{clr,tr}} + \left[\sigma_{SB} T_T^4 - (R)_{\infty,\mathrm{clr,tr}}\right]\left(1 - e^{-k_2 \cdot \mathrm{IWP}}\right) + \mathrm{ACRE_{LW}}\,(\mathrm{CRH}) \ . \tag{B50}$$

It follows that

$$\mathrm{LWCRE}_S = \left[\sigma_{SB} T_T^4 - (R)_{\infty,\mathrm{clr,tr}}\right]\left(1 - e^{-k_2 \cdot \mathrm{IWP}}\right) + \mathrm{ACRE_{LW}}\,(\mathrm{CRH}) \ . \tag{B51}$$

B32 Clear-sky solar radiation in the tropics

The tropical shortwave radiation in the clear sky is parameterized following Kelley et al. (1999). The downward clear sky solar radiation at the surface is given by

$$(S\downarrow)_{S,\mathrm{clrtr}} = (1 - A)\, J_{\mathrm{tr}}(W_{\mathrm{tr}})\, (S\downarrow)_{\infty,\mathrm{tr}} \ . \tag{B52}$$

The upward (reflected) portion of Eq. (98) is then:

$$(S\uparrow)_{S,\mathrm{clr,tr}} = (1 - A)\, J_{\mathrm{tr}}(W_{\mathrm{tr}}) \alpha_S\, (S\downarrow)_{\infty,\mathrm{tr}} \tag{B53}$$

The net radiation at the surface is then:

$$(S)_{S,\mathrm{tr}} = (S\downarrow)_{S,\mathrm{clr,tr}} - (S\uparrow)_{S,\mathrm{clr,tr}} + \mathrm{SWCRE_S} \tag{B54}$$

The clear sky shortwave radiation at the top of the atmosphere is given by:

$$(S\uparrow)_{\infty,\mathrm{clr,tr}} = \alpha_S\, (1 - A)\, J^2_{\mathrm{tr}}(W_{\mathrm{tr}})\, (S\downarrow)_{\infty,\mathrm{tr}} \tag{B55}$$

The net shortwave clear-sky radiation at the top of the atmosphere is then:

$$(S)_{\infty,\mathrm{clr,tr}} = (S\downarrow)_{\infty,\mathrm{tr}} - (S\uparrow)_{\infty,\mathrm{clr,tr}} \tag{B56}$$

Moving on, the all-sky shortwave radiation at the TOA is:

$$(S)_{\infty,\mathrm{clr,tr}} = (S)_{\infty,\mathrm{clr,tr}} + \mathrm{SWCRE}_{\infty} \tag{B57}$$

B4 Simple parameterization of tropical cloud radiative effects

We assume that each CRE is of the form

$$y = \alpha \left(e^{\beta x^2} - 1\right) \ , \tag{B58}$$

where y is a shorthand for the CRE, x is a shorthand for the CRH, α is a parameter to be determined, and $\beta \equiv \ln 2$. The form of (B58) ensures that $y = 0$ when $x = 0$, $y = \alpha$ when $x = 1$, and $\partial y / \partial x = 0$ for $x = 0$.

We parameterize the surface longwave CRE, the surface shortwave CRE, the TOA longwave CRE, and the TOA shortwave CRE by using (B58) with appropriate choices for α. The longwave and shortwave values of ACRE are obtained by adding or subtracting the surface and TOA CREs.

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