Math188 - HW #1

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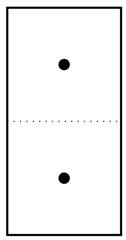
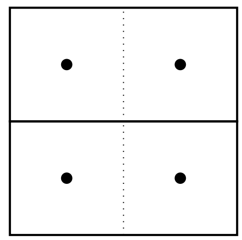
2026.01.12

1.

2.



Configuration A



Configuration B

I prove an equivalent identity for when $n \geq 2$:

$$f_n = \sum_{i=0}^{n-2} f_i + 1$$

Using the language of domino tiles, the definition $f_n = f_{n-1} + f_{n-2}$ can be understood as the sum of two cases from which the nth column can be filled: either through the use of a single vertical tile (configuration A) – counted by f_{n-1} – or the use of a two horizontal tiles stacked on top of each other (configuration B) – counted by f_{n-2}.

Similarly, f_{n-1} can be decomposed into the sum of two cases: f_{n-2}, which counts the number of ways to fill the $(n-1)$th column using configuration A, and f_{n-3}, which counts the number of ways to fill the $(n-1)$th column using configuration B.

Continuously decomposing the number of ways to fill the kth column using configuration A, where $k \geq 2$, into

number of ways to fill the $(k-1)$th column using configuration A
+ number of ways to fill the $(k-1)$th column using configuration B

yields the equality

$$f_n = f_0 + f_1 + f_1 + f_2 + f_3 + \ldots + f_{n-2}$$

$f_1 = 1$, so the equality can be rewritten as

$$f_n = \sum_{i=0}^{n-2} f_n + 1$$

as desired.

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Collaboration Acknowledgement

I used claude.ai for help with typesetting, particularly for the figures.