Power, Cost and Capacity Optimization of Belt Conveyor System

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***Abstract*—This paper presents a multidisciplinary and multi- objective optimization and post optimality analysis of a belt conveyor employed in material handling systems. It provides a global optimization metric covering the two most widely used codes of belt manufacturing to conform to the distinct unit systems used worldwide. The cost function for the belt conveyor system is developed and passed through the optimizers like the reduced gradient method and the genetic algorithm for minimization. The second objective function in the form of the capacity of conveyance is introduced for maximization and both functions are treated with Pareto Front to locate optimal points in the objective space. The resultant points are then filtered to conform to the standards of the industrial belt conveyors to produce a set of distinct parameters. A set suitable for the application and the layout then can be selected to optimize the economy and production of the manufacturing plant. The post optimization analysis of the variables and the parameters determines the most sensitive parameters towards the design. The effect of the variance in the constraints on the objective space is given by the active constraints. The paper provides a specific template for the sand conveyance which can be translated to multiple applications and materials.**

Keywords—Belt Conveyor, Power, Optimization, SQP, Genetic Algorithm, Multi- objective, Post Optimality Analysis

I. INTRODUCTION

A material handling system involves the transport, storage and control of various materials by different procedures and techniques utilizing specialized equipment. A robust material handling system is one of the most crucial necessities of any manufacturing plant. One of the most commonly used methods of conveyance of the bulk materials is a belt conveyor. The belt conveyor offers advantages such as safety of operation, wide variety of conveyance capacities, reliability and customizability. The belt conveyors are especially important when it comes to reducing the labor and minimizing the energy consumption. Moreover the belt conveyor systems provide environmentally safe mode of transportation of the materials as compared to other modes of transport. Another important aspect of a belt conveyor is the flexibility in the design. The design parameters of the belt can be constantly improved based on the application, environment, availability of resources and production requirements. There are various codes used to design a belt conveyor worldwide. The most universally accepted methods include the code developed by the Conveyor Equipment Manufacturers Association henceforth referred to as the CEMA code and the code developed by the International Organization for Standardization henceforth referred to as the ISO code. The details of the codes are explained in the subsequent sections. The CEMA code follows the American unit system while the ISO code follows the System International (IS) units.

This paper provides an elaborate procedure to optimize various parameters associated with the design of the belt conveyor system. The paper displays the impact of the optimized design vector on the minimization of the cost of the conveyor system. It also emphasizes the fact that even a small step towards optimal output can scale up to be a great reduction in the cost and a great increment in the capacity of the belt conveyor. The paper uses the application of the sand transportation but it can be translated to other applications easily by substituting the application specifics as described in the following sections. Furthermore it provides a global metric to optimization of the belt conveyor as it covers both the CEMA and ISO systems to be able to be digested by any reader worldwide.

There has been a lot of research carried out in the domain of energy conservation of a belt conveyor. The [1] gives a detailed review of energy management in long belt conveyor systems. The mathematical model of a conveyor system has been developed by [2]. The [3] discusses the modelling of the belt conveyor based on the energy efficiency. The cost efficiency and audit methodology is proposed by [4] focusing mainly on reduction in electrical power. However there has not been a lot of work in the design of a belt conveyor through the perspective of multidisciplinary optimization involving machine design, material science, structural design accompanied by energy utilization. Also the correlation among the prominent codes of design of a belt conveyor has not been studied with respect to the optimization of the parameters..

II. FORMULATION

A. Problem Statement

Minimize power and cost and maximize conveying capacity for the belt conveyor system for sand.

B. CEMA

CEMA [5] calculates the required tension in the belt to overcome the forces that may cause a slag in the belt that would hinder the motion of the belt as well as the material. Therefore, CEMA addresses the resistances caused from friction between belt and idlers, load and flexure resistance of material of the belt and the material that is being conveyed, forces due to elevation, skirt board friction and other accessories with which the conveyor system has been installed. The following problem formulation uses CEMA to derive the value of power required to convey sand at a distance of 3000 feet and then calculate the cost of power along with the cost of the conveyor system. The cost of the system mainly includes the cost of power consumption, the cost of the idlers and the cost of the belt itself.

TABLE 1. INEQUALITY CONSTRAINTS FOR CEMA

Number	Variable	Inequality Constraint
1	Speed	$200 \leq V \leq 250$
2	Weight of material per unit belt length	$0 \leq W_m \leq 300$
3	Idler Spacing	$2 \leq S_i \leq 4$
4	Weight of belt per unit belt length	$4 \leq W_B \leq 35$
5	Conveying capacity	$50 \leq Q \leq 100$
6	Idler Frictional factor	$0.2 \leq K_x \leq 2$
7	Force on belt and load flexure factor	$0.01 \leq K_y \leq 0.04$
8	Length of the conveyor	$500 \leq L \leq 3000$
9	Effective Tension	$T_o \leq T_e$
10	Belt Width	$18 \leq b \leq 96$
11	Idler Diameter	$4 \leq d \leq 7$

TABLE 2. EQUALITY CONSTRAINTS FOR CEMA

No	Quantity	Equality Constraint
1	Effective Tension	$T_e = LK_t\left(K_x + K_y W_b + 0.015 W_b\right) + W_m\left(LK_y \pm H\right) + T_p + L_b\left(2C_s h_s^2 + 6\right)$
2	Power	$P = T_e . V/33000$
3	Motor Power	$P_M = P/\eta$
4	Weight of the material	$W_m = \dfrac{33.33 \times Q}{V}$
5	Idler friction factor	$K_x = 0.00068(W_m + W_b) + \dfrac{A_i}{S_i}$
6	Minimum Tension	$T_o = 4.2 S_i (W_m + W_b)$
7	Material depth	$h_s = 0.1 \times b$
8	Capacity	$Q = \left(\dfrac{0.445b - 0.9}{\sin(\alpha)}\right)^2 \times \left(\dfrac{\pi\alpha}{180} - \dfrac{\sin(2\alpha)}{2}\right) \times \dfrac{\rho}{144} \times V/36.746$

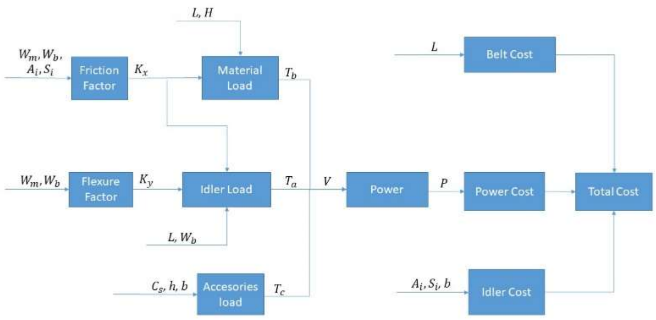

Fig.1 Modules in CEMA

W_b, W_m, A_i, S_i						
Friction Factor	W_b, W_m					
K_x	**Flexure Factor**	L, K_x, K_y, W_b				
	K_y	**Idler Load**	L, W_m, K_y, H			
		T_a	**Material Load**	C_s, h, b		
			T_b	**Accessories Load**	T_a, T_b. T_c	L, S_i, ρ
				T_c	**Power**	P
					P	**Cost**

Fig. 2 N^2 Diagram for CEMA

Design vector is as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} V \\ S_i \\ W_b \\ L \\ K_y \\ b \end{bmatrix}$$

The formulation for the optimization problem is as follows:

$$J = \frac{9450(x_4+10)}{x_4} + 0.014x_1x_4x_6^2 + 0.016x_3x_4 + x_3x_5 + \frac{1.5}{10^6}(2.56x_6-5.18)^2 - \frac{3}{2x_2} + \frac{2.22x_4x_5}{10^{10}}(2.56x_6+994.82)^2 + \frac{2232.4x_6}{x_2}$$

$$200 \le x_1 \le 250$$

$$2 \le x_2 \le 4$$

$$4 \le x_3 \le 35$$

$$250 \le x_4 \le 3000$$

$$0.01 \le x_5 \le 0.04$$

$$18 \le x_6 \le 96$$

$$4.2x_3(x_2 + 0.0022(2.56x_6-5.18)^2) - \frac{2.75x_4x_6^2}{10^4} - 0.015x_3x_4 + x_3x_5 + \frac{1.51(2.56x_6-5.18)^2}{10^6} - \frac{3}{2x_2} + 0.0022(2.56x_6-5.18)^2 - 1000 \le 0$$

C. ISO

ISO [6] estimates the resistance offered by the system in moving the material at a specific rate. It takes into account the primary resistance which is the friction force created due to the mass of belt, material, carrying and returning idlers, secondary resistance, slope resistance and the special resistance. The equations of the resistances are empirical in terms of conveying capacity and belt velocity. The values of the factors in the equations differ for each application. After the power required to convey the material is calculated from the ISO code, the cost function is designed similar to CEMA.

TABLE 3. INEQUALITY CONSTRAINTS FOR ISO

Number	Variable	Inequality Constraint
1	Speed	$1.016 \le V \le 1.524$
2	Weight of material per unit belt length	$0 \le M_g \le \infty$
3	Weight of carrying idler per unit length	$38.5 \le M_{ro} \le 77$
4	Weight of returning idler per unit length	$12.8 \le M_{ru} \le 25.67$
5	Weight of belt per unit belt length	$5.92 \le M_b \le 51.81$
6	Conveying capacity	$50 \le T \le 100$
7	Length of belt	$152.4 \le L \le 914.4$
8	Primary Resistance	$0 \le F_H \le \infty$
9	Secondary Resistance	$0 \le F_N \le \infty$
10	Special Resistance	$0 \le F_{st} \le \infty$
11	Belt Width	$0.45 \le b \le 2.44$
12	Belt Sag	$4.2 \times \frac{3000}{M_{ro}} \times \frac{47}{914.4} \le (F_H + F_N + F_{st})$

TABLE 4. EQUALITY CONSTRAINTS FOR ISO

No	Quantity	Equality Constraint
1	Primary Resistance	$F_H = 0.25 \times L(M_{ro} + M_{ru} + (2M_b + M_g)cos\delta)$
2	Secondary Resistance	$F_N = \frac{TV}{3.6} + \frac{T^2}{20736b^2}$
3	Special Resistance	$F_{st} = 0.1478\frac{T^2}{V^2} + 0.0006\frac{T}{V} + 50$
4	Power	$P = \left(\frac{(F_H + F_N + F_{st})V}{1000} \times \frac{914.4}{L}\right)$
5	Weight of the material	$M_g = \frac{T}{3.6V}$

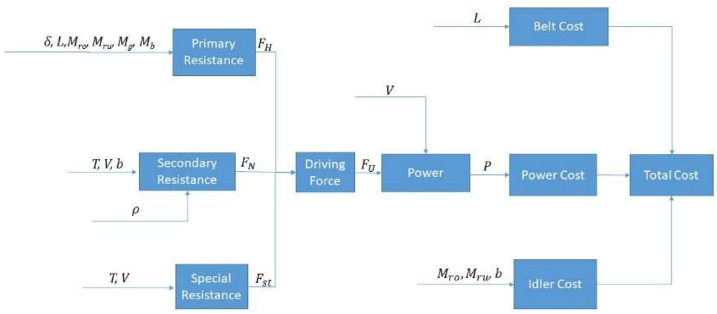

Fig. 3 Modules in CEMA

δ, L, M_{ro}, M_{ru}, M_g, M_b	T, V, b				
Primary Resistance	ρ				
F_H	**Secondary Resistance**	T, V			
	F_N	**Special Resistance**	F_H, F_N, F_{st}		
		F_{st}	**Driving Force**	F_U	M_{ro}, M_{ru}, L, ρ
			F_U	**Power**	P
				P	**Cost**

Fig. 4 N[2] Diagram for ISO

Design vector is as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} V \\ M_{ro} \\ M_{ru} \\ M_b \\ T \\ b \\ L \end{bmatrix}$$

The formulation for the optimization problem is as follows:

$$J = 457.2(x_2 + x_3) + \frac{10972.8x_7 + 36539.4}{x_7} + \frac{1}{x_5} + 27.432x_1\left(\frac{x_5}{x_1}\right)^2 + \frac{x_5^2}{20736x_6^2} + 0.278x_1x_5 + 0.014x_7\left(\frac{x_2 + x_3 + 2x_4 + 5x_5}{x_1}\right) + \frac{0.824\frac{x_5}{x_1} + 96.7942}{12500x_7}$$

$$1.106 \leq x_1 \leq 1.524$$

$$9.36 \leq x_2 \leq 25.7$$

$$12.8 \leq x_3 \leq 25.67$$

$$5.82 \leq x_4 \leq 51.81$$

$$50 \leq x_5 \leq 100$$

$$0.45 \leq x_6 \leq 2.44$$

$$152.4 \leq x_7 \leq 914.4$$

$$\frac{647.63(x_2 + x_3)}{x_2} + \frac{x_5^2}{20736x_6^2} - 0.278x_1x_5 - 0.024\left(\frac{x_5}{x_1}\right)^2 - 0.014\left(\frac{x_2 + x_3 + 2x_4 + 5x_5}{x_1}\right) - 0.824\frac{x_5}{x_1} - 96.7942 \leq 0$$

III. DESIGN EXPLORATION

Non-linear functions have multiple local optimum points. These kinds of complex systems generally have a design vector of higher dimensions therefore, it is extremely difficult to figure out the global optimum out of all the local optima. That is the main difficulty associated with dimensionality and it becomes more difficult as the dimension of the design vector increases. Due to this, the optimizers often get trapped in the local minima and while there are many ways to deal with this problem, the computational cost in doing so increases at a tremendous rate which is undesirable.

A quick and cheap solution to this problem is design space exploration. It is performed before the optimizer is used. Every variable is changed and its effect on the objective function is observed and subsequently, inferences are made based on statistical analysis of the data gathered via design space exploration. There are numerous methods to perform this exploration such as matrix designs, full and half factorial designs, parameter study, Orthogonal Arrays and Latin Hypercubes. Owing to the complexity and non-linearity of the objective function and dimension of the design space, full or half factorial designs are not suitable for this problem. One of the most convenient methods is the Orthogonal Array. It gives a considerably less number of experiments for high dimensional design vectors and levels of each variable in the design vector.

For the design space exploration, CEMA has six variables and ISO has five variables. Each of these variables is assigned five levels uniformly spaced across the feasibility region. The best match for this design is the L25 matrix which can accommodate a maximum of 6 variables with 5 levels each, giving 25 experimental settings. Following are the tables for variables and their levels for CEMA and ISO.

TABLE 5. VARIABLES AND LEVELS FOR DOE FOR CEMA

Level	Length of belt (L)	Feed rate (Q) (tph)	Belt speed (fpm)	Width of belt (in)	Idler Spacing (S_i) (ft)	Roller diameter (in)
1	500	50	200	18	3	4
2	600	62.5	300	36	3.5	5
3	1000	75	350	42	4	6
4	1500	87.5	400	60	4.5	7
5	3000	100	500	84	5	8

TABLE 6. VARIABLES AND LEVELS FOR DOE FOR ISO

Level	V (m/s)	M_{ro} (kg/m)	T (tph)	b (m)	L (m)
1	1.016	38	50	0.45	152.4
2	1.143	44	62.5	0.762	182.88
3	1.27	52	75	1.068	304.8
4	1.397	61	87.5	1.524	457.2
5	1.524	77	100	2.438	914.4

After calculating the value of the objective function for all the 25 experiments, main effects of each level of all the variables were calculated and subsequently, a combination of best design variables was derived for both ISO and CEMA.

For CEMA, L=3000, Q=50, V=200, b=18, Si=4 and d=6 gives the best design.

For ISO, V=1.106, Mro=38, T=50, b=1.524 and L=914.4 give the best design.

These points are considered to be the best design out of all the combinations available in the orthogonal array method. Nothing can be said regarding the optimality of these points. These points cater to the need of a good starting point for the optimizer since starting points are extremely crucial for the optimizer to converge to the global optimal point.

IV. GRADIENT-BASED OPTIMIZER

These types of optimizers use the gradient information of the function at each iteration and decide upon which direction to move forward. These methods work well when the design vector is small, but when the dimension of the design vector increases, these methods turn out to be quite expensive. This happens primarily because the optimizer has to calculate gradient and hessian (in some methods) at each iteration which requires a large amount of computational power.

Numerous methods exist to make an approximation of second-order derivatives and other terms that require high computational power. Numerical methods employ these kinds of estimation techniques and try to converge to the optimum points, such as Newton and Quasi-Newton methods. Various other numerical techniques also exist, namely, Interior Point Method, Sequential Quadratic Programming, Gradient Descent and Steepest Descent.

The fmincon function [7] provided by Matlab is used to optimize the cost. This function uses algorithms like SQP, Interior Point, Active-set and Trust Region Reflective. The default algorithm is Interior Point. In this case, Interior Point and SQP algorithms are used since both the algorithms have the ability to optimize non- linear functions with nonlinear and linear constraints. While both the algorithms converged to the same point, SQP showed more efficiency in the terms of iteration required to converge to the optimum point. Therefore, SQP is chosen to be the optimizing algorithm for cost. After employing the algorithm of SQP and observing the results, a multi-start approach is used to check for multiple local minima. The optimizer always converged to the same point which means that there are no local minima according to the gradient based method SQP. SQP is run for both the formulations and its results are shown below.

TABLE 7. RESULT FROM FMINCON FOR CEMA

No.	Variable	Value
1	Belt Speed (*fpm*)	250
2	Idler Spacing(*ft*)	4
3	Weight of the Belt(*lbs/ ft*)	4
4	Length of the Belt(*ft*)	3000
5	K_y	0.01
6	Width of the Belt(*inches*)	18
	Cost($)	19528

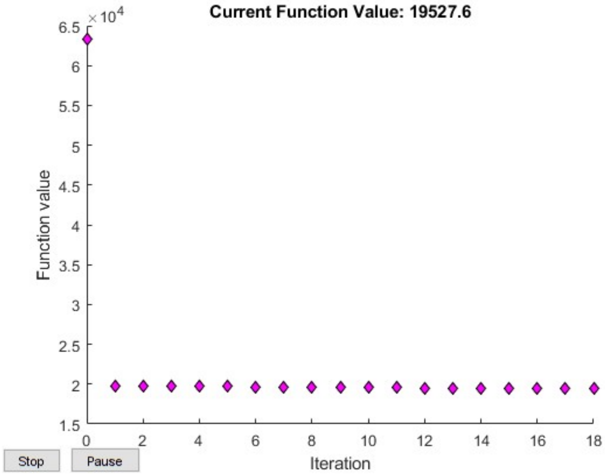

Fig. 5 Convergence History for CEMA in fmincon

TABLE 8. RESULT FROM FMINCON FOR ISO

No.	Variable	Value
1	Belt Speed (*m/s*)	1.016
2	Mass of Carrying Idler(*kg/m*)	9.36
3	Feed Rate(*tph*)	50
4	Width of the Belt (*m*)	914.4
5	Length of the Belt(*m*)	0.46
	Cost($)	21145

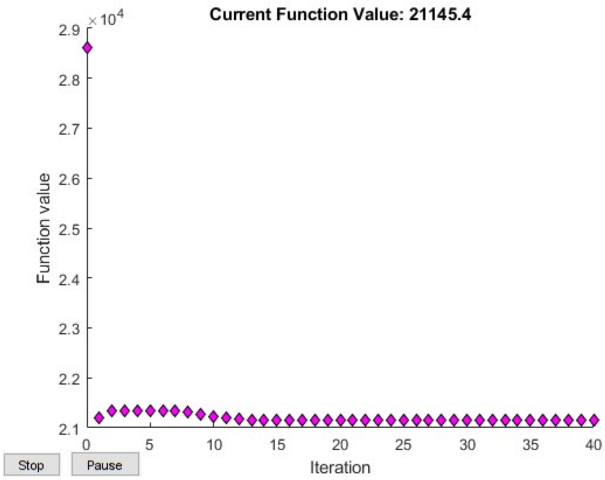

Fig. 6 Convergence History for ISO in fmincon

More methods for optimization are available that can verify the results of gradient based method. Whether the point rendered by the gradient based methods is global optimum or not can be tested out by Heuristic Methods.

V. HEURISTIC OPTIMIZER

Heuristic methods are used to handle large, complex and nonlinear problems. The main advantage of using Heuristic Methods is that they do not get trapped into the local optima and target the global optimum instead. Although, it is not guaranteed that Heuristic Techniques will always give the global optimum, the result from them can be fed to a Gradient Based Optimizer to find the global optimum as the result of Heuristics would be fairly close to the global optimum, if not the global optimum itself. There are quite a few heuristic techniques that can be used to optimize such complex problems, like Genetic Algorithm, Simulated Annealing, Particle Swarm and Tabu Search. Initially, Simulated Annealing was tried which gave opposing results from that of SQP. Thus, it was concluded that Simulated Annealing cannot be used in this problem since gradient based methods are more reliable.

Subsequently, Genetic Algorithm [8] was employed which worked perfectly well. In Genetic Algorithm, there is no information needed regarding the gradient of the function at any iteration. Apart from this, GA works with multiple points at a time in terms of population, giving the advantage of multi-start. This improves the reliability of the results significantly. Also, it works on stochastic methods of selection, thus using the randomness to reach the result. Furthermore, it does not get trapped in the local minima and therefore, results from SQP can also be verified. The results from GA are shown below which are obtained after tuning parameters of Genetic Algorithm for speed and accuracy of convergence. Subsequent steps are only carried out for CEMA as there is an abundance of data for validation purposes, but similar steps until the end of this paper can be carried to optimize for ISO as well.

TABLE 9. TUNED PARAMTERS FOR GA

No.	Parameter	Value
1	Population Size	200
2	Initial Population	[200,2,4,250,0.01,18;250,4,35,3000 ,0.04,96]
3	Elite Count	10
4	Mutation Function	Adaptive feasible
5	Initial Penalty	100
6	Stall generations	4
7	Function Tolerance	1e-10

TABLE 10. RESULT FROM GA FOR CEMA

No.	**Variable**	**GA**
1	Belt Speed (*fpm*)	200.78
2	Idler Spacing(*ft*)	3.99
3	Weight of the Belt (*lbs/ft*)	4.699
4	Length of the Belt (*ft*)	3000
5	Flexure Factor	0.011507
6	Width of the Belt (*inches*)	18
	Cost($)	**19527.65**

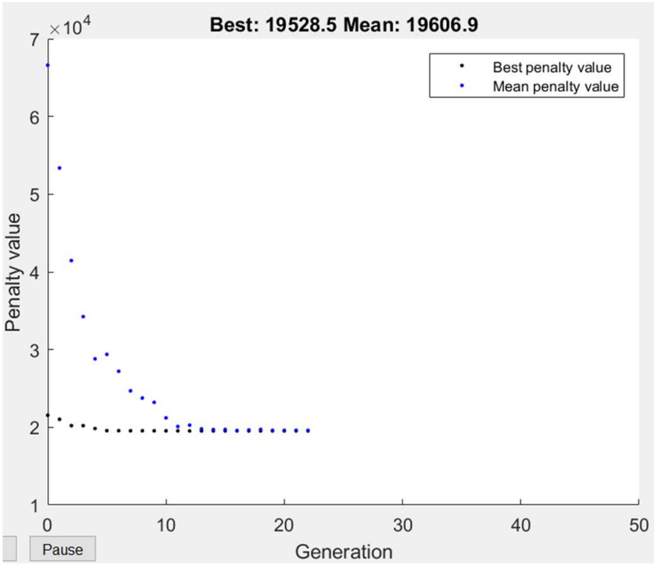

Fig. 7 Convergence History from GA for CEMA

The function value and the value of variables have a close resemblance with that of SQP and thus, the point can be concluded as a global optimum.

VI. MULTI-OBJECTIVE OPTIMIZATION

Another important objective to optimize the performance of a belt conveyor system is the capacity of conveyance of the sand. The problem now converts to a multi- objective domain. The two objectives are mutually opposing as one wants to minimize the cost and at the same time keep the highest possible conveying capacity.

Also it is important to scale the objective functions before combining them as they differ greatly in their magnitudes. The scaling can be achieved by determining the maximum value of the cost and the capacity functions and then dividing them with the two objectives respectively.

This is formulated as follows:

$$Objective\ 1 = J_{cost}$$
$$Objective\ 2 = J_{capacity} = Q$$
$$J_{Total} = J_{cost} + (-J_{capacity})$$

J_{Total} Is the combined objective function. The negative sign indicates the mutually opposing nature of the objectives.

The goal is to minimize the in order to reduce the cost and increase the capacity. It is a tradeoff between cost and the capacity and the designer has to decide which objective gets a higher preference. The multi-objective design methods are used to determine the weights of the scaled objective function in case of scalarization methods. However, a dominant solution derived may not necessarily be an efficient solution. Thus it is important to pass the objective space through Pareto Front [9] to locate non dominating points on the objective space. The points on the Pareto Front are non-dominant and optimal points for the combined objective function.

The resulting Pareto Front is as follows:

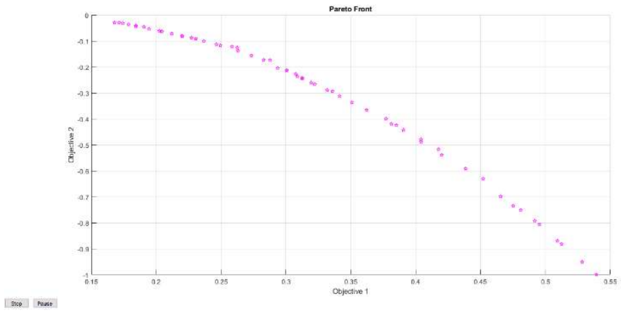

Fig. 8 Pareto Front

However, the optimal set of points obtained from the Pareto Front will not necessarily conform to all the design criteria involved in the belt conveyor systems. Thus each point is tested for the capacity limits of the sand conveyance and for the limitations of the resistance factor the belt. The following set of five points lie on the Pareto Front and follow the norms of the CEMA code.

TABLE 11. FEASIBLE VALUES FROM PARETO FRONT

Speed(fpm)	Idler Spacing (ft)	Weight of the belt (lbs)	Length of the belt (ft)	Flexure Factor	Width (in)	Capacity (tph)
250	4	20	400	0.035	30	85
250	4	15	800	0.035	30	85
250	4	25	1400	0.029	24	53
250	4	30	500	0.034	24	53
250	4	10	2000	0.027	30	85

The values in the table follow the tabular results from the CEMA code.

VII. SCALING OF THE VARIABLES

It is important to scale the variables in an optimization problem that deals with real engineering problems. Often the units of variables vary from one another by a large magnitude. Thus scaling of variables ensures an optimum output.

The scaling process involves determining diagonal entries of an ill conditioned Hessian Matrix of the objective function with respect to the variables and then transforming the variables in order to improve the condition of the Hessian.

The diagonal entries of the Hessian are as follows:

$$\begin{bmatrix} H_{22} \\ H_{44} \\ H_{66} \end{bmatrix} = \begin{bmatrix} 1.255 \times 10^{3} \\ 7 \times 10^{-6} \\ 1.2 \times 10^{-3} \end{bmatrix}$$

Only the significant magnitudes are considered for scaling. The three variables which had the highest sensitivity require scaling. The scaling is performed to have the variables lie in the same range of magnitudes.

The formula for scaling:

$$H_{ii} \approx \frac{1}{x_i^2}$$

$$H_{ii} \approx 10^{m}$$

$$x_i \approx 10^{-[\frac{m}{2}]} \times x_i$$

The diagonal entries of the Hessian after scaling are as follows:

$$\begin{bmatrix} H_{22} \\ H_{44} \\ H_{66} \end{bmatrix} = \begin{bmatrix} 12.5 \\ 0.07 \\ 0.12 \end{bmatrix}$$

Which has entries lying in similar magnitudes improving the condition of the Hessian.

The objective function is modified by introducing the scale factors and the results obtained by the gradient based optimization are as follows:

TABLE 12. OPTIMAL OUTPUT AFTER SCALING

No.	Variable	Value
1	Belt Speed (*fpm*)	250
2	Idler Spacing(*ft*)	4
3	Weight of the Belt(*lbs/ ft*)	4
4	Length of the Belt(*ft*)	3000
5	Flexure Factor	0.01
6	Width of the Belt(*inches*)	18
	Cost($)	19528

The output does not change after scaling which is expected as the bounds on the variables are fixed and for those bounds and constraints this is the only optimal solution. This solution also conforms to the industrial standards of belt manufacture.

VIII. SENSITIVITY ANALYSIS

The post analysis of an optimization problem is very important. It provides an insight to the variable and parameter space and their effects on the cost and capacity functions. The active constraints demonstrate the dominance of a constraint on the objective space.

A. SENSITIVITY OF THE VARIABLES

First the gradient of the objective function with respect to all the independent variables was found out. The individual numerical value for each variable was determined at the optimal point. Then the normalization procedure was carried out by finding out the value of the cost function and using the following formula:

$$S_N = \frac{x_{opt}}{J(x_{opt})} \nabla J$$

Where $\boldsymbol{J}$ is the objective function, $\boldsymbol{x_{opt}}$ is the optimal vector and $\boldsymbol{S_N}$ is the sensitivity.

The results are expressed as follows:

TABLE 13. SENSITIVITY OF VARIABLES

Variable	Sensitivity
V	1.35E-05
S_i	-0.514
W_b	7.30E-06
L	-1.60E-03
K_y	5.50E-06
b	0.515

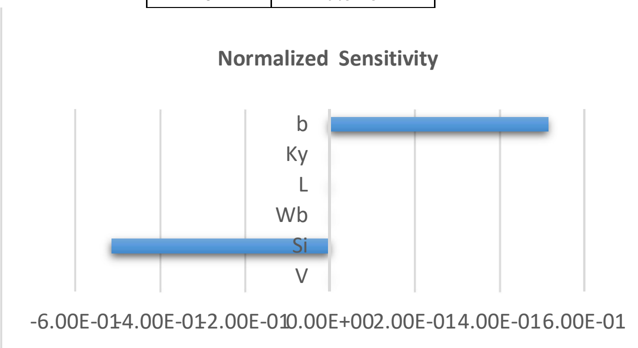

Fig. 9 Tornado chart of variable sensitivities

The post optimality analysis shows that the driving factors in the optimization are the belt width 'b' and the Idler spacing 'S_i'. Also length has a negligible effect on the optimization. Since the positive trend of belt width will increase the belt tension, resulting in higher power consumption and cost. Moreover with bigger spacing between the idlers, the number of idlers will decrease significantly cutting down the cost considerably. Thus it shows the negative trend. The other variables will not affect the power or the idler count and hence are not sensitive.

B. PARAMETER SENSITIVITY

The parameters considered for this analysis are as follows:

1. Temperature correction factor(k_t)=1 for average temperature ranges
2. The angle of surcharge of the material to be conveyed(alpha) = 10° for sand
3. Density of the material conveyed= 100 lbs per cubic feet for sand
4. Idler factor depending on the idler diameter(A_i)=1.5
5. The tension produced due to pulleys= 1000 lbs for the system chosen
6. Skirt board friction factor(C_s)=0.1378 for sand

First the gradient of the objective function with respect to all the parameters was found out. The individual numerical value for each parameter is as given above.

Then the normalization procedure was carried out by finding out the value of the cost function and using the following formula:

$$P_N = \frac{p_{opt}}{J(p_{opt})} \nabla J$$

Where J is the objective function, p_{opt} is the optimal vector and P_N is the sensitivity.

TABLE 14. PARAMETER SENSITIVITY

Parameter	Sensitivity
k_t	-2.60E-03
alpha	0.0417
density	-4.08E-02
A_i	-9.88E-09
T_p	2.60E-05
Cs	6.30E-06

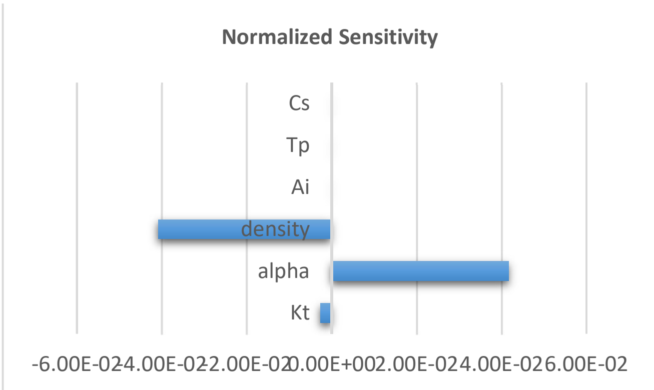

Fig. 10 Tornado Chart for Parameter Sensitivities

The parameter sensitivity analysis shows that the driving parameters are the angle of surcharge and the density of the material conveyed. The temperature coefficient also affects the cost. If the density of the sand is decreased due to environmental or chemical factors the cost will increase. Also if the angle of surcharge increases then the cost will be affected drastically. As far as the temperature coefficient is concerned it changes only in when the temperatures are below. In most of the material handling plants the temperatures are maintained above that.

C. OBSERVATIONS

The driving parameters are the angle of surcharge and the density of the material conveyed. The temperature coefficient also affects the cost.

If the density of the sand is decreased due to environmental or chemical factors the cost will increase.

Also if the angle of surcharge increases then the cost will be affected drastically.

As far as the temperature coefficient is concerned it changes only in when the temperatures are below-15℉. In most of the material handling plants the temperatures are maintained above that.

D. ACTIVE CONSTRAINTS

The optimization package 'fmincon' provides a value 'Lambda' of format of a structure which gives the information about the active and inactive constraints. Since there are no linear or non- linear equality and linear inequality constraints, those fields are empty sets. The only non- linear inequality constraint is inactive since it has a Lagrange Coefficient of zero. For the lower bounds, three constraints are active which correspond to the weight of the belt which has a negligible value, the factor of flexural resistance has a small value and the most dominant is the width of the belt. For the upper bounds, the upper bound of idler spacing is very dominant and the length of the belt has a negligible value.

When the dominant constraints are changed by ±10% the change in the output observed was as follows:

TABLE 15. ACTIVE CONSTRAINTS SENSITIVITY

Constraint	Optimal Cost	Change
LB by -10%	18411	5.7 % decrease
LB by 10%	20644	5.7 % increase
UB by -10%	19403.7	0.64 % decrease
UB by 10%	21884	12% increase

LB- Lower Bound, UB- Upper Bound

The table given above shows that the cost is very sensitive towards the lower bounds of the above mentioned variable. However, increasing the lower bound of the belt width is not desirable. The lowering of the lower bound is not possible due to the belt specification guidelines. The change is not appreciable for decreasing the upper bound of the idler spacing and lowering it considerably increases the cost. This is obvious since lesser idler spacing will increase the number of idlers there by increasing the cost significantly.

IX. CONCLUSION

From all the results that are obtained by the various input vectors spanning inside and outside the bounds all converge to a single set of independent variables. This proves that it is the global optimum. Moreover, both the codes i.e. CEMA and ISO provide the convergence for the same objective function. This underscores the likelihood of global minima. Furthermore, the convergence of the Genetic Algorithm pointing towards the same minimum value supports the above mentioned fact.

The multi- objective optimization explores the tradeoff between cost and capacity function and helps to assign importance to each function by the method of finding non-dominating points on the Pareto Front. The post filtering of the points gives an optimal set of distinct combinations which can be used for different factory layouts and which conform to the codes developed.

The post optimality analysis shows the sensitivity of the design towards the idler spacing and the width of the belt. It also contradicts with the intuition of the high dependency of the length of the belt on the objective function. The parameter analysis provides the important fact of the effect of the material conveyed and the application on the belt conveyor design optimization. The active constraints showcase the effect of change of a constraint on the cost function.

The paper gives a streamlined process of design of a belt conveyor system for sand conveyance. The future work can include an introduction of a discrete function to choose between various materials to be conveyed to increase the versatility of the optimization. The function will provide the material properties for the selected materials which are considered as parameters in this paper.

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