11. We know that $\sin(0) = 0$ and $\sin(\pi) = 0$.

    Thus, 0 and π have the same image.

    So, f is many-one.

    Range $(f) = [-1, 1] \subset R$. Hence, f is into.

    So, f is neither one-one nor onto.

12. $f(n_1) = f(n_2) \Rightarrow n_1^2 + n_1 + 1 = n_2^2 + n_2 + 1$

    $$\Rightarrow (n_1^2 - n_2^2) + (n_1 - n_2) = 0$$
    $$\Rightarrow (n_1 - n_2)(n_1 + n_2 + 1) = 0$$
    $$\Rightarrow n_1 - n_2 = 0 \Rightarrow n_1 = n_2.$$

    $\therefore$ f is one-one.

    But, $f(n) = 1 \Rightarrow n^2 + n + 1 = 1 \Rightarrow n^2 + n = 0$

    $$\Rightarrow n(n+1) = 0 \Rightarrow n = 0 \text{ or } n = -1.$$

    And, none of 0 and -1 is a natural number.

    Thus, $1 \in N$ has no pre-image in N.

    $\therefore$ f is into.

14. Dom $(f) = R$. Also, $y = x^2 + 1 \Rightarrow x = \sqrt{y-1}$.

    x is defined when $y - 1 \geq 0$, i.e., $y \geq 1$.

    $\therefore$ range $(f) = \{y \in R : y \geq 1\}$.

15. g is not a function, since 1 has two images under g.

16. When x is real, $1 + x^2 \neq 0$. So, dom $(f) = R$.

    $$y = \frac{x^2}{(1+x^2)} \Rightarrow x^2(1-y) = y \Rightarrow x = \sqrt{\frac{y}{1-y}}.$$

    For x to be real, $\dfrac{y}{(1-y)} \geq 0$ and $(1-y) \neq 0$.

    $\therefore$ range $(f) = \{y \in R : 0 \leq y < 1\}$.

    Also, 1 and -1 have the same image $\left(\frac{1}{2}\right)$.

Composition of Functions

Let $f : A \to B$ and $g : B \to C$ be two given functions. Then, the composition of f and g, denoted by $g \circ f$ is the function, defined by

$$(g \circ f) : A \to C : (g \circ f)(x) = g\{f(x)\} \ \forall \ x \in A.$$

Clearly, dom $(g \circ f)$ = dom (f).

Also, $g \circ f$ is defined only when range $(f) \subseteq$ dom (g).

REMARK $(f \circ g)$ is defined only when range $(g) \subseteq$ dom (f).
And, dom $(f \circ g)$ = dom (g).