

JOURNEY TO A SUCCESSFUL IN-SEM EXAM

Total Marks:30

UNIT-1

10
Topics

1st MILESTONE

1. Introduction to Engineering Mechanics
2. Terminology in Engineering Mechanics
3. Force System
4. Composition & Resolution of Force
5. Different Laws in Mechanics

2nd MILESTONE

6. Coplanar non-concurrent force system
7. Moment of a force
8. Varignon's Theorem of Moments
9. Couple
10. Equivalent Force Couple System

UNIT-2

10
Topics

1st MILESTONE

1. Centre of gravity
2. Centroid
3. Centroid of a Line
4. Moment of Inertia
5. Radius of Gyration

2nd MILESTONE

6. Introduction to Friction
7. Types of Friction
8. Laws of Friction
9. Analysis of Friction
10. Belt Friction

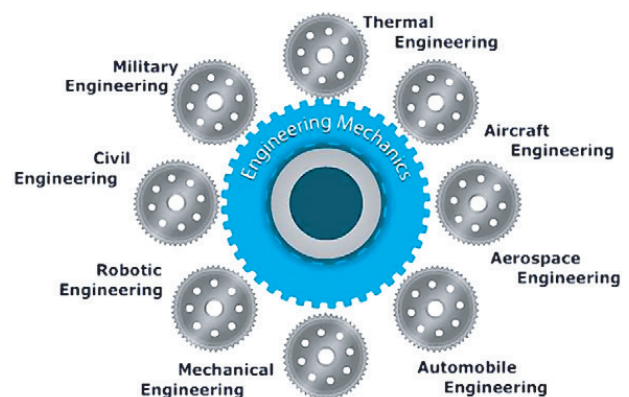
1. Introduction to Engineering Mechanics

Engineering mechanics is the application of scientific and mathematical concepts of mechanics to studying moving and stationary systems under different forces. Engineers are keen to use the laws of mechanics to actual field problems. As this subject is derived from fundamental, we must have our fundamental concept strong. Let us first talk of the prerequisites for this course. Basic understanding of high school physics and good mathematical calculation ability is required to master this subject. We will need scientific calculator to practice a lot of problem.

We will be studying mechanics in two parts- static and dynamics. Statics is the branch of mechanics that deals with the study of objects at rest. Objects at rest may or may not be under the influence of forces. Dynamics is the branch of mechanics that deals with the study of objects in motion and the forces causing such motion.



Contributor to engineering mechanics



Engineering mechanics in different fields

2. Terminology in Engineering Mechanics

As a subject, mechanics is vast and it has laws and theory to explain almost every event happening around us. To make our study simple we will first define the terms related to engineering mechanics which are used again and again.

Rigid body

Body which does not deform or whose deformation is neglected is known as rigid body. For our subject we will be only dealing with rigid body.

Mass

Mass of a rigid body represents the quantity of matter it holds. The mass will be fixed unless some part of rigid body is removed. In space the body may feel weightless but mass remains constant. SI unit of mass is kilogram (kg).

Weight

Weight of a rigid body is the force with which the gravity pulls the mass. In space the gravitational pull will change so the weight of body will also change. SI unit of weight is newton (N)

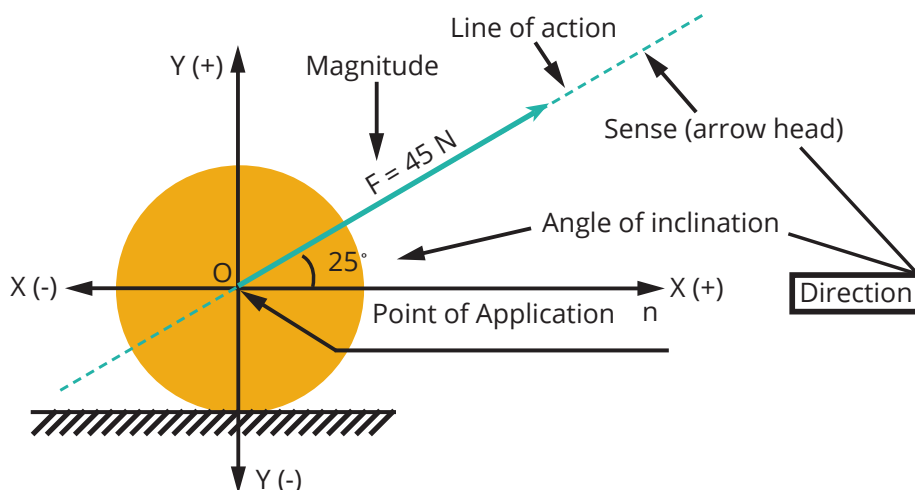
Convert mass to weight by multiplying mass by 9.81 m/s^2

Example: $8 \text{ kg} = (8 \times 9.81) \text{ N} = 78.48 \text{ N}$

Force

Force is the action of one body on another. A force tends to move the body in the direction of its action. The unit of force is Newton (N). In order to define force in complete sense, we need following characteristics of a force -

1. Magnitude
2. Direction
3. Point of application
4. Line of action









Line of Action

A force can be seen as a segment of an indefinitely long line. To each force a reference line is associated, which is referred to as its line of action. A reference axis is used in order to describe the orientation of this line. The Figure shows that a line of action extending along the force.


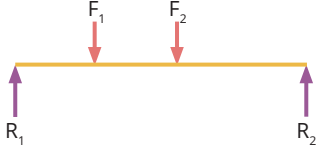
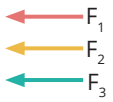
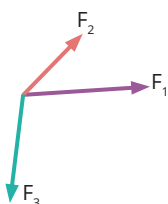
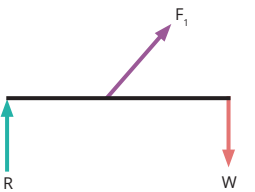
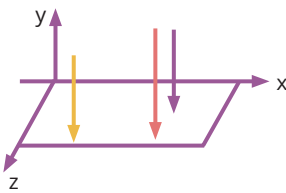
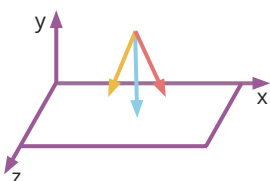
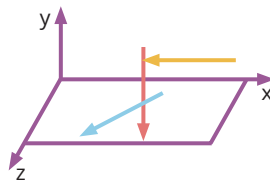
Effects of a Force

A force when acting on a particle or a body may produce internal or external effects. Following are some effects of force or force system on particle or body.

-  It may change the state of particle or body. i.e. if a body is at rest, the force may bring it in motion. If the body is already in motion, the force may accelerate it or may stop it.
-  It may produce the deformation in the rigid body. i.e. change the shape, size of the body.
-  It may retard the motion of a body.
-  It may bring the body to in equilibrium or stable state.
-  It may create rotational effect in the body.
-  It may create internal stresses in the body.

3. Force System

When several forces act simultaneously on a rigid body a force system is formed.

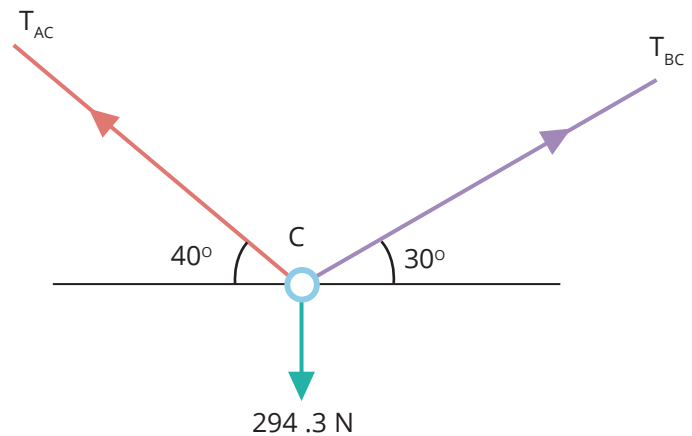
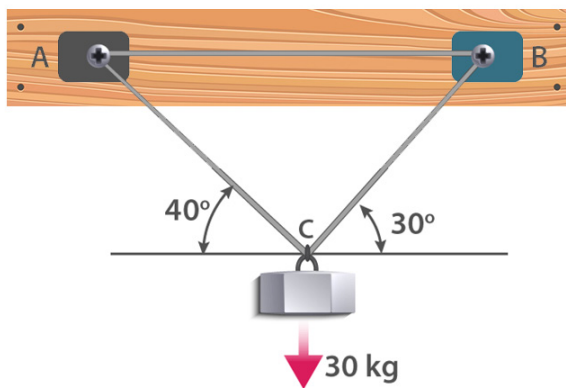
| Forces System | Characteristics | Examples | Line Diagram |
|------------------------------------|--|--|---|
| Collinear forces | Line of action of all the forces act along the same line | Forces on a rope in a tug of war |  |
| Coplanar parallel forces | All forces are parallel to each other and lie in a single plane | System of forces acting on a beam subjected to vertical loads (including reactions) |  |
| Coplanar like parallel forces | All forces are parallel to each other, lie in a single plane and are acting in the same direction. | Weight of a stationary train on a rail when the track is straight. |  |
| Coplanar concurrent forces | Line of action of all forces pass through a single point and forces lie in the same plane | Forces on a rod resting against a wall |  |
| Coplanar nonconcurrent forces | All forces do not meet at a point, but lie in a single plane | Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity |  |
| Non-coplanar parallel forces | All forces are parallel to each other, but not in same plane. | The weight of benches in a classroom. |  |
| Non-coplanar concurrent forces | All forces do not lie in the same plane, but their lines of action pass through a single point. | A tripod carrying a camera. |  |
| Non-coplanar non-concurrent forces | All forces do not lie in the same plane, but their lines of action do not pass through a single point. | Forces on a moving bus. |  |

Co-Planar Concurrent Force System

Coplanar: lies in the same plane

Concurrent: line of action passes through single point

When line of action of forces passes through a single point and also lies in the same plane, then the force is known as Coplanar concurrent force system.

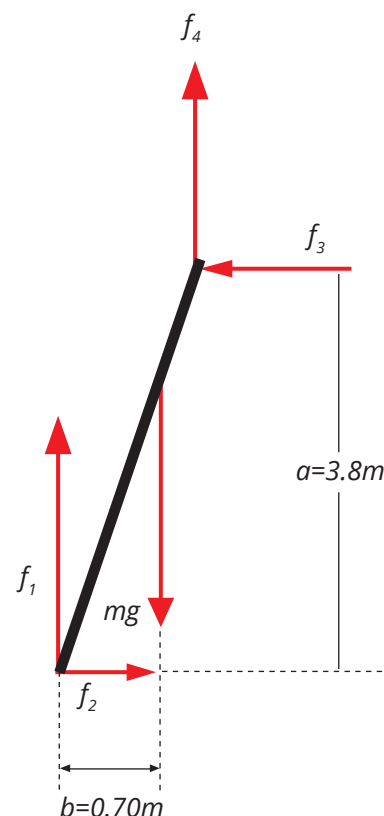


Co-Planar Non-concurrent Force System

Coplanar: Forces lie in the same plane

Non-Concurrent: Line of action of forces do not pass through a single point

In coplanar non-concurrent force system, all the forces lie in the same plane, but their line of action do not pass through a single point.



4. Composition & Resolution of Force

Resultant

In a force system a number of forces are acting simultaneously on a body; then it is possible to find a single force which could replace them. This single force, which would produce the same effect as produced by all given forces is called as resultant (R).

Composition

The process of finding the resultant force which will have the same effect as that of number of forces acting on a body together.

Resolution

The Procedure of splitting the up the single force into number of components without changing the effect of that force on the body. Resolution is the method of composition or method of finding the resultant.

5. Different Laws in Mechanics

Newtons Laws of Motion

Newton's 1st Law

It states that an object will remain in its state of rest or in its state of uniform motion in a straight line unless and until it is acted upon by an external force.

Newton's 2nd Law

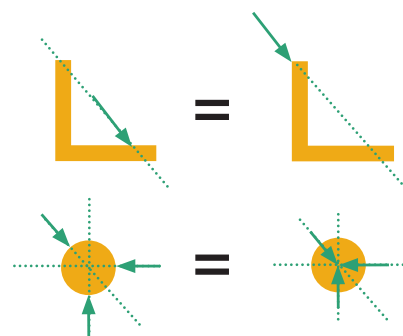
The rate of change of momentum of an object is directly proportional to the force applied and it take place in the direction of force.

Newton's 3rd Law

It states that when one object exerts a force on second object, the second object simultaneously exerts a force equal in magnitude and opposite in direction on the first body i.e. to every action there is equal and opposite reaction.

Law of Transmissibility of Force

The state of a rigid body remains unaltered, if force acting on a body is replaced by another force of the same magnitude and direction, but acting anywhere along the line of action of the replaced force.



Triangle Law of Force

This law is also now as orthogonal resolution or perpendicular resolution of force. The most common two-dimensional resolution of a force vector is into rectangular components also known as perpendicular resolution of force.

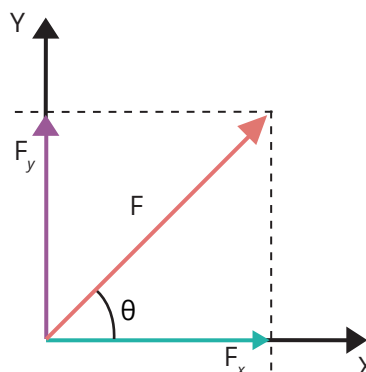
Let i and j be the unit vectors of force \mathbf{F} in the direction of x and y , the force can be represented as-

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$


Component of force along X - direction: $F_x = F \cos \theta$

Component of force along Y - direction: $F_y = F \sin \theta$

θ is the angle made by F with $+X$ axis as shown in figure

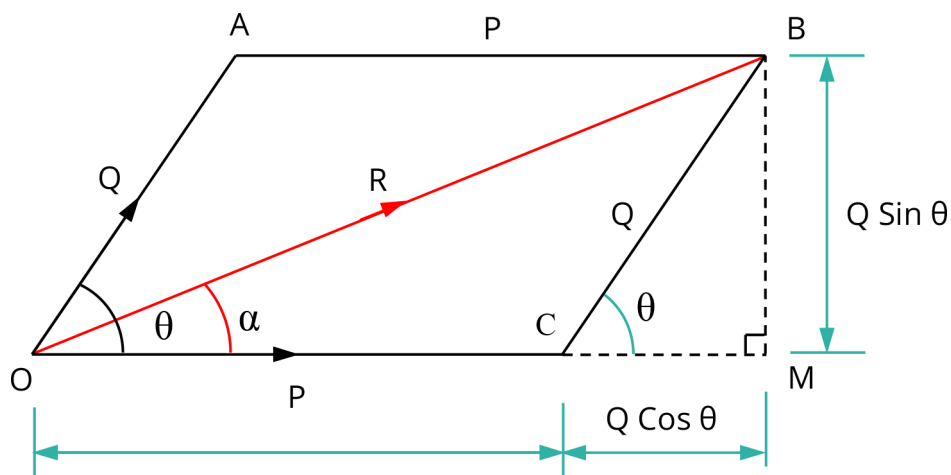


Parallelogram Law of Force

 This law states that if two forces are acting simultaneously on a body at a point; they can be represented in magnitude and directions by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram which passes through the point of intersection of the two forces represents their resultant in magnitude and direction.



Consider two forces P & Q acting at a point are represented by two sides OC and OA of parallelogram $OACB$. Then their Resultant (R) is represented by Diagonal OB passing through point of intersection of two forces.



Let,

θ = Angle between the two forces P & Q

α = Angle between the force P & Resultant R

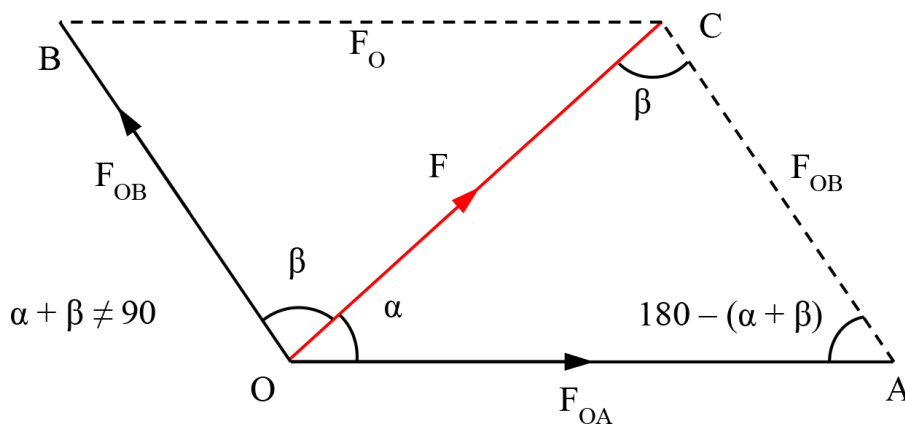
| | |
|--|------------------------------|
| $\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$ | ----- Magnitude of resultant |
| $\therefore \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$ | ----- Direction of resultant |

Sine Rule

It states that the ratio of two sides of an oblique triangle is in same proportion as that of the ratio of sine of their opposite angle. OR The ratio of two sides of triangle will be equal to the ratio of sine of their opposite angle.

🚀 Simple procedure to resolve a force into two non-perpendicular directions is given below.

- 1) Construct a parallelogram by keeping original given force (F) along the diagonal & two components along two adjacent sides of parallelogram (passing through same point).
- 2) Find out 3 angles of any one triangle.
- 3) Apply sine rule in that triangle.



🚀 Let a force F is to be resolved along the two direction OA & OB which are not perpendicular. Then a parallelogram is constructed as shown above, by keeping the original force along the diagonal.

🚀 By applying the sine rule in ΔOAC ,

$$\therefore \frac{F_{OB}}{\sin \alpha} = \frac{F_{OA}}{\sin \beta} = \frac{F}{\sin [180 - (\alpha + \beta)]}$$

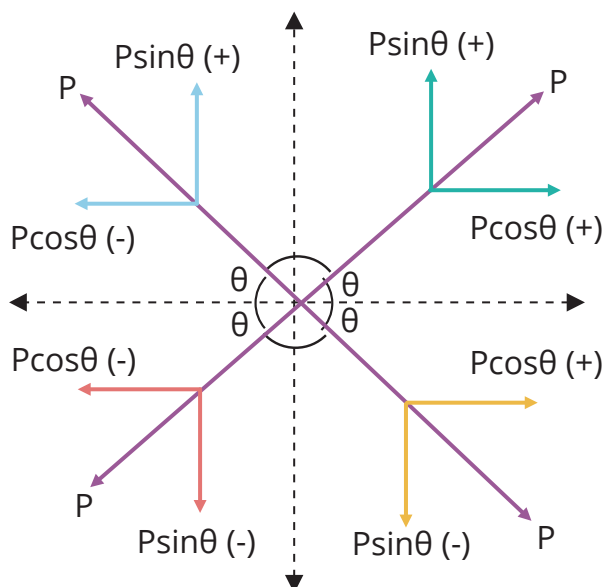
$$\therefore \frac{F_{OB}}{\sin \alpha} = \frac{F_{OA}}{\sin \beta} = \frac{F}{\sin (\alpha + \beta)}$$

$$\therefore F_{OA} = \frac{F \sin \beta}{\sin (\alpha + \beta)} \quad \text{-----Component along OA}$$

$$\therefore F_{OB} = \frac{F \sin \alpha}{\sin (\alpha + \beta)} \quad \text{-----Component along OB}$$

STEPS & FORMULA

Problem Based on Co-Planer Concurrent Force System



1) Resolve all the forces horizontally and find the algebraic sum of all the horizontal components i.e. $\sum F_x$

2) Resolve all the forces vertically and find the algebraic sum of all the vertical components i.e. $\sum F_y$

3.) Resultant = $\sqrt{(\sum F_x)^2 + (\sum F_y)^2}$

4.) $\alpha = \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right|$

α is the angle made by 'R' with the horizontal.

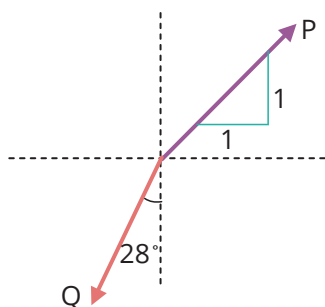
Exercise - 1.1

Co-Planer Concurrent Force System

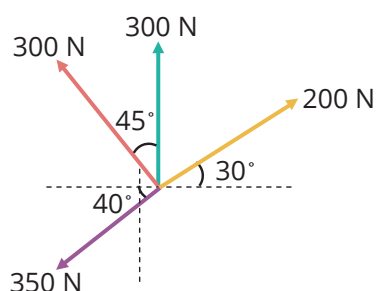
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for detailed
solution.



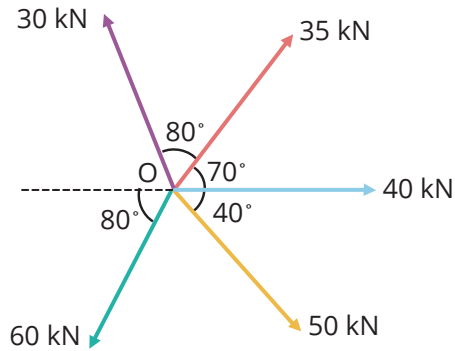
Q 1. Find components of forces P and Q in horizontal and vertical directions. Refer to the figure given below



Q 2. Find the magnitude of the resultant and its location of the following forces acting at a point O

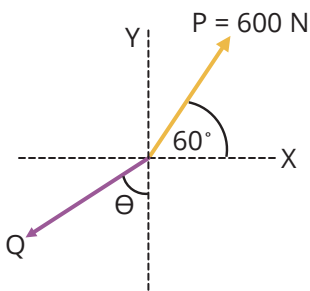


Q 3. Find the resultant of following force system

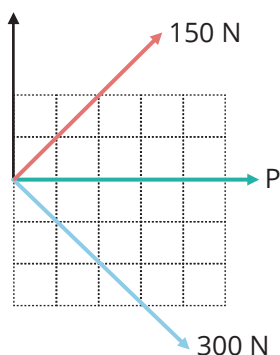


Q 4. Two planar forces are: $3i+4j$ and $-2i+j$. Add these forces and find out the angle which the resultant makes with the x-axis.

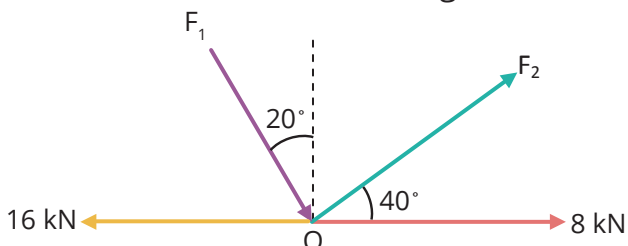
Q 5. The resultant of two forces P and Q is 1200 N horizontally leftward. Determine the force Q and the corresponding angle θ for the system of forces.



Q 6. Determine the magnitude of force P so that the resultant of the force system is vertical and hence find magnitude of resultant.

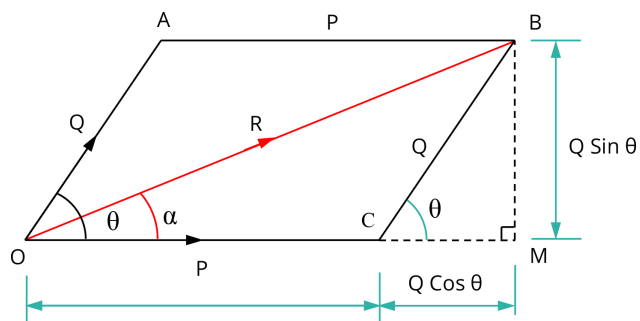


Q 7. Resultant of four forces acting at O is 3 kN vertically upward. Compute F_1 and F_2



STEPS & FORMULA

Problem Based on Parallelogram Law and Sine Rule

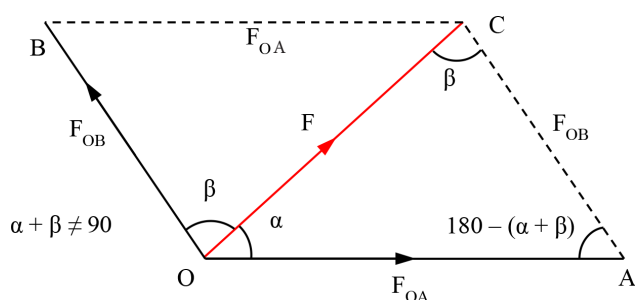


$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\therefore \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

θ = Angle between the two forces P & Q

α = Angle between the force P & Resultant R



$$\therefore \frac{F_{OB}}{\sin \alpha} = \frac{F_{OA}}{\sin \beta} = \frac{F}{\sin [180 - (\alpha + \beta)]}$$

$$\therefore \frac{F_{OB}}{\sin \alpha} = \frac{F_{OA}}{\sin \beta} = \frac{F}{\sin (\alpha + \beta)}$$

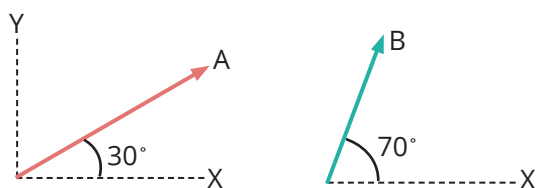
Exercise - 1.2

Parallelogram Law and Sine Rule

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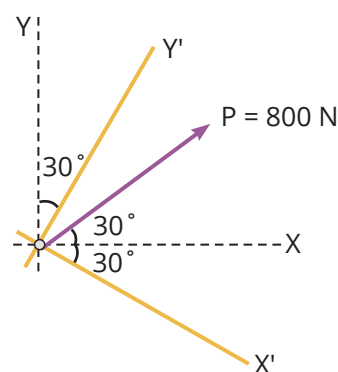


Q 1. Two force vector \vec{A} and \vec{B} are shown in figure, the magnitude of which are $\vec{A}=60\text{N}$ and $\vec{B}=100\text{N}$. Find the resultant $\vec{R} = \vec{A} + \vec{B}$.

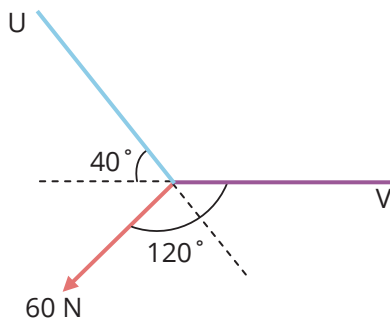


Q 2. A force $P = 800\text{ N}$ is shown in figure

- Find the y component of P with respect to x and y axis
- Find the y' component of P with respect to x' and y' axis
- Find the y component of P with respect to x' and y axis
- Find the y' component of P with respect to x and y' axis

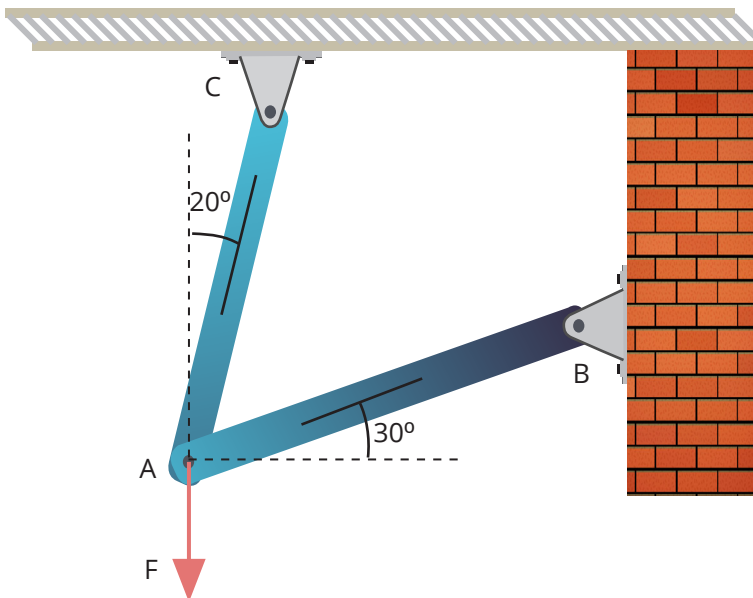


Q 3. Resolve the 60 N force in to components acting along the u and v axes and determine the magnitudes of the components.



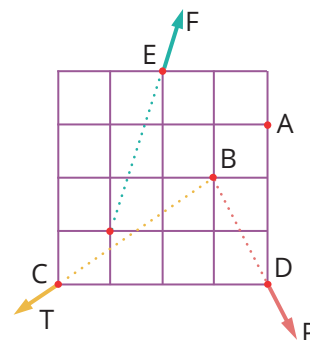
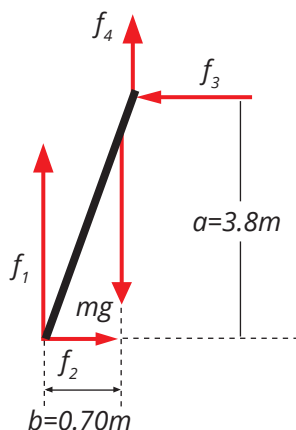
Q 4. The angle between the two concurrent forces is 90° and their resultant is 2500 N. The resultant makes an angle of 46° with one of the forces, determine the magnitude of each force.

Q 5. The magnitude of vertical force F shown in Fig. is 8000 N. Resolve F into components parallel to the bars AB and AC .



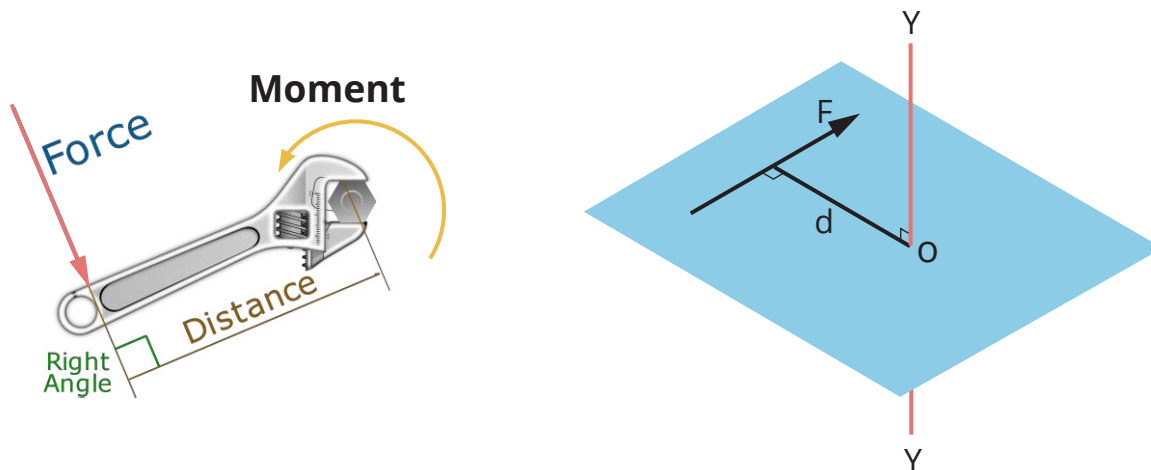
6. Coplanar Non-concurrent Force System

In coplanar non-concurrent force system, all the forces lie in the same plane, but their line of action do not pass through a single point. In this system, forces will be separated by some distance. Example of man on ladder is best example of coplanar non-concurrent force system.



7. Moment of a Force

- 🚀 Just as force has a tendency to translate the body, moment has a tendency to rotate the body about the point.
- 🚀 Moment of force about a point is the measure of the rotation effect of force.
- 🚀 The moment 'M' of force F about point 'O' is equal to the product of 'F' and 'd'.



🚀 Mathematically, Moment of Force is expressed as- $M_O = F \times d$

Where,

F = Force applied

d = Perpendicular distance between the Center of Bolt and the Line of Action of Force.

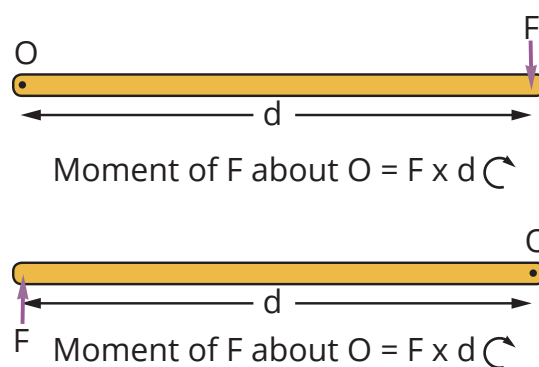
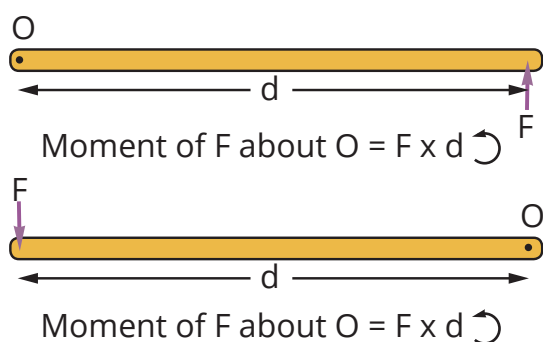
🚀 S I Unit of Moment : N.m

Important Point About Moment

1. Moment is reference dependent; it will change as soon we change our reference about which moment is taken.
2. As moment is the rotational effect, it can be clockwise or anticlockwise.
3. Like forces moment can be added or subtracted
4. The moment is zero when the force is passing through the point from where the moment is to be taken

Sign Conventions

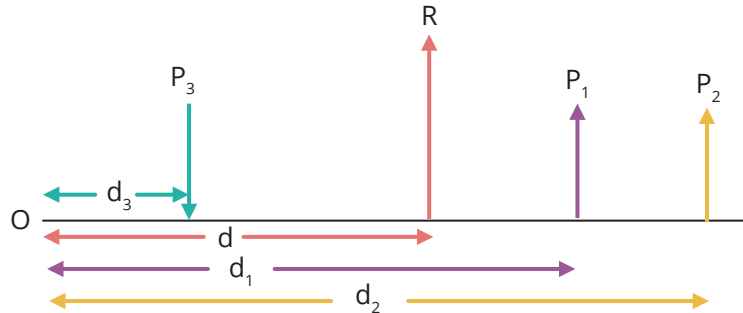
If we consider clockwise moment as positive, Anticlockwise will be taken as negative and vice versa.



8. Varignon's Theorem Of Moments

The algebraic sum of the moments of all the forces about a particular point is equal to the moment of their resultant about the same point.

Consider a coplanar non-concurrent force system



Let 'R' be the resultant of three forces, and as shown in the figure.

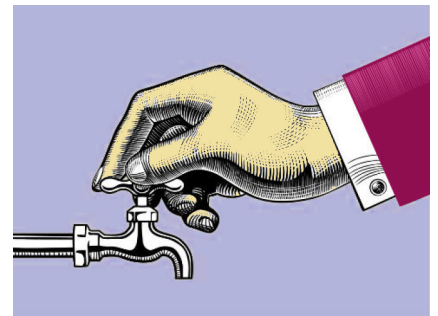
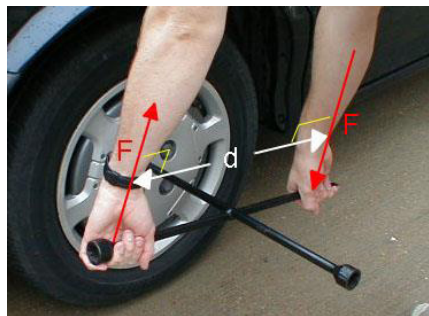
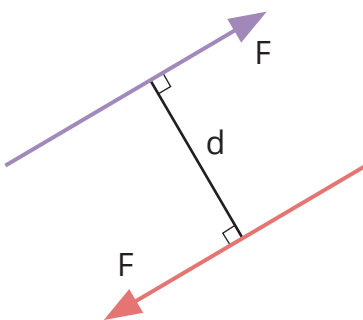
Varignon's Theorem of Moments

Moment about point 'O' is: $MR_O = \sum MF_O$

$$-(R \times d) = -(P_1 \times d_1) - (P_2 \times d_2) + (P_3 \times d_3)$$

9. Couple

- 🚀 Two parallel forces which are equal in magnitude and opposite in direction and separated by a finite distance is said to be a couple.
- 🚀 Both the Forces are Equal in Magnitude
- 🚀 Both the Forces are Parallel and Opposite in Direction
- 🚀 Moment Couple $M = F \times d$
- 🚀 Couple can be clockwise or anti clockwise.



Characteristics of a Couple

1. The sum of two forces along any direction is always zero
2. The sum of moments of two forces about a given point is not zero
3. The couple tends to rotate the body in a particular direction
4. A couple cannot be balanced by a single force. To balance the couple another couple needs to be applied.
5. Couple can rotate the body but can not translate it.

STEPS & FORMULA

Problem Based on Moment and Varignon's Theorem

- 1) Resolve all the forces horizontally and find the algebraic sum of all the horizontal components i.e. $\sum F_x$
- 2) Resolve all the forces vertically and find the algebraic sum of all the vertical components i.e. $\sum F_y$
- 3) Resultant = $\sqrt{(\sum F_x)^2 + (\sum F_y)^2}$
- 4) $\alpha = \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right|$ α is the angle made by 'R' with the horizontal.
- 5) Apply Varignon's theorem of moments (V.T.M.)
- 6) Moment of all the forces = Moment of the resultant

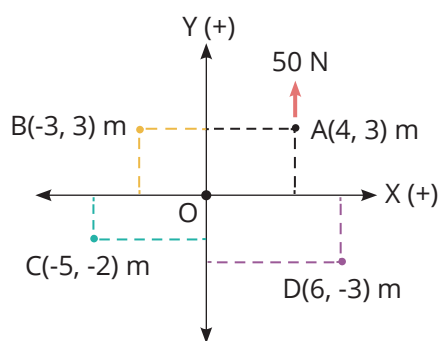
Exercise - 1.3

Moment and Varignon's Theorem

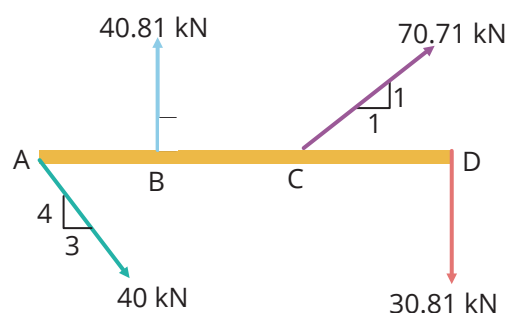
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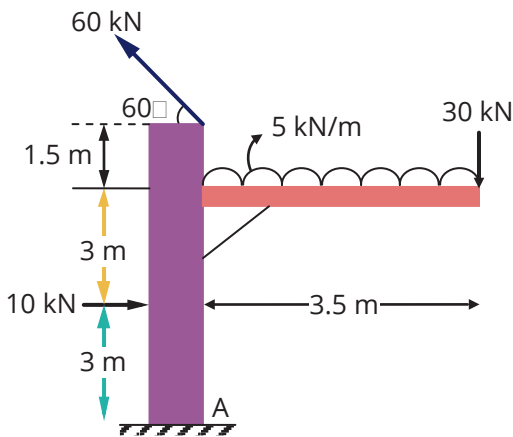
Q 1. Find the moment of the force 50 N about point A, B, C, D and O respectively shown in figure.



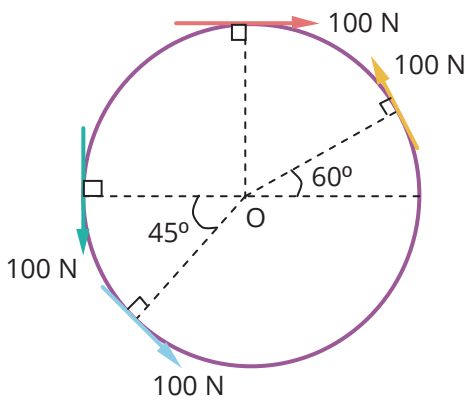
Q 2. A coplanar force system acts on a link AD as shown in figure for which AB = BC = CD = 2m. Determine resultant completely with respect to A (P)



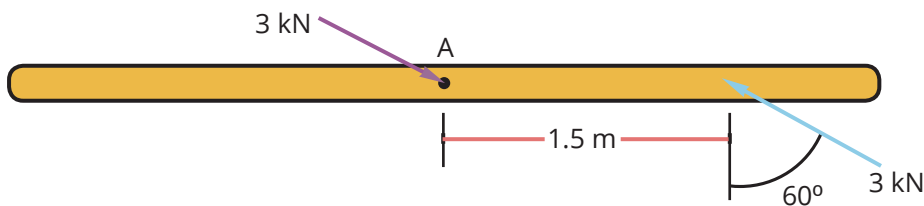
Q 3. The system of forces acting on a frame is as shown in figure. Calculate the magnitude and direction of the resultant. Also find the position of resultant with respect to point 'A'.



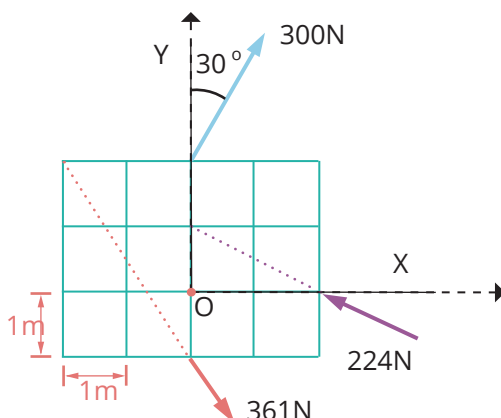
Q 4. Four forces act tangentially to a 8 cm diameter circle as shown in figure. What will be magnitude and location of resultant with respect to 'O'.



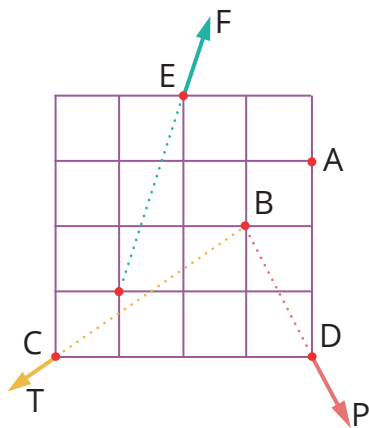
Q 5. What is the moment that the couple below exerts about point A?



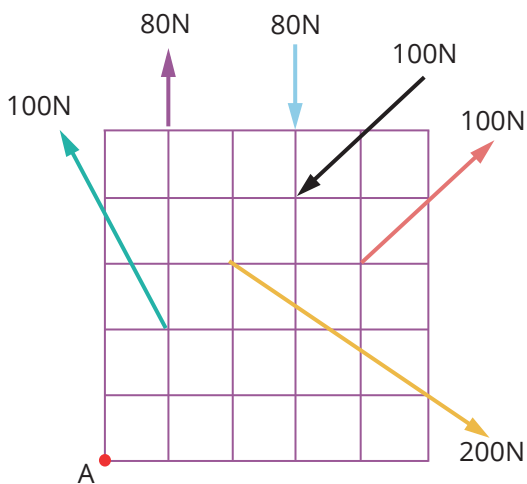
Q 6. Determine the resultant of the force system shown in below diagram and its x and y intercepts.



Q 7. The three forces shown in below diagram are required to cause a horizontal resultant acting through point A. If $F = 316\text{N}$, determine the values of P and T .



Q 8. Determine the resultant moment about point A of the system of forces shown in below diagram. Each square is 1m on a side.



10. Equivalent Force Couple System

🚀 Every force acting on a body have two effects-

- 1.) Produces push or pull effect
- 2.) Produces turning effect (Moment)

🚀 Two force systems are equivalent if they result in the same resultant force and the same resultant moment.

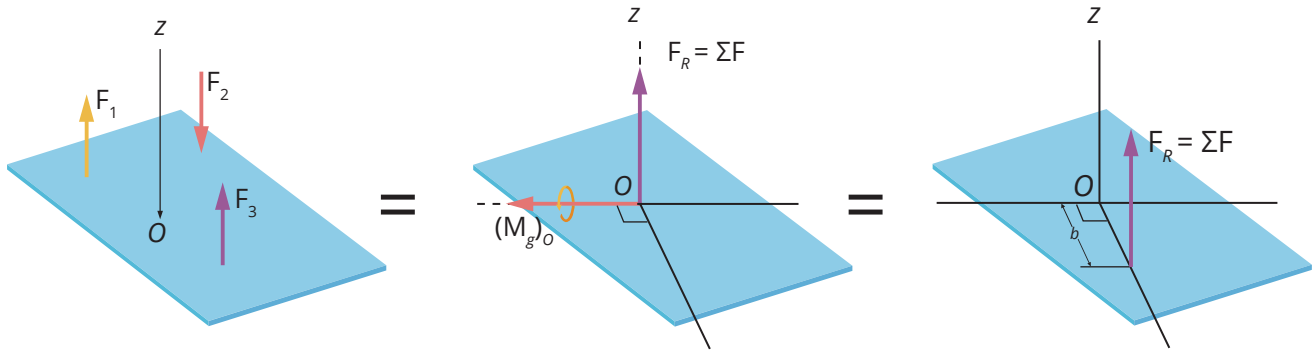
$$F \text{ for system 1} = F \text{ for system 2}$$

$$M_O \text{ for system 1} = M_O \text{ for system 2}$$

— The two force systems are equivalent



Three cases below represent equivalent force system.



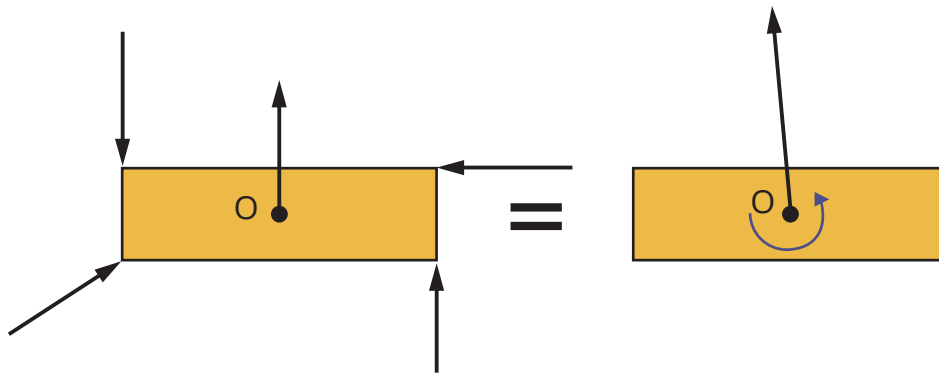
Consider 3 force acting on a rectangular plate

3 forces are replaced by a single force and a moment about point O .

3 forces are replaced by a single force at point P



Any set of forces on a body can be replaced by a single force and a single couple acting that is statically equivalent to the original set of forces and moments. This set of an equivalent force and a couple is known as the equivalent force couple system.



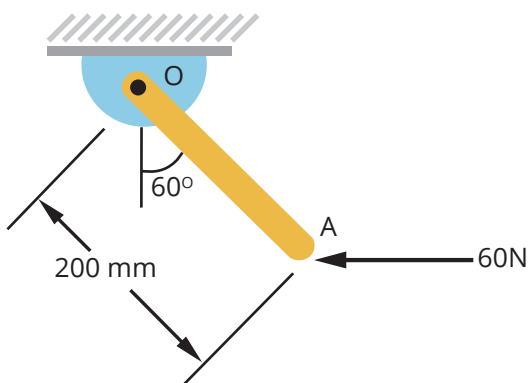
Exercise - 1.4

Equivalent Force Couple System

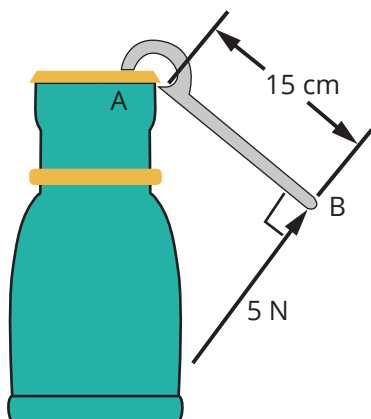
Scan using
FORTFLAG app
for detailed
solution.



Q 1. The 60-N force acts at point A on the lever as shown. Replace the force at A by a force and couple moment acting at point O that will have an equivalent effect.

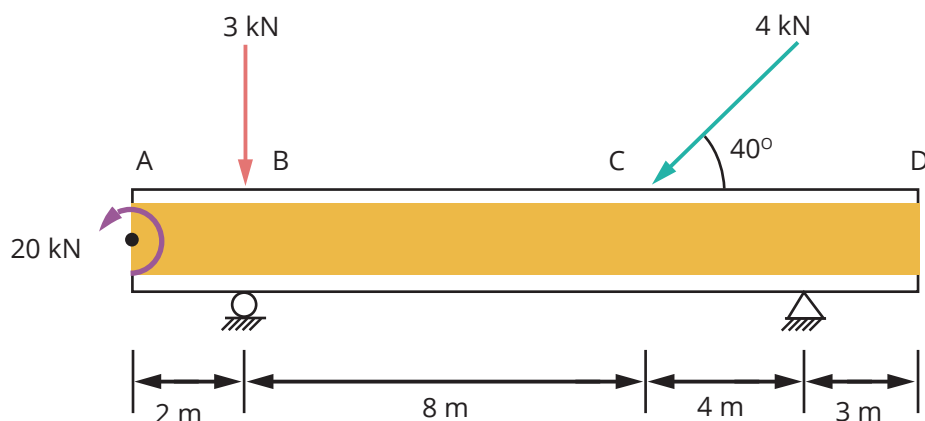


- Q 2.** Replace the 5 N force acting on the end of the bottleopener by an equivalent force and couple moment acting on the underside of the bottle cap at A. Use your results to explain how a bottle opener works.

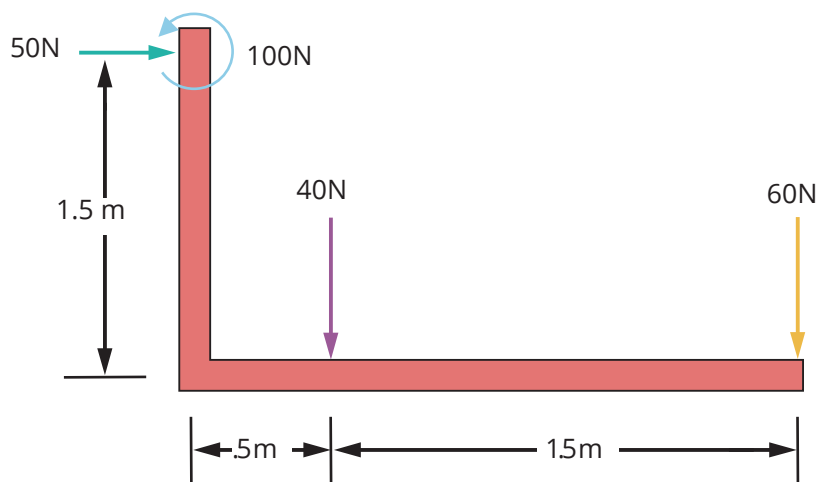


- Q 3.** A force system consists of a clockwise couple of $480 \text{ N}\cdot\text{m}$ plus a 240 N force directed up to the right through the origin of X and Y axes at $\theta_x = 30^\circ$. Replace the given system by an equivalent single force and compute the intercepts its line of action with the X and Y axes.

- Q 4.** Replace the forces and couple moment by a single force and specify where it acts.



- Q 5.** Find the equivalent force couple system for the forces shown below about point A.

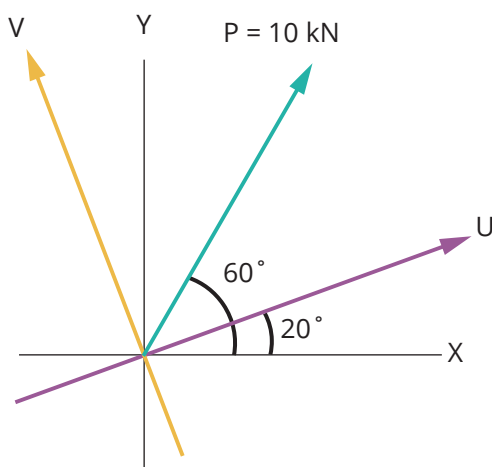


Assignment - 1

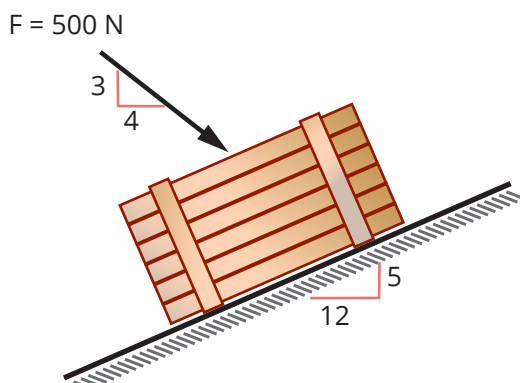
1. Define the following

- Non-concurrent force system
- Law of transmissibility of force
- Varignon's theorem
- equivalent force couple system

2. Find the components in the x, y, u and v directions of the force $P = 10 \text{ kN}$ shown in Fig.

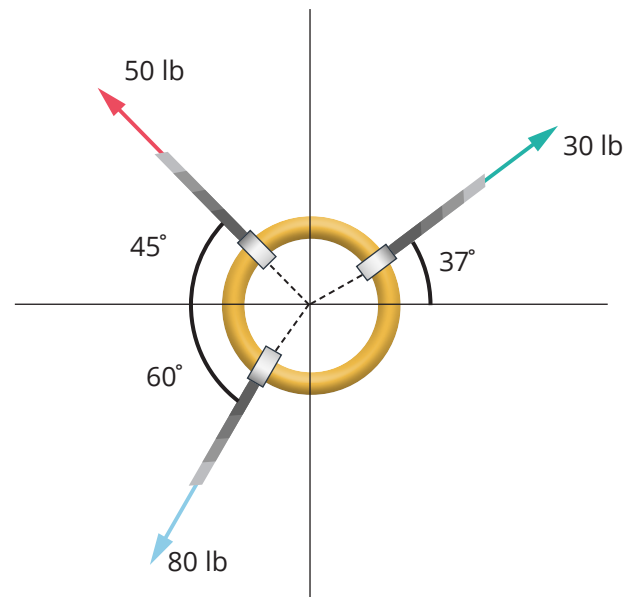


3. A block is resting on an incline of slope 5:12 as shown in Fig. P-007. It is subjected to a force $F = 500 \text{ N}$ on a slope of 3:4. Determine the components of F parallel and perpendicular to the incline.

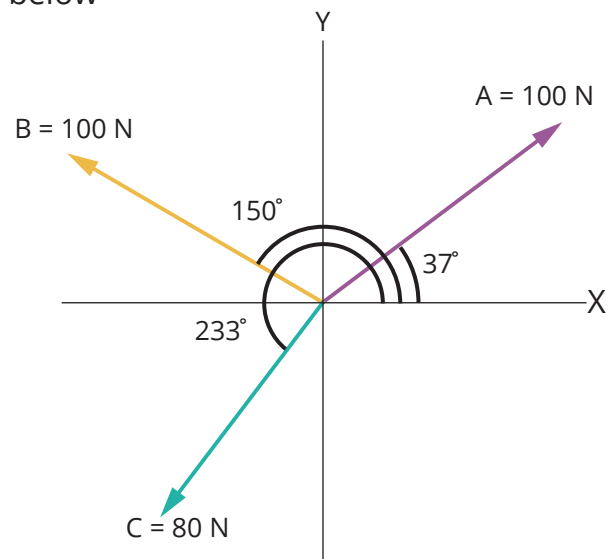


@@The angle between the two concurrent forces is 90° and their resultant is 2500 N . The resultant makes an angle of 46° with one of the forces, determine the magnitude of each force.

4. Three ropes are tied to a small metal ring. At the end of each rope three students are pulling, each trying to move the ring in their direction. If we look down from above, the forces and directions they are applying are shown in Fig. Find the net force on the ring due to the three applied forces.

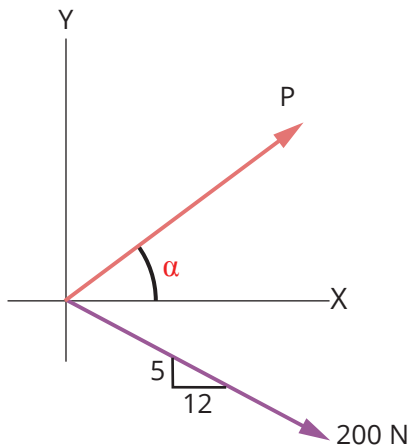


5. Three vectors A, B, and C are shown in the figure below. Find one vector (magnitude and direction) that will have the same effect as the three vectors shown in Fig. below

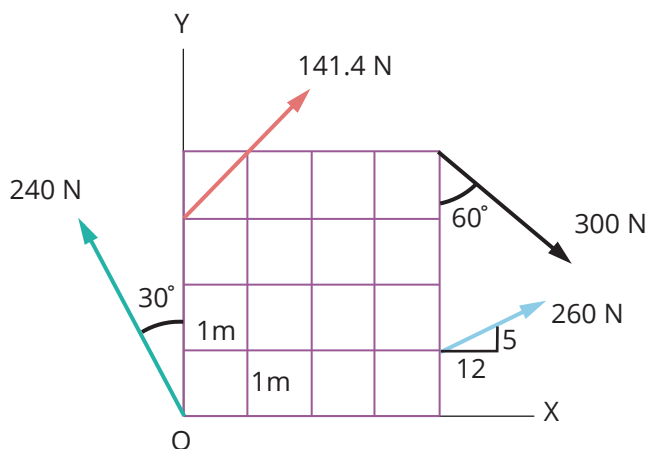


6. From Fig., P is directed at an angle α from x -axis and the 200 N force is acting at a slope of 5 vertical to 12 horizontal.

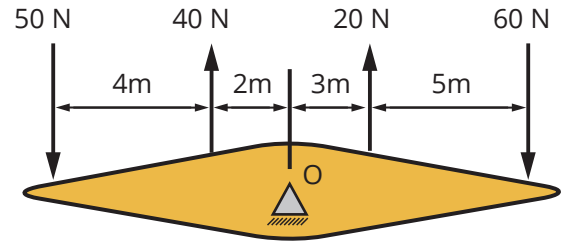
- Find P and α if the resultant is 500 N to the right along the x -axis.
- Find P and α if the resultant is 500 N upward to the right with a slope of 3 horizontal to 4 vertical.
- Find P and α if the resultant is zero.



7. Completely determine the resultant with respect to point O of the force system shown in Fig.



8. Determine the resultant of the four parallel forces acting on the rocker arm of Fig.



9. Replace the force at A by an equivalent force and couple moment at point O .

