Exercise 2

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1

We can look at three core cases: when correlation between variables is big and positive, big and negative and when there are no obvious correlation. From the Figure 1 we can easily observe, that when we change correlation between two variables the bivariate distribution changes respectively. If $\mathrm{corr}(x_1, x_2) = 1$ – observations are grouped around line $x_1 = x_2$.

2

R for number of simulations; number of reps is number of reps.

```
> r<-1000
> number_of_reps <- c(5, 15, 50, 5000)
```

Bernoulli:

```
> mean_ber <- matrix(0, nrow = r, ncol = 4)
> for (j in 1:r){
+   for (i in 1:4){
+     gen_ber <- rbinom(number_of_reps[i], 1, 1/3)
+     mean_ber[j,i] <-   sum(gen_ber, na.rm = FALSE)/number_of_reps[i]
+   }
+ }
> plot(density(mean_ber[,1]))
> plot(density(mean_ber[,2]))
> plot(density(mean_ber[,3]))
> plot(density(mean_ber[,4]))
```

Binomial:

```
> mean_binom <- matrix(0, nrow = r, ncol = 4)
> for (j in 1:r){
+   for (i in 1:4){
+     gen_binom <- rbinom(number_of_reps[i], 5, 1/3)
+     mean_binom[j,i] <-   sum(gen_binom, na.rm = FALSE)/number_of_reps[i]
```

```
+   }
+ }
> plot(density(mean_binom[,1]))
> plot(density(mean_binom[,2]))
> plot(density(mean_binom[,3]))
> plot(density(mean_binom[,4]))
```

Cauchy:

```
> mean_cauchy <- matrix(0, nrow = r, ncol = 4)
> for (j in 1:r){
+   for(i in 1:4){
+     gen_cauchy <- rcauchy(number_of_reps[i])
+     mean_cauchy[j,i] <- sum(gen_cauchy, na.rm = FALSE)/number_of_reps[i]
+   }
+ }
> plot(density(mean_cauchy[,1]))
> plot(density(mean_cauchy[,2]))
> plot(density(mean_cauchy[,3]))
> plot(density(mean_cauchy[,4]))
```

Out of Figures 2, 3 we can observe that the more observations we have in our sample density, the more likely sample mean behave normally. The Cauchy, as it was already said is counter example – its tails are wide enough so it has neither mean nor variance.

3

Matr - initial matrix. This is triangular matrix with determinant equals to 1, so only "a" element changes dering inversion. We can observe it in the following exercise:

```
> Matr <- matrix(c(1,0,100500,1), nrow = 2, ncol = 2)
> a = c(seq(0,100, by=10))
> for (i in a){
+   Matr[1,2] <- i
+   print(solve(Matr))
+ }
```

```
     [,1] [,2]
[1,]    1    0
[2,]    0    1
     [,1] [,2]
[1,]    1  -10
[2,]    0    1
     [,1] [,2]
[1,]    1  -20
```

```
[2,]    0    1
     [,1] [,2]
[1,]    1  -30
[2,]    0    1
     [,1] [,2]
[1,]    1  -40
[2,]    0    1
     [,1] [,2]
[1,]    1  -50
[2,]    0    1
     [,1] [,2]
[1,]    1  -60
[2,]    0    1
     [,1] [,2]
[1,]    1  -70
[2,]    0    1
     [,1] [,2]
[1,]    1  -80
[2,]    0    1
     [,1] [,2]
[1,]    1  -90
[2,]    0    1
     [,1] [,2]
[1,]    1 -100
[2,]    0    1
```

If "a" is very big:

```
> solve(matrix(c(1,0,100000,1), nrow = 2, ncol = 2))
```

```
     [,1]    [,2]
[1,]    1 -1e+05
[2,]    0  1e+00
```

4

a)

```
> M1 <- matrix(c(1,2,3,1), nrow = 2, ncol = 2)
> solve(M1)
```

```
     [,1] [,2]
[1,] -0.2  0.6
[2,]  0.4 -0.2
```

Yes.
b)

```
> M2 <- matrix(c(1,2,-1,-2), nrow = 2, ncol = 2)
```

No.
c)

```
> M3 <- matrix(c(1,2,2,1,1,0), nrow = 3, ncol = 2)
> N <-matrix(c(0,2,1,3,1,4), nrow = 2, ncol = 3)
> MN <- M3%*% N
> MN

     [,1] [,2] [,3]
[1,]    2    4    5
[2,]    2    5    6
[3,]    0    2    2
```

d)

$$Tr = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

e)

```
> Tr <- matrix(c(1,1,1,0,2,-1), nrow = 2, ncol = 3)
> Tr

     [,1] [,2] [,3]
[1,]    1    1    2
[2,]    1    0   -1

> tTr <- t(Tr)
> tTr

     [,1] [,2]
[1,]    1    1
[2,]    1    0
[3,]    2   -1
```

f)

```
> v <- c(1,2)
> A <- matrix(c(1,2,0,1),ncol = 2, nrow = 2)
> t(v)%*%A%*%v

     [,1]
[1,]    9
```

g)

```
> Z <- matrix(c(1,2,3,1,0,0,0,1,0), nrow = 3, ncol = 3)
> rank(Z)
```

```
[1] 6.0 8.0 9.0 6.0 2.5 2.5 2.5 6.0 2.5
```

```
> solve(Z)
```

```
     [,1] [,2]       [,3]
[1,]    0    0  0.3333333
[2,]    1    0 -0.3333333
[3,]    0    1 -0.6666667
```

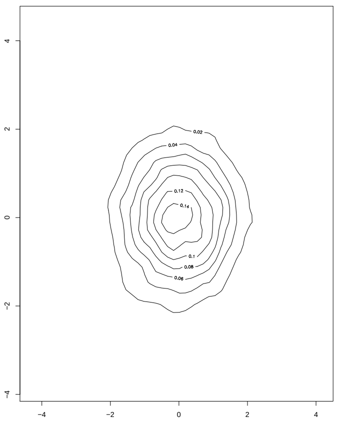
h)

```
> solve(Z)
```

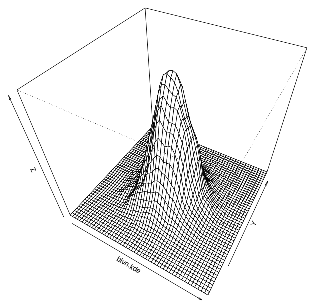
```
     [,1] [,2]       [,3]
[1,]    0    0  0.3333333
[2,]    1    0 -0.3333333
[3,]    0    1 -0.6666667
```

```
> solve(Z)%*%Z
```

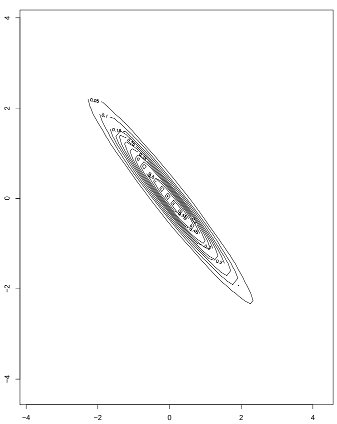
```
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```



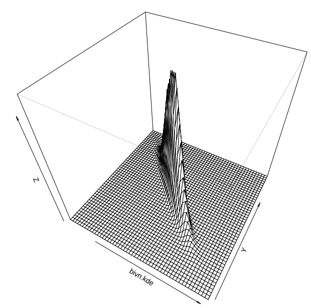
(a) $\mathrm{C}(x_1, x_2) = 0$



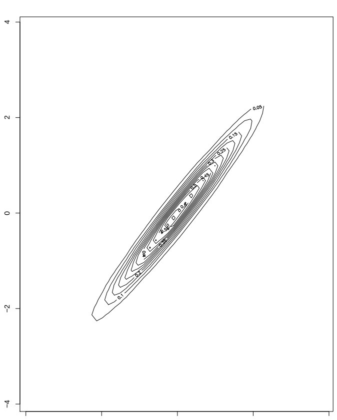
(b) $\mathrm{C}(x_1, x_2) = 0$; 3D



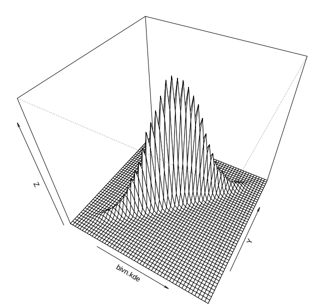
(c) $\mathrm{C}(x_1, x_2) =$ -1



(d) $\mathrm{C}(x_1, x_2) =$ -1, 3D

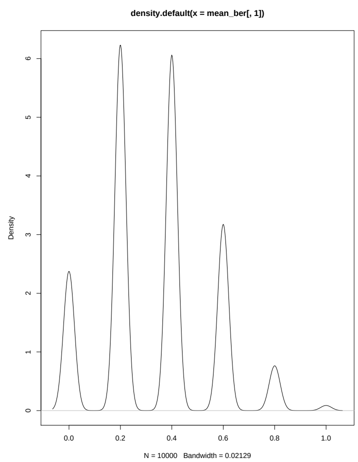


(e) $\mathrm{C}(x_1, x_2) = 1$

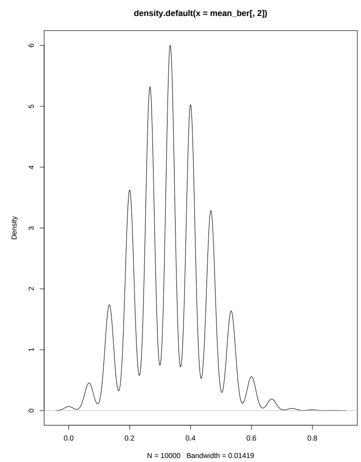


(f) $\mathrm{C}(x_1, x_2) = 1$, 3D

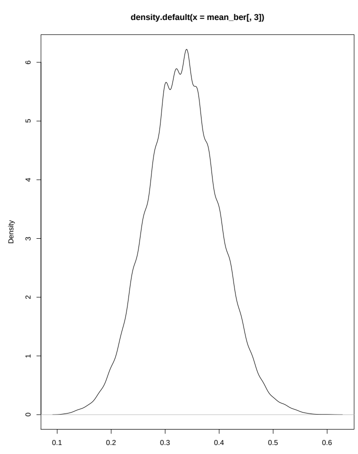
Figure 1: Different correlations



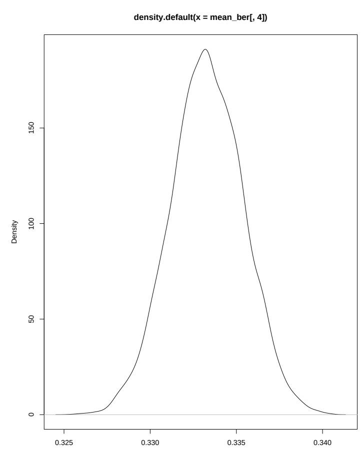
(a) Bernoulli, 5 reps



(b) Bernoulli, 15 reps

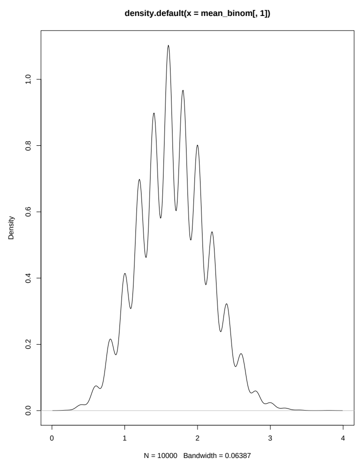


(c) Bernoulli, 50 reps

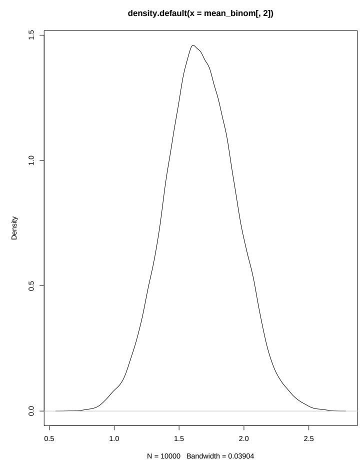


(d) Bernoulli, 5000 reps

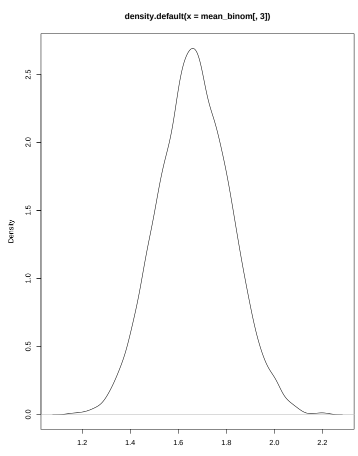
Figure 2: Bernoulli



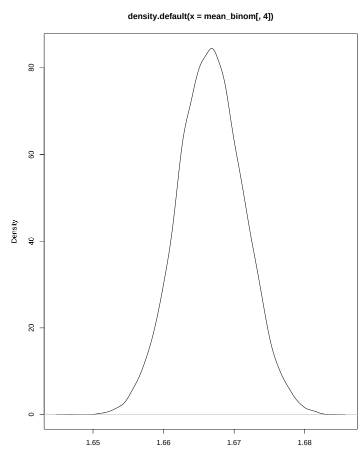
(a) Binomial, 5 reps



(b) Binomial, 15 reps

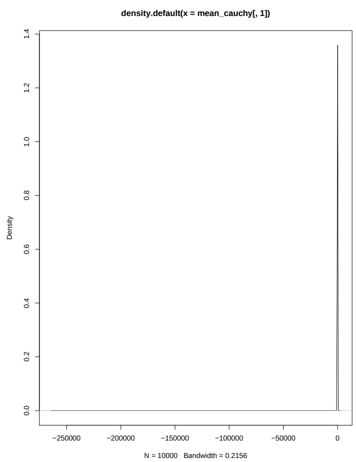


(c) Binomial, 50 reps

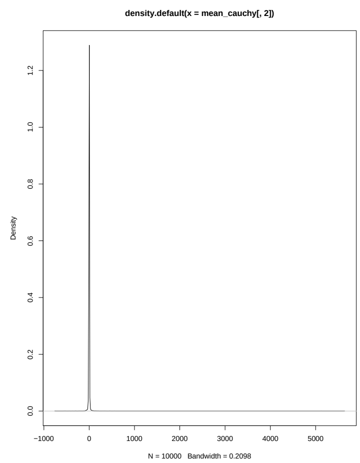


(d) Binomial, 5000 reps

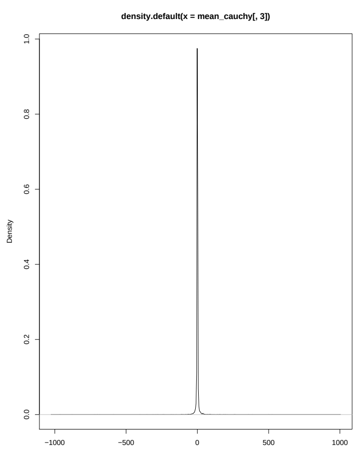
Figure 3: Binomial



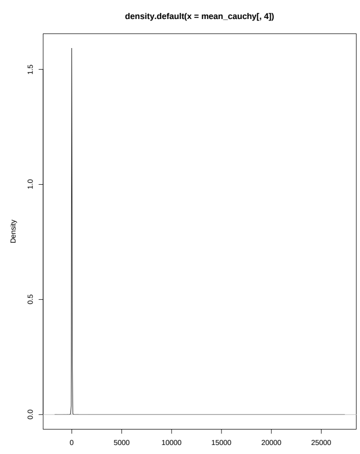
(a) Cauchy, 5 reps



(b) Cauchy, 15 reps



(c) Cauchy, 50 reps



(d) Cauchy, 5000 reps

Figure 4: Cauchy