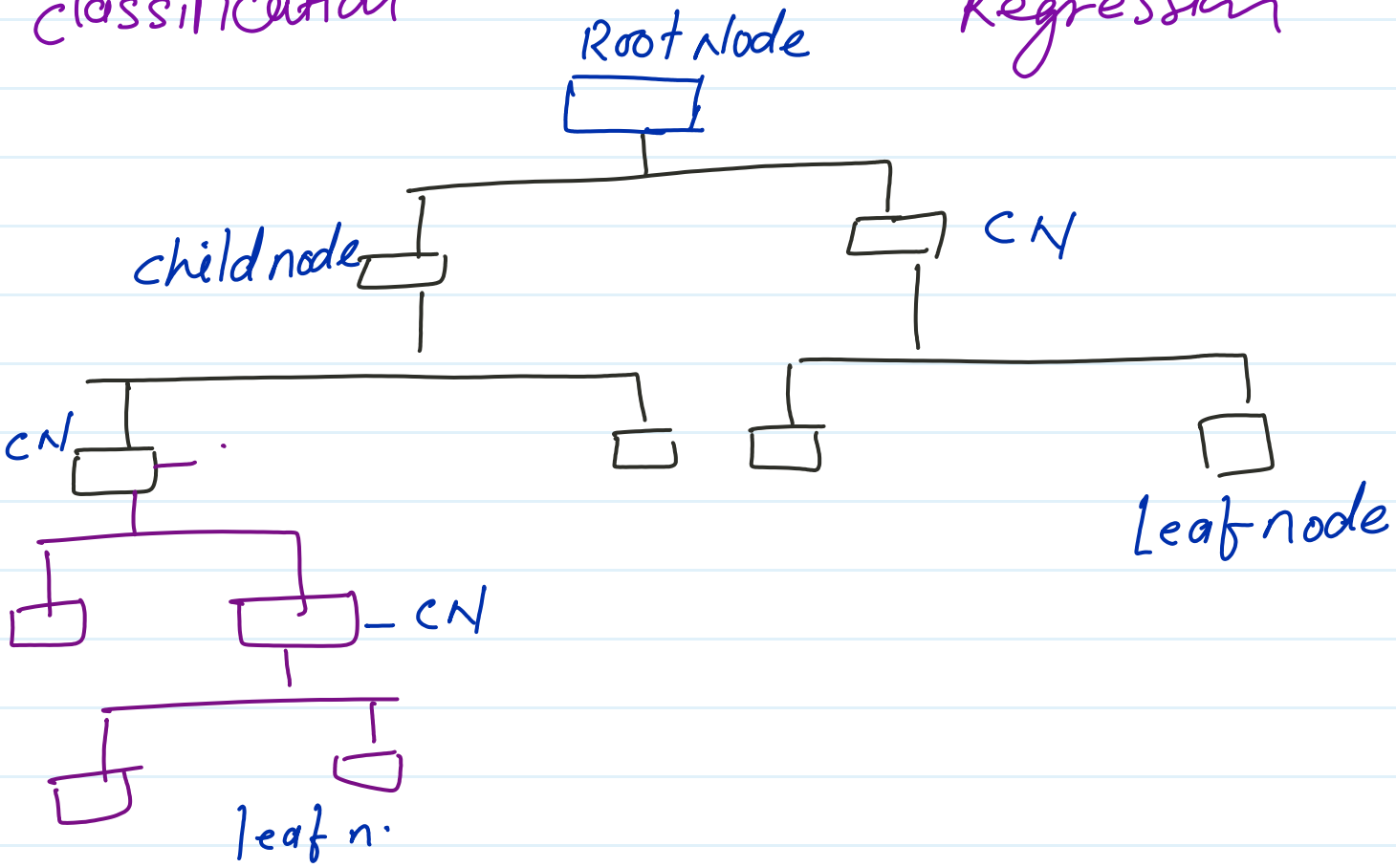
Decision tree

- classification
- Regression



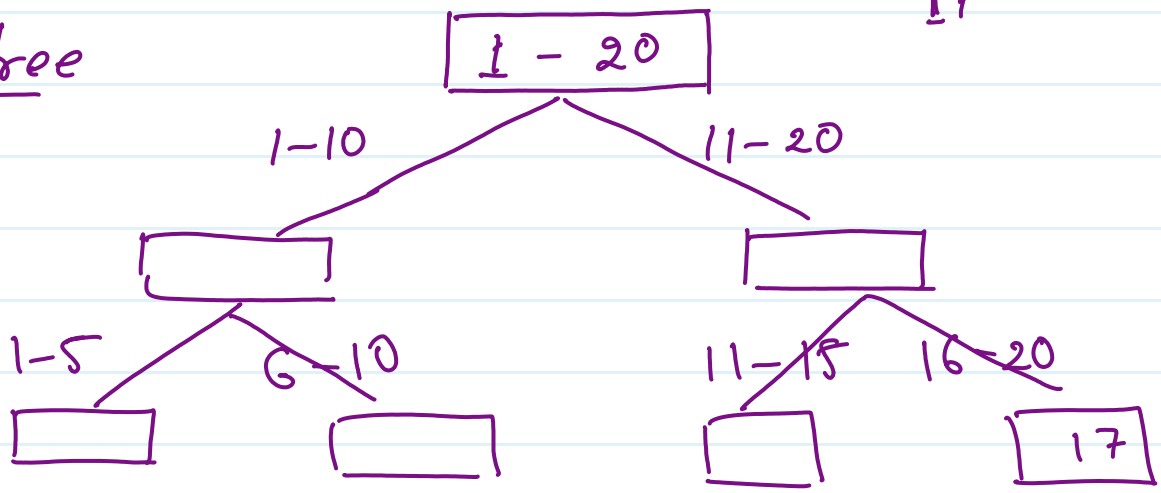
method of solve decision Tree algorithm –

1. ID3 — Entropy, Information Gain
2. Cart — Gini Index

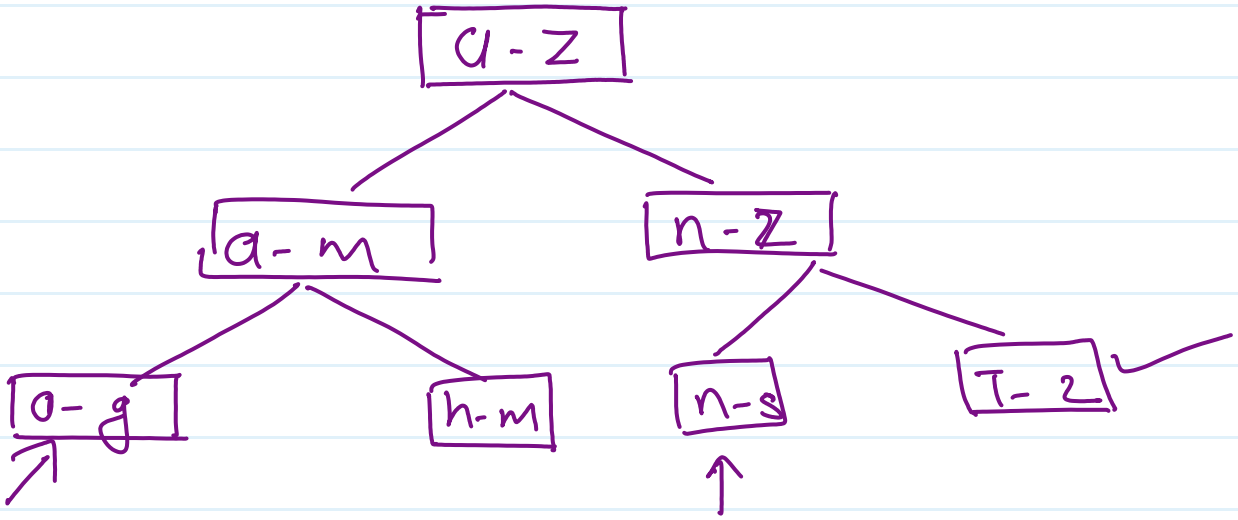
Dataset [5, 6, 3, 11, 10, 2, 3, 7, 8]

17

Binary tree



[Red, yellow, purple, blue, white, green, black]



Entropy OR Gini Index

Information Gain

✡ In DT we do not required to label categorical data into numerical

x_1 x_2 x_3

✡ Entropy and Gini Index → purity split in dataset

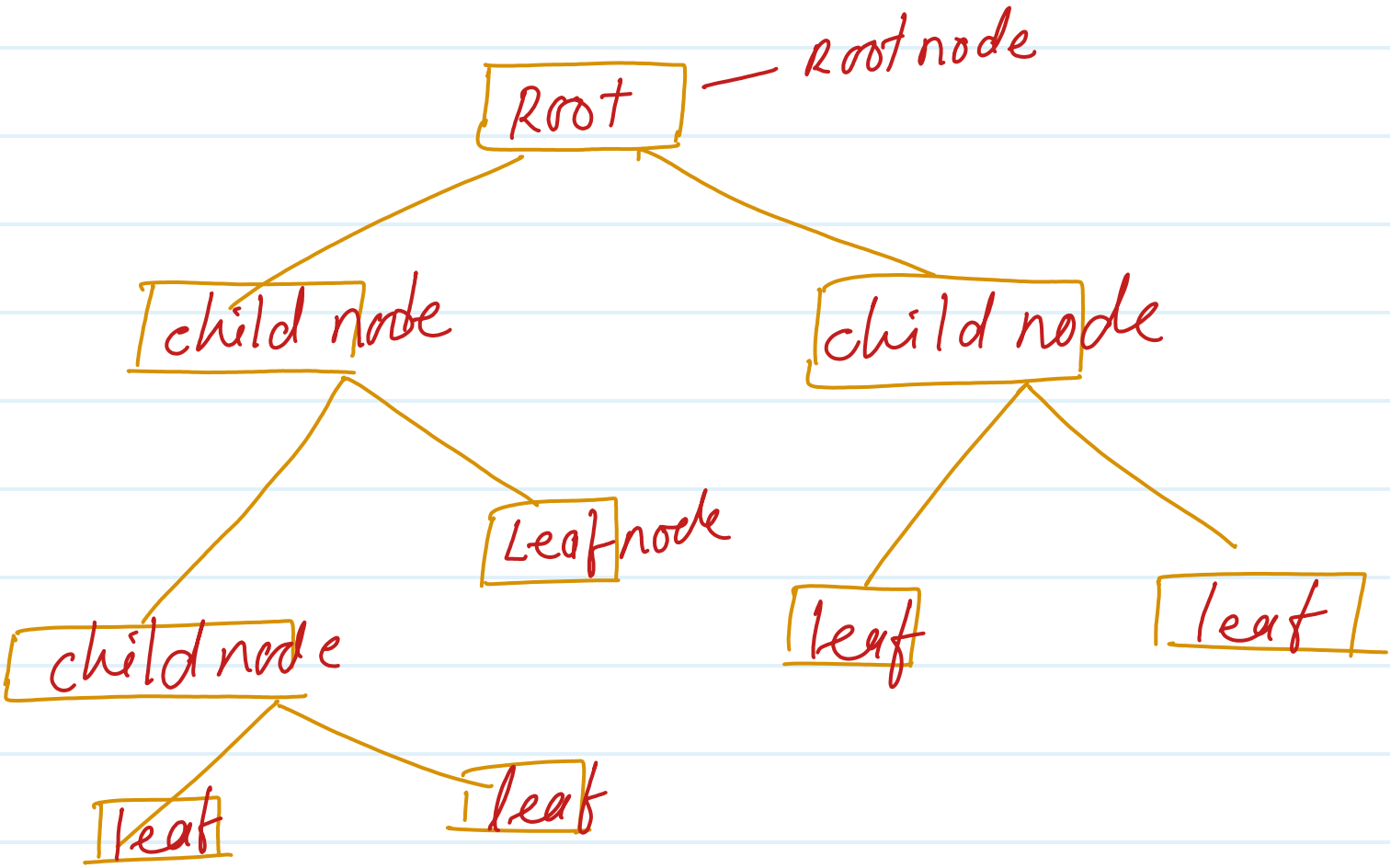
✡ Information Gain → DT feature split

class:	weight	height	o/p	obese/Noobese.
	60	160	ob	
	70	170	NO	
	80	180	ob	
	90	190	NO	
	100	200	NO	

(11) DT Regresser

Regression we use standard deviation / MSE / MAE

weight	height	BMI
60	160	21
70	170	22
80	180	20
85	190	23
90	195	24

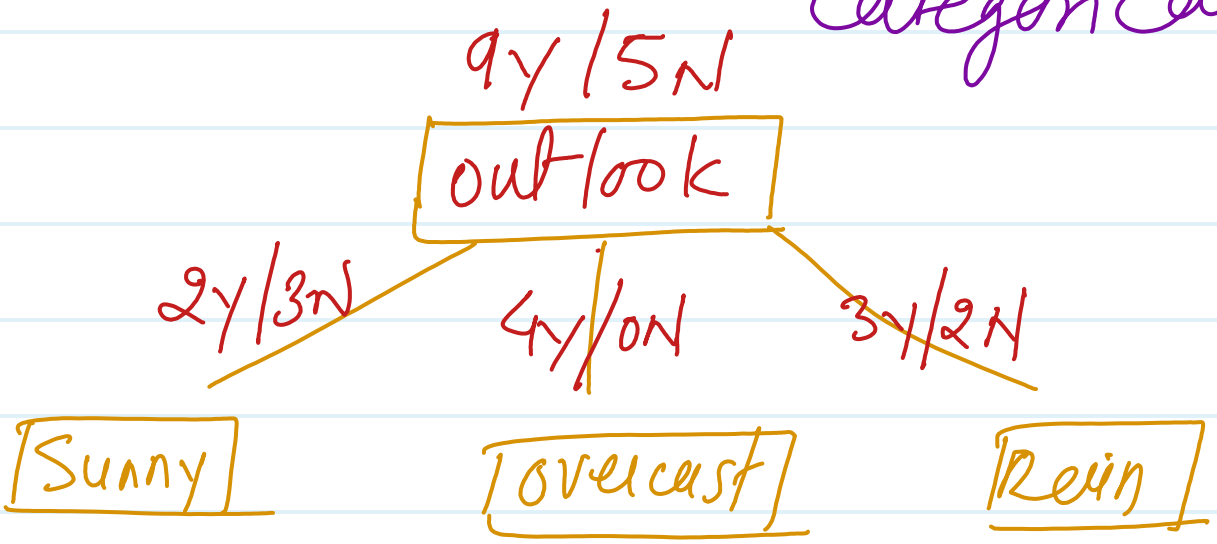


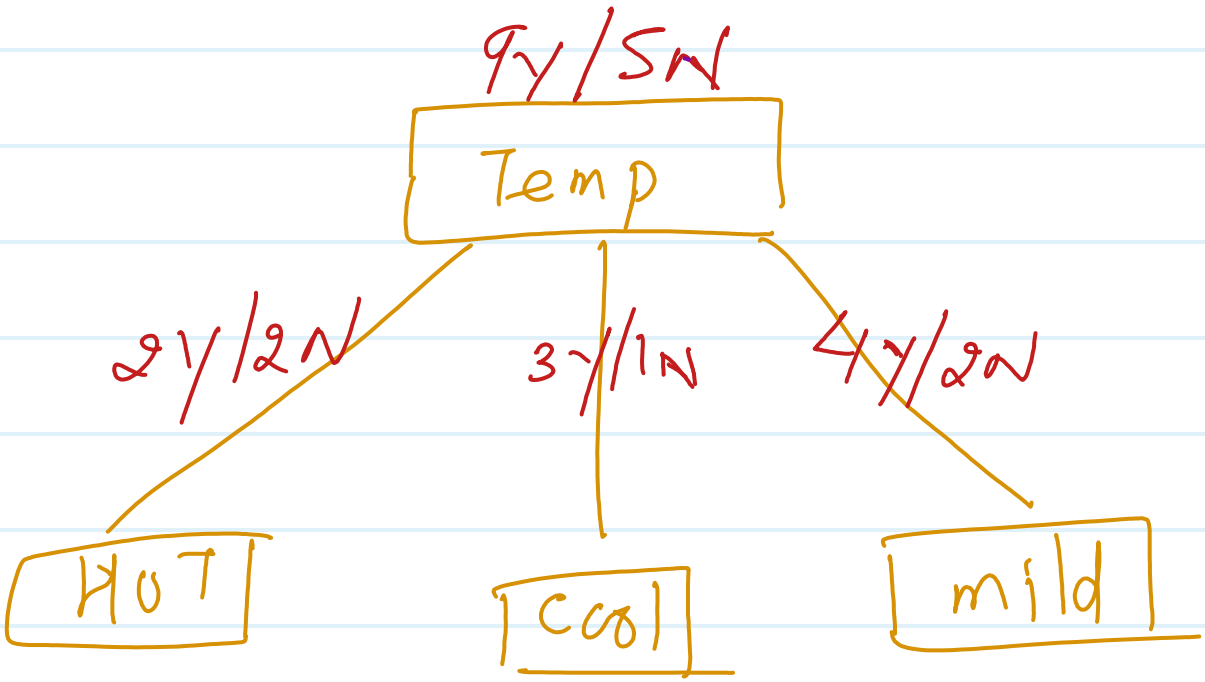
Decision Tree Classifier

Tennis

outlook	Temp	humidity	wind	play
sunny	H	High	weak	N
Sunny	H	H	strong	N
overcast	H	H	W	Y
rain	M	H	W	Y
rain	C	Normal	W	Y
rain	C	N	S	N
overcast	C	N	W	Y
sunny	M	H	W	N
Sunny	C	N	W	Y
rain	M	N	W	Y
Sunny	M	N	S	Y
overcast	M	high	S	Y
overcast	H	N	W	Y
rain	M	H	S	N

- Feature can be numeric and categorical
- Output can be numeric and categorical





ⓐ Entropy $H(S)$

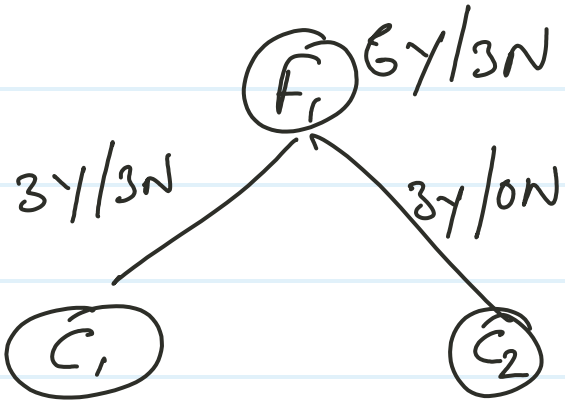
⇒ Formula (Binary class)

$$H(S) = -P_{yes} \log_2(P_{yes}) - P_{No} \log_e(P_{No})$$

multiclass

$$H(S) = -P_{C_1} \log_2(P_{C_1}) - P_{C_2} \log_2(P_{C_2}) - P_{C_3} \log_2(P_{C_3})$$

Example



F_1 F_2 F_3 o/p

$$C_1 \Rightarrow H(S) = -\frac{3}{6} \log\left(\frac{3}{6}\right) - \frac{3}{6} \log \frac{3}{6}$$

$\Rightarrow$ 1 impure split

$$C_2 \Rightarrow H(S) = -\frac{3}{3} \log \frac{3}{3} - \frac{0}{3} \log \frac{0}{3}$$

$\Rightarrow$ 0 pure split

For the pure split of feature

pure entropy should be zero (0)

for impure split = 1

⑤ Gini index (Impurities)

main formula -

$$G.I. = 1 - \sum_{i=1}^{n} (p)^2$$

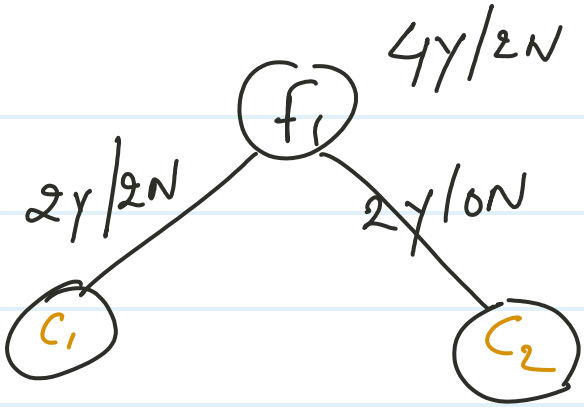
binary class.

$$G.I. = 1 - \sum_{i=1}^{n} \left[(P_{c_1})^2 + (P_{c_2})^2 \right]$$

multiclass

$$G.I. = 1 - \sum_{i=1}^{n} \left[(P_{c_1})^2 + (P_{c_2})^2 + (P_{c_3})^2 + \cdots \right]$$

Example



$C_1 \Rightarrow G.I. = 1 - \left[\left(\frac{2}{4}\right)^2 + \left(\frac{2}{4}\right)^2\right]$

$= 0.5$ ✓

$C_2 \Rightarrow G.I. = 1 - \left[\left(\frac{2}{2}\right)^2 + \left(\frac{0}{2}\right)^2\right]$

$= 0$ ✓

Range of entropy = 0 to 1

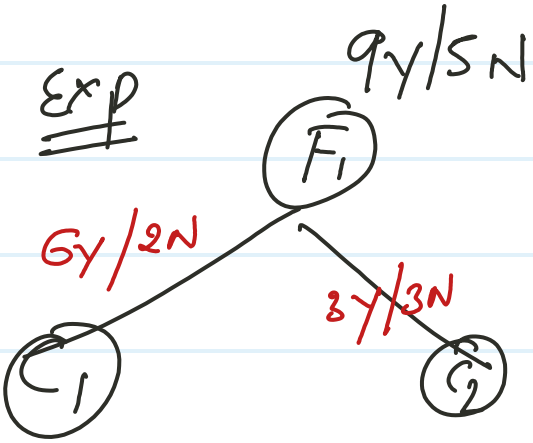
Gini impurity (index) = 0 to 0.5

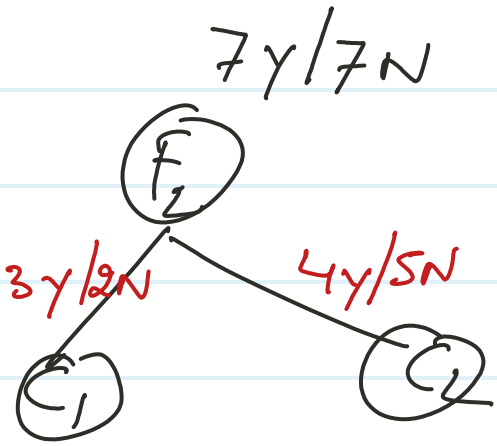
(c) Information Gain

formula –

$$gain(S, f_i) = H(S) - \sum_{i=1}^{n} \frac{|S_v|}{|S|} H(S_v)$$

Exp





⇒ For F_1 ⇒

$$H(S) = -\frac{9}{14}\log\frac{9}{14} - \frac{5}{14}\log\frac{5}{14}$$

$$\boxed{H(S) = 0.94}$$

$$C_1 \Rightarrow H(S) = -\frac{6}{8}\log\frac{6}{8} - \frac{2}{8}\log\frac{2}{8}$$

$$\boxed{H(S) = 0.81}$$

$C_2 \Rightarrow$

$$H(S) = -\frac{3}{3}\log\frac{3}{3} - \frac{0}{3}\log\frac{0}{3}$$

$$\boxed{H(S) = 1}$$

gain of $F_1 \Rightarrow$

$$\text{gain}(S, F_1) = 0.94 - \left[\frac{8}{14} \times 0.81 + \frac{6}{14} \times 1\right]$$

$$\simeq \text{gain}(S, F_1) = 0.049$$

$F_2 \rightarrow H(S) = -\frac{7}{7}\log\frac{7}{7} - \frac{7}{7}\log\frac{7}{7}$

$$= 0$$

$C_1 \rightarrow H(S) = -\frac{3}{5}\log\frac{3}{5} - \frac{2}{5}\log\frac{2}{5}$

$$= 0133 + 0.159$$

$$\boxed{= 0.29}$$

$$C_2 \Rightarrow H(S) = -\frac{4}{9}\log\frac{4}{9} - \frac{5}{9}\log\frac{5}{9}$$

$$= 0.014$$

$$f_2 \text{ gain}(S, F_2) = 0 - \left[\frac{5}{14} \times 0.29 + \frac{9}{14} \times 0.019\right]$$

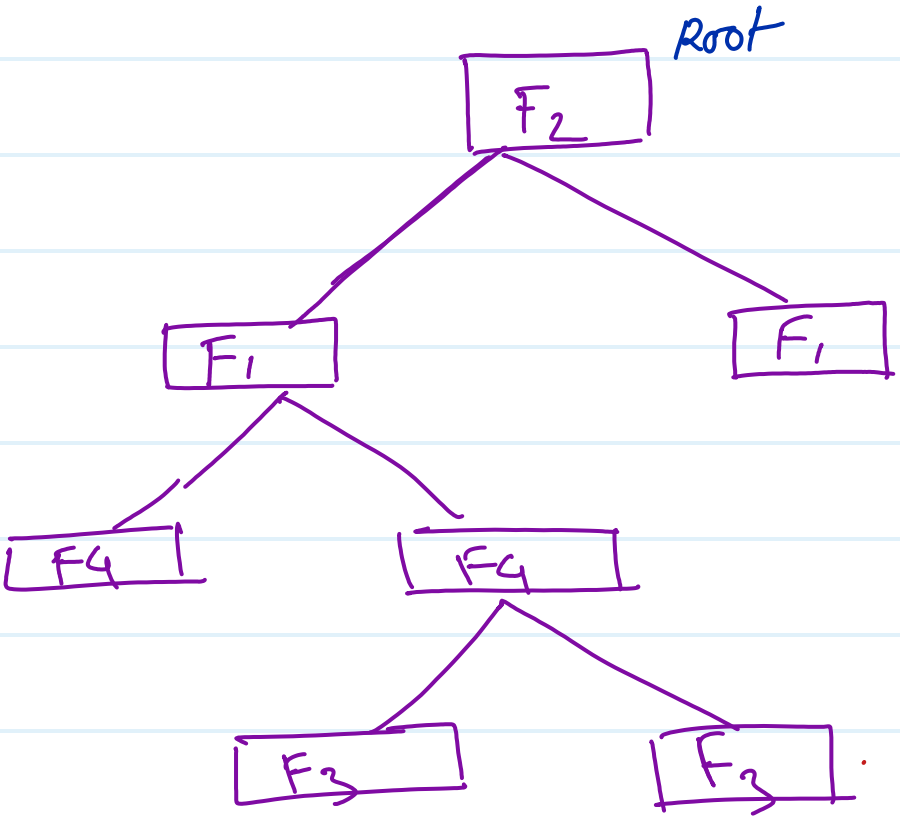
$$= 0 - [0.10 + 0.009]$$

$$= -0.10$$

F_1	F_2	F_3	F_4
0.49	0.56	0.025	0.10

Since F_2 has highes value of information gain's among the all feature so that it will be our root node.

F_1	F_2	F_3	F_4
0.49	0.56	0.025	0.10



=

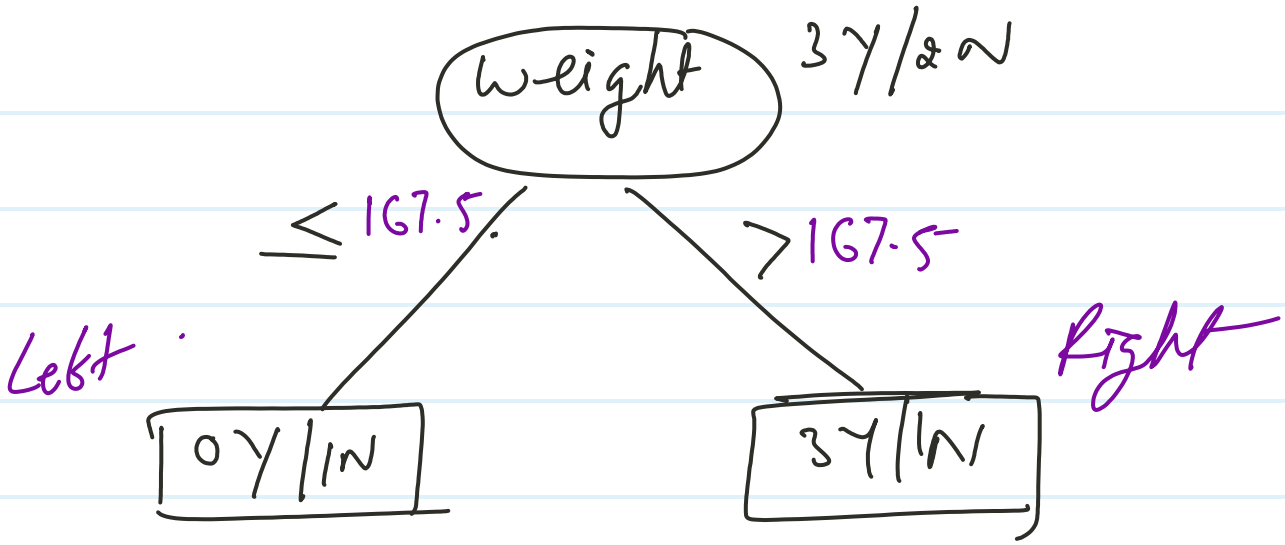
✡ Independent analysis befor making DT

build DT with numerical feature

weight	heart De.
220	Y
180	Y
225	Y
190	N
155	N

weight		Heart
155		N
	167.5	
180		Y
	185	
190		N
	204	
220		Y
	222.4	
225		Y

with respect to every point avg. value need to find out gini index / Entropy



$$\text{Gini impurity} = 1 - \sum_{i=1}^{n} p_i^2$$

$$\text{gini (Left)} = 0$$

$$\text{gini (Right)} = 1 - \left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]$$

$$= 0.375$$

$$\text{Information gain} = \text{G.I.[Root]} - \sum_{\text{Value}} \frac{|S_v|}{|S|} \text{G.I.[child]}$$

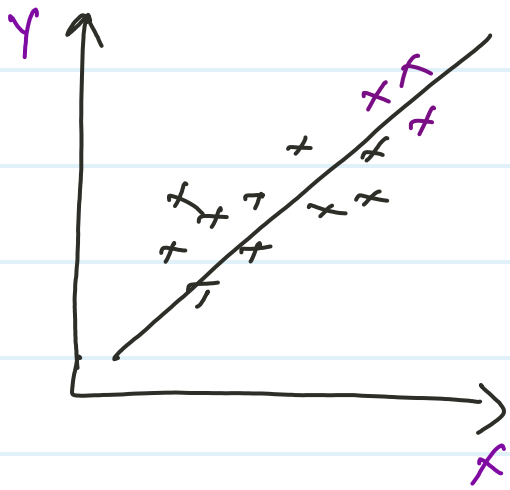
$$\text{G.I. [Root]} = 0.48$$

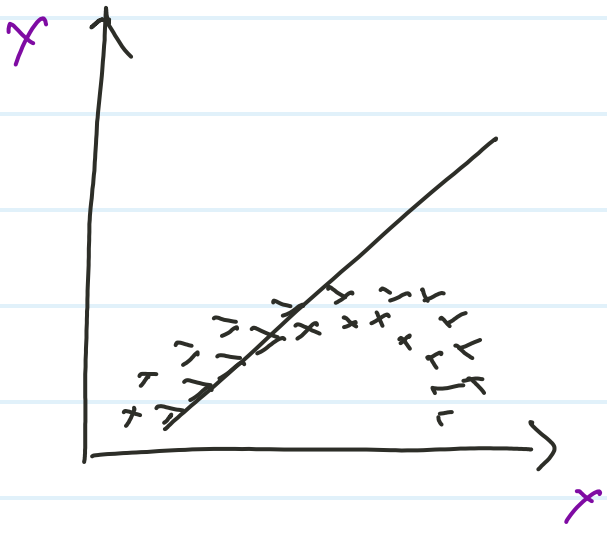
$$\text{I.G. [167.5]} = 0.48 - \left[\frac{1}{5} \times 0 + \frac{4}{5} \times 0.375\right]$$

$$\text{I.G. [167.5]} = 0.18$$

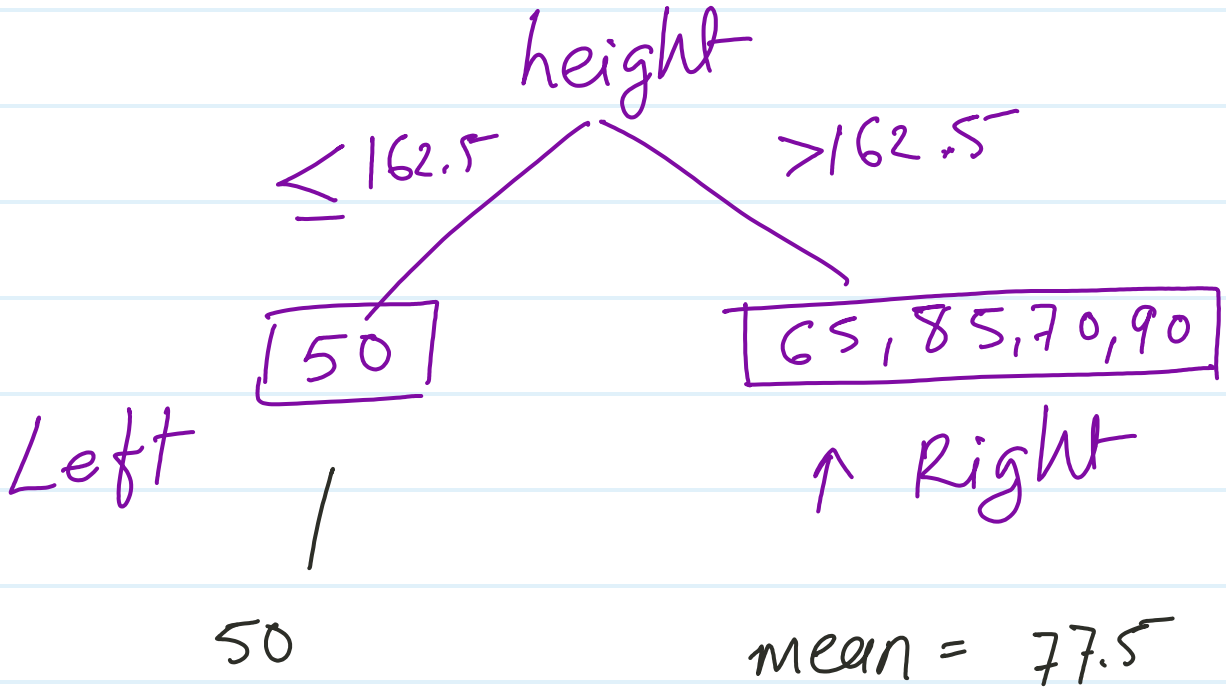
Information Gain should be high and Gini Index should be low.

DT Regression



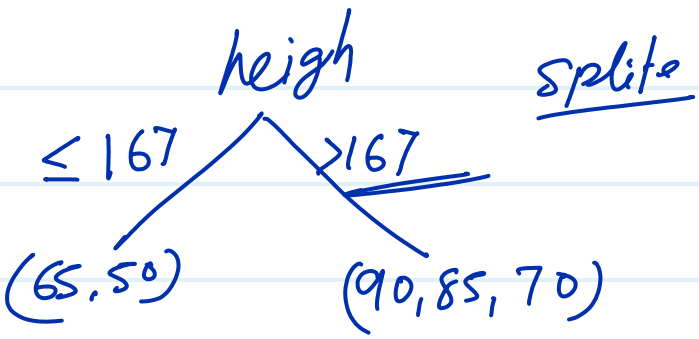


height	weight
160	50
162.5	
165	65
167.5	
170	85
172.5	
175	70
177.5	
180	90



50

mean = 77.5



Ex

	height	weight
162.5 <	165	65
	160	50
167.5 <	180	90
172.5 <	170	85
177.5 <	175	70

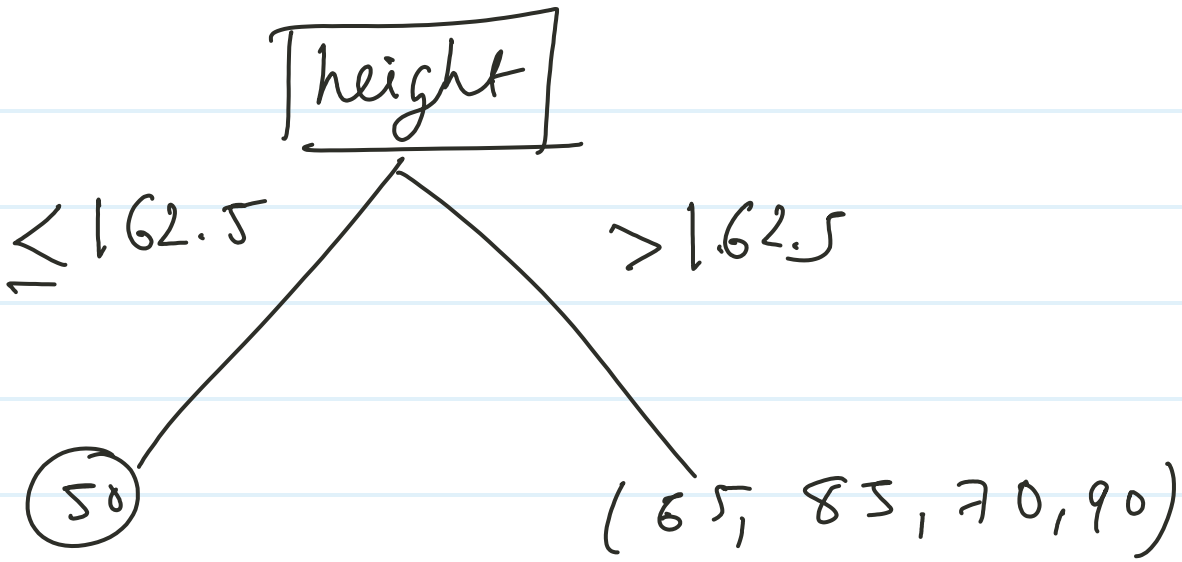
160 > 165 > 170 > 175 > 180

Regression problem weight calculated with respect to height

① Step - Short the value of height column (x feature)

② step - Find Adjcent Avg. value blw data point

③ step - Find Information gain with help of entropy and Gini Index.



mean = 77.5

Regression -

(1) mean = 77.5

(2) MSE, RMSE, MAE

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y-\hat{y})^2$$

overall mean = $\frac{50+65+85+70+90}{5} = 72.$

height (variance) = $\frac{(72-50)^2+(72-65)^2+(72-85)^2+(72-70)^2+(72-90)^2}{5}$

height (variance) = 206

$$var(Right) = \frac{(77.5-65)^2 + (77.5-85)^2 + (77.5-70)^2 + (77.5-90)^2}{4}$$

var (Right) = 106.25

var (left) = 50

Reduction in variance

$$= var(root) - \sum_{i=1}^{n} w_i \times var[child]$$

$$= 206 - \left[\frac{1}{5} \times 0 + \frac{4}{5} \times 106.25\right]$$

Reduction variance = 121

We calculate MSE for all the datapoint whichever is less will be threshold..

height (x_1)	gender (x_2)	weight (y)
160	M	65
165	F	70
170	M	80
175	M	90
180	F	100

From height / Gender, choose root node

for height mse = 55.5

for gender mse = 53

so value of gender mse is less it will be our root node. ✓

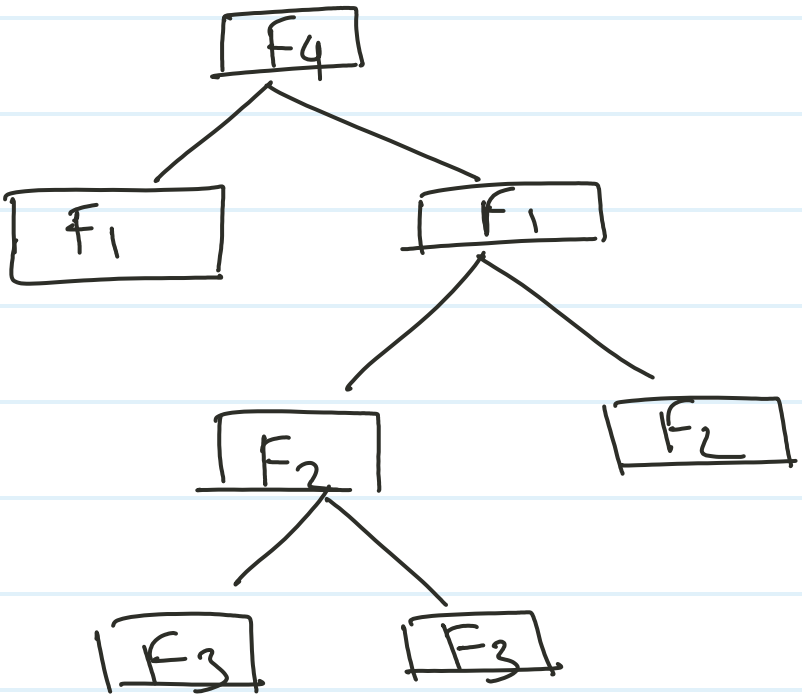
F_1	F_2	F_3	F_4	Target (y)
–	–	–	–	
–	–	–	–	
–	–	–	–	
–	–	–	–	
–	–			

$F_1 \Rightarrow$ MSE = 49

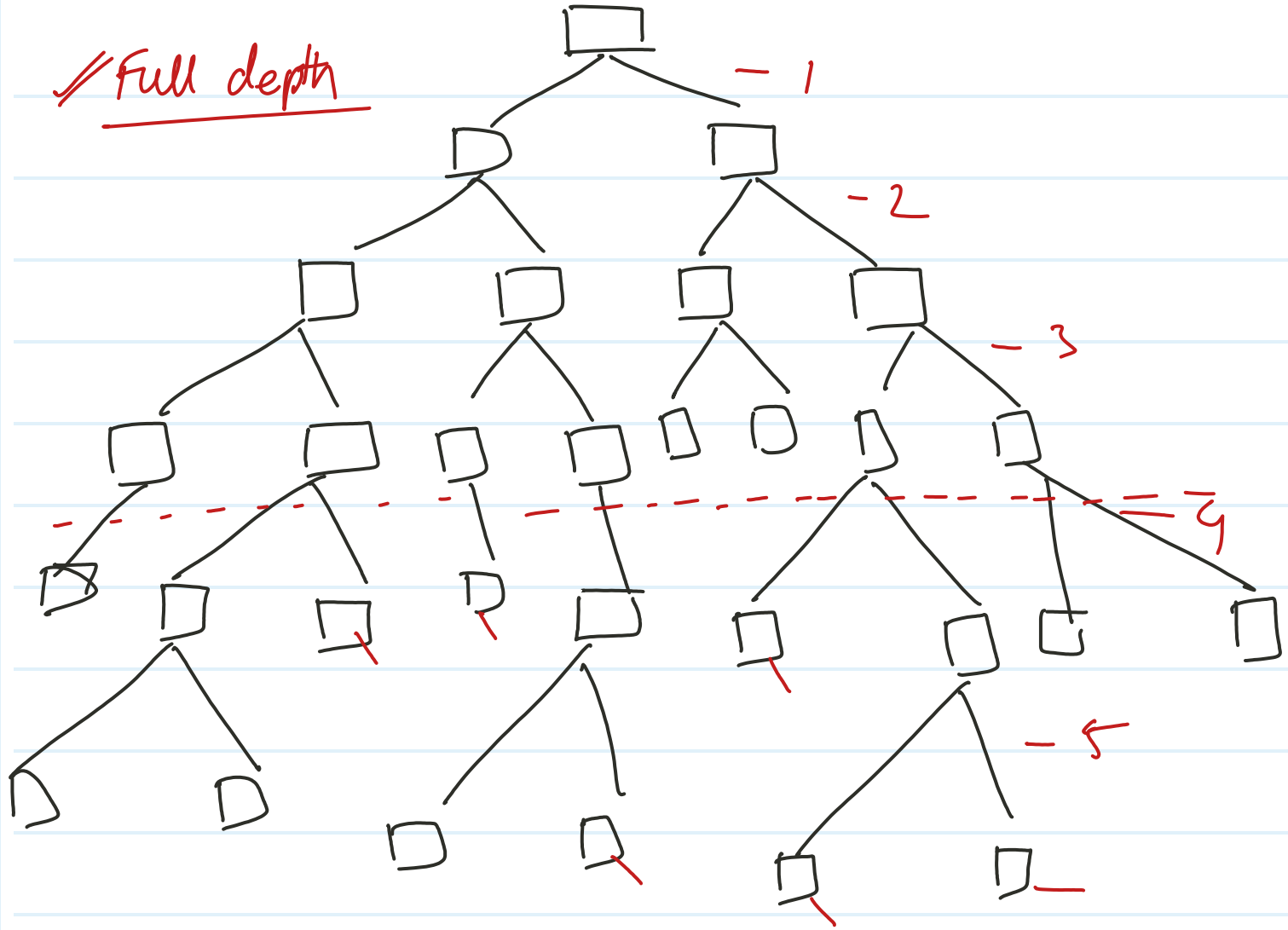
$F_2 \Rightarrow$ MSE = 55

$F_3 \Rightarrow$ MSE = 60

$F_4 \Rightarrow$ MSE = 43



// Full depth



Pre-pruning and post-pruning

max-depth	= 5	[0, 5, 6, 8, 10]
mini-sample-leaf	= 10	[5, 6, 8, 10, 12]
min-sample-split	= 8	[4, 6, 8, 10, 12]
max-feature	= 6	[2, 4, 6, 8, 10]

These 4 hyperparameters selected for pre-pruning before build DT. algorithms.

Post pruning =>

① make DT till end
② cut DT. using ccp value.
③ ccp value is nothing but threshold for gini / entropy.

ccp value is responsible for depth of Tree.. If ccp is less, the depth will be less.
High ccp value the depth will be more.

ccp = [0.4, 0.5, 0.6, 0.07]

For model training either we can use pre-pruning or post-pruning.

① When we have large dataset at this time we use pre-pruning.

② When we have small dataset at this time we use postpruning.

Why we use post or pre-pruning!?

⇒ To avoid model from overfitting.