VOILA! TILITY

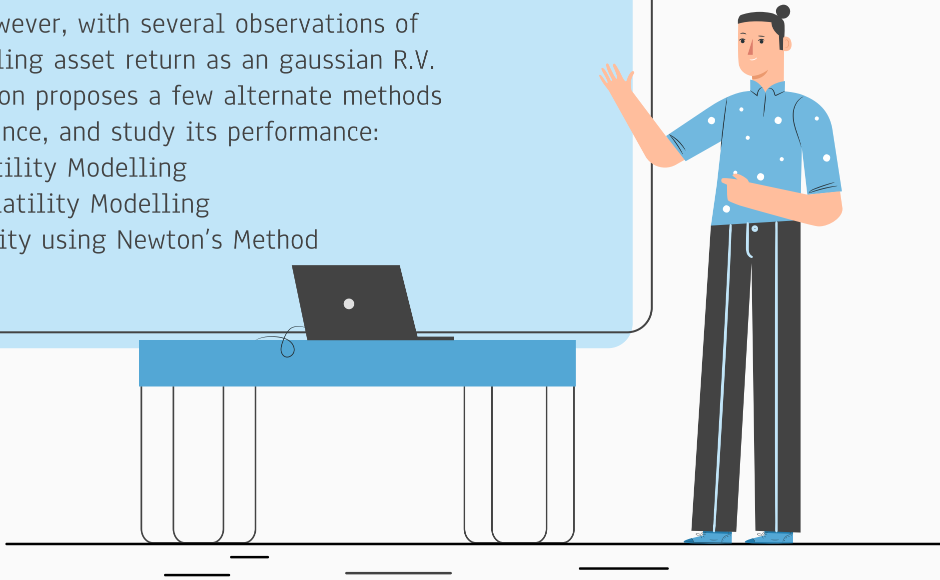
A proof-of-concept for alternate asset volatility estimator

Author: Jayson Cheung (SID 20868240)

INTRODUCTION AND MOTIVATION

By taking the i.i.d and gaussian assumption of log asset return, we can simply use the traditional unbiased estimators for mean and variance to model asset return in the next day. However, with several observations of real life log-return distributions, modelling asset return as an gaussian R.V. has its shortcoming. Thus this presentation proposes a few alternate methods in modelling the parameter of variance, and study its performance:

A. GARCH Volatility Modelling
B. Stochastic Volatility Modelling
C. Option-Implied Volatility using Newton's Method



GARCH(1,1) METHOD

$$\sigma_t^2 = \omega + \alpha_1 w_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$w_t = \sigma_t z_t$$

OBSERVED LIMITATION

- Volatility clustering: Volatility of the asset displays patterns and trend over time [1]
- Thus we shall consider the variance conditional to prior information
- Modelling volatility of tomorrow requires historical component

PROPOSED SOLUTION

- Using GARCH(1,1) to model volatility

Forecasted volatility contains the 1) conditional component and 2) autoregressive component

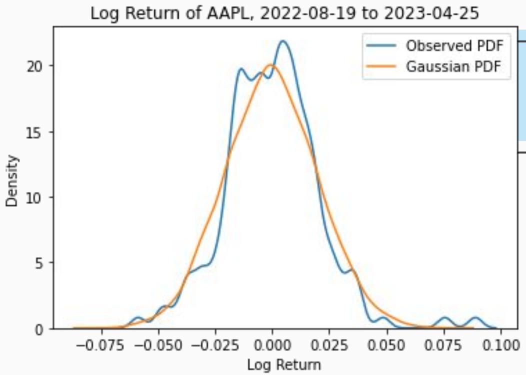
- Fitting of model: Maximum-Likelihood Estimator of parameters for alpha and beta

$$\ln L(\theta|\mathbf{r}) = -\frac{T}{2}\ln(2\pi) - \sum_{t=1}^{T}\ln(\sigma_t^2(\theta)) - \frac{1}{2}\sum_{t=1}^{T}\frac{(r_t-\mu)^2}{\sigma_t^2(\theta)}.$$

$$\sigma_t^2 = \sigma_t^2(\theta) = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2(\theta) = \omega + \alpha_1 (r_{t-1}-\mu)^2 + \beta_1 \sigma_{t-1}^2(\theta)$$

$$\theta = (\mu, \omega, \alpha_1, \beta_1)'$$

[1] “Volatility Clustering in Financial Markets: Empirical Facts and Agent-Based Models” by Rama Cont



STOCHASTIC METHODS

OBSERVED LIMITATION

- Volatile movements prominent in equity markets [2]
- Non-stationarity of asset return: Market goes through different phases and conditions, and it exhibits different characteristics over time [3]
- Other financial data features such as heavy tails and skewness is not well described by the gaussian model

PROPOSED SOLUTION

- Combining the concept of non-stationarity and volatility clustering, we consider volatility follows a random process
- SV Model: stochvol in R, using the default zero mean model
- Rolling estimate, training set as the data of past 30 days

[2] "The Impact of News on the Stock Market: A Cross-Country Analysis" by Michael Bleaney and Paul Mizen (2015)
[3] "Distribution and statistics of the Sharpe Ratio" by Eric Benhamou (2021)

IMPLIED VOLATILITY - INCORPORATING MARKET INFORMATION

OBSERVED LIMITATION

- Market sentiment, arbitrage and other factors play a major role in driving asset return
- And could not be easily represented in closed form
- Complex market structure, e.g. option and futures trading, swap contracts…
- Efficient Market: Market reveals more information than you observed

PROPOSED SOLUTION

- Due to the complexity of actually pricing an American Option, we simplify our study by treating as an European Option and apply the BSM, calculating the implied volatility
- Newton-Rhapston's Algorithm: [4]

1. Pick an initial guess for the first iteration i=0
2. Compute the pde w.r.t std, i.e. vega of the contract
3. Update new result by:
4. If the new update - old or computed price-market price smaller than a tolerance, quit the loop and take the latest calculate as the final result

$$\frac{\delta_c}{\delta\sigma_i}$$

$$\sigma_{i+1} = \sigma_i - \frac{c(\sigma_i) - c_m}{\frac{\delta_c}{\delta_{\sigma i}}}$$

[4] Inspired by https://quantpy.com.au/black-scholes-model/implied-volatility-with-the-newton-raphson-method/

EVALUATING VARIOUS METHOD OVER TIME

EVALUATION METHOD

- Test for Bias: Calculate the sum of absolute difference between forecast and realized variance:

$$\sum_{i=0}^{n} |\widehat{\sigma}_i^2 - \sigma_i^2|$$

  And taking the mean of it
- Test for Precision: Mean Squared Error of the estimates

TEST SETUP

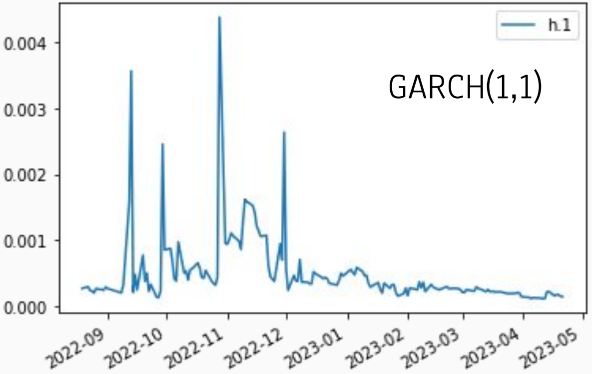
- Equity to be evaluated on: AAPL
- AAPL Call Option: AAPL230421C00165000
- Evaluation Period: 2022-08-20 to 2023-04-24 (171 trading days)
- Assumption: No transaction fee for the american options
- Lookback period for sample variance (n-1): 30 days

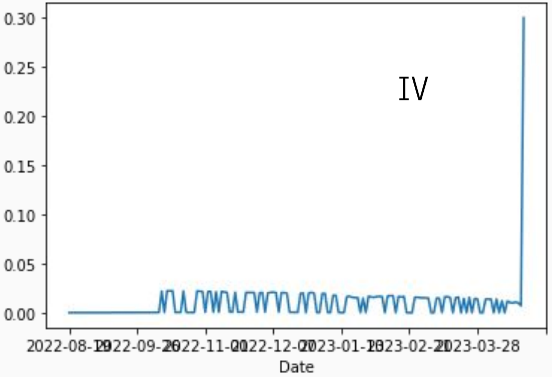
Link to code and data used:

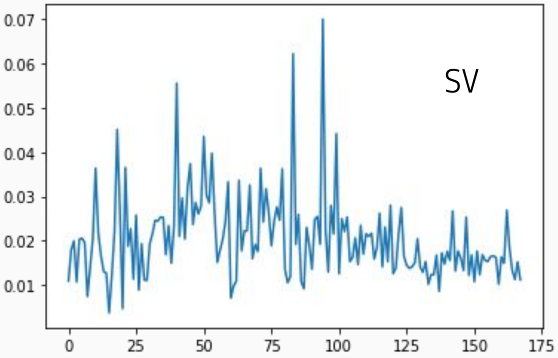
RESULTS

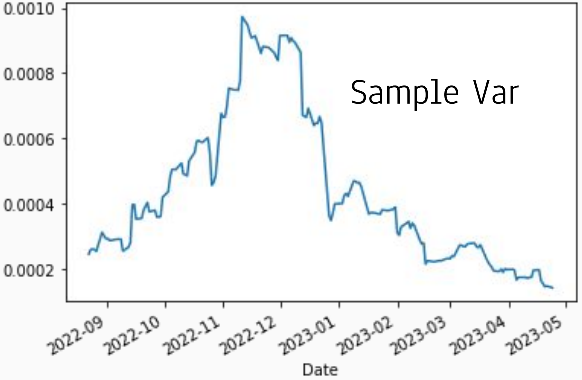
METHOD	Bias	Precision
GARCH(1,1)	1.972e-4	2.988e-7
SV	2.00e-2	9.173e-5
OPTION-IV	1.68e-2	5.83e-3
SAMPLE VAR	1.985e-5	5.298e-8

EVALUATION









LIMITATIONS AND FINDINGS

Link to code and data used:

A

The lack of sufficient sample size for GARCH made its prediction more biased, and not very precise. Besides, alternate methods in tuning look-back periods as a parameter can also be considered

B

The result in implied volatility means that the Black Scholes Model is not behaving well. Indeed, the algorithm does not converge for ⅓ of the total test data. Besides, other factors such as moneyness would affect the market price. Thus, alternate methods such as MCMC should be considered

C

The SV model needs to seek alternate methods in fitting, especially other methods such as constant mean, AR(k) or general Bayesian Model will be more adapted to financial data