

$$\left(\sum_{k=1}^{2n-1} \frac{1}{k} (1 - \cos k\theta) \right)^2 + \left(\sum_{k=1}^{2n-1} \frac{\sin k\theta}{k} \right)^2 < 4 \left(\sum_{k=1}^n \frac{1}{2k-1} \right)^2$$

A SHARP TRIGONOMETRIC DOUBLE INEQUALITY AND BRANNAN'S CONJECTURE: THE CASE $r=1$

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01. BACKGROUND

In 1973, *D. A. Brannan*

conjectured that if we let

$$\frac{(1 + zx)^\alpha}{(1 - x)^\beta} = \sum_{m=0}^{\infty} A_m(\alpha, \beta, z) x^m$$

Where $\alpha > 0$, $\beta > 0$, and $z = e^{i\theta}$, $0 \leq \theta \leq 2\pi$,
then

$$|A_m(\alpha, \beta, z)| \leq A_m(\alpha, \beta, 1)$$

for all odd integer m . Here, A_m refers
to the coefficient of the m th order term
in the polynomial.

01. BACKGROUND

Recently, in an attempt to prove the conjecture for $0 < \alpha < 1$ and $\beta = 1$, *R.W. Barnard et al.* reformulates inequality into finding the largest r satisfying

$$|A_m(\alpha, \beta, z)| \leq A_m(\alpha, \beta, r)$$

In the paper, the authors try to show that $r \leq 1$ for all odd m . In particular, they successfully show that the inequality above holds for $0 < r \leq 1/2$.

01. BACKGROUND

R.W. Barnard et al. reduce the case where $1/2 < r \leq 1$ into proving the following conjecture, expressed in term of the trigonometric form as

$$\left(\sum_{k=1}^{2n-1} \frac{r^k}{k} (1 - \cos k\theta) \right)^2 + \left(\sum_{k=1}^{2n-1} \frac{r^k}{k} \sin k\theta \right)^2 < 4 \left(\sum_{k=1}^n \frac{1}{2k-1} \right)^2$$

for $1/2 < r \leq 1$, $n \in \mathbb{N}$ and notably $\theta \in [0, \pi)$

Therefore in this paper, we give a partial answer to the above conjecture for $r = 1$

02. METHODOLOGY

$$\left(\sum_{k=1}^{2n-1} \frac{1}{k} (1 - \cos k\theta)\right)^2 + \left(\sum_{k=1}^{2n-1} \frac{\sin k\theta}{k}\right)^2 < 4 \left(\sum_{k=1}^n \frac{1}{2k-1}\right)^2$$

for $n \geq 5$ and $\theta \in [0, 2\pi/3]$

METHODOLOGY

LEMMA 3.1

$$-\frac{2929}{1260} + \frac{6079}{1260}x + \frac{8063}{1260}x^2 - \frac{9}{4}x^3 - x^4 < 0.$$

LEMMA 3.3

$$-(1 + \cos \theta)^2 \sum_{k=1}^5 \frac{1}{2k-1} + \frac{1}{16}(1 + \cos \theta)^4 + \left(\frac{\pi - \theta}{2} + \frac{3}{10} \right)^2 < 0.$$
$$\theta \in \left[0, \frac{\pi}{2} \right]$$

LEMMA 3.2

$$-(1 + \cos \theta)^2 \sum_{k=1}^5 \frac{1}{2k-1} + \frac{1}{16}(1 + \cos \theta)^4 + \left(\frac{\pi - \theta}{2} + \frac{1}{8} \right)^2 < 0.$$
$$\theta \in \left[\frac{\pi}{2}, \frac{2\pi}{3} \right]$$

LEMMA 3.4

$$\left(\sum_{k=1}^{2n-1} \frac{1}{k} (1 - \cos k\theta) \right)^2 + \left(\sum_{k=1}^{2n-1} \frac{\sin k\theta}{k} \right)^2 < 4 \left(\sum_{k=1}^n \frac{1}{2k-1} \right)^2.$$
$$\theta \in \left[0, \frac{2\pi}{3} \right]$$

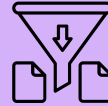
03. APPLICATIONS



Engineering



Signal processing



Digital filters

**THANK
YOU**