CSE508 Network Security

10/3/2017

Public Key Cryptography

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Public Key Cryptography

Many algorithms with different purposes

One common property: pair of values, one public and one secret

Session key establishment

Exchange messages to create a shared secret key

Encryption

Anyone can encrypt a message using a recipient's public key

Only the recipient can decrypt a message using their private key

No shared secret! Private key (secret) is stored only at one side

Digital signatures

Sign a message with a private key

Diffie–Hellman Key Exchange

Allows two parties to jointly establish a shared secret key over an insecure communication channel

- The established key can then be used to encrypt subsequent communication using a symmetric key cipher
- "New Directions in Cryptography" by Whitfield Diffie and Martin Hellman, 1976

Based on the discrete logarithm problem

$$3^{29} \bmod 17 \xrightarrow{\textit{easy}} ??$$

$$3^{\color{red}{??}} \bmod 17 \xleftarrow{\textit{hard}} 12$$

Diffie–Hellman Key Exchange

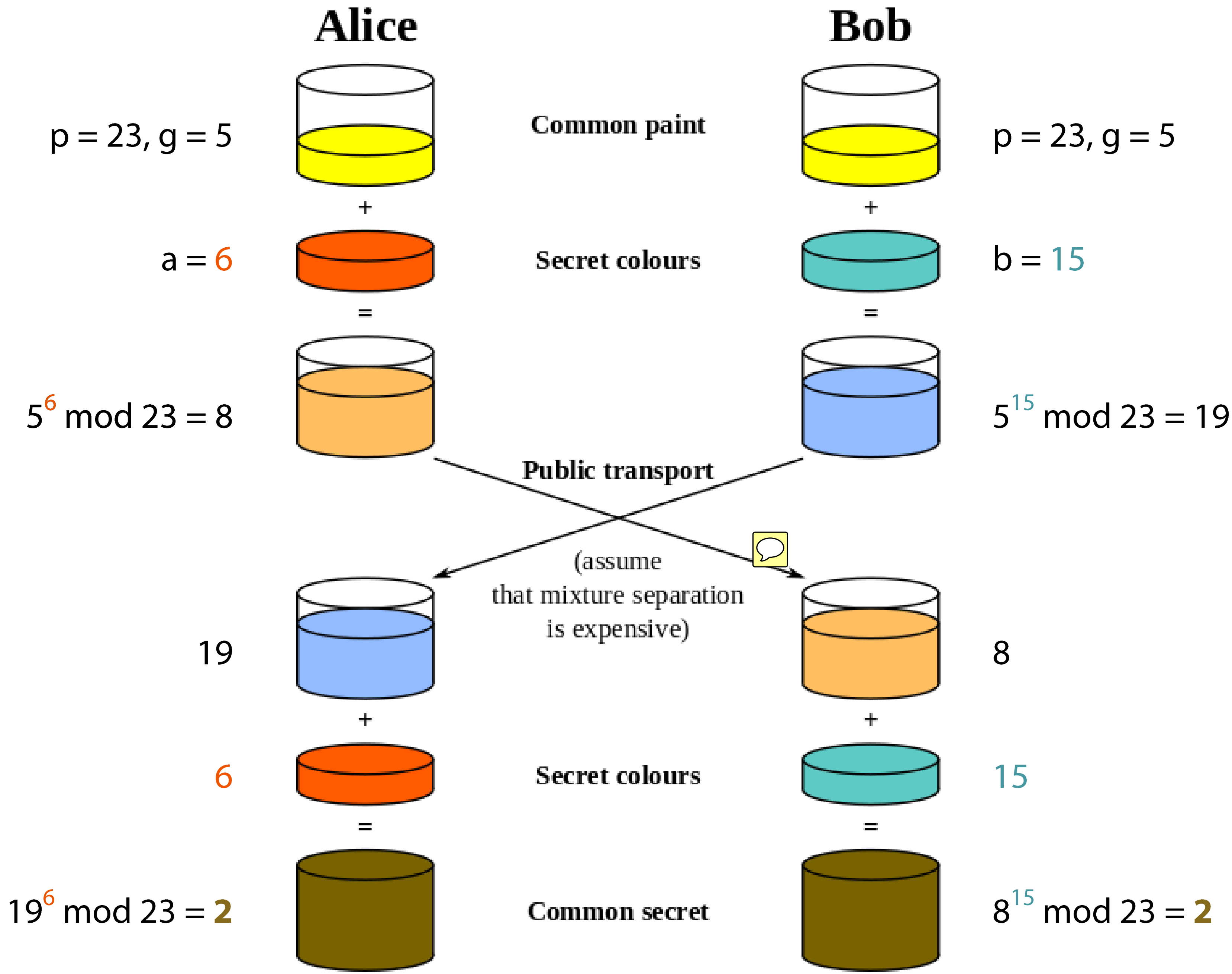
Alice and Bob agree on a large (at least 1024 bit) prime number p and a base g – *both public*

- p is usually of the form $2q+1$ where q is also prime
- g is a generator of the multiplicative group of integers modulo p (for every x coprime to p there is a k such that $g^k \equiv x \bmod p$)

Alice picks a secret (private) large random number a and sends to Bob $g^a \bmod p$

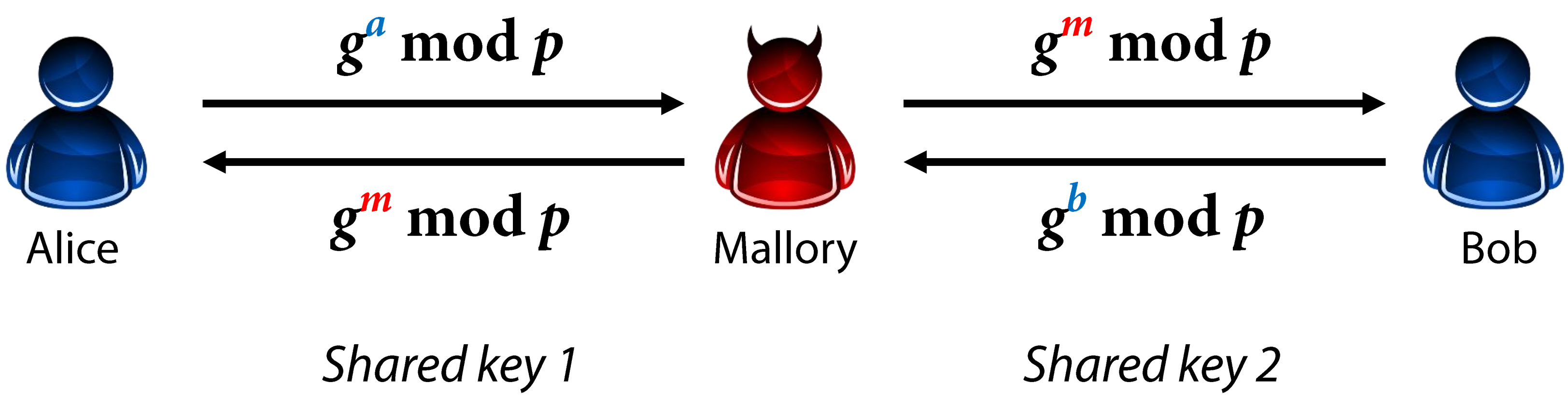
Bob picks a secret large random number b and sends to Alice $g^b \bmod p$

$$\left.\begin{array}{l}\text{Alice calculates } s = (g^b \bmod p)^a = g^{ba} \bmod p \\ \text{Bob calculates } s = (g^a \bmod p)^b = g^{ab} \bmod p\end{array}\right\}\ \textbf{\textit{shared key}}$$


Alice
Bob
$p = 23, g = 5$
Common paint
$p = 23, g = 5$
+
+
$a = 6$
Secret colours
$b = 15$
=
=
$5^6 \bmod 23 = 8$
$5^{15} \bmod 23 = 19$
Public transport
(assume
that mixture separation
is expensive)
19
8
+
+
6
Secret colours
15
=
=
$19^6 \bmod 23 = \mathbf{2}$
Common secret
$8^{15} \bmod 23 = \mathbf{2}$

Man-in-the-Middle Attack

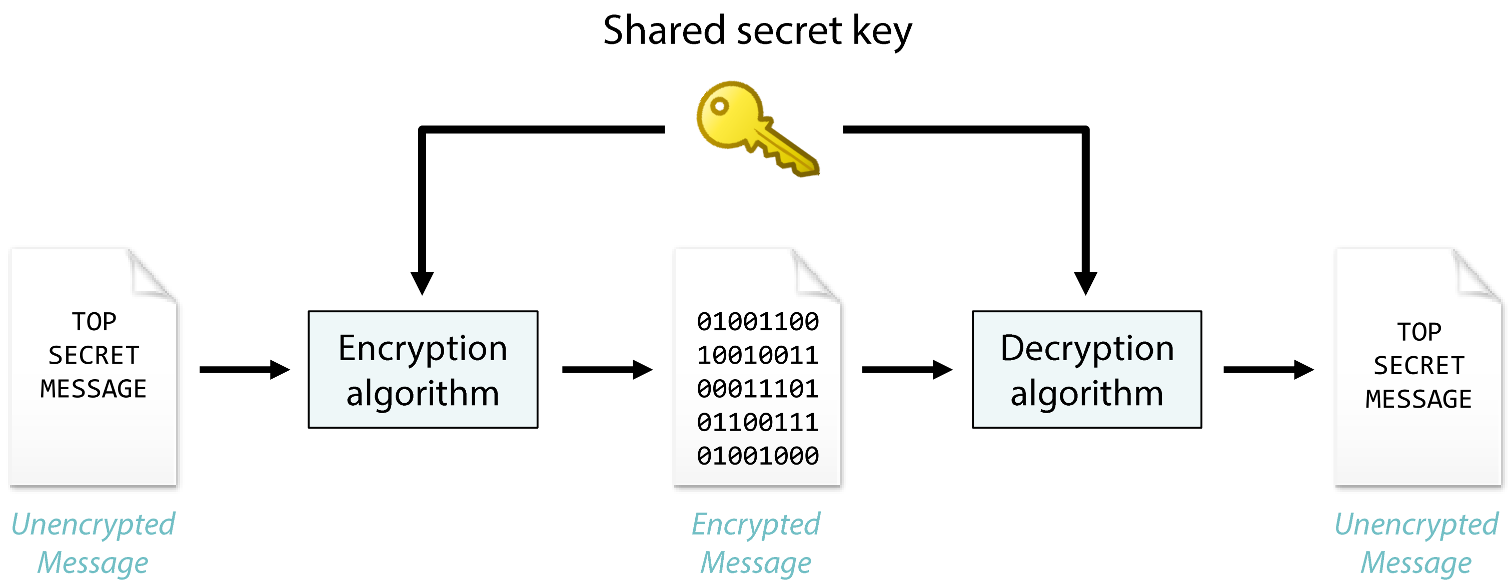
Alice and Bob share no secrets



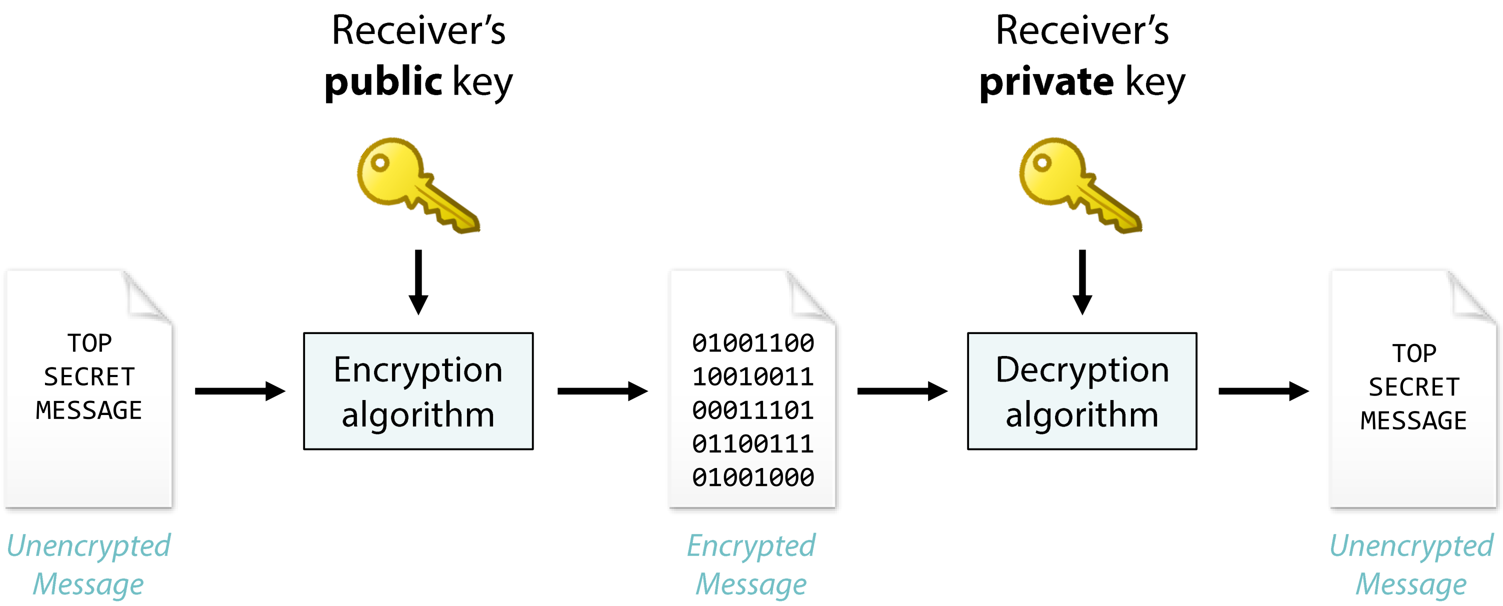
Mallory actively decrypts and re-encrypts all traffic

- **No authentication:** Alice and Bob assume that they communicate directly
- General problem: *need for a root of trust*

Symmetric Key Cryptography



Public Key Cryptography



Advantages

No shared secrets

Only private keys need to be kept secret, but they are never shared

Easier key management

No need to transmit any secret key beforehand

For n parties, n key pairs are needed (instead of $n(n-1)/2$ shared keys)

Provides both secrecy *and authenticity*

Disadvantages

More computationally intensive

Encryption/decryption operations are 2–3 orders of magnitude slower than symmetric key primitives

About one order of magnitude larger keys

Key generation is more difficult

RSA

Named after its inventors: Rivest, Shamir, Adleman

Based on the assumption that factoring large numbers is hard

- Relatively easy to find two large prime numbers p and q
- No efficient methods are known to factor their product N

Variable key length

- Largest (publicly known) factored RSA modulus is 768-bit long
- That was in 2009: it took 2 years and many hundreds of machines
- It is believed that 1024-bit keys may already (or in the near future) be breakable by a sufficiently powerful attacker

2048-bit keys should be the absolute minimum

- For now (next decade or two)…

RSA

Choose two distinct large prime numbers p and q

Let $n = pq$ (modulus)

Select e as a relative prime to $(p - 1)(q - 1)$

Calculate d such that $de \equiv 1 \bmod (p - 1)(q - 1)$

Public key = (e, n)

Private key = d

To encrypt m, calculate $c \equiv m^e \bmod n$

- Plaintext block must be smaller than the key length

To decrypt c, calculate $m \equiv c^d \bmod n$

- Ciphertext block will be as long as the key

RSA in Practice

RSA calculations are computationally expensive

- Two to three orders of magnitude slower than symmetric key primitives ➔ use RSA in combination with a symmetric key

Sending an encrypted message:

- Encrypt message with a random symmetric key
- Encrypt the symmetric key with recipient's public key
- Transmit both the encrypted message and the encrypted key

Setting up an encrypted communication channel:

- Negotiate a symmetric key using RSA
- Use the symmetric key for subsequent communication

PKCS: Public-Key Cryptography Standards (#1–#15)

- Make different implementations interoperable
- Avoid various known pitfalls in commonly used schemes

Forward Secrecy

Threat: capture encrypted traffic now, use in the future

- Private keys may be compromised later on (e.g., infiltrate system)
- A cryptanalytic breakthrough may be achieved

FS: Ensure that even if current keys are compromised, past encrypted traffic cannot be compromised

- Generate random secret keys without using a deterministic algorithm
- Cannot read old messages
- Cannot forge a message and claim that it was sent in the past

Support

- IPsec, SSH, Off-the-Record messaging (OTR)
- TLS (Diffie–Hellman instead of RSA key exchange)

Note a panacea

- Ephemeral keys may be kept in memory for hours
- Server could be forced to record all session keys
- TLS session resumption needs careful treatment

Elliptic Curve Cryptography

Proposed in 1985, but not used until 15 years later

Relies on the intractability of a different mathematical problem: *"elliptic curve discrete logarithm"*

Main benefit over RSA: shorter key length

- E.g., a 256-bit elliptic curve public is believed to provide comparable security to a 3072-bit RSA public key

Endorsed by NIST

- Key exchange: elliptic curve Diffie–Hellman (ECDH)
- Digital signing: elliptic curve digital signature algorithm (ECDSA)

Commercial National Security Algorithm Suite and Quantum Computing FAQ

Q: What is the Commercial National Security Algorithm Suite?

A: The Commercial National Security Algorithm Suite is the suite of algorithms identified in CNSS Advisory Memorandum 02-15 for protecting NSS up to and including TOP SECRET classification. This suite of algorithms will be incorporated in a new version of the National Information Assurance Policy on the Use of Public Standards for the Secure Sharing of Information Among National Security Systems (CNSSP-15 dated October 2012). The Advisory

Algorithm	Usage
RSA 3072-bit or larger	Key Establishment, Digital Signature
Diffie-Hellman (DH) 3072-bit or larger	Key Establishment
ECDH with NIST P-384	Key Establishment
ECDSA with NIST P-384	Digital Signature
SHA-384	Integrity
AES-256	Confidentiality

Cryptographic Hash Functions

Hash functions that are considered practically impossible to invert

Properties of an ideal cryptographic hash function

- Easy to compute the hash value for any given message
- Infeasible to generate a message that has a given hash
- Infeasible to modify a message without changing the hash
- Infeasible to find two different messages with the same hash

Many-to-one function: *collisions can happen*

Cryptographic Hash Function Properties

Pre-image resistance

Given a hash value h it should be computationally infeasible to find any input m such that $h = \text{hash}(m)$

Example: break a hashed password

Second pre-image resistance

Given m_1 it should be computationally infeasible to find m_2 such that $m_1 \neq m_2$ and $\text{hash}(m_1) = \text{hash}(m_2)$

Example: forge an existing certificate

Collision Resistance

It should be computationally infeasible to find two different inputs m_1 and m_2 such that $\text{hash}(m_1) = \text{hash}(m_2)$ (collision)

Example: prepare two contradicting versions of a contract

Birthday Paradox

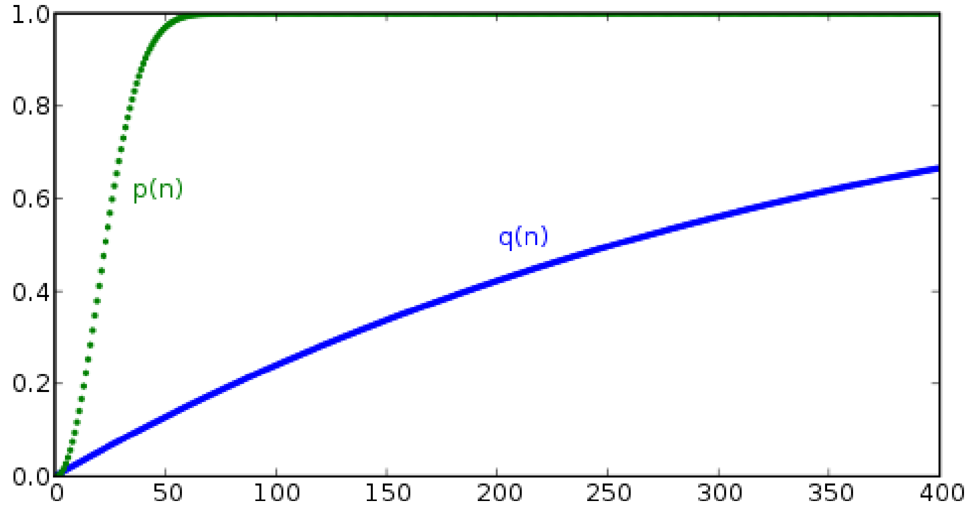
How many people does it take before the odds are 50% or better of having…

…another person with the same birthday as *you*? **253**

Second pre-image resistance

…two people with the same birthday? **23**

Collision resistance



Uses of Cryptographic Hash Functions

Data integrity

Digital signatures

Message authentication

User authentication

Timestamping

Certificate revocation management

Common Hash Functions

MD5: 128-bit output

1993: Boer and Bosselaers, “pseudo-collision” of the MD5 compression function: 2 different IVs which produce an identical digest

1996: Dobbertin, collision of the MD5 compression function

2004: Wang, Feng, Lai, and Yu, collisions for the full MD5

2005: Lenstra, Wang, and de Weger, construction of two X.509 certificates with different public keys but same hash

2008: Sotirov, Stevens, Appelbaum, Lenstra, Molnar, Osvik, de Wege, creating rogue CA certificates

Use it? NO, it’s unsafe

SHA-1: 160-bit output

2005: Rijmen and Oswald, attack on a reduced version of SHA1 (53 out of 80 rounds)

2005: Wang, Yao, and Yao, an improvement, lowering the complexity for finding a collision to 2^{63}

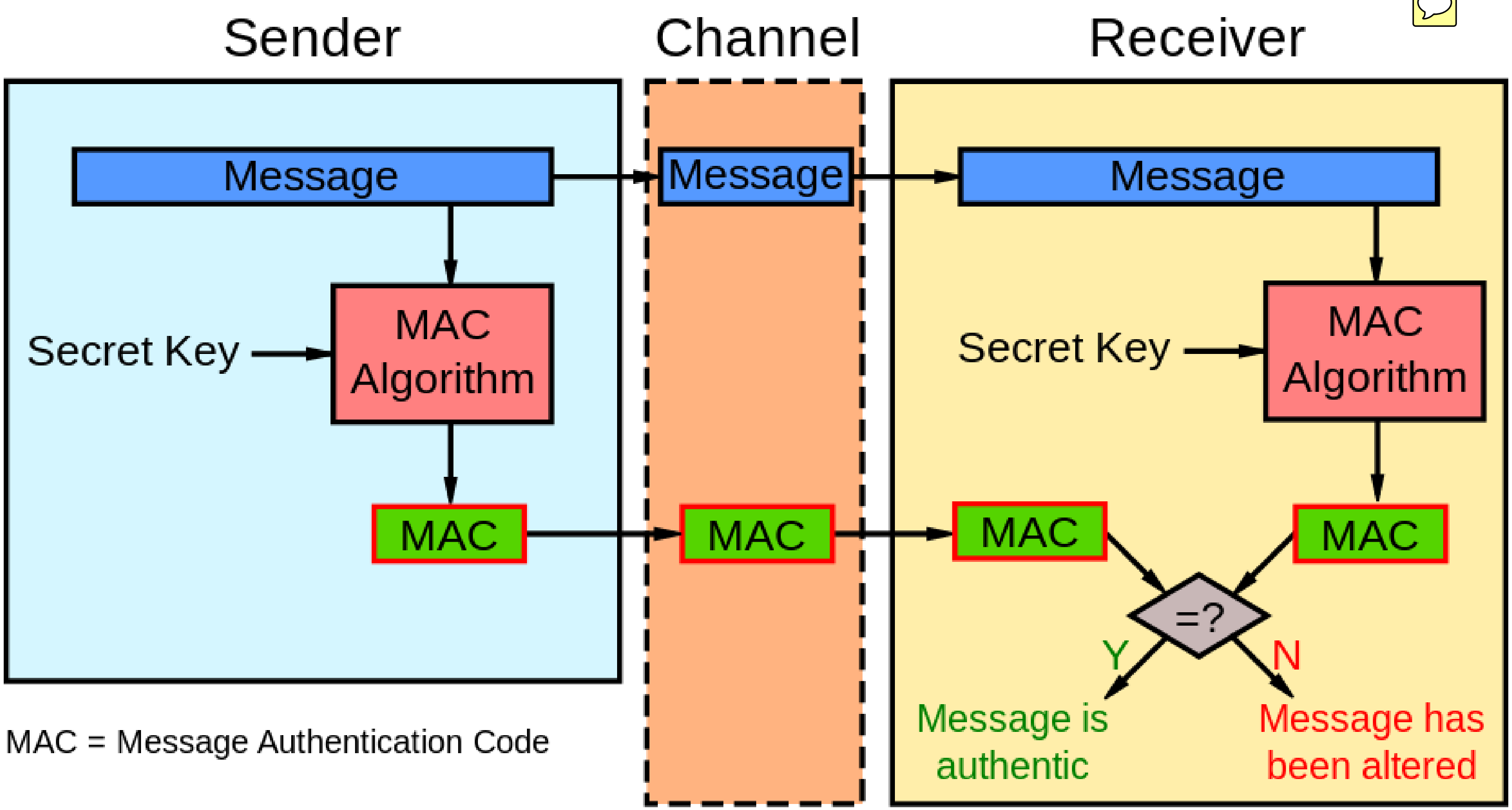
2006: Rechberger, attack with 2^{35} compression function evaluations

2015: Stevens, Karpman, and Thomas, freestart collision attack

Use it? Use SHA-256 or better instead

Message Authentication Codes (MACs)

Verify both message integrity and authenticity



https://commons.wikimedia.org/wiki/File:Message_Authentication_Code.svg

MAC = H(*key* || *message*)

|| denotes concatenation

Problem: easy to append data to the message without knowing the key and obtain another valid MAC

Length-extension attack: calculate $H(m_1 \| m_2)$ for an attacker-controlled m_2 given only $H(m_1)$ and the length of m_1

Keyed-hash message authentication code (HMAC)

$$HMAC(K, m) = H(\,(K \oplus opad) \,||\, H(K \oplus ipad \,||\, m)\,)$$

opad/ipad: outer/inner padding

Impossible to generate the HMAC of a message without knowing the secret key

Double nesting prevents various forms of length-extension attacks

Order of Encryption and MACing

Encrypted data usually must be protected with a MAC

- Encryption alone protects only against passive adversaries

Different options:

MAC-and-Encrypt $E(P) \| M(P)$

- No integrity of the ciphertext

MAC-then-Encrypt $E(P \| M(P))$

- No integrity of the ciphertext (have to decrypt it first)

Encrypt-then-MAC $E(P) \| M(E(P))$

- **Preferable option – *always MAC the ciphertext***

Digital Signatures

Use RSA backwards:

- Sign (encrypt) with the private key
- Verify (decrypt) with the public key

Ownership of a private key turns it into a *digital signature*

- Anyone can verify that a message was signed by its owner
- *Non-repudiation*

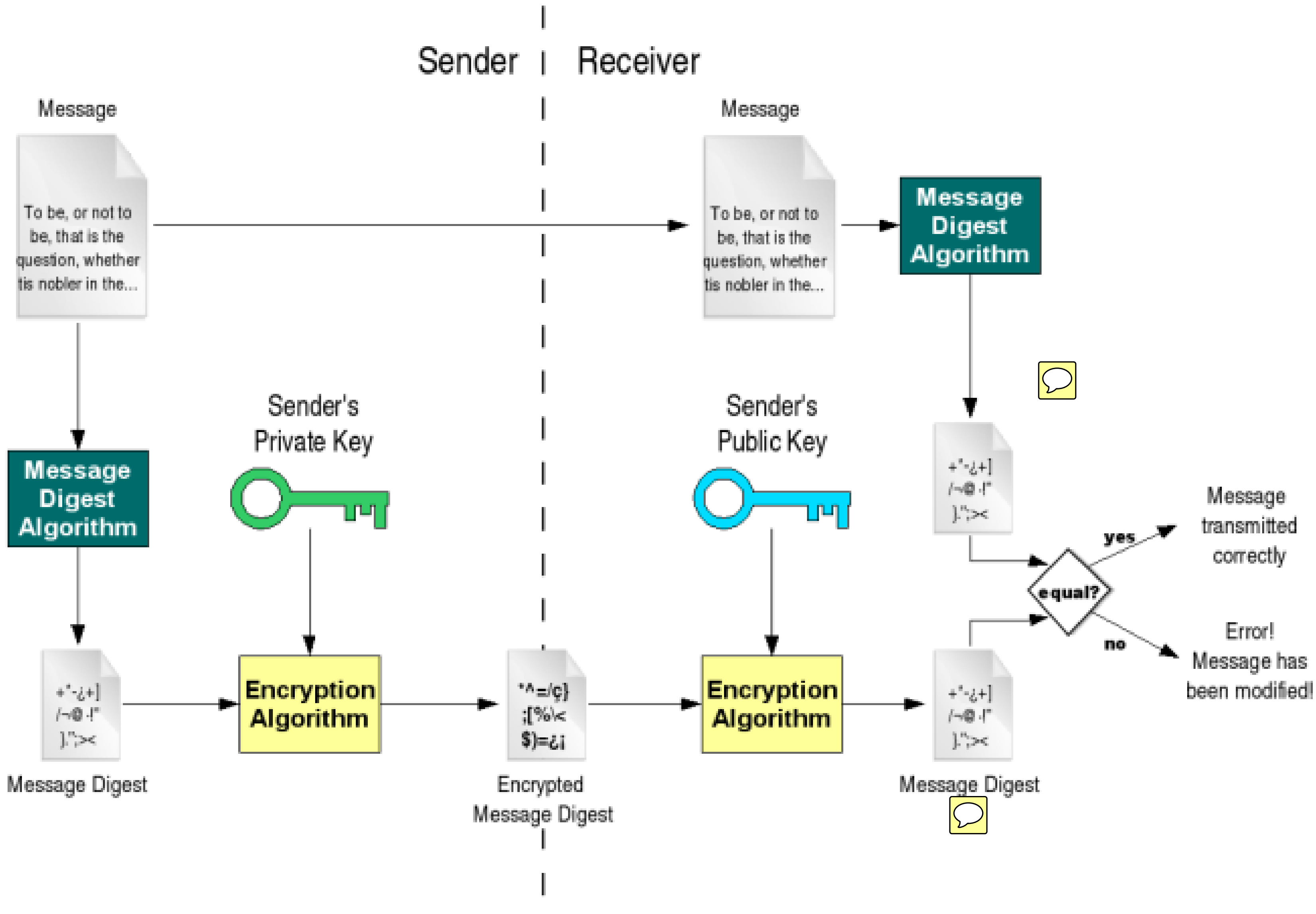
Again, too expensive to sign the whole message

- Calculate a cryptographic hash of the message and sign the hash

What if a private key was stolen or deliberately leaked?

- All the signatures (past and future) of that signer become suspect
- The signer might know which signatures were issued legitimately, but there is no way for the verifier to distinguish them

Digital Signatures



Hashes vs. MACs vs. Digital Signatures

	Hash	**MAC**	**Signature**
Integrity	✓	✓	✓
Authentication		✓	✓
Non-repudiation			✓
Keys	*None*	*Symmetric*	*Asymmetric*

Public Key Authenticity

Authentication without confidence in the keys used is pointless

Need to gain confidence or proof that a particular public key is authentic

- It is correct and belongs to the person or entity claimed
- Has not been tampered with or replaced by an attacker

Different ways to establish trust

- TOFU: trust on first use (e.g., SSH)
- Web of trust – decentralized trust model (e.g., PGP)
- PKI: public key infrastructure (e.g., SSL)
- (subject of future lecture)

Adi Shamir: *Crypto is typically bypassed, not penetrated*