1. Which of the following are correct?

a. False |= True.

Correct. False entails every sentence as it has no models while True is entailed by every sentence as it has all the models. Hence False Entails True.

b. True |= False.

Incorrect. False has no models and entails every sentence while True has all the model and is entailed by every sentence. Hence False Cannot Entail True.

c. $(A \wedge B) \models (A \Leftrightarrow B)$.

Correct. LHS means A and B both are true in m and RHS means either both A and B are True or False. Hence LHS Entails RHS.

d. $A \Leftrightarrow B \models A \vee B$.

Incorrect. LHS has two models where A and B both are True and where A and B both are False, but RHS does not satisfy as it does not have a model where both are False. Hence LHS does not entail RHS

e. $A \Leftrightarrow B \models \neg A \vee B$.

Correct**.** LHS has two models where A and B both are True and where A and B both are False, and RHS also has both these two models as well as the model where A is False, and B is True. Hence LHS entails RHS.

f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$.

Correct. RHS is only False when both A and B are True, and C is False in which case LHS is also False. Apart from that in all other cases LHS is present in RHS. Hence LHS entails RHS.

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

$(C \vee (\neg A \wedge \neg B))$

$= ((C \vee \neg A) \wedge (C \vee \neg B))$ distributivity of $\vee$ over $\wedge$

$= ((\neg A \vee C) \wedge (\neg B \vee C))$ commutativity of $\vee$

$= ((A \Rightarrow C) \wedge (B \Rightarrow C))$ by implication elimination

Hence LHS $\equiv$ RHS.

h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.

Correct. LHS has an And condition or a Conjunct between its two part while RHS does not have it. This allows RHS to have more models than LHS. Hence LHS entails RHS.

i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.

Incorrect. The only difference between LHS and RHS is the disjunct and ¬C before ¬D. Thus, lets analyze the disjunct. Removing the disjunct from LHS eliminates some models, i.e. RHS has fewer models than LHS. Hence LHS does not entail RHS.

j. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable.

Correct. A statement is called satisfiable if it is True in some model. If A is True and B is False, the term before conjunct is True and so is the term after conjunct. This condition also makes the statement True. Hence it is satisfiable.

k. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable.
Correct. A sentence is satisfiable if it is True in some model. The term after conjunct is same as $A \Rightarrow B$ and if that is True implies the term before conjunction will be True. Hence the statement is satisfiable.

l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.
Correct. $A \Rightarrow B$ (part 1) has two models and two non-models while $(A \Leftrightarrow B) \Leftrightarrow C$ (part 2) also has two models and two non-models. Models in part 1 satisfies models in part 2 (can be verified by truth table) so does non-models in part 1. Hence the statement is Correct.

2. According to some political pundits, a person who is radical (R) is electable (E)if he/she is conservative (C), but otherwise is not electable.
a. Which of the following are correct representations of this assertion?
(i) $(R \land E) \Longleftrightarrow C$
Incorrect representation, as this sentence among other meanings also mean all conservatives are Radial and Electable, which is not True.

(ii) $R \Rightarrow (E \Longleftrightarrow C)$
Correct representation , as this a radical person is electable only if the person is conservative , which is True.

(iii) $R \Rightarrow ((C \Rightarrow E) \lor \neg E)$
Incorrect representation, as simplifying the above equation we get $\neg R \lor \neg C \lor E \lor \neg E$, which is always true as either E or $\neg E$ is True. But this is not what the statement represents.

b. Which of the sentences in (a) can be expressed in Horn form?
Horn form is a disjunction of literals of which at most one is positive.
All the sentences in a can be expressed in Horn form.
Statement i can be put in Horn Form as follows:

$(R \land E) \Longleftrightarrow C$
$= ((R \land E) \Rightarrow C) \land (C \Rightarrow (R \land E))$ biconditional elimination
$= ((R \land E) \Rightarrow C) \land (C \Rightarrow R) \land (C \Rightarrow E)$ splitting over $\land$

Statement ii can be put in Horn Form as follows:

$R \Rightarrow (E \Longleftrightarrow C)$
$= R \Rightarrow ((E \Rightarrow C) \land (C \Rightarrow E))$ biconditional elimination
$= \neg R \lor ((\neg E \lor C) \land (\neg C \lor E))$ implication elimination
$= (\neg R \lor \neg E \lor C) \land (\neg R \lor \neg C \lor E)$ distributivity of $\lor$ over $\land$

Statement iii can be put in Horn Form as follows:

As showed in part iii of (a), statement iii is always True. Thus, True $\Rightarrow$ True.

3. Consider the following sentence: $[(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \Rightarrow [(Food \land Drinks) \Rightarrow Party]$.

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

Q:3·a) Statement: S:

$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$.

The statement has 3 variable so let's make a truth table for it.

Food	Party	Drinks	Statement
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

From the truth table we can see the statement is always True.

A statement is said to be valid if it is True for all models.

Hence the statement is valid.

b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

Date. / /

Q.3. b)

LHS

$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)]$

$\equiv (\neg Food \vee Party) \vee (\neg Drinks \vee Party)$

implication elimination

$\equiv (\neg Food \vee Party \vee \neg Drinks \vee Party)$

$\equiv \neg Food \vee \neg Drinks \vee Party$

Duplicate elimination

RHS:

$[Food \wedge Drinks] \Rightarrow Party$

$\equiv \neg(Food \wedge Drinks) \vee Party$

implication elimination

$\equiv (\neg Food \vee \neg Drinks) \vee Party$

De morgan's Law

$\equiv (\neg Food \vee \neg Drinky \vee Party)$

Both sides are same in CNF,
Thus we can say original sentence of form $P \Rightarrow P$, which means it is valid for any P.

Thus it coroborrates a).

c. Prove your answer to (a) using resolution.

Q3: c)

If we need to prove that a sentence is Valid we need to prove its negation is unsatisfiable.

The procedure to its is negate the sentence convert it to CNF & use resolution for proving contradiction.

Let us use results obtained in part b.

$\neg[[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]]$

$= [(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \wedge \neg[(Food \wedge Drinks) \Rightarrow Party]$

implication elimination

$= (\neg Food \vee \neg Drinks \vee Party) \wedge Food \wedge Drinks \wedge \neg Party$

(from (b))

> As we can see all clauses resolve against first clause, which inturn leaves an empty clause

Hence Proved -

4. This exercise uses the function MapColor and predicates In(x, y), Borders(x, y),and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.

(i) In (Paris ∧ Marseilles, France).

In this logical expression the conjunction is used inside the term. Thus the expression is syntactically invalid and therefore meaningless.

(ii) In(Paris, France) ∧ In(Marseilles, France).

This logical correctly expresses the English sentence (by using conjunction between terms), as it means Paris is in France and Marseilles is in France.

(iii) In(Paris, France) ∨ In(Marseilles, France).

This expression suggests either Paris is in France or Marseilles is in France which does not convey the meaning of the sentence. Hence the expression is syntactically valid but does not express meaning of the English sentence.

b. There is a country that borders both Iraq and Pakistan.

(i) ∃ c Country(c) ∧ Border (c, Iraq) ∧ Border (c, Pakistan).

This expression means there exists a country such that it borders Iraq and it also borders Pakistan. Hence this correctly expresses the English sentence.

(ii) ∃ c Country(c) ⇒ [Border (c, Iraq) ∧ Border(c, Pakistan)].

Here the implication used in expression is existential. Thus, though the expression is syntactically valid it does not express the meaning of the English sentence.

(iii) [∃ c Country(c)] ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)].

In this expression the variable c is used outside its quantifier which makes the expression invalid. Thus, the expression is syntactically invalid and therefore meaningless.

(iv) ∃ c Border(Country(c), Iraq ∧ Pakistan).

In this logical expression the conjunction is used inside the term. Thus the expression is syntactically invalid and therefore meaningless.

c. All countries that border Ecuador are in South America.

(i) ∀cCountry(c) ∧ Border(c,Ecuador) ⇒ In(c, SouthAmerica).

This expression means if there exists a country and it has border with Ecuador then that country is in SouthAmerica. Hence it correctly expresses the English sentence.

(ii) ∀ c Country(c) ⇒ [Border(c,Ecuador) ⇒ In(c, SouthAmerica)].

This expression is effectively same as the expression in (i) i.e. there exists a country , it has border with Ecuador and it is in SouthAmerica. Hence it correctly expresses the English sentence.

(iii) ∀ c [Country(c) ⇒ Border(c,Ecuador)] ⇒ In(c, SouthAmerica).

Here the implication used in LHS expression is an existential as all the non-countries in RHS are being sanctioned. Thus, though the statement is syntactically valid it does not express the meaning of the English sentence.

(iv) ∀cCountry(c) ∧ Border(c,Ecuador) ∧ In(c, SouthAmerica).

In this expression conjunction is used as the main connective for the universal quantifier. Thus the expression is syntactically invalid and therefore meaningless.

d. No region in South America borders any region in Europe.

(i) ¬[∃ c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)].

This expression means that there exists no two countries in SouthAmerica and Europe respectively which shares a border. Hence it correctly expresses the English sentence.

(ii) ∀ c, d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒¬Borders(c, d)].

This expression means if there exists a country c in SouthAmerica and d in Europe implies they do not share a border. Hence it correctly expresses the English sentence.

(iii) ¬∀ c In(c, SouthAmerica) ⇒∃d In(d, Europe) ∧¬Borders(c, d).

This logical expression means that there exists some country in SouthAmerica which borders every country in Europe. Thus though the expression is syntactically valid it does not express the meaning of the English sentence.

(iv) ∀ c In(c, SouthAmerica) ⇒∀d In(d, Europe) ⇒¬Borders(c, d).

This logical expression means borders do not exist between two countries c and d if c is in SouthAmerica and d is in Europe.

e. No two adjacent countries have the same map color.

(i) ∀ x, y ¬Country(x) ∨¬Country(y) ∨¬Borders(x, y) ∨ ¬(MapColor(x)= MapColor(y)).

This expression correctly expresses the English Sentence.

(ii) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y) ∧¬(x = y)) ⇒ ¬(MapColor(x)= MapColor(y)).

This expression correctly expresses the English Sentence.

(iii) ∀ x, y Country(x) ∧ Country(y) ∧ Borders(x, y) ∧ ¬(MapColor(x)= MapColor(y)).

In this expression conjunction is used as the main connective for the universal quantifier. Thus the expression is syntactically invalid and therefore meaningless.

(iv) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y)) ⇒ MapColor(x != y).

This expression uses inequality inside a term hence the expression is syntactically invalid and therefore meaningless.