A Quick and Dirty Bayesian Leslie Matrix Model of Norway HarpEast Data

Jay Ver Hoef

Marine Mammal Laboratory
NOAA Fisheries Alaska Science Center
Seattle, Washington and Fairbanks, Alaska, USA

Population Values

$$
\begin{aligned}
A_t &= \text{Number of Nonpups in year } t \\
H_t^A &= \text{Harvest of Nonpups in year } t \\
P_t &= \text{Number of Pups in year } t \\
H_t^P &= \text{Harvest of Pups in year } t \\
N_t &= A_t + P_t
\end{aligned}
$$

A Simple Two-age Leslie Matrix Model

Boveng, P.L., Ver Hoef, J.M., Withrow, D.E., and London, J.M. 2018. A Bayesian Analysis of Abundance, Trend and Population Viability for Harbor Seals in Iliamna Lake, Alaska. Risk Analysis 38(9): 1988-209 DOI:10.1111/risa.12988

$$\begin{aligned} A_t &= \delta_{t-1}(A_{t-1} - H^A_{t-1}) + \kappa_{t-1}(P_{t-1} - H^P_{t-1}), \\ P_t &= c_{t-1}\phi_{t-1}(A_{t-1} - H^A_{t-1}), \end{aligned}$$

$$\mathbf{n}_t = \begin{pmatrix} P_t \\ A_t \end{pmatrix}, \ \mathbf{h}_t = \begin{pmatrix} H^A_t \\ H^P_t \end{pmatrix}, \ \mathbf{M}_t = \begin{pmatrix} 0 & c_t\phi_t \\ \kappa_t & \delta_t \end{pmatrix}$$

$$\mathbf{n}_t = (\mathbf{n}_{t-1} - \mathbf{h}_{t-1})\,\mathbf{M}_{t-1}$$

$c_t = \exp(-\rho N_t/1000)$

$c_t =$ Exponential decay in fecundity with increased N_t

Parameters

P_1 : Abundance of Pups in Year $t = 1$

A_1 : Abundance of Nonpups in Year $t = 1$

δ_t : Adult Mortality Rate in Year t

κ_t : Pup Mortality Rate, from Survey to Survey, in Year t

ϕ_t : Fecundity Rate for all Adults, to Survey Time, in Year t

ρ : Controls Exponential Decay in Fecundity as Population $\uparrow$

Priors

$[P_1] = \pi(P_1) \sim \mathrm{N}(300{,}000,\ \ 50{,}000^2)$

$[A_1] = \pi(A_1) \sim \mathrm{N}(1{,}000{,}000,\ \ 100{,}000^2)$

$[\boldsymbol{\delta}] = \pi(\mathrm{logit}(\delta_t)) \sim \mathrm{N}(2.2, 0.5^2)\ \forall\ t;\ \mathrm{logit}^{-1}(2.2) = 0.90$

$[\boldsymbol{\kappa}] = \pi(\mathrm{logit}(\kappa_t)) \sim \mathrm{N}(1.4, 0.5^2)\ \forall\ t;\ \mathrm{logit}^{-1}(1.4) = 0.80$

$[\boldsymbol{\phi}] = \pi(\mathrm{logit}(\phi_t)) \sim \mathrm{N}(-0.4, 0.5^2)\ \forall\ t; \mathrm{logit}^{-1}(-0.4) = 0.40$

$[\rho] = \pi(\log(\rho)) \sim \mathrm{N}(-4.0, 0.5^2)$

Data

H_t^A and H_t^P for all years
p_t an estimate of pup abundance in year $t \in \mathcal{S}$
s_t a standard error of pup abundance in year $t \in \mathcal{S}$
$\mathcal{S}$ is an index set of sampled years from T total years
$\mathcal{T}$ is the index set of years $t = 1, \ldots, T$

Likelihood

$$[p_t \mid P_t, s_t, \boldsymbol{\delta}, \boldsymbol{\kappa}, \boldsymbol{\phi}, \rho, P_1, A_1, \{\mathbf{h}_t; t \in \mathcal{T}\}] \sim \mathrm{N}(P_t, s_t)$$

where P_t is obtained from

$$\mathbf{n}_t = (\mathbf{n}_{t-1} - \mathbf{h}_{t-1})\,\mathbf{M}_{t-1};\ t \in \mathcal{T}$$

where $\{\mathbf{n}_t; t \in \mathcal{T}\}$ depends on $\boldsymbol{\delta}, \boldsymbol{\kappa}, \boldsymbol{\phi}, \rho, P_1, A_1$

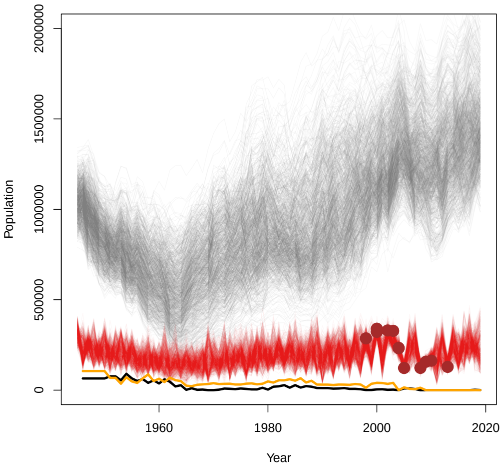
Posterior

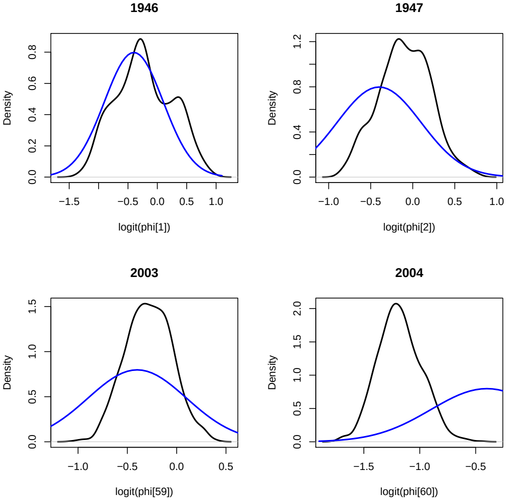
$$[P_1, A_1, \boldsymbol{\delta}, \boldsymbol{\kappa}, \boldsymbol{\phi}, \rho \mid \{(p_t, s_t); t \in \mathcal{S}\}, \{\mathbf{h}_t; t \in \mathcal{T}\}] \propto$$
$$\prod_{\mathcal{S}} [p_t \mid P_t, s_t, \boldsymbol{\delta}, \boldsymbol{\kappa}, \boldsymbol{\phi}, \rho, P_1, A_1, \{\mathbf{h}_t; t \in \mathcal{T}\}][P_1][A_1][\boldsymbol{\delta}][\boldsymbol{\kappa}][\boldsymbol{\phi}][\rho]$$

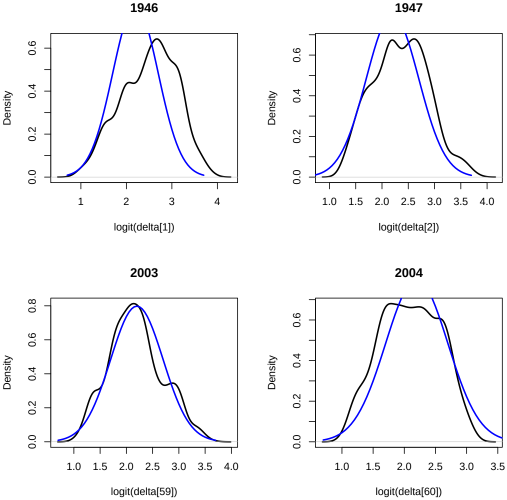
Posterior of $\{\mathbf{n}_t; t \in \mathcal{T}\}$ is constructed from posteriors of $P_1, A_1, \boldsymbol{\delta}, \boldsymbol{\kappa}, \boldsymbol{\phi}$ and ρ.

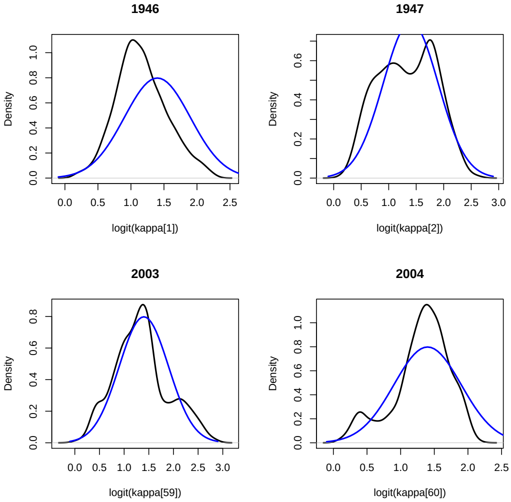
Use MCMC with Metropolis sampling to obtain sample from posterior distribution.

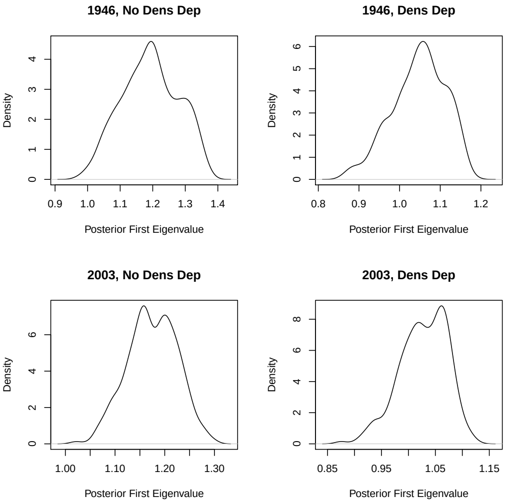
Custom code written in R, uses batch sampling for $\boldsymbol{\delta}, \boldsymbol{\kappa}, \boldsymbol{\phi}$, with a burnin of 10,000 samples, followed by 100,000 where only each 100th sample was saved, yielding 1,000 samples from posterior.


0
500000
1000000
1500000
2000000
Population
1960
1980
2000
2020
Year


1946
Density
0.0
0.2
0.4
0.6
0.8
−1.5
−0.5
0.0
0.5
1.0
logit(phi[1])
1947
Density
0.0
0.4
0.8
1.2
−1.0
−0.5
0.0
0.5
1.0
logit(phi[2])
2003
Density
0.0
0.5
1.0
1.5
−1.0
−0.5
0.0
0.5
logit(phi[59])
2004
Density
0.0
0.5
1.0
1.5
2.0
−1.5
−1.0
−0.5
logit(phi[60])


1946
Density
0.0
0.2
0.4
0.6
1
2
3
4
logit(delta[1])
1947
Density
0.0
0.2
0.4
0.6
1.0
1.5
2.0
2.5
3.0
3.5
4.0
logit(delta[2])
2003
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.5
2.0
2.5
3.0
3.5
4.0
logit(delta[59])
2004
Density
0.0
0.2
0.4
0.6
1.0
1.5
2.0
2.5
3.0
3.5
logit(delta[60])


1946
1947
2003
2004
Density
logit(kappa[1])
logit(kappa[2])
logit(kappa[59])
logit(kappa[60])


1946, No Dens Dep
1946, Dens Dep
2003, No Dens Dep
2003, Dens Dep
Density
Posterior First Eigenvalue
0 1 2 3 4
0.9 1.0 1.1 1.2 1.3 1.4
0 1 2 3 4 5 6
0.8 0.9 1.0 1.1 1.2
0 2 4 6
1.00 1.10 1.20 1.30
0 2 4 6 8
0.85 0.95 1.05 1.15