Equivalence for Row-Standardized CAR models

The CAR model (7.7), written in the form with an overall variance parameter σ^2, weight matrix $\mathbf{W}$, and autocorrelation parameter ρ, is

$$\mathbf{\Sigma} = \sigma^2 (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{K}_{\mathrm{CAR}}.$$

Recall from Section 7.3 that

$$\mathbf{D} = \mathrm{diag} \left(\frac{1}{w_{1+}}, \frac{1}{w_{2+}}, \dots, \frac{1}{w_{n+}} \right) = [\mathrm{diag}(\mathbf{W1})]^{-1},$$

and the row-standardized CAR model is

$$\mathbf{\Sigma} = \sigma^2 (\mathbf{I} - \rho \overline{\mathbf{W}})^{-1} \mathbf{D},$$

where recall that $\overline{\mathbf{W}} = \mathbf{DW}$. Then the row-standardized CAR model can also be written as

$$\begin{aligned}
\mathbf{\Sigma} &= \sigma^2 (\mathbf{I} - \rho \mathbf{DW})^{-1} \mathbf{D} \\
&= \sigma^2 [\mathbf{D}^{-1}(\mathbf{I} - \rho \mathbf{DW})]^{-1} \\
&= \sigma^2 (\mathbf{D}^{-1} - \rho \mathbf{W})^{-1} \\
&= \sigma^2 (\mathrm{diag}(\mathbf{W1}) - \rho \mathbf{W})^{-1}.
\end{aligned}$$

This last formula is a good computing formula for $\mathbf{\Sigma}^{-1}$.

This result is in,

Ver Hoef, J.M., Peterson, E. E., Hooten, M. B., Hanks, E. M., and
Fortin, M.-J. 2018. Spatial Autoregressive Models for Statistical
Inference from Ecological Data. Ecological Monographs, 88: 36{59.

although I saw it somewhere else (and don't recall exactly where).