Analog Electronics

Assignment - 02

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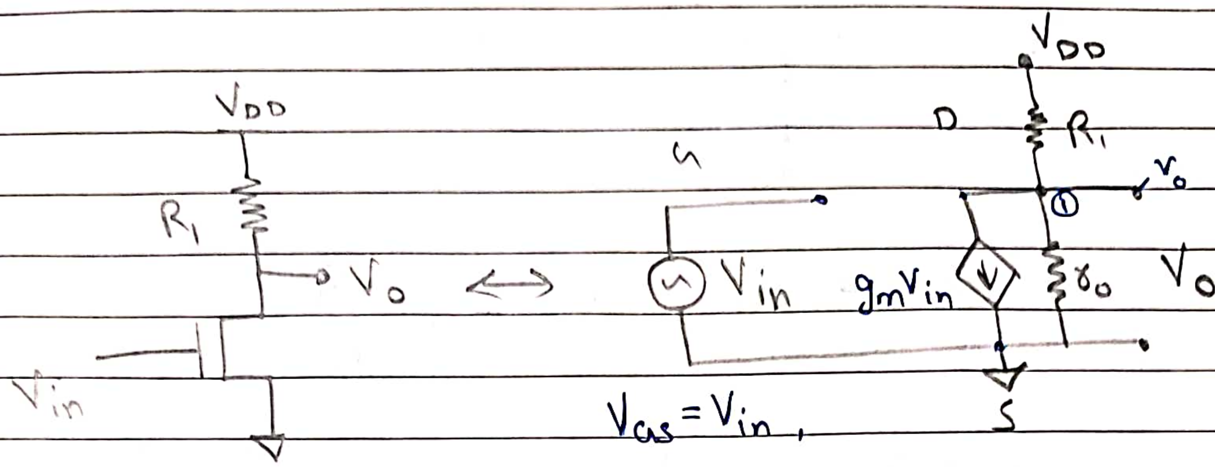
EE22BTECH11025

①



$V_{GS} = V_{in}$,

we are considering channel length modulation effect,

applying KCL at ①,

$$g_m V_{in} + \frac{V_o - V_{DD}}{R_1} + \frac{V_o}{r_o} = 0$$

$$\Rightarrow V_o\left(\frac{1}{R_1} + \frac{1}{r_o}\right) = -\left(g_m V_{in} - \frac{V_{DD}}{R_1}\right)$$

$$\Rightarrow \frac{V_o}{R_1 \parallel r_o} = -\left(g_m V_{in} - \frac{V_{DD}}{R_1}\right)$$

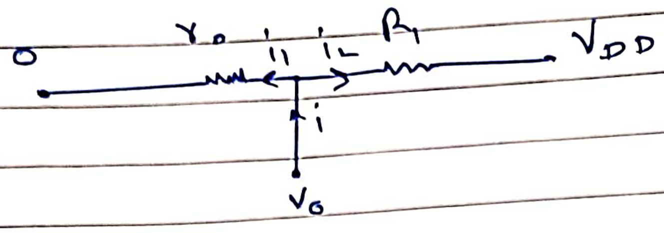
$$\Rightarrow V_o = -\left(g_m V_{in} - \frac{V_{DD}}{R_1}\right)(R_1 \parallel r_o)$$

$$\text{Gain} = \frac{V_o}{V_{in}} = -\left(g_m - \frac{V_{DD}}{V_{in}} \times \frac{1}{R_1}\right)(R_1 \parallel r_o)$$

$$\boxed{\text{gain} = -g_m(R_1 \parallel r_o)}, \boxed{V_{DD} = 0}, \boxed{V_{in} = V_{GS}}$$

i/p impedance = ∞

for o/p impedance



$$i = i_1 + i_2 = \frac{V_o}{r_o} + \frac{V_o - V_{DD}}{R_1}$$

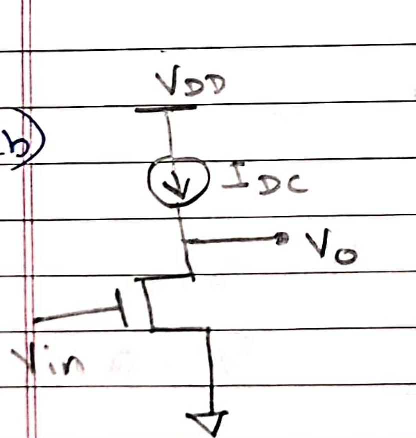
$$\cancel{Z_{o/p} = \frac{V_o}{i} = \frac{1}{r_o} + \frac{1}{R_1} - \frac{1}{R_1} \times \left(\frac{V_{DD}}{V_o}\right)}$$

$$\cancel{= \frac{1}{r_o} +}$$

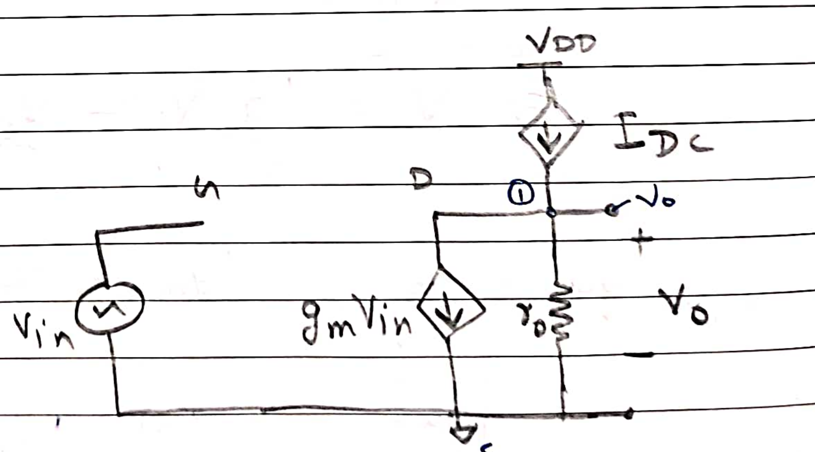
$$Z_{o/p} = \frac{V_o}{i} = \left[\frac{1}{r_o} + \frac{1}{R_1} - \frac{1}{R_1} \times \left(\frac{V_{DD}}{V_o}\right)\right]^{-1}$$

$$Z_{o/p} = \left[\frac{1}{r_o} + \frac{1}{R_1}\left[1 - \frac{V_{DD}}{V_o}\right]\right]^{-1} = \left[\frac{1}{r_o} + \frac{1}{R_1}\right]^{-1}$$

(b)



$\Longleftrightarrow$



$$\boxed{V_{GS} = V_{in}}$$

Applying KCL at ①,

$$-I_{DC} + g_m V_{in} + \frac{V_o}{r_o} = 0$$

$$\Rightarrow \frac{V_o}{r_o} = -\left[g_m V_{in} - I_{DC}\right]$$

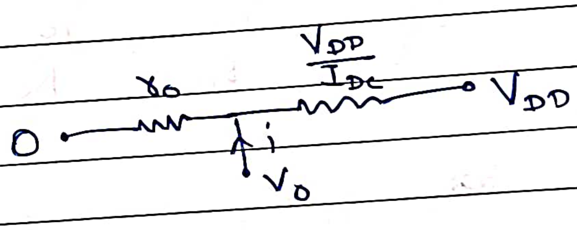
$$V_o = -\left[g_m V_{in} - I_{DC}\right] r_o$$

$$\text{gain} = \frac{V_o}{V_{in}} = -\left[g_m - \frac{I_{DC}}{V_{in}}\right] \times r_o$$

i/p impedance $= \infty$

~~o/p impedance =~~

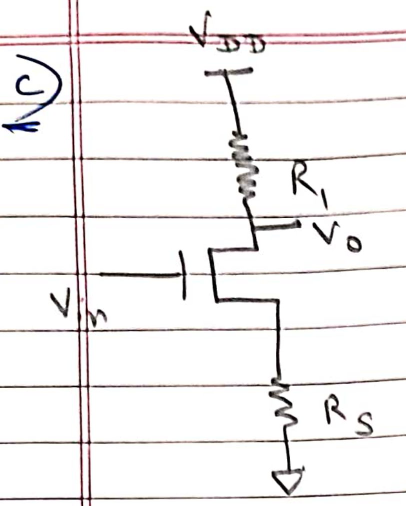
for o/p impedance,



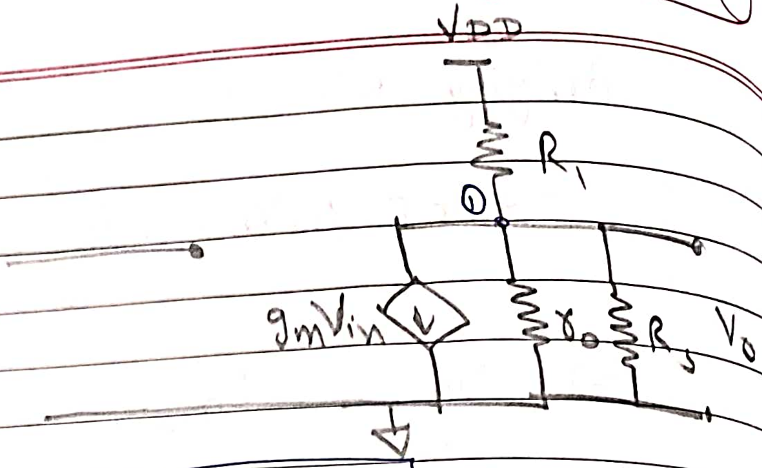
$$Z_{o/p} = \frac{V_o}{i} = \left[\frac{1}{r_o} + \frac{I_{DC}}{V_{DD}}\left[1 - \frac{V_{DD}}{V_o}\right]\right]^{-1}$$

$$= r_o$$

c)



$\Longleftrightarrow$



$$\boxed{V_{GS} = V_{in}}$$

using KCL at ①,

$$\frac{V_o - V_{DD}}{R_1} + g_m V_{in} + \frac{V_o}{r_o} + \frac{V_o}{R_s} = 0$$

$$\Rightarrow \cancel{\frac{V_o - V_{DD}}{R_1} + \frac{V_o}{r_o} +}$$

$$\Rightarrow \frac{V_o}{R_1} + \frac{V_o}{r_o} + \frac{V_o}{R_s} = \frac{V_{DD}}{R_1} - g_m V_{in}$$

$$\Rightarrow \frac{V_o}{R_1 \| r_o \| R_s} = -V_{in}\left(g_m - \frac{V_{DD}}{V_{in}} \times \frac{1}{R_1}\right)$$

$$\Rightarrow \frac{V_o}{V_{in}} = -\left[g_m - \frac{V_{DD}}{V_{in}}\left(\frac{1}{R_1}\right)\right](R_1 \| r_o \| R_s)$$

~~Gain~~

$$\boxed{V_{GS} = V_{in}}$$

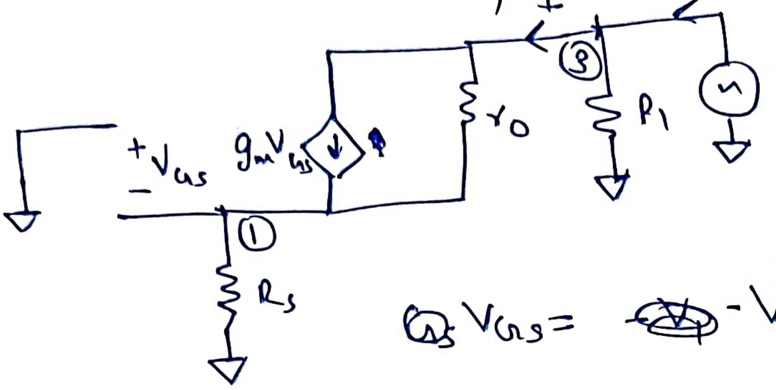
$$\boxed{V_{DD} = 0}$$

$$\text{gain} = -g_m(R_1 \| r_o \| R_s)$$

c) for i/p imependece

$Z_{i/p} = \infty$

for o/p imependence, I V_0 I_o



$\left[-\frac{V_{GS}}{R_s} = +g_m V_{GS} + \frac{V_0 + V_{GS}}{r_o} \right)\right)$

$V_{GS} = -V_1$

$\Rightarrow V_{GS} = -V_0 \left(\frac{R}{R_s + r_o + g_m r_o R_s} \right)$

KCL at ③,

$$I_o = \frac{V_o}{R_1} + I$$

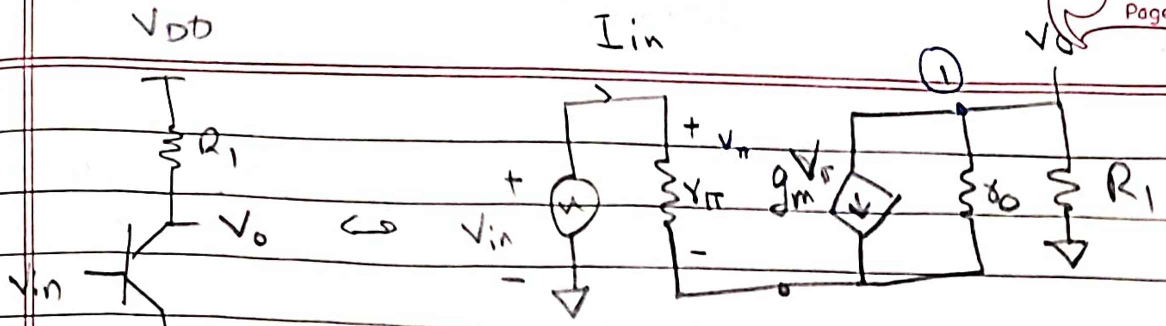
$$I_o = \frac{V_o}{R_1} + g_m V_1 + V_0 + V_{GS}$$

$$I_o = V_o \left(\frac{1}{R_1} - \frac{g_m r_o + 1}{r_o (R_s + r_o + g_m r_o R_s)} + \frac{1}{r_o} \right)$$

$$\frac{V_o}{I_o} = \frac{R_1 r_o (R_s + r_o + g_m r_o R_s)}{r_o (R_s + r_o + g_m r_o R_s) + R_1 (R_s + r_o + g_m r_o R_s) - (g_m r_o + 1) R_s R_1}$$

~~$(g_m r_o + 1) R_s R_1$~~

d)



$$\boxed{V_{in} = V_\pi}$$

KCl at ①,

$$g_m V_\pi + \frac{V_o}{r_o} + \frac{V_o}{R_1} = 0$$

$$g_m V_{in} + \frac{V_o}{r_o \| R_1} = 0$$

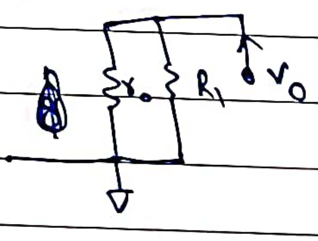
$$\Rightarrow \frac{V_o}{V_{in}} = -g_m (r_o \| R_1)$$

for i/p impedence,

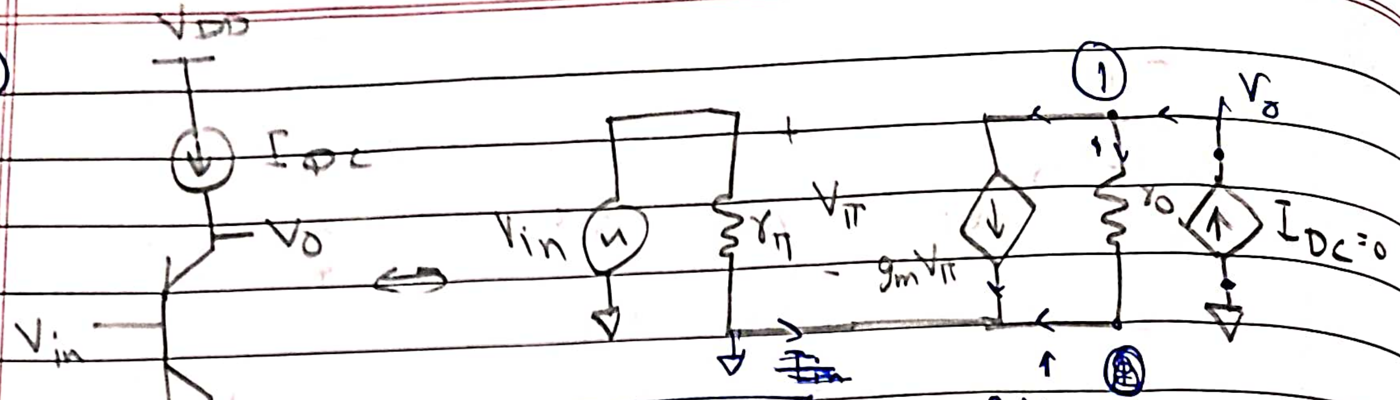
$$Z_{i/p} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{\frac{V_{in}}{r_\pi}} = r_\pi$$

for o/p impedance,

$$Z_{o/p} = r_o \| R_1$$



e)



$$\boxed{V_{in} = V_\pi}$$

KCL at (1),

$$g_m V_{in} + \frac{V_o}{r_o} = 0$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = -g_m r_o}$$

for i/p impedance,

$$Z_{i/p} = \frac{V_{in}}{I_{in}} = r_\pi$$

$$\cancel{I_{in} = g_m V_\pi + I_{DC}}$$

$$\cancel{I_{in} + g_m V_{in} - I_{DC} + g_m V_{in} = 0}$$

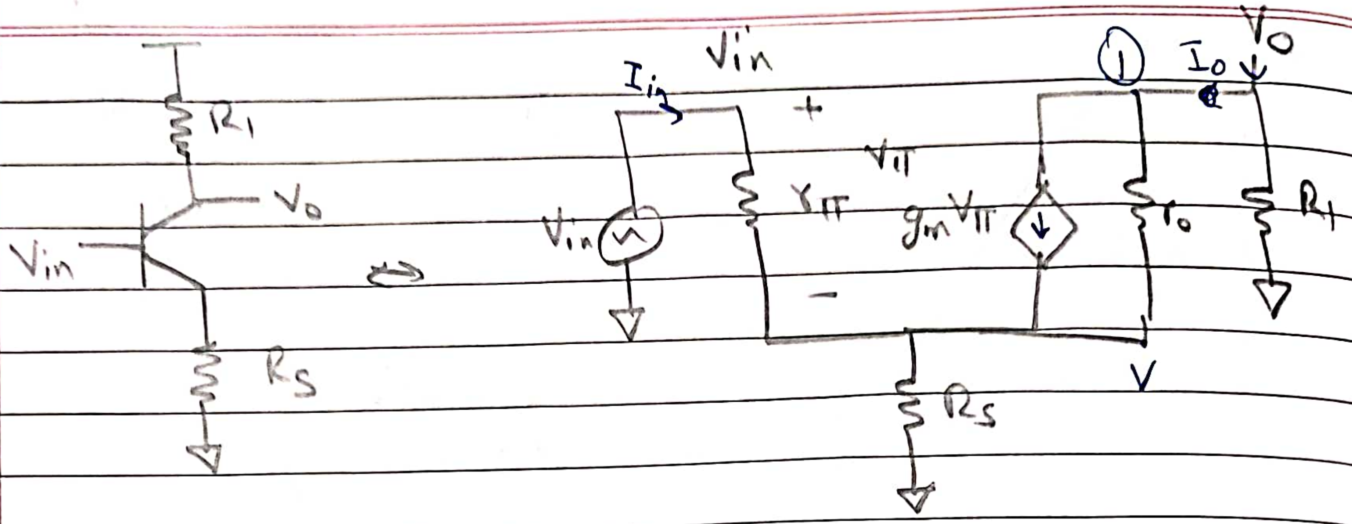
$$\cancel{\Rightarrow I_{in} = I_{DC} - 2 g_m V_{in}}$$

$$\cancel{\Rightarrow Z_{i/p} = \frac{V_{in}}{I_{DC} - 2 g_m V_{in}}}$$

for o/p impedance,

$$Z_{o/p} = r_o$$

f)



$$V_\pi = \frac{r_\pi}{R_s + r_\pi} V_{in} \quad , \quad V = \frac{R_s}{R_s + r_\pi} V_{in}$$

KCl at 1,

$$\frac{V_o - \frac{R_s}{R_s + r_\pi} V_{in}}{r_o} + g_m V_\pi + \frac{V_o}{R_1} = 0$$

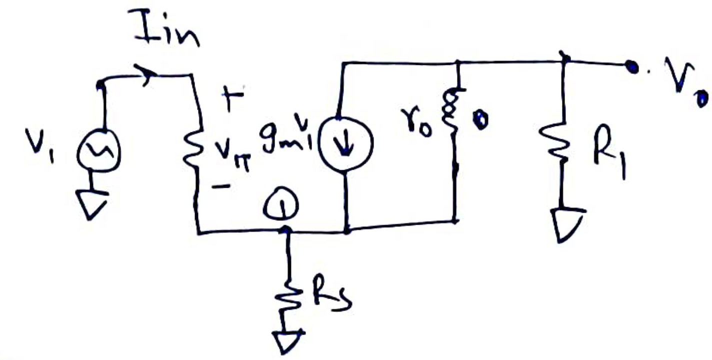
$$\Rightarrow \frac{V_o}{r_o} - \frac{R_s}{(R_s + r_\pi) r_o} V_{in} + g_m \frac{r_\pi}{R_s + r_\pi} V_{in} + \frac{V_o}{R_1} = 0$$

$$\Rightarrow \frac{V_o}{r_o \| R_1} = \left(\frac{R_s}{(R_s + r_\pi) r_o} - g_m \frac{r_\pi}{R_s + r_\pi} \right) V_{in}$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{R_s - g_m r_o r_\pi}{r_o (R_s + r_\pi)}}$$

(f) for i/p impedance,

~~$I_{i/p}$~~



$$\frac{V_{in}-V_\pi}{R_s} - I_{in} - I = 0 \qquad r_\pi I_{in} = V_{GS}$$

$$V_1 = V_{in} - V_\pi$$

Now

$$I_{in}R_s + IR_1 + IR_s + Ir_o - g_m r_\pi I_{in} r_o = 0$$

$$\Rightarrow I_{in} = \frac{I(R_1+R_s+r_o)}{g_m r_\pi r_o - R_s}$$

$$\therefore V_{in} - R_s I = (r_\pi + R_s)I_{in}$$

~~$\Rightarrow V_{in}$~~

$$\boxed{Z_{i/p} = \frac{V_{in}}{I_{in}} = \frac{R_s(g_m r_\pi r_o - R_s)}{R_1 + r_o + R_s} + (r_\pi + R_s)}$$

for o/p impedance,

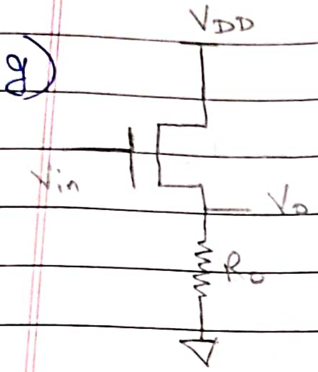
$$I = I_o - \frac{V_o}{R_1}, \quad I = -\frac{V_\pi}{r_\pi} - \frac{V_\pi}{R_s}$$

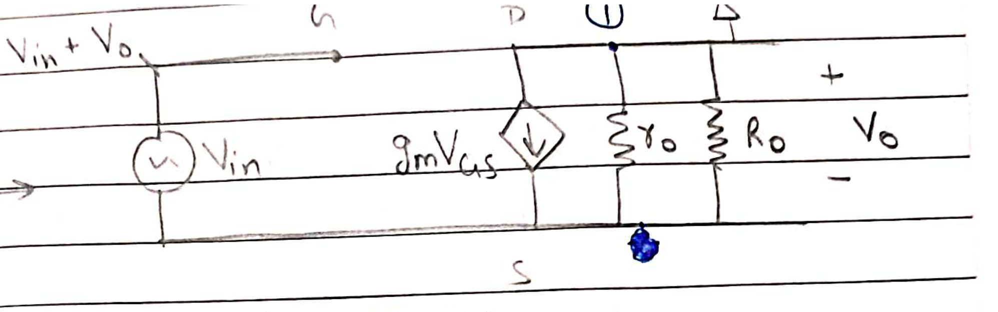
$$V_o - (I - g_m V_\pi)r_o = -V_\pi \qquad \therefore V_\pi = I(r_s \| r_\pi)$$

$$V_o = I\underbrace{\left[r_o + (1+g_m r_o)(r_s \| r_\pi)\right]}_{\boxed{\lambda}}$$

$$V_o = \left(I_o - \frac{V_o}{R_1}\right)\lambda \Rightarrow \boxed{\frac{V_o}{I_o} = \frac{\lambda}{1+\frac{\lambda}{R_1}}}$$

g)





$V_{GS} = V_{in} + V_o$

by KCL at ①,

$\cancel{g_m V_{in} + \frac{V_o}{r_o} + \frac{V_o}{R_o} = 0}$

$\Rightarrow \cancel{\frac{V_o}{V_{in}} = -g_m (r_o \| R_o)}$

~~gain = $-g_m (r_o \| R_o)$~~

~~i/p impedance =~~

~~o/p impedance = $r_o \| R_o$~~

$g_m(V_{in}+V_o) + \frac{-V_o}{r_o} + \frac{-V_o}{R_o} = 0$

$\Rightarrow g_m V_{in} = -\left[g_m - \frac{1}{r_o} - \frac{1}{R_o}\right] V_o$

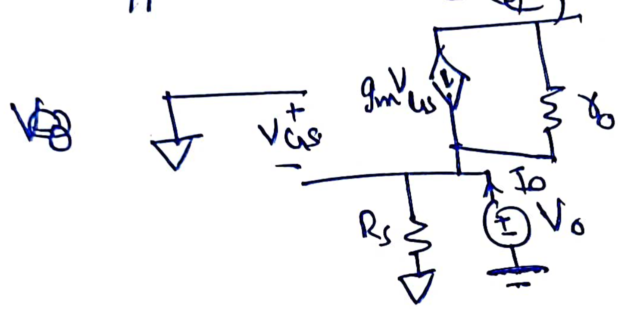
$\Rightarrow \frac{V_o}{V_{in}} = -g_m \left[g_m - \frac{1}{r_o} - \frac{1}{R_o}\right]^{-1}$

gain

(9) for i/p ~~image~~ impedence,

$Z_{i/p} = \infty$

for o/p imeduclece,

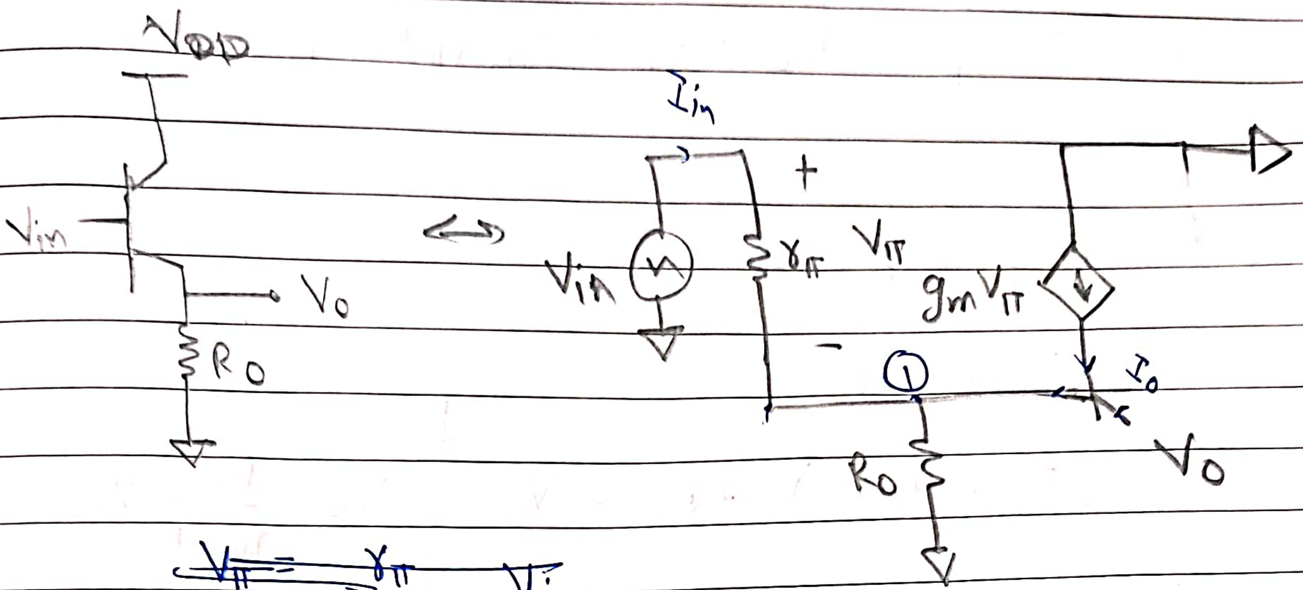


$$\frac{V_o}{R_o} - I_o - g_m V_{GS} + \frac{V_o}{r_o} = 0$$

$$\frac{V_o}{R_o} = I_o - g_m V_o - \frac{V_o}{r_o}$$

$$\Rightarrow \frac{V_o}{I_o} = R_o \| r_o \| (1/g_m)$$

b)



$$\cancel{V_\pi = \frac{r_\pi}{R_O + r_\pi} V_{in}}$$

$\cancel{V_o =}$ $\boxed{V_{in} - V_o = V_\pi}$

KCL at (1),

$$\frac{V_o - V_{in}}{r_\pi} + \frac{V_o}{R_o} - g_m V_\pi = 0$$

$$\Rightarrow \frac{V_o}{r_\pi} + \frac{V_o}{R_o} + g_m V_o - g_m V_{in} - \frac{V_{in}}{r_\pi} = 0$$

$$\Rightarrow \frac{V_o}{V_{in}} = \left(\frac{1}{r_\pi} + g_m\right)\left(r_\pi \,\|\, R_o \,\|\, 1/g_m\right.$$

h) for i/p impedence,

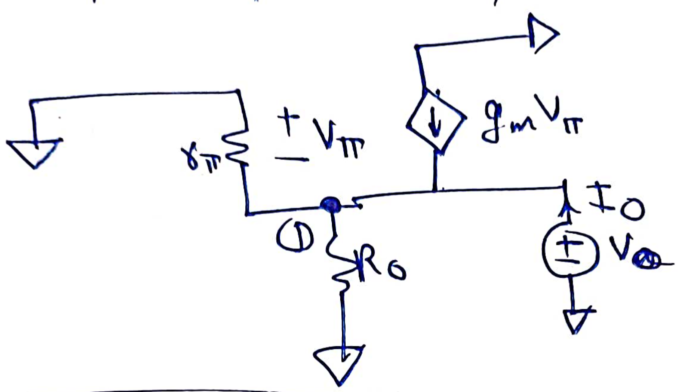
$$V_\pi = V_{in} - V_o = \frac{\left(\frac{1}{R_o}\right) V_{in}}{R_o \parallel r_\pi \parallel (1/g_m)}$$

$$\frac{V_{in}}{I_{in}} = \frac{V_{in}}{V_\pi / r_\pi} = \cancel{\frac{1}{r_\pi}\left(\frac{1}{R_o} + \frac{1}{r_\pi} + g_m\right)}$$

$$\boxed{\cancel{Z_{i/p} = \frac{R_o}{r_\pi} \cdot \frac{R_o}{R_o \parallel r_\pi \parallel (1/g_m)}}}$$

$$z_{i/p} = \frac{R_o r_\pi}{R_o \parallel r_\pi \parallel (1/g_m)}$$

for o/p impedence, $V_{in} = 0$,

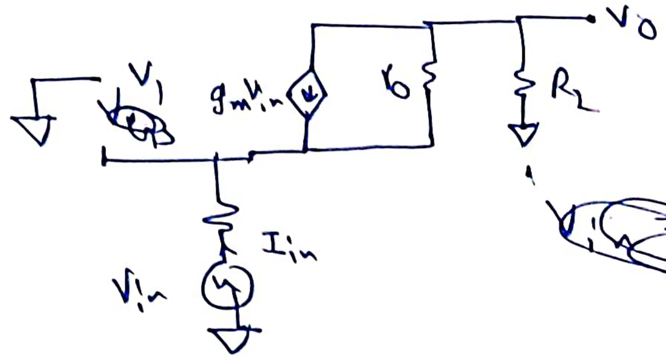


$$\boxed{V_o + V_\pi = 0}$$

$$\frac{V_o}{R_o} - \frac{V_\pi}{r_\pi} - g_m V_\pi - I_o = 0$$

$$\Rightarrow \frac{V_{in}}{I_o} = \left(R_o \parallel r_\pi \parallel 1/g_m\right)$$

i)



$$\boxed{I_{in} = \frac{V_o}{R_2}}$$

$$\frac{-V_1 - V_{in}}{R_1} + \frac{-V_1 - V_o}{r_o} = (g_m V_1)$$

$$\Rightarrow \quad -I_{in} + \frac{V_{in}}{r_o} - \frac{I_{in} R_1}{r_o} - \frac{V_o}{r_o} = -g_m V_{in} + g_m I_{in} R_1$$

$$\Rightarrow \quad I_{in}\left(-1 - \frac{R_1}{r_o} - g_m R_1\right) - \frac{V_o}{r_o} = V_{in}\left(-g_m - \frac{1}{r_o}\right)$$

$$\therefore \quad -\frac{V_o}{R_2}\left(1 + \frac{R_1}{r_o} - g_m R_1\right) - \frac{V_o}{r_o} = V_{in}\left(g_m + \frac{1}{r_o}\right)$$

$$\Rightarrow \quad \boxed{\frac{V_o}{V_{in}} = \frac{g_m + 1/r_o}{\left(\frac{1}{R_2} + \frac{R_1}{R_2 r_o} + \frac{g_m R_1}{R_2} + \frac{1}{r_o}\right)}}$$

for i/p (Z)

$$Z_{i/p} = \frac{V_{in}}{I_{in}} = \frac{R_2 V_{in}}{V_o}$$

$$= \frac{R_2\left[\frac{1}{R_2} + \frac{R_1}{R_2 r_o} + \frac{g_m R_1}{R_2} + \frac{1}{r_o}\right]}{\left[g_m + \frac{1}{r_o}\right]}$$

for o/p (Z)

$$\frac{-V}{R_1} + \frac{-V_1 - V_o}{r_o} = g_m V_1$$

$$\Rightarrow \quad I_o = \left[\frac{V_o}{R_2 \| r_o}\right] - V_1\left(g_m + \frac{1}{r_o}\right)$$

$$\boxed{V_1 = \frac{V_o}{r_o} \times \quad R_1 \| r_o \| 1/g_m}$$

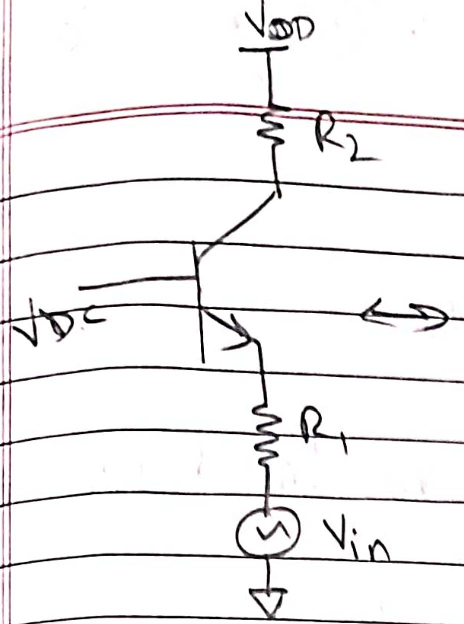
$$\cancel{I_o = v_o [}$$

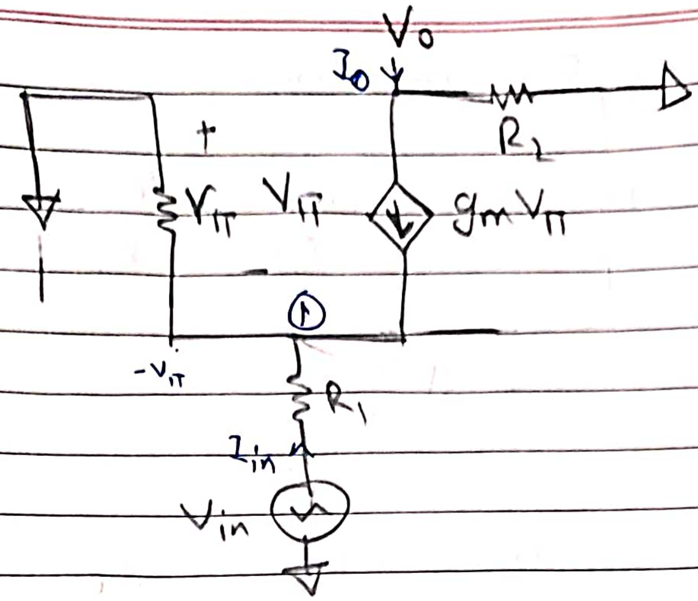
$$I_o = \frac{V_o}{R_2 \| r_o} - \frac{v_o\left(g_m + \frac{1}{r_o}\right)}{r_o}\left(R_1' \| r_o \| 1/g_m\right)$$

$$\cancel{\frac{V_o}{I_o} = R_1 \| 1/g_m \| r_o}$$

$$\frac{V_o}{I_o} = \frac{\dfrac{r_o}{R_1 \| 1/g_m \| r_o}}{\dfrac{r_o}{R_2}\left[\dfrac{1}{R_1} + g_m + \dfrac{1}{r_o}\right] + \dfrac{r_o}{R_1}}$$

i)





$$\boxed{V_o = -(g_m V_\pi) R_2}$$

KCL at 1,

$$\cancel{\frac{V_{in}+V_\pi}{R_1}} - \frac{V_{in}+V_\pi}{R_1} + \frac{-V_\pi}{r_\pi} - g_m V_\pi = 0$$

$$\Rightarrow \frac{V_{in}+V_\pi}{R_1} + \frac{V_\pi}{r_\pi} + g_m V_\pi = 0$$

$$\Rightarrow \boxed{V_{in} = -V_\pi\left(1 + g_m R_1 + \frac{R_1}{r_\pi}\right)}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{g_m R_2}{1 + g_m R_1 + \frac{R_1}{r_\pi}}}$$

for i/p impedence,

$$I_{in} = \frac{V_{in}+V_\pi}{R_1} \Rightarrow Z_{i/p} = \frac{V_{in}}{I_{in}} = \frac{1 + g_m R_1 + \frac{R_1}{r_\pi}}{\left(g_m + \frac{1}{r_\pi}\right)}$$

~~for o/p impedence,~~

o/p impedence,

$$I_o = \cancel{\frac{V_o}{I_o}} \parallel \frac{V_o}{R_2} + g_m V_\pi$$

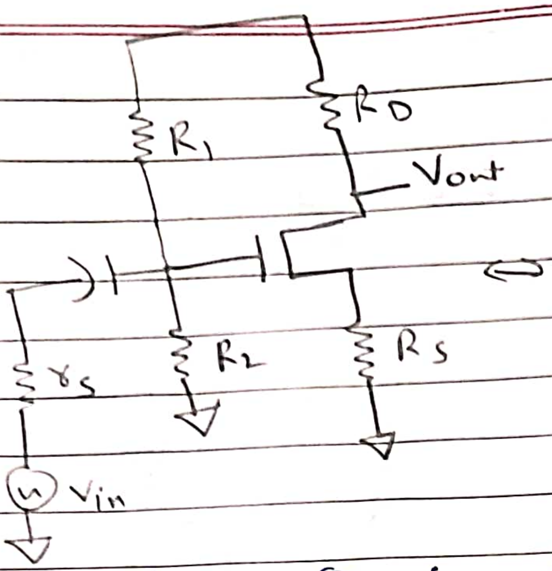
$$I_o = \frac{V_o}{R_2}$$

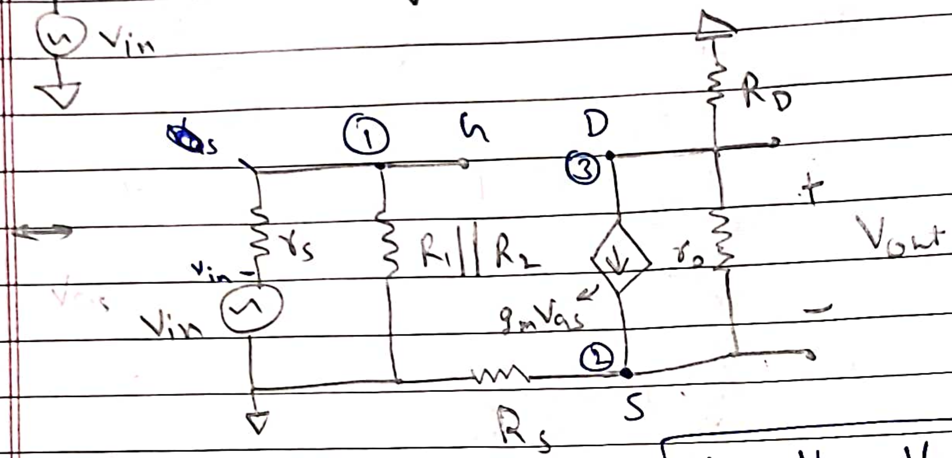
$$-V_\pi = (r_\pi \parallel R_1) g_m V_\pi$$

$$\Rightarrow V_\pi = 0$$

$$\Rightarrow \frac{V_o}{I_o} = \boxed{R_2 = Z_{o/p}}$$

k)





$$\boxed{V_G - V_S = V_{GS}}$$

$$\boxed{V_D - V_S = V_{DS} = V_{out}}$$

KCL at 1,

$$\frac{V_G - V_{in}}{r_s} + \frac{V_G}{R_1 || R_2} = 0$$

$$\Rightarrow \quad V_G = \frac{V_{in}}{r_s}\left(\frac{1}{r_s} + \frac{1}{R_1||R_2}\right)^{-1}$$

KCL at 2,

$$-g_m V_{GS} + \frac{V_S}{R_S} + \frac{V_S - V_D}{r_o} = 0$$

$$\Rightarrow -g_m V_{GS} + \frac{V_S}{R_S} - \frac{V_O}{r_O} = 0$$

$$\Rightarrow V_S = \left(g_m V_{GS} + \frac{V_O}{r_O}\right) R_S$$

$$V_G - V_S = V_{GS} = \frac{V_{in}}{r_s}\left(\frac{1}{r_s} + \frac{1}{R_1||R_2}\right)^{-1} - \left(g_m V_{GS} + \frac{V_o}{r_o}\right) R_S$$

$$V_{GS} = \frac{V_{in}}{r_s}\left(\frac{R_1 + R_2 + r_s R_1 R_2}{r_s(R_1+R_2)}\right)^{-1} - \left(g_m V_{GS} + \frac{V_o}{r_o}\right) R_S$$

$$\Rightarrow (1 + g_m R_S)\, V_{GS} = \frac{V_{in}}{r_s} \times \frac{r_s(R_1+R_2)}{R_1+R_2+r_s R_1 R_2} - \frac{V_o}{r_o} R_S$$

$$\Rightarrow \cancel{V_{GS} = V_{in}(R_1+R_2)}$$

$$\Rightarrow V_{GS} = \frac{V_{in}}{1 + r_s(R_1||R_2)} - \frac{V_o}{r_o} R_S$$

KCL at 3,

$$g_m V_{GS} + \frac{V_o}{r_o} + \frac{V_D}{R_D} = 0$$

$$\Rightarrow g_m V_{GS} + \frac{V_o}{r_o} + \frac{V_o + V_s}{R_D} = 0$$

$$\Rightarrow g_m V_{GS} + \frac{V_o}{r_o||R_D} + \frac{R_S}{R_D}\left(g_m V_{GS} + \frac{V_o}{r_o}\right) = 0$$

$$\Rightarrow g_m\left(1 + \frac{R_S}{R_D}\right) V_{GS} + \frac{V_o}{r_o||R_D} + \frac{V_o R_S}{r_o R_D} = 0$$

$$\Rightarrow g_m\left(1+\frac{R_S}{R_D}\right)\left[\frac{V_{in}}{1+\gamma_s(R_1||R_2)}-\frac{V_o}{r_o}R_s\right]$$

$$+\frac{V_o}{r_o||R_D}+\frac{V_oR_s}{r_oR_D}=0$$

$$\Rightarrow g_m\left(1+\frac{R_s}{R_D}\right)\frac{V_{in}}{1+\gamma_s(R_1||R_2)}$$

$$= \cancel{\left[g_m\left(1+\frac{R_s}{R_D}\right)+\right.}$$

$$=-V_o\left[\frac{1}{r_o||R_D}+\frac{R_s}{r_oR_D}-g_m\left(1+\frac{R_s}{R_D}\right)\frac{R_s}{r_o}\right]$$

$$\frac{V_o}{V_{in}}=\frac{-g_m\left(1+\frac{R_s}{R_D}\right)\frac{1}{1+\gamma_s(R_1||R_2)}}{\frac{1}{r_o||R_D}+\frac{R_s}{r_oR_D}-g_m\left(1+\frac{R_s}{R_D}\right)\frac{R_s}{r_o}}$$

$Z_{i/p}=r_s+R_1||R_2$

~~$Z_{o/p}=r_o$~~

(k) for o/p impedance;

$$g_m V_{GS} = \frac{V_G - V_o}{R_O} + \frac{V_G}{R_S}$$

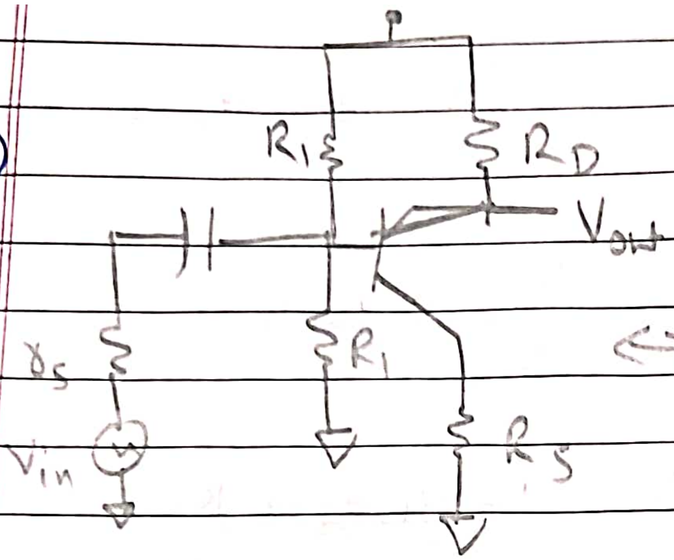
$$V_G \left(\boxed{\frac{V_G}{r_o \| R_S \| 1/g_m} = \frac{V_o}{r_o}} \right.$$

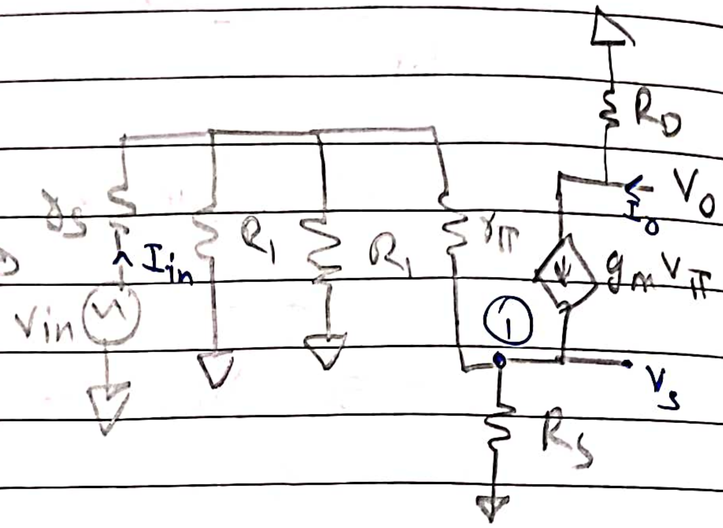
$$I_o - \frac{V_o}{R_D} - \frac{V_o - V_G}{r_o} - g_m V_1 = 0$$

$$\Rightarrow I_o = \frac{V_o}{R_D} + \frac{V_o}{r_o} - \frac{V_G}{r_o \| 1/g_m}$$

$$\Rightarrow \frac{V_o}{I_o} = \frac{\dfrac{r_o}{r_o \| R_S \| 1/g_m}}{\dfrac{1}{R_D}\left(\dfrac{1}{r_o \| R_S \| 1/g_m}\right) + \dfrac{1}{R_S}}$$

L)





$$V_o = -(g_m V_\pi) R_D$$

$$V_s = (I_\pi + g_m V_\pi) R_s \,, \quad \boxed{V_\pi = I_\pi r_\pi}$$

$$\boxed{V_s = I_\pi R_s (1 + g_m r_\pi)}$$

Now, $I_\pi = \dfrac{V_{in} - V_s}{R_1 || R_2 + r_s + r_\pi}$

$$\Rightarrow \frac{V_s}{\cancel{R_s}} = \left(\frac{V_{in} - V_s}{R_1 || R_2 + r_s + r_\pi}\right)(1 + g_m r_\pi) R_s$$

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$$\boxed{V_s\left(\frac{1}{R_s}+\frac{1+g_m r_\pi}{R_1||R_2+r_s+r_\pi}\right)=\frac{V_{in}(1+g_m r_\pi)}{R_1||R_2+r_s+r_\pi}}$$

Now,

$$-\frac{V_o}{R_D}=g_m\left(\frac{(V_{in}-V_s)r_\pi}{R_1||R_2+r_s+r_\pi}\right)$$

$$\frac{V_o}{R_D}=g_m\left(V_{in}-\frac{V_{in}(1+g_m r_\pi)R_s}{R_1||R_2+r_s+r_\pi+(1+g_m r_\pi)R_s}\right)$$

$$\boxed{\frac{V_o}{V_{in}}=-g_m\left(1-\frac{R_s(1+g_m r_\pi)}{R_1||R_2+r_s+r_\pi+(1+g_m r_\pi)R_s}\right)}\ A_v$$

for i/p (Z),

$$\cancel{V_{in}=I_{in}}$$

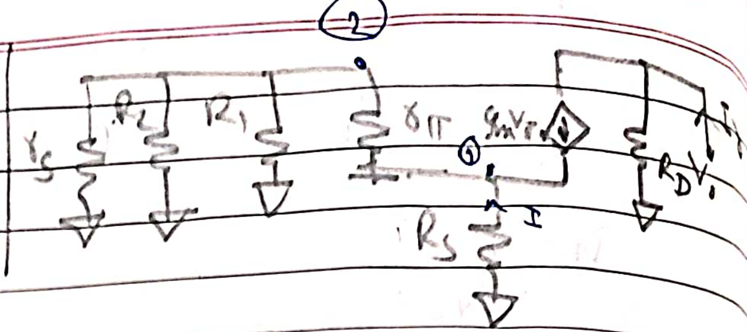
$$\frac{V_{in}-V_s}{R_1||R_2+r_s+r_\pi}=I_{in}$$

$$V_s=\frac{V_{in}(1+g_m r_\pi)R_s}{R_1||R_2+r_s+r_\pi+(1+g_m r_\pi)R_s}$$

$$V_{in}-I_{in}(R_1||R_2+r_s+r_\pi)=\frac{V_{in}(1+g_m r_\pi)R_s}{R_1||R_2+r_s+r_\pi+R_s(1+g_m r_\pi)}$$

$$\Rightarrow\frac{V_{in}}{I_{in}}=\frac{R_1||R_2+r_s+r_\pi}{1-\frac{R_s(1+g_m r_\pi)}{R_1||R_2+r_s+r_\pi+R_s(1+g_m r_\pi)}}=Z_{i/p}$$

for o/p impedence,



KCl at 1,

$$\frac{V_\pi}{r_\pi} + g_m V_\pi = I$$

$$\Rightarrow \boxed{\cancel{V_\pi = 0}}$$

KCl at 2

$$\cancel{(V_1 + V_\pi)}$$

$$\frac{V_1 + V_\pi}{R_S \| R_2 \| R_1} + \frac{V_\pi}{r_\pi} = 0$$

$$\Rightarrow \frac{I R_S + V_\pi}{R_S \| R_2 \| R_1} = -\frac{V_\pi}{r_\pi}$$

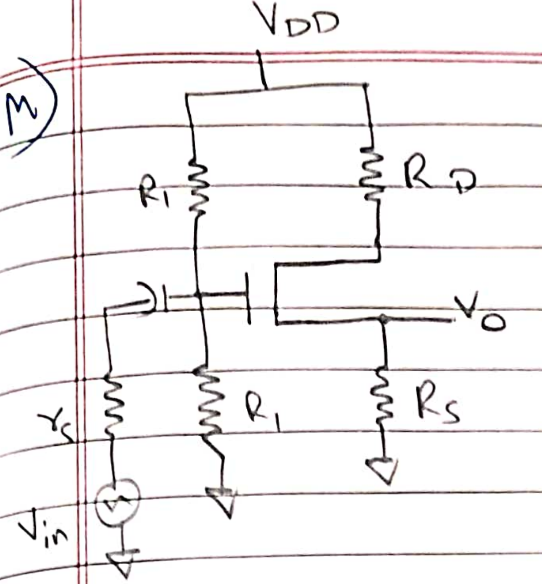
$$\Rightarrow \left[V_\pi R_S \left[\frac{1}{r_\pi} + g_m \right] + V_\pi \right] \left[\frac{1}{R_S \| R_2 \| R_1} \right] = \frac{-V_\pi}{r_\pi}$$

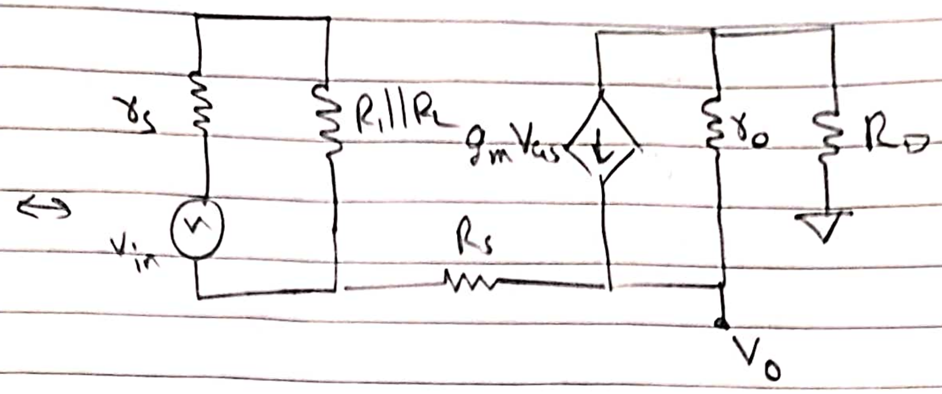
$$\Rightarrow \cancel{V_\pi \left[R_S \right.}$$

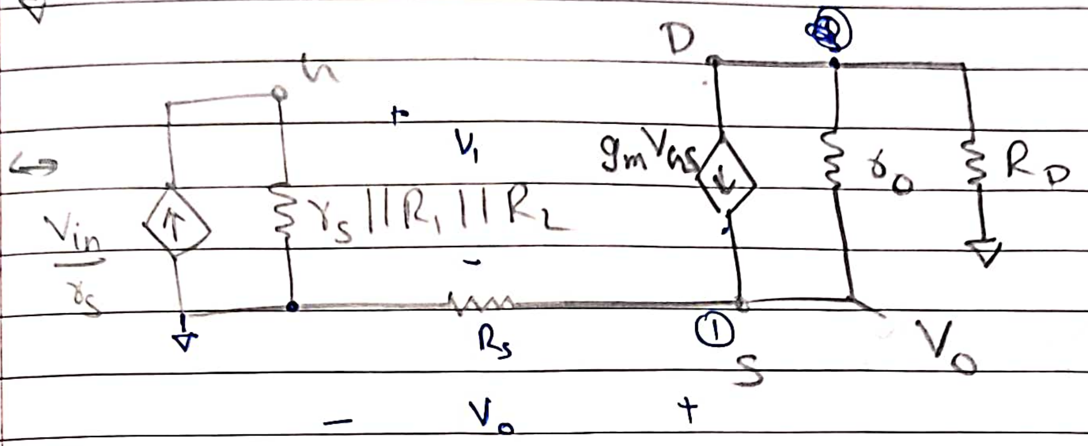
$$\Rightarrow V_\pi = 0$$

$$R_{out} = \frac{V_o}{I_o} = R_D$$

M)







$$V_{GS} = \frac{V_{in}}{r_s}(r_s \,||\, R_1 \,||\, R_2) - V_o$$

KCL at (1),

$$-g_m V_{GS} + \frac{V_o}{r_o+R_D} + \frac{V_o}{R_s} = 0$$

$$-g_m\left(\frac{V_{in}}{r_s}\right)(r_s \,||\, R_1 \,||\, R_2) + g_m V_o + \frac{V_o}{r_o+R_D} + \frac{V_o}{R_s} = 0$$

$$\Rightarrow \quad \frac{V_o}{V_{in}} = \left(g_m + \frac{1}{r_o+R_D} + \frac{1}{R_s}\right)^{-1} \frac{g_m}{r_s}(r_s \,||\, R_1 \,||\, R_2)$$

$$= \frac{\left[\left(\frac{1}{g_m}\right) || (r_o+R_D) \,||\, R_s\right] g_m (r_s \,||\, R_1 \,||\, R_2)}{r_s}$$

for i/p (Z)

$$Z_{i/p} = \frac{V_{in}}{I_{in}} = r_s + R_1 || R_2$$

for o/p (Z)

$$\boxed{V_a = 0}$$

$$\boxed{V_s + V_o = 0}$$

$$\left(I_o - \frac{V_o}{R_s}\right) + g_m V_1 + \frac{V_s - V_D}{r_o} = 0$$

$$\Rightarrow \cancel{V_s = V_o \left[r_o || \right.}$$

$$\boxed{V_s = \frac{V_o}{r_o || 1/g_m || R_s} r_o - I_o r_o}$$

Now

$$\frac{V_s}{R_D} + \frac{V_s - V_a}{r_o} + g_m V_1 = 0$$

$$\Rightarrow V_s + \frac{V_s - V_a}{r_o} + g_m V_1 + \frac{V_s - V_D}{r_o} = 0$$

$$\cancel{V_o \left(\frac{1}{r_o} + g_m + \right.}$$

$$\frac{V_o}{r_o || g_m || R_s} (r_o) \times \frac{1}{R_D || r_o} - \frac{1}{r_o || 1/g_m} = I_o r_o \left(\frac{1}{R_D} + \frac{1}{r_o}\right)$$

$$\frac{V_o}{I_o} = \frac{\left(1 + \frac{r_o}{R_s}\right)}{\left[(r_o || R_s || 1/g_m)\left(1 + \frac{r_o}{R_D}\right) - (r_o || 1/g_m) \right]}$$

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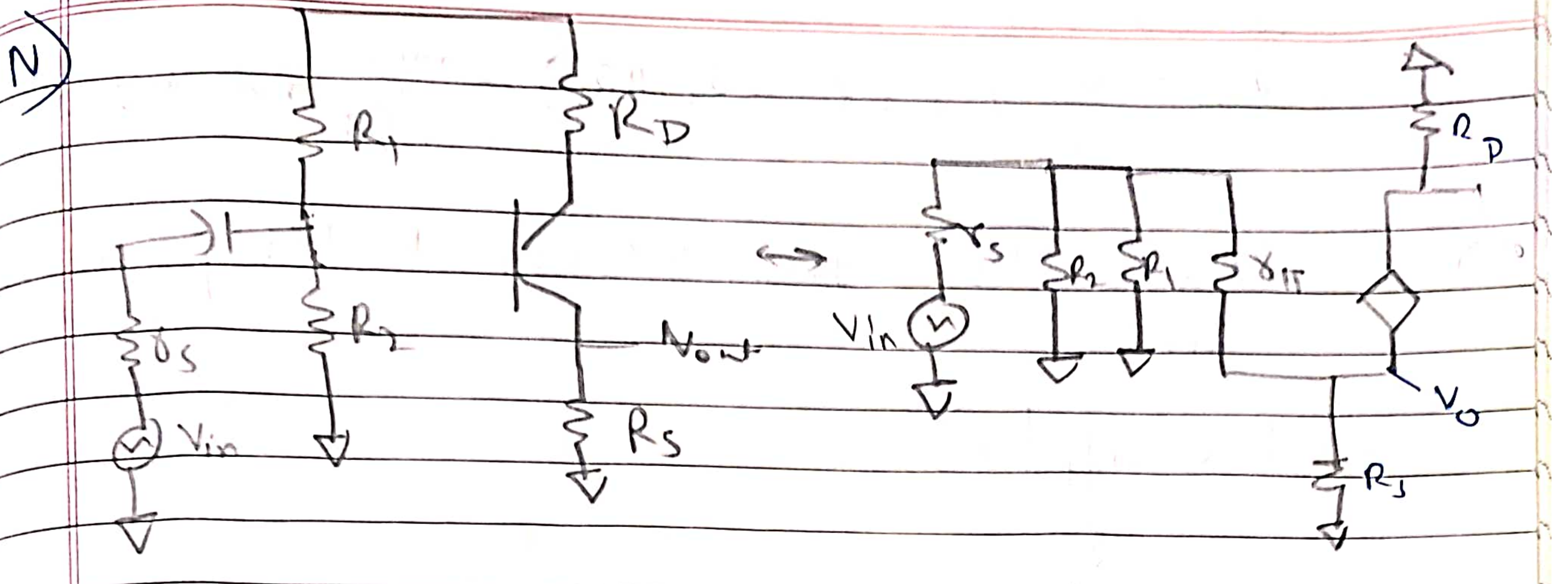
N)



$$\frac{V_D}{R_D} + g_m V_\pi = 0$$

$$\frac{V_o}{R_S} - \frac{V_\pi}{r_\pi} - g_m V_\pi = 0$$

$$\frac{V_o}{R_S} = \frac{V_\pi}{r_\pi}(1 + g_m r_\pi)$$

$$V_{in} - \frac{V_\pi}{r_\pi}\left(r_s + r_\pi + R_1 \| R_2\right) = V_o$$

$$\frac{V_o}{R_S} = \frac{(V_{in} - V_o)(1 + g_m r_\pi)}{r_s + r_\pi + R_1 \| R_2}$$

$$V_o\left(\frac{1}{R_S} + \frac{1 + g_m r_\pi}{r_s + r_\pi + (R_1 \| R_2)}\right) = \frac{V_{in}(1 + g_m r_\pi)}{(r_s + r_\pi + R_1 \| R_2)}$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{R_S(1 + g_m r_\pi)}{r_s + r_\pi + (R_1 \| R_2) + R_S(1 + g_m r_\pi)}}$$

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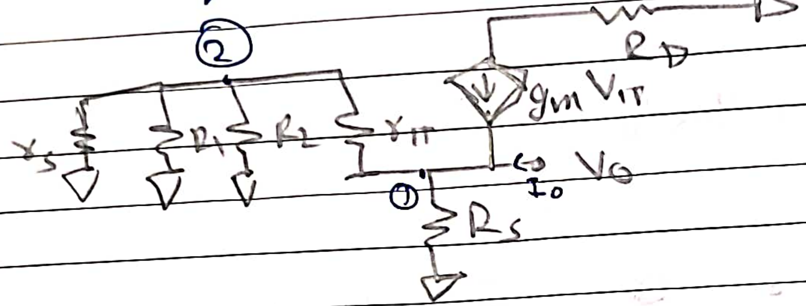
for i/p (Z)

$$V_{in} - \frac{V_\pi}{r_\pi}\left(r_s + r_\pi + R_1 || R_2\right) = \frac{V_\pi}{r_\pi} R_s\left(1 + g_m r_\pi\right)$$

$$\frac{V_{in}}{I_{in}} = R_s\left(1+g_m r_\pi\right) + r_s + r_\pi + R_1 || R_2$$

$$\frac{V_\pi}{r_\pi} = I_{in}$$

for o/p (Z)



KCl at ①,

$$\frac{V_\pi}{r_\pi} + g_m V_\pi + I_0 - \frac{V_o}{R_s} = 0$$

$$\Rightarrow V_\pi = \left(\frac{V_o}{R_s} + I_o\right)\left(r_\pi || 1/g_m\right)$$

KCL at ②,

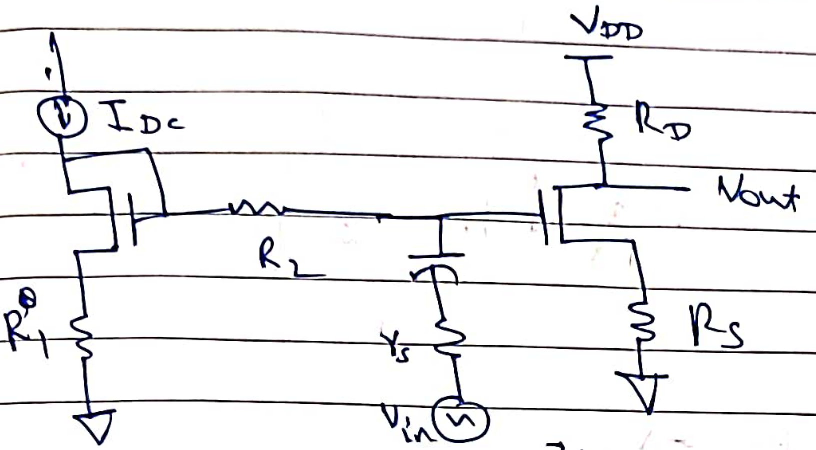
$$\cancel{\left(V_o + V_\pi\right)\left(r_s ||\right.}$$

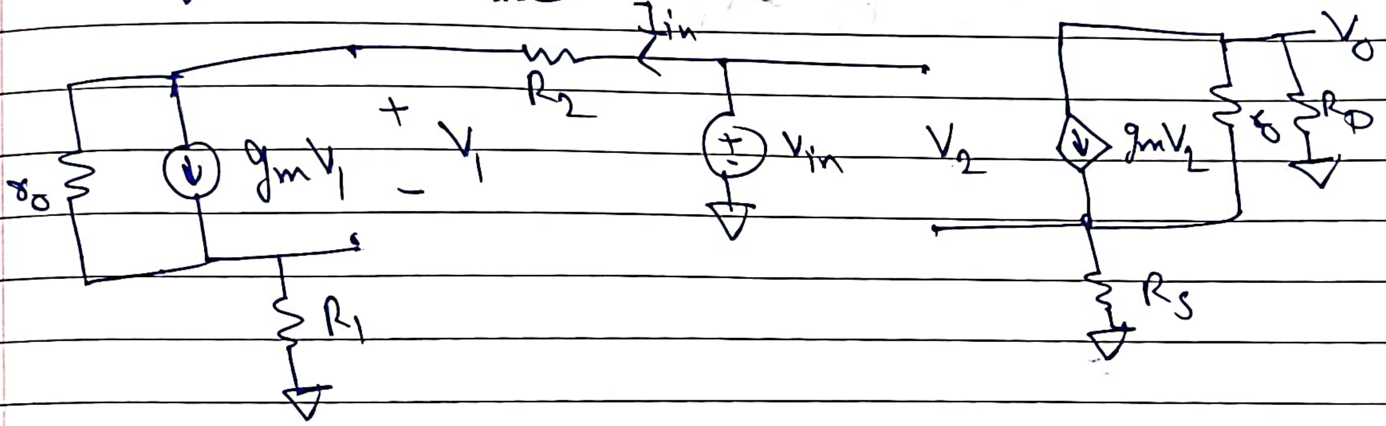
$$\frac{V_o + V_\pi}{r_s || R_1 || R_2} + \frac{V_\pi}{r_\pi} = 0$$

$$\cancel{\frac{V_o}{I_o} = \left(R_s\left(r_s || R_2 || R_1\right)\left(\right.\right.}$$

$$\frac{V_o}{I_o} = \frac{\dfrac{R_S}{r_s \parallel R_2 \parallel R_1 \parallel r_\pi}}{\dfrac{R_S\left(g_m + 1/r_\pi\right) + 1}{r_s \parallel R_2 \parallel R_1} + \dfrac{1}{r_\pi}} = Z_{o/p}$$

D)





$\cancel{I_{in} = g_m V_1 + V_o}$

$$V_{in} - I_{in} R_2 - V_1 - I_{in} R_1 = 0$$

$$\Rightarrow V_{in} - I_{in} R_2 - \frac{V_{in}}{g_m r_o} - \frac{I_{in}}{g_m r_o}(R_1 + R_2) - \frac{I_{in}}{g_m} = 0$$

$$\Rightarrow V_{in}\left(1 + \frac{1}{g_m r_o}\right) = I_{in}\left((R_1 + R_2)\left(1 + \frac{1}{g_m r_o}\right) + \frac{1}{g_m}\right)$$

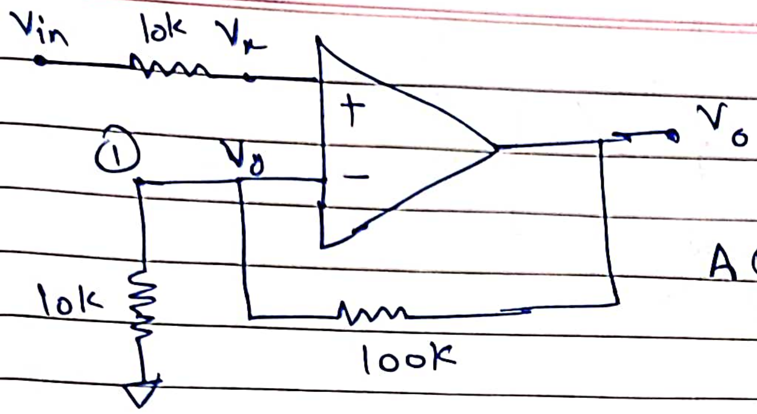
$$Z_{i/p} = R_{in} = \frac{(R_1 + R_2)\left(1 + \frac{1}{g_m r_o}\right) + \frac{1}{g_m}}{1 + \frac{1}{g_m r_o}}$$

$$g_m V_{in} + \frac{g_m V_0 R_S}{R_D} = -\frac{V_0 R_S}{R_D r_0} - \frac{V_0}{r_0} - \frac{V_0}{R_D}$$

$$\Rightarrow V_0\left[-\frac{g_m R_S}{R_D} + \frac{R_S}{R_D r_0} + \frac{1}{r_0} + \frac{1}{R_D}\right] = -g_m V_{in}$$

$$\frac{V_0}{V_{in}} = \frac{-g_m}{\left(\frac{R_S}{R_D}\right)\left[g_m + \frac{1}{r_0}\right] + \frac{1}{r_0} + \frac{1}{R_D}}$$

2)



$$A = \frac{V_o}{V_x - V_y}$$

for ideal case,

$$V_x = V_y$$

$$V_x = V_{in} = V_y$$

KCL at 1,

$$\frac{V_{in}}{10} + \frac{V_{in} - V_o}{100} = 0$$

$$\Rightarrow V_{in} + \frac{V_{in}}{10} - \frac{V_o}{10} = 0$$

$$\Rightarrow 11V_{in} = V_o$$

$$\Rightarrow \frac{V_o}{V_{in}} = 11$$

$$\Rightarrow g_{ideal} = 11$$

i) for non ideal case, $A = 10$

$$A.V_x \neq V_y$$

$$V_x = V_{in}$$

KCL at (1),

$$\frac{V_y}{1\cancel{0}} + \frac{V_y - V_o}{10\cancel{0}} = 0$$

$$\Rightarrow 11 V_y = V_o$$

$$\Rightarrow V_y = \frac{V_o}{11}$$

$$A = \frac{V_o}{V_x - V_y}$$

$$\Rightarrow 10 = \frac{V_o}{V_{in} - \frac{V_o}{11}}$$

$$\Rightarrow V_{in} - \frac{V_o}{11} = \frac{V_o}{10}$$

$$\Rightarrow V_{in} = V_o\left(\frac{1}{10} + \frac{1}{11}\right)$$

$$\Rightarrow \frac{V_o}{V_{in}} = \frac{110}{21}$$

$$\Rightarrow g_{NI} = \frac{110}{21}$$

$$\% \text{error} = \frac{g_I - g_{NI}}{g_I} \times 100$$

$$= \frac{11 - \frac{110}{21}}{g_I} \times 100$$

$$= 52.38\%$$

ii) $A = 100$

$$100 = \frac{V_o}{V_{in} - \frac{V_o}{11}}$$

$$\Rightarrow \cancel{100V_{in}} - V_{in} - \frac{V_o}{11} = \frac{V_o}{100}$$

$$\Rightarrow V_{in} = \left(\frac{1}{100} + \frac{1}{11}\right) V_o$$

$$\Rightarrow \frac{1100}{111} = \frac{V_o}{V_{in}} = g_{NI}$$

$$\therefore \% \text{ err} = \frac{g_I - g_{NI}}{g_I} \times 100$$

$$= \frac{11 - \frac{1100}{111}}{11} \times 100$$

$$= 9.91\%$$

iii) $A = \frac{V_o}{V_{in} - \frac{V_o}{11}}$, $A = 1000$

$$\Rightarrow V_{in} - \frac{V_o}{11} = \frac{V_o}{A}$$

$$\Rightarrow V_{in} = \left(\frac{1}{11} + \frac{1}{A}\right) V_o$$

$$\Rightarrow \frac{11A}{11+A} = \frac{V_o}{V_{in}} = g_{NI}$$

$$g_{NI} = \frac{11 \times 1000}{11 + 1000}$$

$$= 10.88$$

$$\% \text{ err} = \frac{11 - 10.88}{11} \times 100$$

$$= 1.09\%$$

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iv) $A = 10^4$

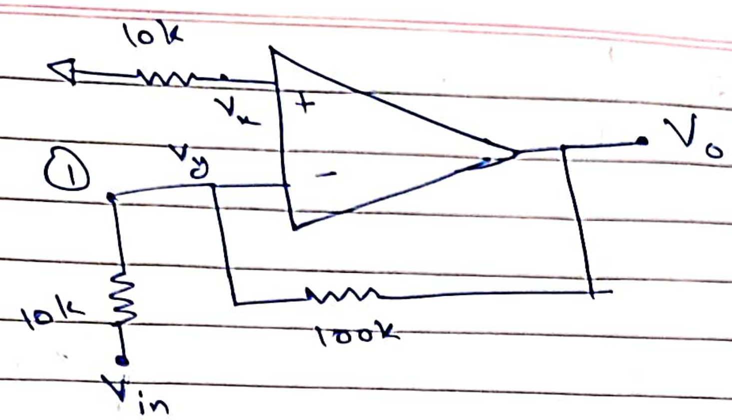
$$g_{NI} = \frac{11A}{11+A}$$

$$= \frac{11\times10^4}{11+10^4} = 10.98791$$

$$\%\ err = \frac{11-10.98791}{11}\times100$$

$$= 0.1099\ \%$$

b)



for ideal case,

$$V_x = V_y = 0$$

KCL at 1,

$$\frac{0 - V_{in}}{10} + \frac{0 - V_o}{100} = 0$$

$$\Rightarrow \quad 10V_{in} = -V_o$$

$$\Rightarrow \quad \frac{V_o}{V_{in}} = g_I = -10$$

for non ideal case,

$$V_x = 0$$

KCL at 1,

$$\frac{V_y - V_{in}}{10} + \frac{V_y - V_o}{100} = 0$$

$$\Rightarrow \quad V_y\left(\frac{1}{1\cancel{0}} + \frac{1}{10\cancel{0}}\right) = \frac{V_{in}}{1\cancel{0}} + \frac{V_o}{10\cancel{0}}$$

$$\Rightarrow \quad V_y \times \frac{11}{10} = V_{in} + \frac{V_o}{10}$$

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$$A = \frac{V_o}{0 - V_y}$$

$$-A V_y = V_o$$

$$\Rightarrow -A \times \frac{10}{11}\left(V_{in} + \frac{V_o}{10}\right) = V_o$$

$$\Rightarrow V_{in} + \frac{V_o}{10} = \frac{-11 V_o}{10 A}$$

$$\Rightarrow V_{in} = -\left(\frac{1}{10} + \frac{11}{10A}\right) V_o$$

$$\Rightarrow V_{in} = -\left(\frac{A+11}{10A}\right) V_o$$

$$\frac{V_o}{V_{in}} = \frac{-10A}{11+A} = g_{NI}$$

i) $A = 10$, $g_{NI} = \frac{-100}{21} = -5.24$

$$\% \text{ err} = \cancel{5.24}\ \frac{(-10) - (-\cancel{5.24}\ 4.76)}{-10} \times 100$$

$$= \cancel{52.24\%}\ \cancel{47.62\%}\ 52.4\%$$

ii) $A = 100$, $g_{NI} = \frac{-1000}{111} = -9$

$$\% \text{ err} = 10\%$$

iii) $A = 10^3$, $g_{NI} = \frac{-10 \times 10^3}{1011} = \cancel{-9.88}\ -9.89$

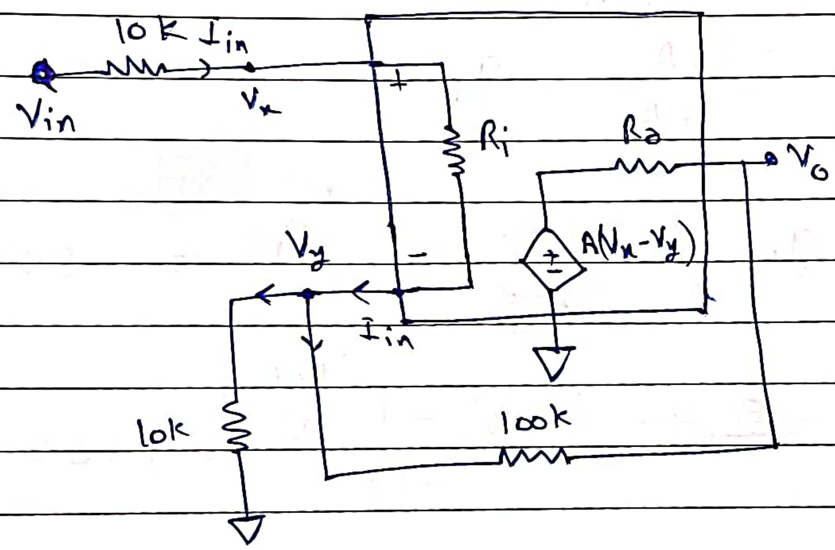
$$\%\,err = \frac{-10 - (-\cancel{10.88}\ 9.89)}{10} \times 100$$

$$= 1.1\%$$

iv) $A = 10^4$, $g_{NI} = \frac{-10 \times 10^4}{10011} = -9.989$

$\%\,err = 0.11\%$

③ a)



$\cancel{V_x = V_i}$ $\quad \frac{V_y - V_{in}}{\cancel{R_i + 10}} + \frac{V_y}{10} + V_y$

$$\cancel{\frac{V_x - V_{in}}{10} + \frac{V_x - V_y}{R_i} = 0}$$

$$I_{in} = \frac{V_x - V_y}{R_i} \Rightarrow \boxed{\frac{V_x}{I_{in}} - \frac{V_y}{I_{in}} = R_i}$$

$$\frac{V_{in} - V_y}{10 + R_i} = I_{in} \Rightarrow \boxed{\frac{V_{in}}{I_{in}} - \frac{V_y}{I_{in}} = 10 + R_i} \qquad \frac{V_{in}}{I_{in}} = Z$$

$$\frac{V_y}{10} - I_{in} + \frac{V_y - A(V_x - V_y)}{100 + R_o} = 0$$

$$\Rightarrow \frac{V_y}{I_{in}} \times \frac{1}{10} - 1 + \left[\frac{V_y}{I_{in}} - \frac{A(V_x - V_y)}{I_{in}}\right]\frac{1}{100 + R_o} = 0$$

$$\Rightarrow \frac{1}{10}\left[Z - 10 - R_i\right] - 1 + \left[Z - 10 - R_i - AR_i\right]\frac{1}{100 + R_o} = 0$$

$$\Rightarrow Z\left(\frac{1}{10} + \frac{1}{100 + R_o}\right) - 1 - \frac{R_i}{10} - 1 - \frac{10}{100 + R_o} - \frac{R_i}{100 + R_o} - \frac{AR_i}{100 + R_o} = 0$$

$$\cancel{\Rightarrow \frac{V_y}{I_{in}}\left(\frac{1}{10} + 1\right) - 1}$$

$$\Rightarrow \frac{V_y}{I_{in}}\left(\frac{1}{10} + \frac{1}{100 + R_o}\right) - 1 - \frac{A(V_x - V_y)}{I_{in}(100 + R_o)} = 0$$

$$\Rightarrow \frac{V_y}{I_{in}} \cdot \frac{110 + R_o}{10(100 + R_o)} - \frac{100 + R_o}{100 + R_o} - \frac{A(V_x - V_y)}{I_{in}(100 + R_o)} = 0$$

$$\Rightarrow (Z - 10 - R_i)(110 + R_o) - (100 + R_o) - \frac{AR_i}{100 + R_o} = 0$$

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$$2 - 10 - R_i = \frac{100 + R_o + \frac{AR_i}{100+R_o}}{110 + R_o}$$

$$2 = \frac{100 + R_o + \frac{AR_i}{100+R_o}}{110 + R_o} + 10 + R_i$$

$$\boxed{Z_{i/p} = \frac{100 + R_o + \frac{AR_i}{100+R_o} + (10+R_i)(110+R_o)}{110+R_o}}$$

for $o/p\,(Z)$,

$$I_{out} = \frac{V_o - A(V_x - V_y)}{R_o} + \frac{V_o - V_y}{100}$$

also,

$$\frac{V_y}{10} + \frac{V_y - V_o}{100} + \frac{V_y}{R_i + 10} = 0$$

$$\Rightarrow V_y\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{R_i+10}\right) = \frac{V_o}{100}$$

$$\frac{V_y - V_x}{R_i} = \frac{V_y}{R_i + 10}$$

$$\Rightarrow \frac{V_y}{10 \| 100 \| (R_i+10)} = \frac{V_o}{100}$$

$$V_x = \left(1 - \frac{R_i}{R_i+10}\right) V_y$$

$$\Rightarrow V_y = \frac{V_o\left(10 \| 100 \| (R_i + 100)\right)}{100}$$

$$I_{out} = \frac{V_o}{R_o} - \frac{A}{R_o} \times \frac{-V_y}{R_i+10} + \frac{V_o}{100} - \frac{V_y}{100}$$

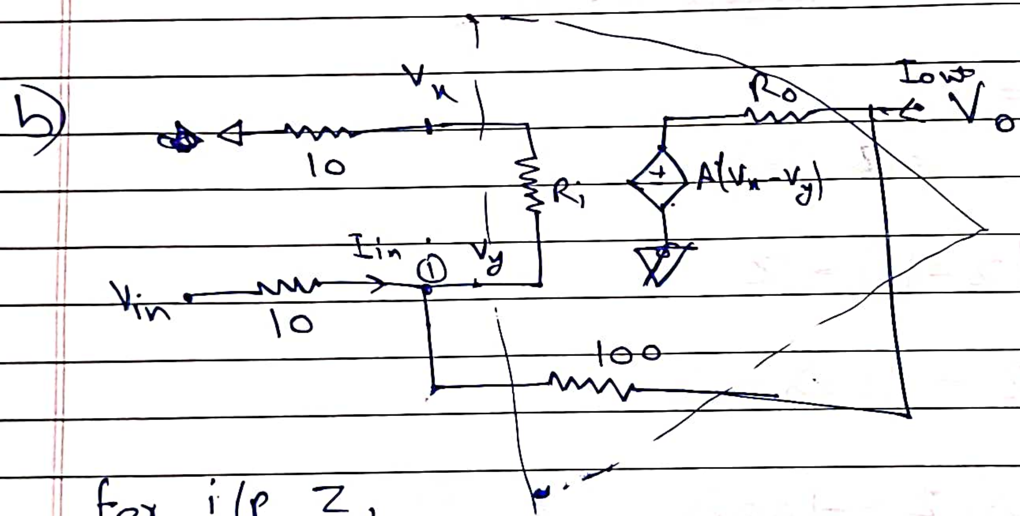
$$= V_o\left(\frac{1}{R_o}+\frac{1}{100}\right) - \cancel{\frac{A}{R_o}}\left(\frac{A}{R_o(R_i+10)} + \frac{1}{10}\right)V_y$$

$$\Rightarrow I_{out} = \frac{V_o}{R_o \| 100} - \frac{10A+1}{10R_o(R_i+10)} \times \frac{V_o\left(10\|100\|(R_i+100)\right)}{100}$$

$$\Rightarrow \frac{I_{out}}{V_o} = \frac{1}{R\|(100)} - \frac{10A+1}{10R_o(R_i+10)} \times \frac{\left(10\|100\|(R_i+100)\right)}{100}$$

$$\Rightarrow Z_{o/p} = \left[\frac{1}{R\|(100)} - \frac{10A+1}{10R_o(R_i+10)} \times \frac{10\|100\|(R_i+100)}{100}\right]^{-1}$$

b)



for i/p Z,

$$\frac{V_y - V_{in}}{10} = -I_{in} \quad , \quad \frac{V_y}{R_i+10} = \frac{V_x}{10} \Rightarrow V_x = \frac{10V_y}{R_i+10}$$

$$\frac{V_y - V_{in}}{10} + \frac{V_y - 0}{R_i+10} + \frac{V_y - A(V_x-V_y)}{100+R_o} = 0$$

$$\Rightarrow V_y\left(\frac{1}{10}+\frac{1}{R_i+10}+\frac{1+A}{100+R_o}\right) - \frac{A}{100+R_o}\times\frac{10V_y}{R_i+10} = \frac{V_{in}}{10}$$

$$\Rightarrow \cancel{\phi V_y / 10}$$

$$\Rightarrow \frac{V_y}{10 \,\|\, (R_i+10) \,\|\, \left(\frac{100+R_o}{A+1}\right)} - \frac{10 A V_y}{(100+R_o)(R_i+10)} = \frac{V_{in}}{10}$$

$$\Rightarrow \underbrace{\frac{1}{10 \,\|\, (R_i+10) \,\|\, \left(\frac{100+R_o}{A+1}\right)} - \frac{10 A}{(100+R_o)(R_i+10)}}_{\lambda} = \frac{V_{in}}{10(V_{in} - 10 I_{in})}$$

$$\Rightarrow \lambda = \frac{V_{in}}{10(V_{in} - 10 I_{in})}$$

$$\Rightarrow \lambda = \frac{Z}{10(Z - 10)}$$

$$\Rightarrow 10\lambda(Z-10) = Z$$

$$\Rightarrow 10\lambda Z - 100\lambda = Z$$

$$\Rightarrow (1 - 10\lambda) Z = -100\lambda$$

$$\Rightarrow \boxed{Z_{i/p} = \frac{100\lambda}{10\lambda - 1}}, \quad \boxed{\lambda = \frac{1}{10 \,\|\, (R_i+10) \,\|\, \left(\frac{100+R_o}{A+1}\right)} - \frac{10 A}{(100+R_o)(R_i+10)}}$$

for o|p (Z),

$$I_{out} = \frac{V_o - A(V_x - V_y)}{R_o} - \frac{V_o - V_y}{100}$$

$$\frac{V_x}{10} = \frac{V_x - V_y}{R_i}$$

$$\frac{V_x}{10} = \frac{V_y}{R_i + 10}$$

$$V_x = \frac{V_y}{R_i + 10} \times 10$$

$$\frac{V_y - V_o}{100} + \frac{V_y - 0}{R_i + 10} + \frac{V_y - 0}{10} = 0$$

$$\frac{V_y}{100 \| (R_i + 10) \| 10} = \frac{V_o}{100}$$

$$I_{out} = \frac{V_o}{R_o} - \frac{V_o}{100} - \frac{A V_x}{R_o} + V_y \left(\frac{1}{R_o} + \frac{1}{100} \right)$$

$$I_{out} = \frac{V_o}{R_o \| 100} - \frac{10 A V_y}{(R_i + 10) R_o} + \frac{V_y}{R_o \| 100}$$

$$I_{out} = \frac{V_o}{R_o \| 100} + \left(\frac{-10 A}{R_o (R_i + 10)} + \frac{1}{R_o \| 100} \right) \frac{V_o}{100} \left(100 \| 10 \| (R_i + 10) \right)$$

$$\frac{I_{out}}{V_{out}} = \frac{1}{R_o \| 100} + \left[\frac{1}{(R_o) \| 100} - \frac{10 A}{R_o (R_i + 10)} \right] \frac{100 \| 10 \| (R_i + 10)}{100}$$

$$Z_{o|p} = \left[\frac{1}{(R_o \| 100)} + \left[\frac{1}{R_o \| (100)} - \frac{10A}{R_o (R_i + 10)} \right] \frac{100 \| 10 \| (R_i + 10)}{100} \right]^{-1}$$