Assignment-1

EE:23010 Probability and Random Process
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Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Question(1.2.4): Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \tag{2}$$

Show that $\mathbf{A}, \mathbf{G}$ and $\mathbf{D}$ are collinear.

Solution: In order to verify the above equation we first need to find G,

G is the intersection of BE and CF
so, using the parametric form given in 1.1.4

Equation of BE,

$$\mathbf{x} = \mathbf{B} + k\mathbf{m} \tag{3}$$

where

$$\mathbf{m} = \mathbf{E} - \mathbf{B} \tag{4}$$

is the direction vector of BE.

Equation of CF,

$$\mathbf{x} = \mathbf{C} + k\mathbf{m} \tag{5}$$

where

$$\mathbf{m} = \mathbf{F} - \mathbf{C} \tag{6}$$

is the direction vector of CF.

Now, Solving for (3) and (5)

We have

$$\mathbf{x} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{7}$$

Thus,

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{8}$$

Now,We know that $\mathbf{E}$ and $\mathbf{F}$ are midpoints of CA and AB respectively.

$$\mathbf{E} = \frac{\mathbf{C} + \mathbf{A}}{2}, \mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{9}$$

$$\mathbf{E} = \frac{\begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{2}, \mathbf{F} = \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -6 \end{pmatrix}}{2} \tag{10}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} \frac{5}{2} \\ \frac{-7}{2} \end{pmatrix} \tag{11}$$

Using section formula for BGE from 1.2.1.1

$$\frac{k\mathbf{E} + \mathbf{B}}{k+1} = \mathbf{G} \tag{12}$$

$$\frac{k\begin{pmatrix} -1 \\ -3 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}}{k+1} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{13}$$

Solving for k in (13) we have,
k = 2

Thus,G divides BE in the ratio of 2:1

$$\frac{BG}{GE} = 2 \tag{14}$$

Similarly,we can verify for CGF and AGD
Hence,

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \tag{15}$$