Assignment-1

EE:23010 Probability and Random Processes

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I. Question 1.2.4

Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \tag{1}$$

II. Solution

In order to verify the above equation we first need to find $\mathbf{G}$. $\mathbf{G}$ is the intersection of BE and CF Using the value of $\mathbf{G}$ from (1.2.3).

$$\implies \mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2}$$

Also,We know that $\mathbf{D}, \mathbf{E}$ and $\mathbf{F}$ are midpoints of BC, CA and AB respectively from 1.2.1.

$$\implies \mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix}, \implies \mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \implies \mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \tag{3}$$

1) Taking direction vector of BG and GE

$$\mathbf{G} - \mathbf{B} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \tag{4}$$

$$\mathbf{E} - \mathbf{G} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{5}$$

$$\tag{6}$$

2) Taking direction vector of GF and CG

$$\mathbf{F} - \mathbf{G} = \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} \tag{7}$$

$$\mathbf{G} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{8}$$

$$\tag{9}$$

3) Taking direction vector of AG and GD

$$\mathbf{G} - \mathbf{A} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{10}$$

$$\mathbf{D} - \mathbf{G} = \begin{pmatrix} \frac{-3}{2} \\ \frac{1}{2} \end{pmatrix} \tag{11}$$

Using the equation, (4), (5), (7), (8), (12), (13). Taking norm of BG, GE, GF, CG, AG, GD

$$\|\mathbf{G} - \mathbf{B}\| = \sqrt{2^2 + (-6)^2} = \sqrt{40} \tag{12}$$

$$\|\mathbf{E} - \mathbf{G}\| = \sqrt{1^2 + 3^2} = \sqrt{10} \tag{13}$$

$$\|\mathbf{F} - \mathbf{G}\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{\sqrt{26}}{2} \tag{14}$$

$$\|\mathbf{G} - \mathbf{C}\| = \sqrt{1^2 + 5^2} = \sqrt{26} \tag{15}$$

$$\|\mathbf{G} - \mathbf{A}\| = \sqrt{(-3)^2 + 1^2} = \sqrt{10} \tag{16}$$

$$\|\mathbf{D} - \mathbf{G}\| = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{10}}{2} \tag{17}$$

Now,

1) from equation (12) and (13),

$$\frac{BG}{GE} = \frac{\|\mathbf{G} - \mathbf{B}\|}{\|\mathbf{E} - \mathbf{G}\|} = \frac{\sqrt{40}}{\sqrt{10}} = 2 \tag{18}$$

2) from equation (14) and (15),

$$\frac{CG}{GF} = \frac{\|\mathbf{G} - \mathbf{C}\|}{\|\mathbf{F} - \mathbf{G}\|} = \frac{\sqrt{26}}{\frac{\sqrt{26}}{2}} = 2 \tag{19}$$

3) from equation (16) and (17),

$$\frac{AG}{GD} = \frac{\|\mathbf{G} - \mathbf{A}\|}{\|\mathbf{D} - \mathbf{G}\|} = \frac{\sqrt{10}}{\frac{\sqrt{10}}{2}} = 2 \tag{20}$$

Hence,

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \tag{21}$$