

Assignment-1

EE:23010 Probability and Random Processes
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I. QUESTION 1.2.4

Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \quad (1)$$

II. SOLUTION

In order to verify the above equation we first need to find \mathbf{G} . \mathbf{G} is the intersection of BE and CF Using the value of \mathbf{G} from (1.2.3).

$$\Rightarrow \mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (2)$$

Also, We know that \mathbf{D}, \mathbf{E} and \mathbf{F} are midpoints of BC, CA and AB respectively from 1.2.1.

$$\Rightarrow \mathbf{D} = \begin{pmatrix} -7 \\ 1 \\ 2 \end{pmatrix}, \Rightarrow \mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \Rightarrow \mathbf{F} = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} \quad (3)$$

Taking direction vector of BG, GE, GF, CG, AG, GD

$$\mathbf{G} - \mathbf{B} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \quad (4)$$

$$\mathbf{E} - \mathbf{G} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (5)$$

$$\mathbf{F} - \mathbf{G} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \quad (6)$$

$$\mathbf{G} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (7)$$

$$\mathbf{G} - \mathbf{A} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (8)$$

$$\mathbf{D} - \mathbf{G} = \begin{pmatrix} -3 \\ 7 \\ 1 \\ 2 \end{pmatrix} \quad (9)$$

Using the equation, (4), (5), (6), (7), (8), (9). Taking norm of BG, GE, GF, CG, AG, GD

$$\|\mathbf{G} - \mathbf{B}\| = \sqrt{2^2 + (-6)^2} = \sqrt{40} \quad (10)$$

$$\|\mathbf{E} - \mathbf{G}\| = \sqrt{1^2 + 3^2} = \sqrt{10} \quad (11)$$

$$\|\mathbf{F} - \mathbf{G}\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{\sqrt{26}}{2} \quad (12)$$

$$\|\mathbf{G} - \mathbf{C}\| = \sqrt{1^2 + 5^2} = \sqrt{26} \quad (13)$$

$$\|\mathbf{G} - \mathbf{A}\| = \sqrt{(-3)^2 + 1^2} = \sqrt{10} \quad (14)$$

$$\|\mathbf{D} - \mathbf{G}\| = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{10}}{2} \quad (15)$$

Now,

1) from equation (10) and (11),

$$\frac{BG}{GE} = \frac{\|\mathbf{G} - \mathbf{B}\|}{\|\mathbf{E} - \mathbf{G}\|} = \frac{\sqrt{40}}{\sqrt{10}} = 2 \quad (16)$$

2) from equation (12) and (13),

$$\frac{CG}{GF} = \frac{\|\mathbf{G} - \mathbf{C}\|}{\|\mathbf{F} - \mathbf{G}\|} = \frac{\sqrt{26}}{\frac{\sqrt{26}}{2}} = 2 \quad (17)$$

3) from equation (14) and (15),

$$\frac{AG}{GD} = \frac{\|\mathbf{G} - \mathbf{A}\|}{\|\mathbf{D} - \mathbf{G}\|} = \frac{\sqrt{10}}{\frac{\sqrt{10}}{2}} = 2 \quad (18)$$

Hence,

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \quad (19)$$