Assignment - 01

Control System

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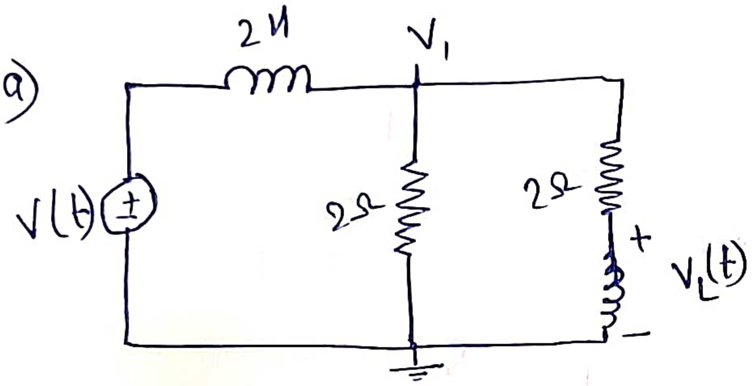
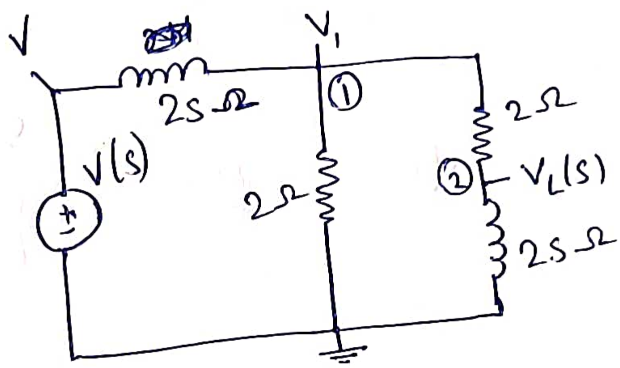
EE22BTECH11025

1

1) a)



$\leftrightarrow$



at node - 1,

$$\frac{V_1 - V}{2s} + \frac{V_1}{2} + \frac{V_1}{2+2s} = 0$$

$$\Rightarrow \cancel{V_1\left(\frac{1}{s}+1\right) = V\left(\frac{1}{8s} - \frac{1}{1+s}\right)}$$

at node - 2,

$$\frac{V_L - V_1}{2} + \frac{V_L}{2s} = 0$$

$$\Rightarrow V_L\left(1+\frac{1}{s}\right) = V_1 \quad -(2)$$

$$\frac{V_1 - V}{s} + \frac{V_1}{1} + \frac{V_1}{1+s} = 0$$

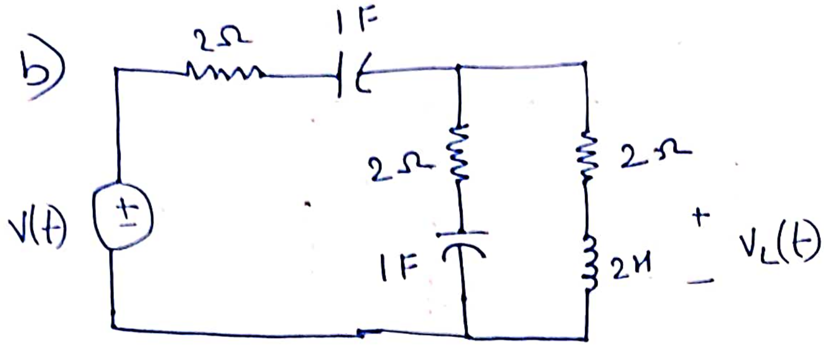
$$\Rightarrow V_1\left(\frac{1}{s} + 1 + \frac{1}{s+1}\right) = \frac{V}{s}$$

$$\Rightarrow \left(1+\frac{1}{s}\right)V_L \times \left(\frac{s+1+s^2+s+s}{s(s+1)}\right) = \frac{V}{s} \quad \text{— from (1)}$$

$$\Rightarrow \frac{(s^2+3s+1)(s+1)}{s^2(s+1)} V_L = \frac{V}{s}$$

$$\Rightarrow \boxed{\frac{V_L}{V} = \frac{s}{s^2+3s+1}}$$

b)



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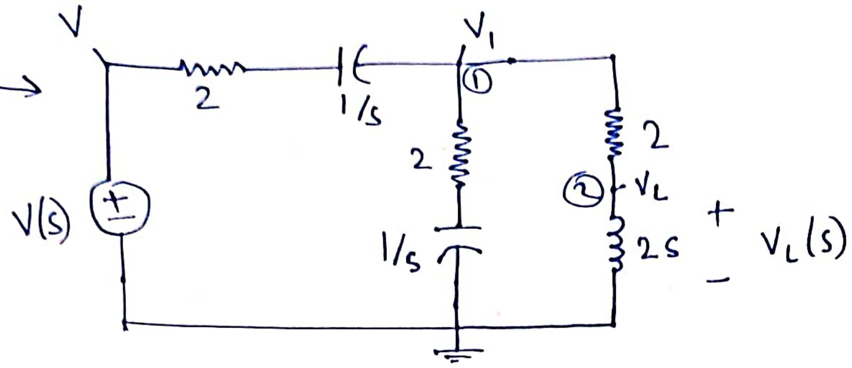


at node - ①

$$\frac{V_1 - V}{\frac{1}{s}+2} + \frac{V_1}{2+\frac{1}{s}} + \frac{V_1}{2+2s} = 0$$

$$\Rightarrow V_1\left(\frac{s}{2s+1} + \frac{s}{2s+1} + \frac{1}{2s+2}\right) = \frac{V\cdot s}{2s+1}$$

$$\Rightarrow V_1\left(\frac{2s(2s+2)+2s+1}{\cancel{(2s+1)}(2s+2)}\right) = \frac{V s}{\cancel{2s+1}}$$

$$\Rightarrow V_L\left(\frac{\cancel{s+1}}{s}\right)\left(\frac{4s^2+6s+1}{2\cancel{(s+1)}}\right) = Vs \quad -\{\text{from } ①\}$$

$$\Rightarrow \boxed{\frac{V_L}{V} = \frac{2s^2}{4s^2+6s+1}}$$ Ans

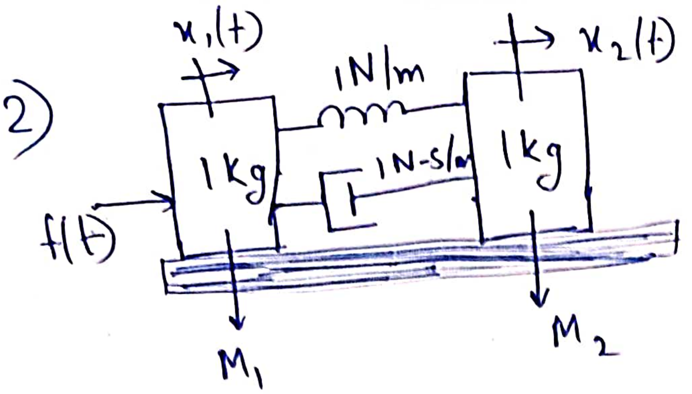
at node - ②

$$\frac{V_L}{2s} + \frac{V_L - V_1}{2} = 0$$

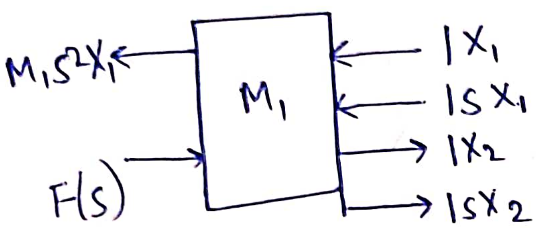
$$\Rightarrow V_L\left(\frac{1}{2s} + \frac{1}{2}\right) = \frac{V_1}{2}$$

$$\Rightarrow V_1 = V_L\left(\frac{s+1}{s}\right) \quad -①$$

2)



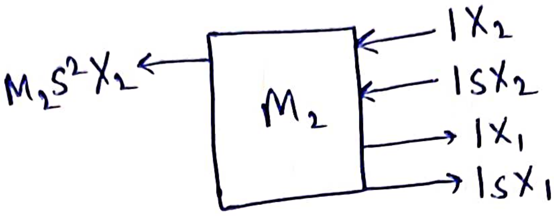
Force on M_1



eqⁿ of motion,

$$F(s) = -(s+1)X_2 + (s^2+s+1)X_1$$

Forces on M_2



eqⁿ of motion,

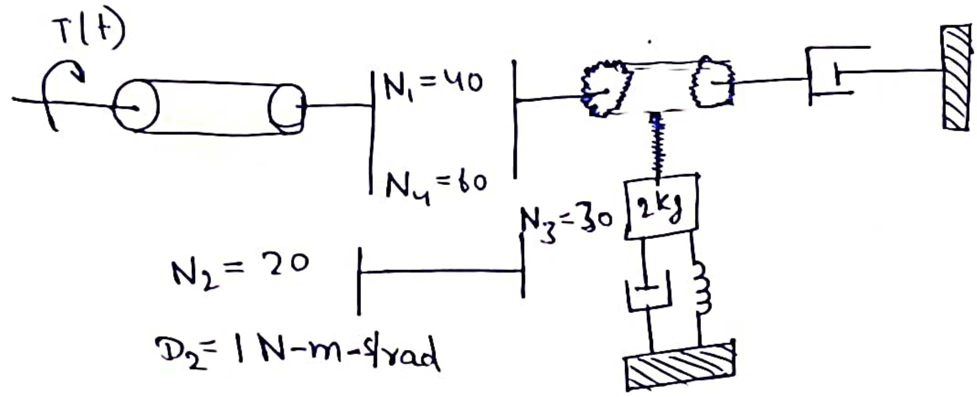
$$(s^2+s+1)X_2 - (s+1)X_1 = 0$$

Solving for $X_2(s)$,

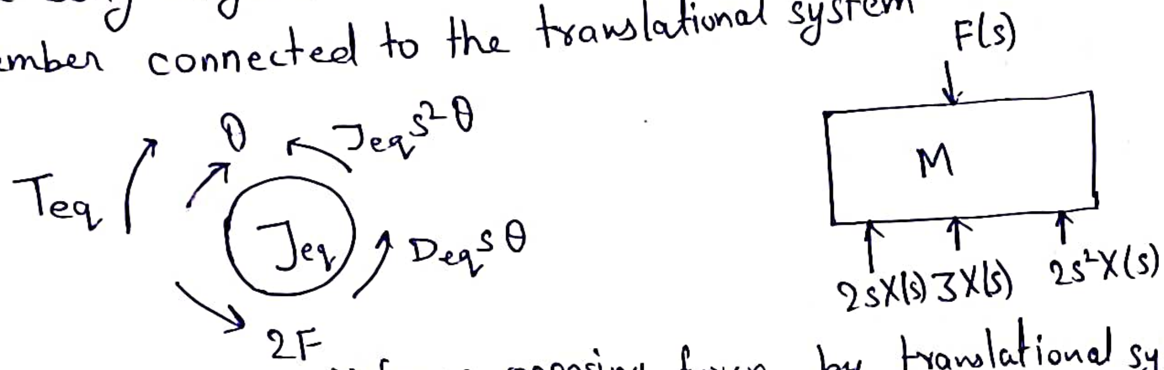
$$X_2(s) = \frac{\begin{vmatrix} s^2+s+1 & F(s) \\ -(s+1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2+s+1) & -(s+1) \\ -(s+1) & (s^2+s+1) \end{vmatrix}} = \frac{(s+1)F(s)}{s^2(s^2+2s+2)}$$

$$\Rightarrow \boxed{\frac{X_2(s)}{F(s)} = \frac{s+1}{s^2(s^2+2s+2)}}$$ Ans

3)



Free body diagram of the translational system and the rotating member connected to the translational system



2F → ~~force~~ opposing force by translational system

$$F(s) = (2s^2 + 2s + 3)X(s) \quad - ①$$

Now, summing torques on the rotating member,

$$(J_{eq}s^2 + D_{eq}s)\theta(s) + 2F(s) = T_{eq}(s) \quad , \quad \theta(s) = \frac{X(s)}{2} \quad - ②$$

$$\Rightarrow \left[\left(\frac{J_{eq}}{2} + 4\right)s^2 + \left(\frac{D_{eq}}{2} + 4\right)s + 6\right]X(s) = T_{eq}(s) \quad \{\text{using } ① \text{ and } ②\}$$

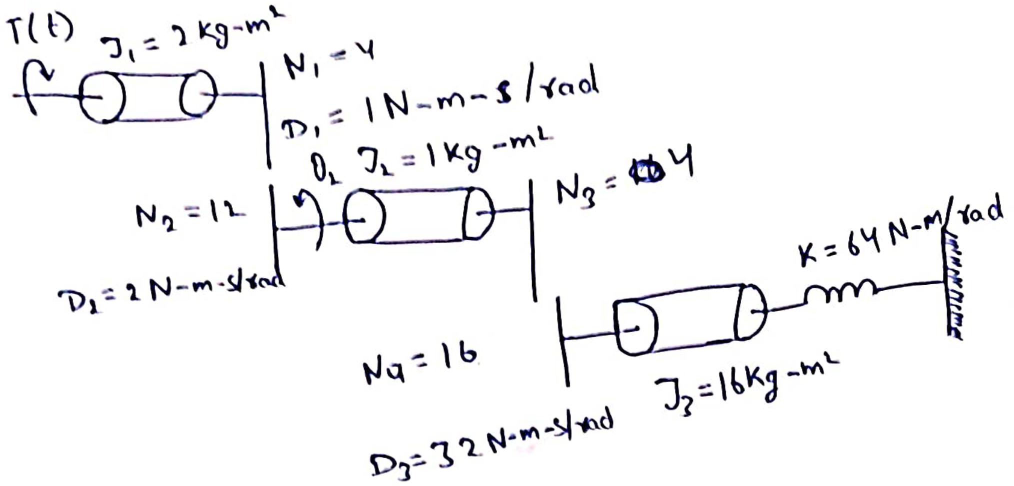
$$\begin{cases} J_{eq} = 3 + 3(4)^2 = 51 \\ D_{eq} = 1(2)^2 + 1 = 5 \\ T_{eq}(s) = 4T(s) \end{cases}$$

$$\Rightarrow \cancel{T_{eq}(s) = \frac{59}{2}}$$

$$\Rightarrow 4T(s) = \left[\frac{59}{2}s^2 + \frac{13}{2}s + 6\right]X(s)$$

$$\Rightarrow \boxed{\frac{X(s)}{T(s)} = \frac{8}{59s^2 + 13s + 12}}$$

4)



$$\frac{N_2}{N_1} T(s) = J_{eq} s^2 \theta_2 + D_{eq} s \theta_2 + K_{eq} \theta_2$$

$$\Rightarrow \frac{N_2}{N_1} T(s) = \left[J_2 + J_1 \left(\frac{N_2}{N_1}\right)^2 + J_3 \left(\frac{N_3}{N_4}\right)^2\right] s^2 \theta_2$$

$$+ \left[D_2 + D_1 \left(\frac{N_2}{N_1}\right)^2 + D_3 \left(\frac{N_3}{N_4}\right)^2\right] s \theta_2$$

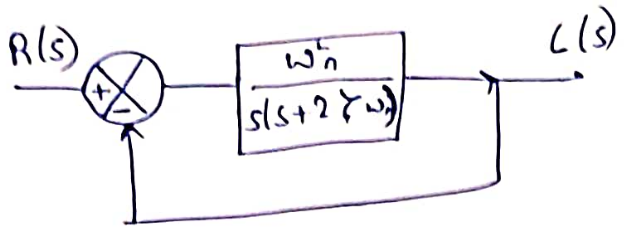
$$+ K \left(\frac{N_3}{N_4}\right)^2 \theta_2$$

$$\Rightarrow 3T(s) = \left[1 + 2(3)^2 + \left(\frac{1}{4}\right)^2 16\right] s^2 \theta_2 + \left[2 + 1(3)^2 + 32\left(\frac{1}{4}\right)^2\right] s \theta_2$$

$$+ 64 \left(\frac{1}{4}\right)^2 \theta_2$$

$$\Rightarrow \boxed{\frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2 + 13s + 4}}$$ Ans

5)



$\cancel{R(s) = u(t)}$ $R(s) = \frac{1}{s}$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \quad , \quad \zeta = 0.6 \, , \omega_n = 5 \text{ rad/sec}$$

$$t_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1 \quad \{\text{curve fitting}\}$$

$$\Rightarrow \boxed{t_r = 1.85344 \text{ seconds}}$$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.7854 \text{ second}}$$

$$\cancel{M_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta}}\right)}$$

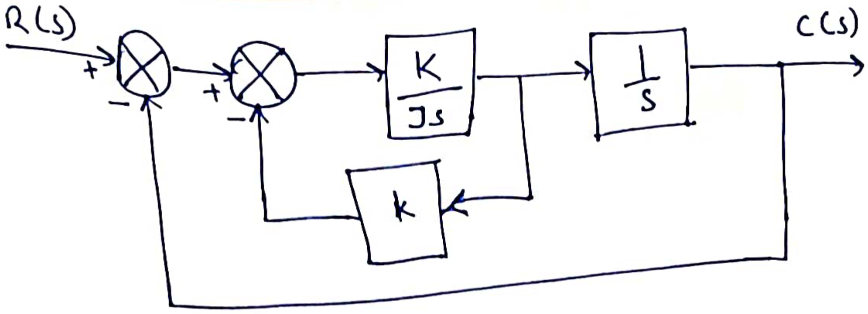
$$\cancel{M_p = \exp\left(\right)}$$

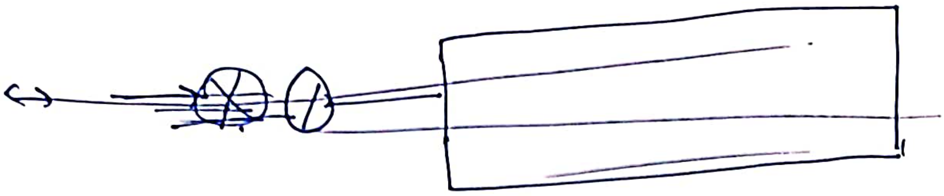
$$M_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) = 0.0948$$

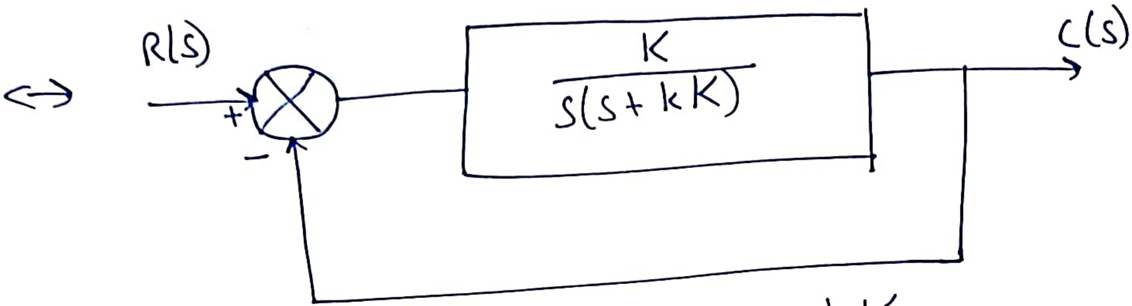
$$t_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{3} = 1.33 \text{ seconds}$$

$$\boxed{t_s \approx 1.333 \text{ seconds}}$$

6)







$\omega_n^2 = K$, $2\zeta\omega_n = kK$

$\Rightarrow \zeta = \frac{k\sqrt{K}}{2}$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + (\ln(\%OS/100))^2}}$$

$$= \frac{-\ln(0.25)}{\sqrt{\pi^2 + (\ln(0.25))^2}} = 0.404$$

$\boxed{\zeta = 0.404} \Rightarrow \frac{k\sqrt{K}}{2} = 0.404$

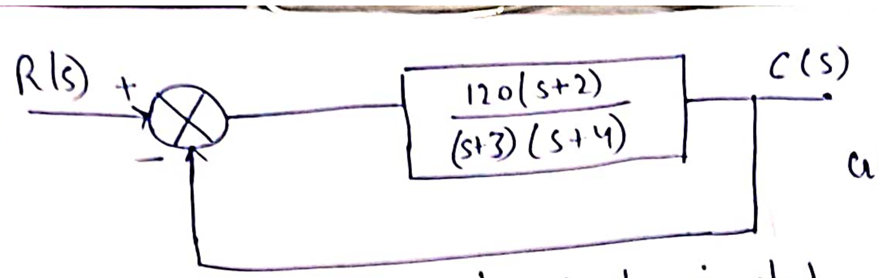
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 2$$

$\Rightarrow \omega_n = 1.72$ rad/sec

$\Rightarrow \boxed{K = \omega_n^2 = 2.95 \text{ rad}^2/\text{sec}^2}$

$k = \frac{2\zeta}{\sqrt{K}} \Rightarrow \boxed{k = 0.47}$

7) a)



$$G(s) = \frac{120(s+2)}{(s+3)(s+4)}$$

Steady ~~st~~ state error is given by,

$$e(\infty) = \lim_{s\to 0} \frac{s R(s)}{1+G(s)}$$

Case-1

$$R(s) = \frac{5}{s}$$

$$e(\infty) = \lim_{s\to 0} \frac{s \times \frac{5}{s}}{1 + \frac{120(s+2)}{(s+3)(s+4)}} = \frac{5}{1 + \frac{120\times 2}{3\times 4}}$$

$\Rightarrow$ ~~$\boxed{e(\infty) = 0.048}$ for, $R(s) = 5/s$~~

$$\Rightarrow \boxed{e(\infty) = 0.24} \text{ for, } R(s) = 5/s$$

Case-2

$$R(s) = \cancel{\frac{5}{s^2}}\ \frac{5}{s^2}$$

$$e(\infty) = \lim_{s\to 0} \frac{s \times \frac{5}{s^2}}{1 + \frac{120(s+2)}{(s+3)(s+4)}} = \lim_{s\to 0} \frac{5}{s + \frac{120 s(s+2)}{(s+3)(s+4)}} = \frac{5}{0} = \infty$$

$$\Rightarrow \boxed{e(\infty) = \infty}, \text{ for } R(s) = \frac{5}{s^2}$$

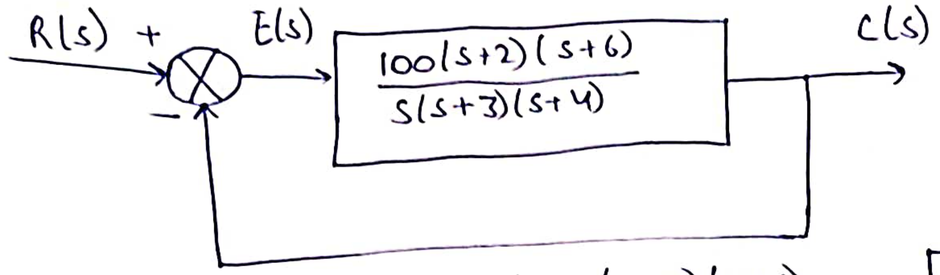
Case-3

$$R(s) = \frac{10}{s^3}$$

$$e(\infty) = \lim_{s\to 0} \frac{s \times \frac{5}{s^3}}{1 + \frac{120(s+2)}{(s+3)(s+4)}} = \infty$$

$$\Rightarrow \boxed{e(\infty) = \infty}, \text{ for } R(s) = \frac{10}{s^3}$$

7) b)



$$G(s) = \frac{100(s+2)(s+6)}{s(s+3)(s+4)}, \quad \boxed{\lim_{s \to 0} G(s) = \infty}$$

steady state error is given by,

$$~~Ef~~ \; e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

Case-1

$$R(s) = \cancel{\tfrac{4}{s}} \; \frac{5}{s}$$

$$e(\infty) = \lim_{s \to 0} \frac{s \times \frac{5}{s}}{1+G(s)} = 0$$

$$\Rightarrow \boxed{e(\infty) = 0}, \text{ for } R(s) = \cancel{4} \; \frac{5}{s}$$

Case-2

$$R(s) = \frac{5}{s^2}$$

$$e(\infty) = \lim_{s \to 0} \frac{s \times \frac{5}{s^2}}{1+G(s)} = \lim_{s \to 0} \frac{5}{s + sG(s)} = \frac{5}{100} = 0.05$$

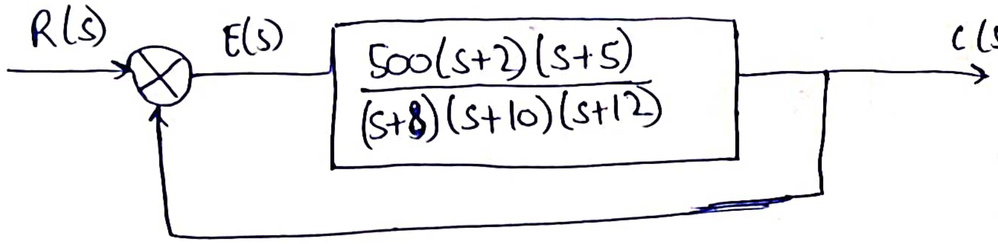
$$\boxed{e(\infty) = 0.05}, \text{ for } R(s) = \frac{5}{s^2}$$

Case-3 $R(s) = 10/s^3$

$$e(\infty) = \lim_{s \to 0} \frac{s \times \frac{10}{s^3}}{1+G(s)} = \frac{10}{0} = \infty$$

$$\boxed{e(\infty) = \infty}, \text{ for } R(s) = \frac{10}{s^3}$$

8) a)



$$K_p = \lim_{s\to 0} G(s) = \frac{500\times 2\times 5}{8\times 10\times 12} = 5.208$$

$$K_v = \lim_{s\to 0} sG(s) = 0$$

$$K_a = \lim_{s\to 0} s^2 G(s) = 0$$

for $R(s) = \frac{1}{s}$,

$$e(\infty) = \frac{1}{1+K_p} = 0.161$$

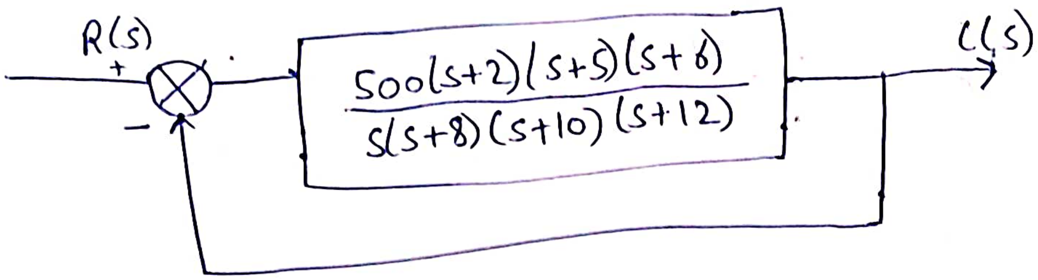
for $R(s) = \frac{1}{s^2}$,

$$e(\infty) = \frac{1}{K_v} = \infty$$

for $R(s) = \frac{2}{s^3}$

$$e(\infty) = \frac{1}{K_a} = \infty$$

8) b)



$$K_p = \lim_{s\to 0} G(s) = \infty$$

$$K_v = \lim_{s\to 0} sG(s) = \frac{500\times 2\times 5\times 6}{8\times 10\times 12} = 31.25$$

$$K_a = \lim_{s\to 0} s^2 G(s) = 0$$

for $R(s) = \frac{1}{s}$,

$\cancel{K_a = e(\infty) = 1}$

$$e(\infty) = \cancel{\lim_{s\to 0}} \frac{1}{1+K_p} = 0$$

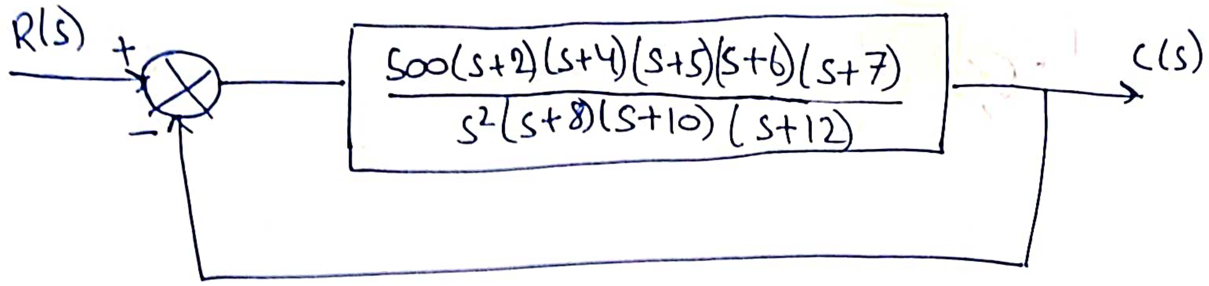
for $R(s) = \frac{1}{s^2}$,

$$e(\infty) = \cancel{\lim_{s\to 0}} \frac{1}{K_v} = 0.032$$

for $R(s) = \frac{2}{s^3}$

$$e(\infty) = \cancel{\lim_{s\to 0}} \frac{1}{K_a} = \infty$$

8) c)



$$K_p = \lim_{s \to 0} G(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s) = \infty$$

$$K_a = \lim_{s \to 0} s^2 G(s) = \frac{500 \times 2 \times 4 \times 5 \times 6 \times 7}{8 \times 10 \times 12} = 875$$

for $R(s) = \frac{1}{s}$,

$$\cancel{e(\infty) = \frac{1}{1+1}}$$

$$\cancel{e(\infty) = li}$$

$$e(\infty) = \frac{1}{1+K_p} = 0$$

for $R(s) = \frac{1}{s^2}$,

$$e(\infty) = \frac{1}{K_v} = 0$$

for $R(s) = \frac{2}{s^3}$,

$$e(\infty) = \frac{1}{K_a} = 1.14 \times 10^{-3}$$