A-1

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EE22BTECH11028

1) $G(s) = \dfrac{75(1+0.2s)}{s(s^2+16s+100)}$

$= \dfrac{0.75\left[1+\frac{s}{5}\right]}{s\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}$, $\omega_n^2 = 100$

$\omega_n = 10$

Break freqency = 5 .1

	$\omega = 0$	$\omega = 5$	$\omega_n = 10$
Pole at 0	-20	-20	20
Pole at -5	0	+20	+20
$\omega_n = 10$			-40
	-20	0	-40

$G(j\omega) = 17.5$ at $\omega = 0.1$

at $\omega_n = 5$.

$17.5 - 20\log(50) = -16.47$

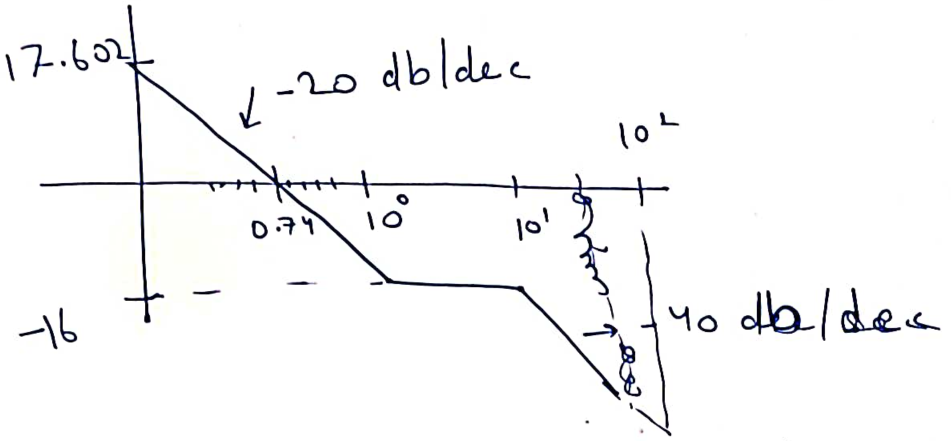
at $\omega_n = 10$

$-16.47 - 0 = -16.47$

$17.5 - 20\log(10x) = 0$

$\Rightarrow x = 0.749$

$\omega_g = 0.749$



Gain margin = 56.47

	0.5	1	50	100
		-45		
Pole at (-8)	+45	-90	-90	0
$\omega_n = 10$	+45	-45	-90	

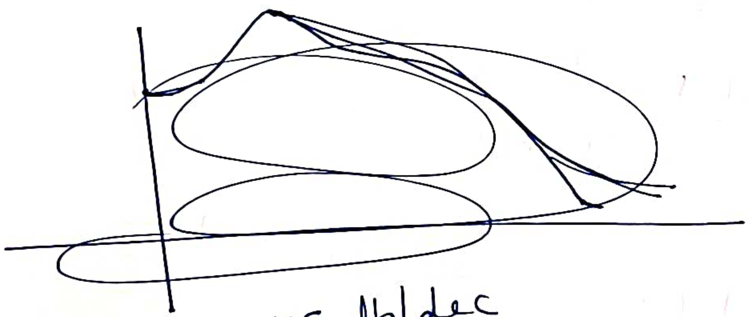
$\omega_n = 0.5$

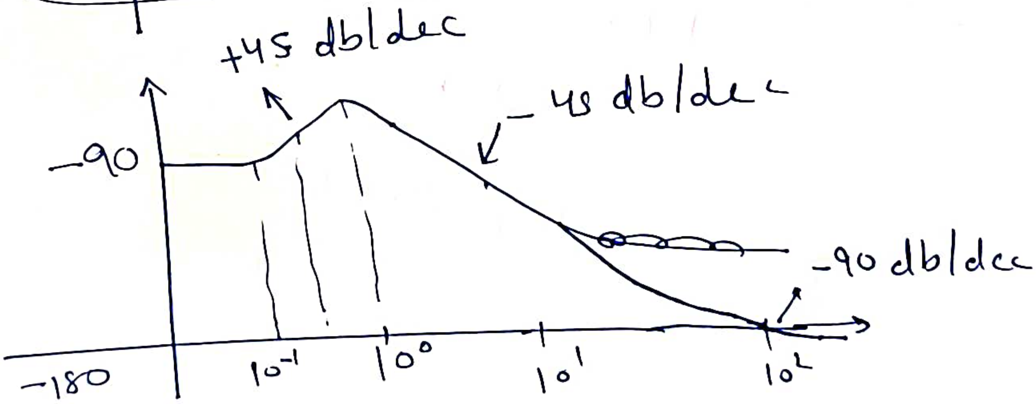
$\hookrightarrow G(j\omega) = -1.5j \quad , \quad \phi = -90°$

at $\omega = 1$, $-90 + 45 \log 2 = -76.45$

at $\omega = 50$, $-76.45 - 45 \log 50 = -152.90$

at $\omega = 100$, $152.9 - 90 \log 2 = -180$





② $G(s) = \dfrac{10}{s(1+\frac{s}{2.5})(s+\frac{s}{10})}$.

Break frequency = 2.5, 10

	0.1	2.5	10	100
Pole at 0	-20	-20	-20	-20
Pole at (-2.5)		-20	-20	-20
Pole at (-10)			-20	-20
	-20	-40	-60	-60

at $\omega = 0.1$

↳ $G(s) = \dfrac{10}{0.1} = 100$

at $\omega = 2.5$

↳ $G(s) = 40 - 20\log\left(\frac{2.5}{0.1}\right) = 12.04$

at $\omega = 10$

↳ $G(s) = 12.04 - 40\log\left(\frac{10}{2.5}\right) = -12.04$

from 10 it decrease with slope -60 db/dec

for gain cross over frequency

$12.04 - 40\log\left(\frac{x}{2.5}\right) = 0$

$x = 4.99\,Hz$ [Gain cross over frequency]

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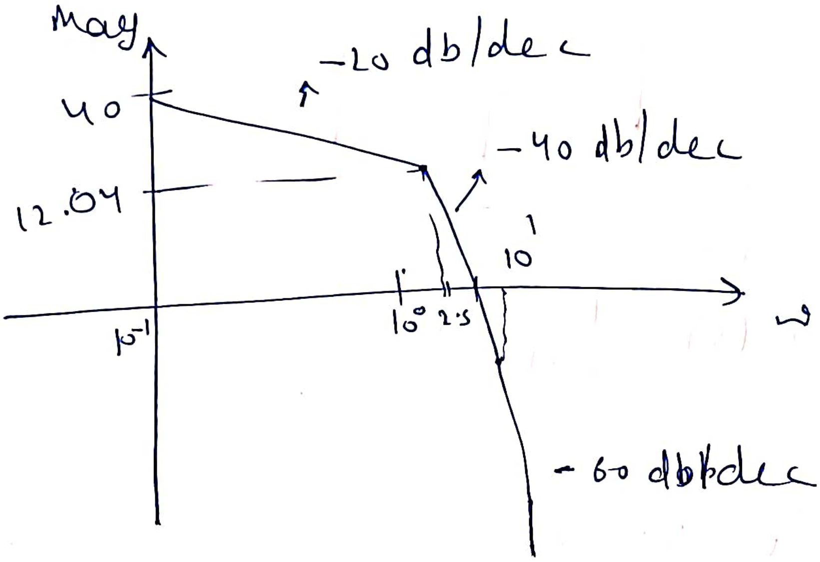


Pole at "0" has no effect

	0.25	1	25	100
Pole = -2.5	-45	-45	0	
Pole = -10	0	-45	-45	0
	-45	-90	-45	0

a

③ $G(s) = \dfrac{20}{s(1+\frac{s}{0.25})(1+\frac{s}{0.35})}$

	$\omega = 0.01$	0.25	0.33	1
Pole at 0	−20	−20	−20	−20
" −0.25	0	−20	−20	−20
" −0.35	0	0	−20	−20
	−20	−40	−60	−60

$\omega = 0.01$

$G(s) = 66.02$

$\omega = 0.25$

$G(s) = 38.08$

$\omega = 0.33$

$G(s) = 33.23$

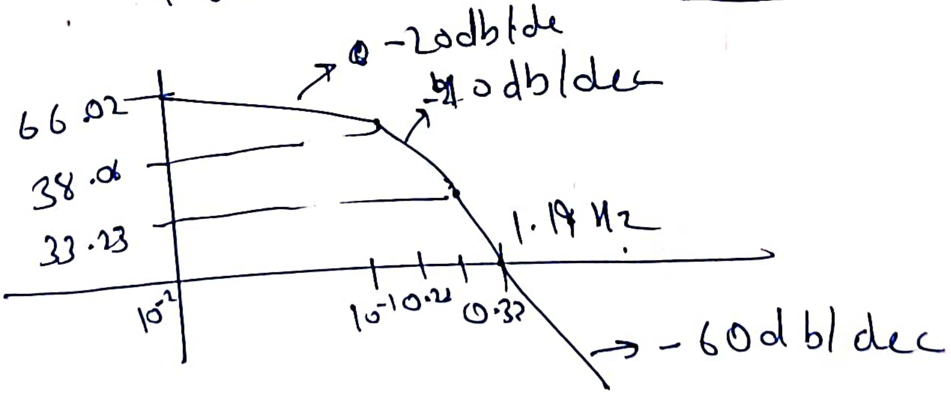
$\omega = 1$

$G(s) = 4.611$

$\omega = 10$

$G(s) = -55.383$

$4.611 - 60 \log x \Rightarrow \boxed{x = 1.19\,Hz}$



17.1

at $\omega = 0.25$

$\angle G(s) = -90$

at $\omega = 1$

$\angle G(s) = -90 - 45 \log \frac{4}{1}$
$= 117.09^\circ$

at $\omega = 25$

$G(s) = -117.09 - 90 \log(25) = -242.4^\circ$

at $\omega = 100$

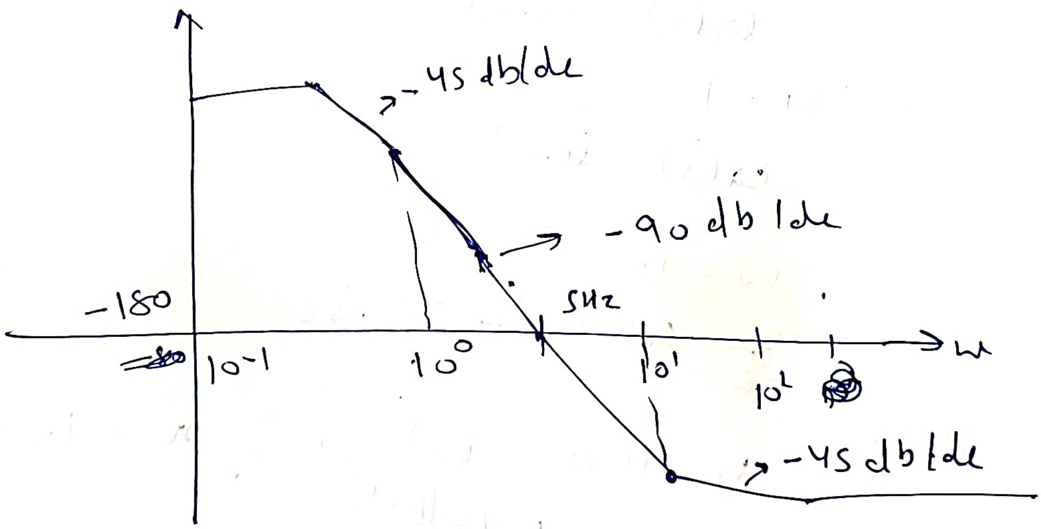
$G(s) = 270^\circ$

After $\omega > 100$ its constant

$-117.09 - 90 \log(x) = -180$

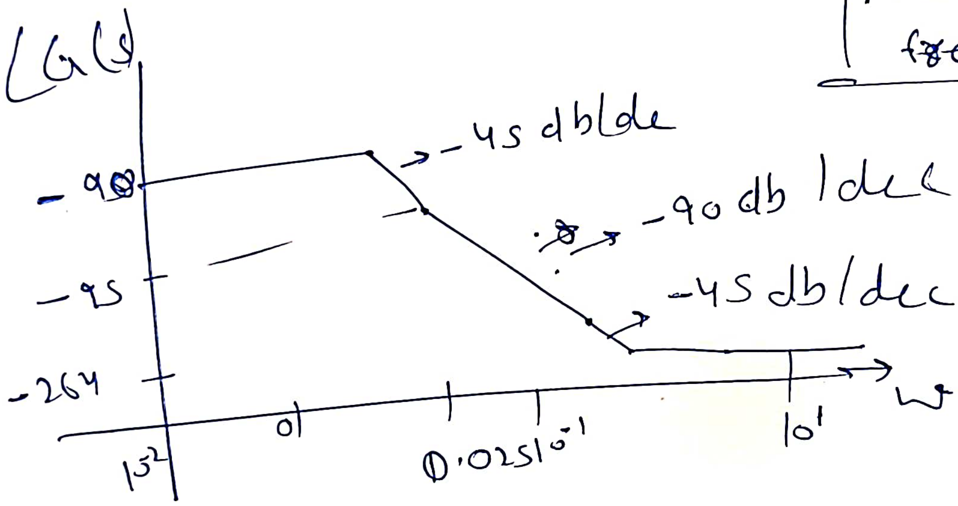
$\Rightarrow x = 5.00003$

↳ Phase cross over freq.

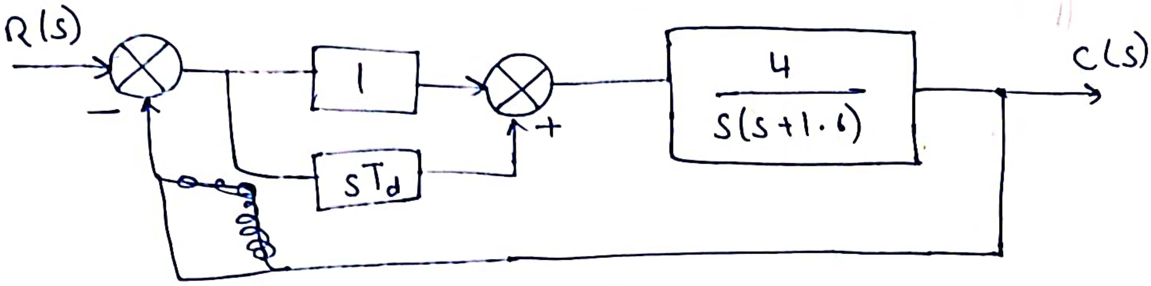


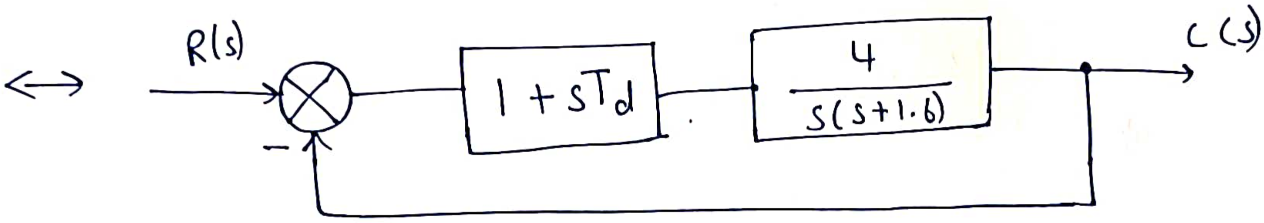
w	0.025	0.033	2.5	3.3
Pole -0.25	-45	-45	0	0
Pole -0.33		-45	-45	0
		-90	-45	0

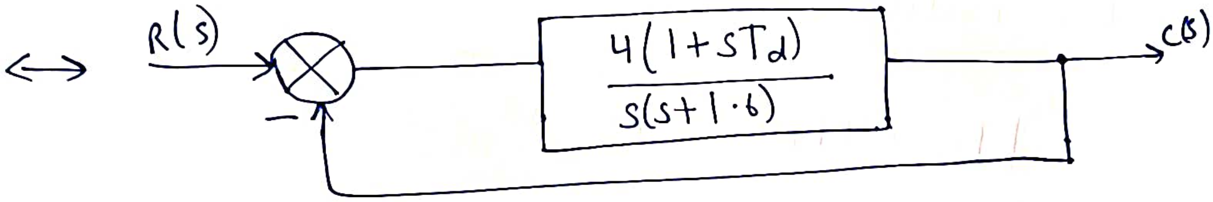
Phase cross over freq. = 0.287Hz

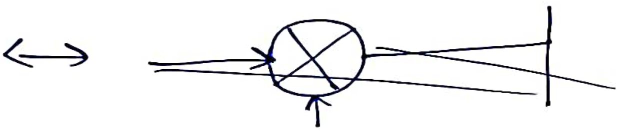


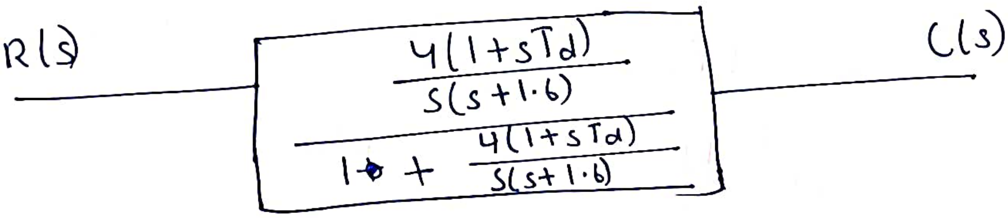
4











$$\frac{C(s)}{R(s)} = \frac{\frac{4(1+sT_d)}{s(s+1.6)}}{1+\frac{4(1+sT_d)}{s(s+1.6)}}$$

~~5.67~~

$$= \frac{4(1+sT_d)}{s(s+1.6)+4(1+sT_d)}$$

$$= \frac{4(1+sT_d)}{s^2+(1.6+4T_d)s+4}$$

for critically damped,

$$(1.6+4T_d)^2 - 16 = 0$$

$$\Rightarrow (1.6+4T_d)^2 = 16$$

$$\Rightarrow 1.6 + 4T_d = \pm 4$$

$$\Rightarrow 4T_d = -5.6 \quad , \quad 4T_d = 2.4$$

$$\Rightarrow \boxed{T_d = -1.4} \quad , \quad \boxed{T_d = 0.6}$$

5) (a)

$$C(s) = \frac{(s+3)}{s(s+2)(s^2+3s+10)}$$

$$= \frac{3}{20} \times \frac{1}{s} - \frac{1}{16} \times \frac{1}{(s+2)} - \frac{1}{80} \times \frac{7s+31}{\left(s+\frac{3}{2}\right)^2 + \frac{31}{4}}$$

$$= \left(\frac{3}{20}\right)\frac{1}{s} - \left(\frac{1}{16}\right)\frac{1}{(s+2)} - \left(\frac{1}{80}\right) \times \frac{7\left(s+\frac{3}{2}\right) + 7.36\sqrt{\frac{31}{4}}}{\left(s+\frac{3}{2}\right)^2 + \frac{31}{4}}$$

∵ The amplitude ~~magnitude~~ of the sinusoids are of the same ~~oth~~ order of magnitude as the residue of the pole at -2, ~~pto~~ pole-zero cancellation cannot be assumed

(b)

$$C(s) = \frac{s+2.5}{s(s+2)(s^2+4s+20)} = \left(\frac{1}{16}\right)\frac{1}{s} - \left(\frac{1}{64}\right)\frac{1}{(s+2)} - \left(\frac{1}{64}\right)\frac{3s+14}{s^2+4s+20}$$

$$= \left(\frac{1}{16}\right)\frac{1}{s} - \left(\frac{1}{64}\right)\frac{1}{(s+2)} - \left(\frac{1}{64}\right)\frac{3(s+2)+2\sqrt{16}}{(s+2)^2+16}$$

∵ The amplitude of the sinusoids are of the same order of magnitude as the residue of the pole at -2, pole-zero cancellation cannot be assumed.

(c) ~~St~~ $C(s) = \dfrac{(s+2.1)}{s(s+2)(s^2+s+5)}$

$$= (0.21)\frac{1}{s} - (0.007)\frac{1}{s+2} - \frac{0.2s+0.21}{s^2+s+5}$$

$$= (0.21)\frac{1}{s} - (0.007)\frac{1}{s+2} - \frac{0.2\left(s+\frac{1}{2}\right)+0.05\sqrt{\frac{19}{4}}}{\left(s+\frac{1}{2}\right)^2+\frac{19}{4}}$$

$\because$ The amplitude of sinusoids are of two order of magnitude larger than the residue of the pole at -2, pole-zero cancellation can be assumed

Now, $2\xi\omega_n = 1$, $\omega_n = \sqrt{5} = 2.236$, $\xi = 0.224$

$$\%OS = \exp\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right) \times 100 = 48.64\%$$

$$T_s = \frac{4}{\xi\omega_n} = 8 \text{ seconds}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1.44 \text{ seconds}$$

$$T_r = 0.55$$

⑥ (a) $T(s) = \dfrac{16}{s^2 + 3s + 16}$

$\omega_n = 4$, $\zeta = 0.375$

$\%OS = \exp\left(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100$

$\Rightarrow \boxed{\%OS = 28.06\%}$

$T_s = \dfrac{4}{\zeta\omega_n} = 2.667 \Rightarrow \boxed{T_s = 2.667 \text{ seconds}}$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.8472 \Rightarrow \boxed{T_p = 0.8472 \text{ second}}$

$\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$

$\cancel{= 1.4}$

$\boxed{T_r = 0.356 \text{ second}}$

(b) $T(s) = \dfrac{0.04}{s^2 + 0.02s + 0.04}$

$\cancel{\omega_n = 0.02}$ $\omega_n = 0.2$, $\zeta = 0.05$

$\%OS = 85.45\%$

$T_p = 15.73$ seconds

$T_s = 400$ seconds

$T_r = 5.26$ seconds

(c) $T(s) = \dfrac{1.05\times10^{7}}{s^2 + 1.6\times10^{3}s + 1.05\times10^{7}}$

$\omega_n = 3240$, $\zeta = 0.247$

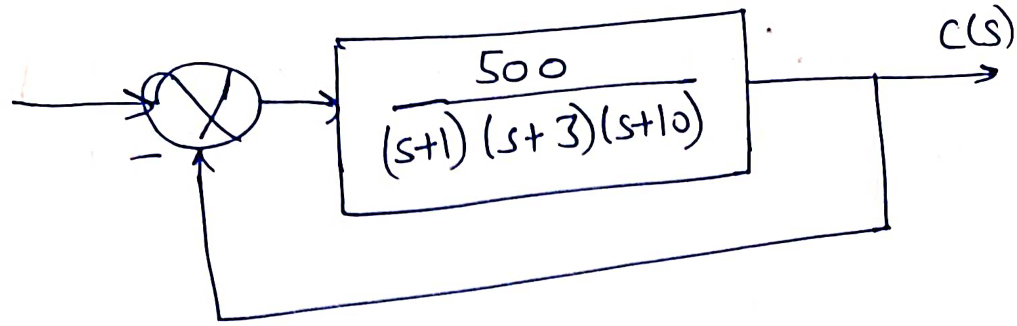
$\%OS = 44.92\%$

$T_s = 0.005$ second

$T_p = 0.001$ second

$T_r = 3.88\times10^{-4}$ second

7



$$G(s) = \frac{500}{(s+1)(s+3)(s+10)}$$

$$|G(j\omega)| = \frac{500}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+9}\,\sqrt{\omega^2+100}} = K \quad \text{(say)}$$

$$\angle G(j\omega) = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{10}$$

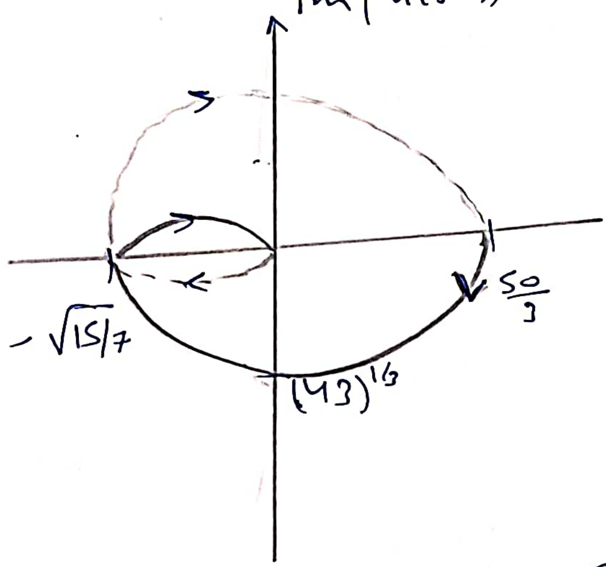
$$G(j\omega) = K\left[(1-j\omega)(3-j\omega)(10-j\omega)\right]$$

$$= K\left[30 - 3j\omega - 10j\omega - \omega^3 - 30j\omega - 3\omega^2 - 10\omega^2 + j\omega^3\right]$$

$$= K\left[(30-14\omega^2) - j(43-\omega^3)\right]$$

~~polo~~ plot crosses real axis at $\omega = \sqrt{\frac{30}{14}}$

Plot crosses img. axis at $\omega = (43)^{1/3}$



	$\lvert G(jw)\rvert$	$\angle G(jw)$
$w=0$	$50/3$	0
$w=\infty$	0	-270

$n=0$, $P=0$

$\therefore$ $Z=0$, system is stable

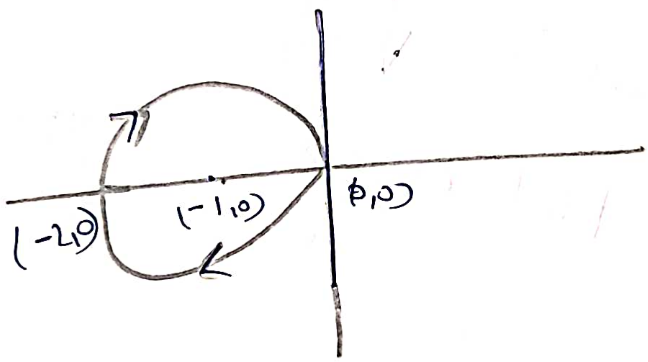
(8) $G(s) = \dfrac{(s+2)}{(s+1)(s-1)}$

$s=jw$,

$G(jw) = \dfrac{2+jw}{(jw+1)(jw-1)}$

$= \dfrac{-(2+jw)}{1+w^2}$

for $w=0$	for $w=\infty$
$(-2,0)$	$(0,0)$



No of encirclement of $(-1,0)$ $=1$

No of zeros in RHP $=1$

$N = P - Z$

$\Rightarrow P = 0$

$\therefore$ There are no poles in RHP

It is stable