Assignment - 3

Control system

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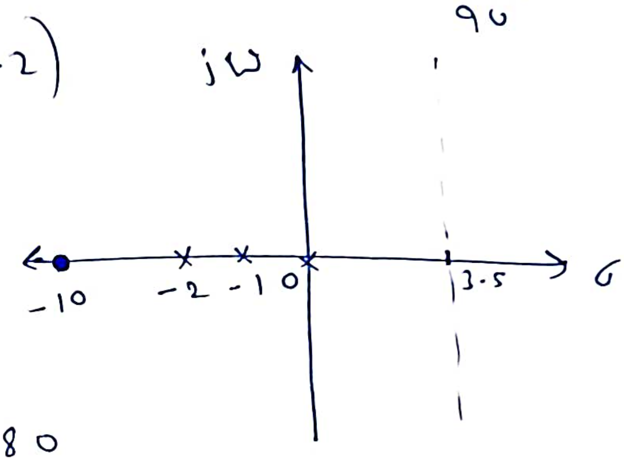
EE22BTECH11025

(1) $G(s)H(s) = \dfrac{k(s+10)}{s(s+1)(s+2)}$

No of poles $\rightarrow$ 3 $(0, -1, -2)$

No of zeros $\rightarrow$ 1 (-10)



No of asymtotes = 2

Angle of asymtotes = $(2k+1)180$
$= -90, 90$

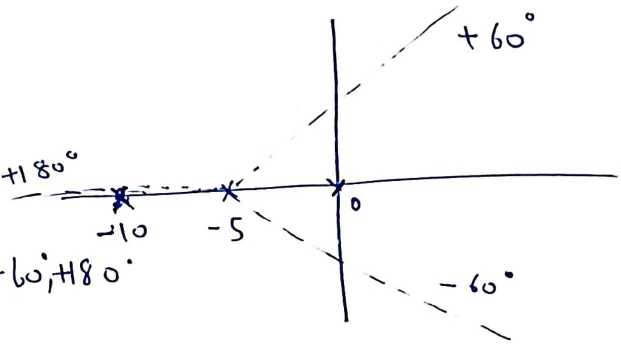
centroid of asymtotes $= \dfrac{\Sigma P - \Sigma Z}{P - Z}$

$= \dfrac{-1-2-(-10)}{2} = 3.5$

(2) $G(s)H(s) = \dfrac{K}{s(s+5)(s+10)}$

No of poles $= 3$ $(0, -5, -10)$

No of zeros $= 0$



No of ~~asy~~ asymtotes $= 3$

Angle of asymtotes $= -60^\circ, +60^\circ, +180^\circ$

centroid of asymtotes $= \dfrac{-5-10}{3} = -5$

Breakaway point is given by,

$$1 + \frac{K}{s(s+5)(s+10)} = 0$$

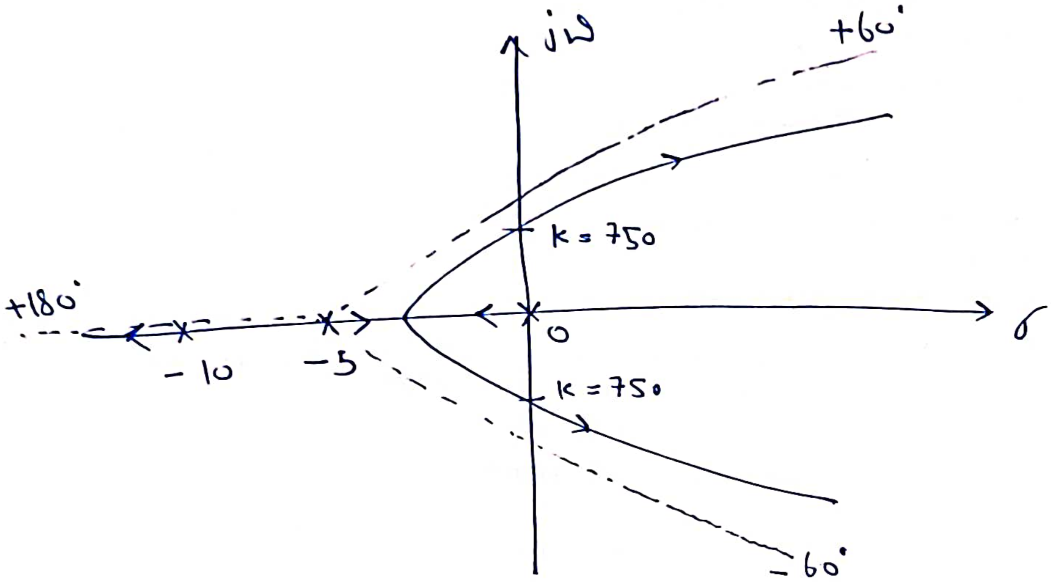
$$\Rightarrow s^3 + 15s^2 + 50s + K = 0$$

$$\boxed{\frac{dK}{ds} = 0}$$

$$\Rightarrow \; -(3s^2+30s+50)=0$$

$s_1 = -2.11$, then $k_{s_1} = 47.61$

$s_2 = -7.88$, then $k_{s_2} = -48.1$



intercept at jω - axis is given by

$s = j\omega$

$$s^3 + 15s^2 + 50s + k = 0$$

$$\Rightarrow (j\omega)^3 + 15(j\omega)^2 + 50(j\omega) + k = 0$$

$$\mathrm{Im}(\;) = 0$$

$$\Rightarrow -15\omega^2 + k + j\omega(50-\omega^2) = 0$$

$$\therefore \; 50-\omega^2 = 0$$

$$\omega = \pm 7.07$$

then, $\boxed{K = 750}$

③ $G(s)H(s) = \dfrac{K}{s(s^2+2s+2)}$

No of poles $= 3$ $\{0, -1 \pm j\}$

No of zero $= 0$

No of asymtotes $= 3$

angle of asymtotes $= -60, +60, +180$

centroid of asymtotes $= -\dfrac{1-1}{3} = -0.667$

for Breakaway point,

$$\frac{dk}{ds} = 0$$

$$1 + G(s)H(s) = 0$$

$$\Rightarrow s^3 + 2s^2 + 2s + k = 0$$

$$\frac{dk}{ds} = 3s^2 + 4s + 2 = 0$$

$$\Rightarrow s_{1,2} = -0.667 \pm 0.47j$$

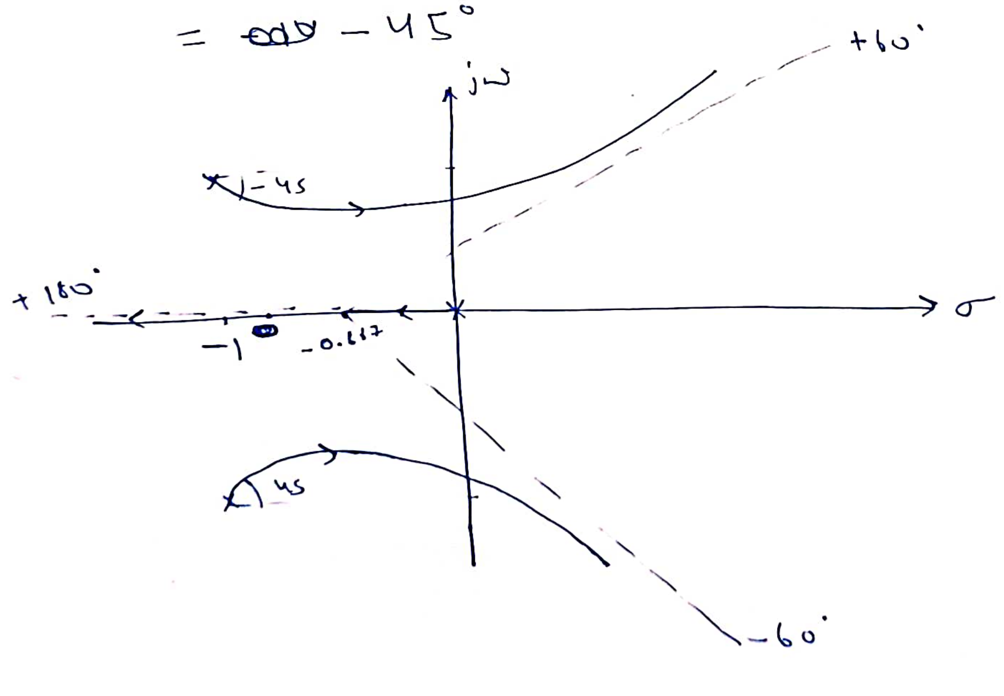
$s_{1,2}$ are complex
$\Rightarrow$ k is complex
but k is real +ve
$\Rightarrow$ so, $s_{1,2}$ are not valid breakaway points

angle of departures,

$$\theta = 180 - \tan^{-1}\left(\frac{1}{0}\right) - \tan^{-1}(-1)$$

$$= 180 - 90 - 135$$

$$= -45^\circ$$



intercept at jw-axis,

$$(jw)^3 + 2(jw)^2 + 2(jw) + k = 0$$

$$\Rightarrow \quad -2w^2 + k + j(-w^3 + 2w) = 0$$

$$w(2 - w^2) = 0 \quad | \quad Im(\ ) = 0$$

$$\Rightarrow \quad w = 0 \quad , \quad w = \pm\sqrt{2}$$

$w = 0$, $k = 0$

$w = \pm\sqrt{2}$, $k = 4$

(4) $G(s)H(s) = \frac{k}{s(s+2)}$

No of poles $= 2 \quad \{0, -2\}$

No of zeros $= 0$

No of asymtotes $= 2$

~~$\theta = (2k+1)180$~~

$\theta = 90°, -90°$

centroid of asymtotes $= -\frac{2}{2} = -1$

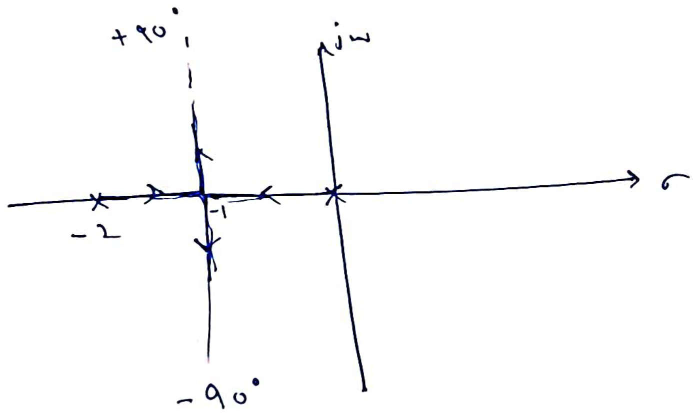
for Breakaway points,

$s^2 + 2s + k = 0$

$\frac{dk}{ds} = -2s - 2 = 0$

$\Rightarrow \boxed{s = -1}$

then, $\boxed{k = 1}$



(5) $G(s)H(s) = \dfrac{K(s+2)(s+3)}{(s+1)(s-1)}$

No of poles = 2 $\{1, -1\}$

No of zeros = 2 $\{-2, -3\}$

No of asymtotes = 0

Breakaway points,

$$1 + \frac{K(s+2)(s+3)}{(s+1)(s-1)} = 0$$

$$\cancel{\Rightarrow (s+1)(s-1) + K(s+2)(s+3) = 0}$$

$$\Rightarrow \quad K = \frac{-(s^2-1)}{s^2+5s+6}$$

$$\frac{dK}{ds} = \frac{(s^2+5s+6)(-2s) - (1-s^2)(2s+5)}{(s^2+5s+6)^2}$$

$$\Rightarrow -2s^3 - 10s^2 - 12s = 2s + 5 - 2s^3 - 5s^2$$

$$\Rightarrow 5s^2 + 14s + 5 = 0$$

$\Rightarrow s_1 = -0.42$ Then, $K = 0.2$

$s_2 = -2.38$ then $K = 19.8$