Module 1

Three-phase Electrical Circuit Analysis

By

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The Overview

General facts

1. A three-phase circuit is basically an electrical circuit of a specific structure
2. Each three-phase element is basically a collection of three simple (i.e., single-phase) elements organized in a certain manner
3. Each constituent simple element of a three-phase element is referred to as one phase of the respective three-phase element
4. A three-phase circuit possesses some distinct properties
5. Although the ordinary AC circuit analysis techniques are also applicable to analyze three-phase circuits, there are some specialized techniques with which a three-phase circuits can be analyzed in a more convenient manner

Elements of a three-phase circuit

1. Three phase source
    - Three phase terminals
    - One or no neutral/ground terminal
2. Three-phase load
    - Three phase terminals
    - One or no neutral/ground terminal

The Overview (Con..)

3. Three phase line
   - Two ports, each with three phase terminals
   - No neutral/ground terminal
4. Three-phase transformer
   - Two ports, each with three phase terminals
   - Two or one or no neutral/ground terminal

Here, we are actually talking about the elementary line concept (i.e., without any shunt branch) as a fundamental component of the three-phase circuit, not about the physical transmission line

Some common terms

1. Star connection and Delta connection
2. Neutral point and neutral line
3. Neutral impedance or grounding impedance
4. Line voltage, line-to-line voltage and phase voltage
5. Line current and phase current
6. Line impedance and phase impedance
7. Balanced element/system and unbalanced element/system
8. Three-phase power

The Overview (Con..)

Utility of the three-phase system

1. A three-phase power supply system is more reliable as well as more convenient to install than the single-phase power supply system because of its modularity
   - If any phase is out, the other phase may continue supplying power to some loads
   - A three-phase transformer can be made simply by externally connecting three separate and smaller single-phase transformers, each with its own protection system

2. Three phase generators and motors are cheaper, smaller in size, simpler in construction, more efficient and easier to operate or control compared to their single-phase counterparts

3. The three-phase generators and motors are usually free from the pulsating torque since the sum of instantons power values over three phases is constant under the balanced condition
   - There should not be any inherent rotor frequency oscillation
   - The machine life is enhanced
   - The stability of parallelly connected generators is improved

The Overview (Con..)

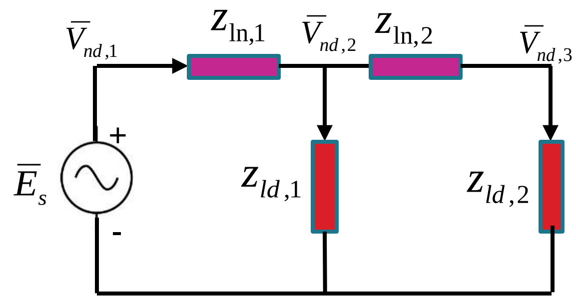
4. The harmonic content of the DC-side voltage output of a single-phase rectifier is much more than the harmonic content of the DC-side voltage output of a three-phase rectifier
   - The ripple factor for a single-phase full-wave diode rectifier is 48%
   - The ripple factor for a three-phase full-wave diode rectifier is 4.27%
   - High ripple factor increases the filtering cost

Things to be learned

1. Matrix modeling of three-phase elements
2. Matrix analysis of three-phase circuits
3. Different three-phase transformer connections
4. Definition and properties of a balanced three-phase circuit
5. Symmetrical component transformation for unbalanced three-phase circuit analysis

Structure of a Three-Phase Circuit

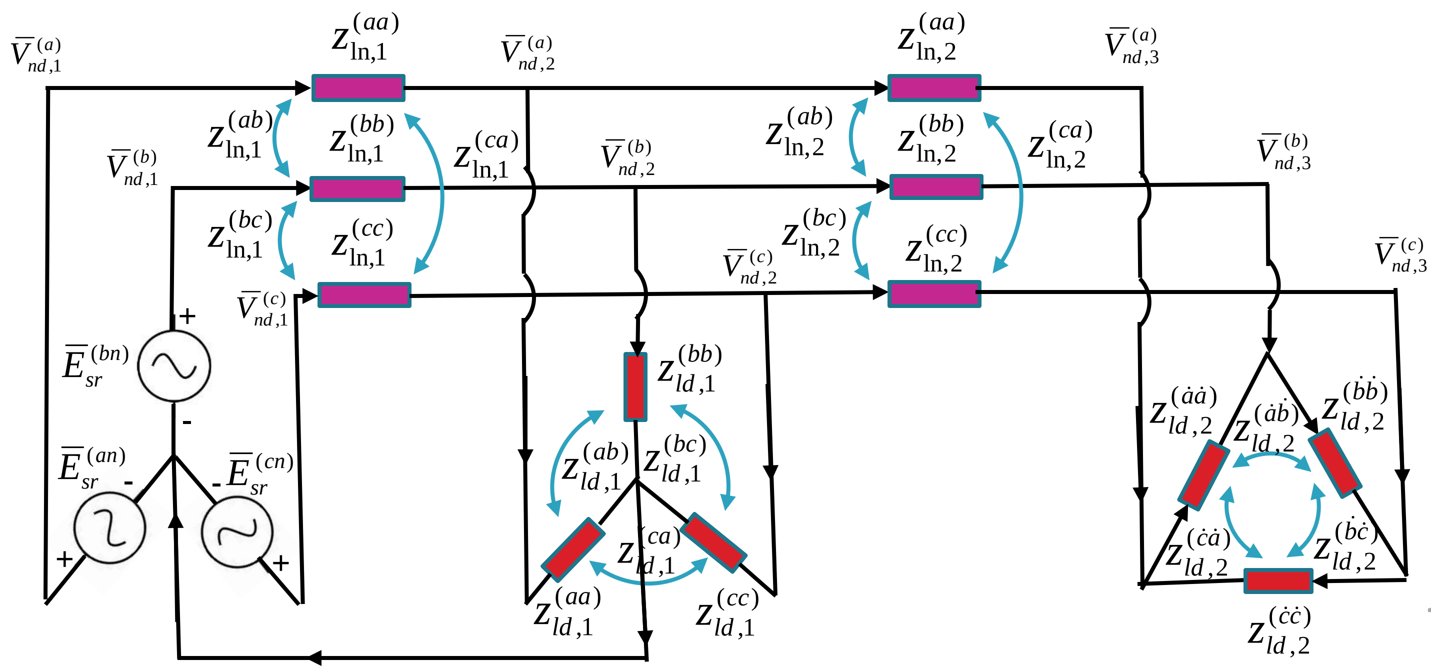
Single-phase circuit:



ln ⟹ Line
ld ⟹ Load
nd ⟹ Node
sr ⟹ Source

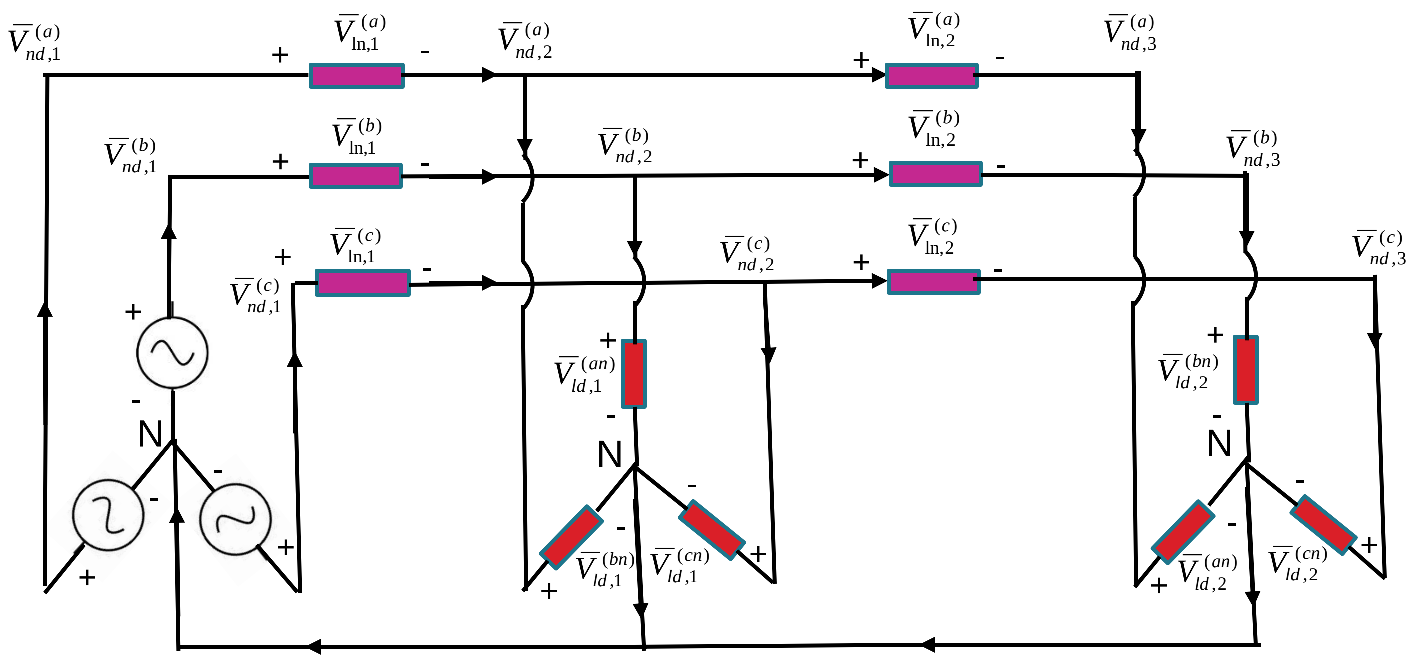
Three-phase circuit:

Circuit 1:



Structure of a Three-Phase Circuit (Con..)

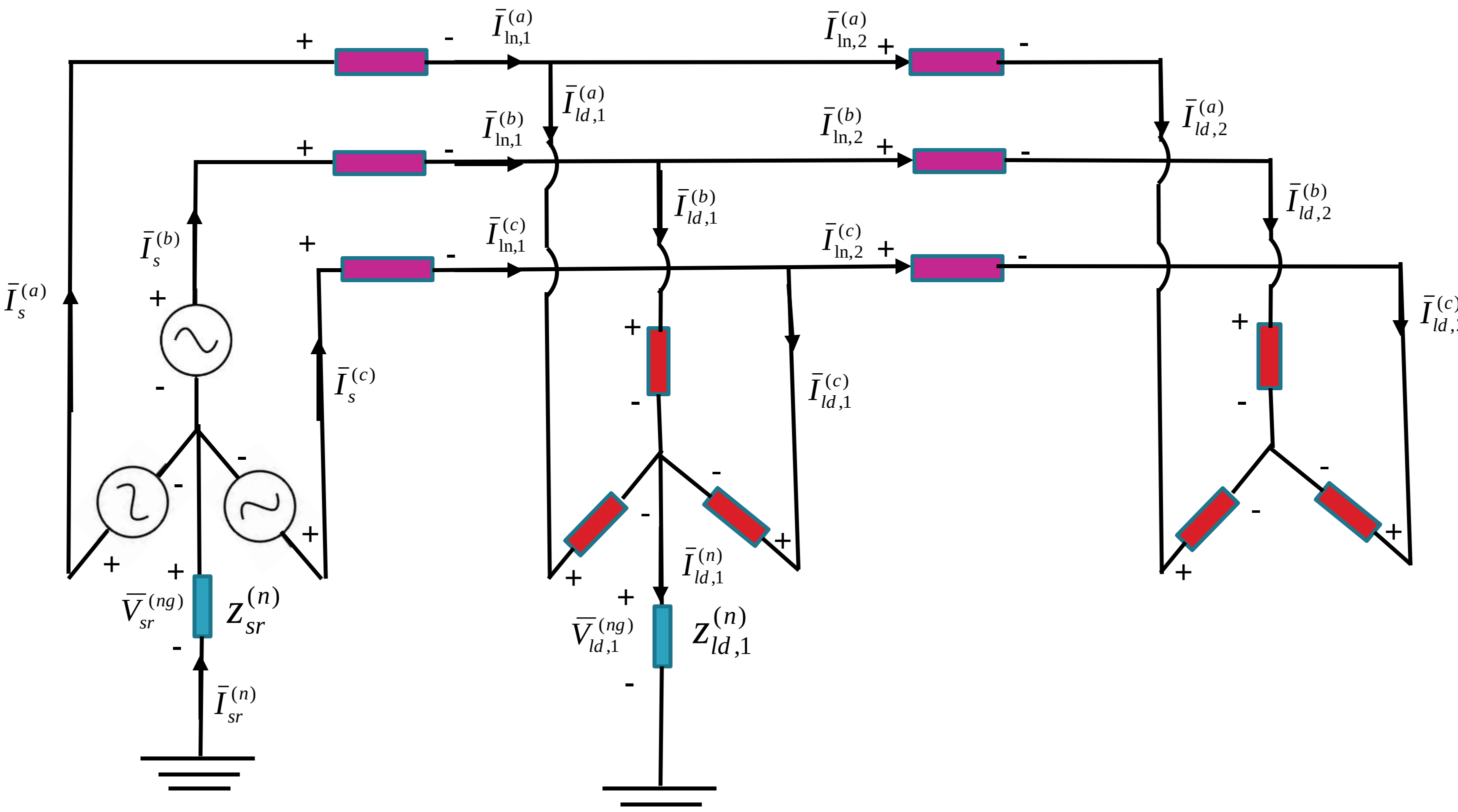
Circuit 2:



N $\Longrightarrow$ Neutral

Structure of a Three-Phase Circuit (Con..)

Circuit 3:



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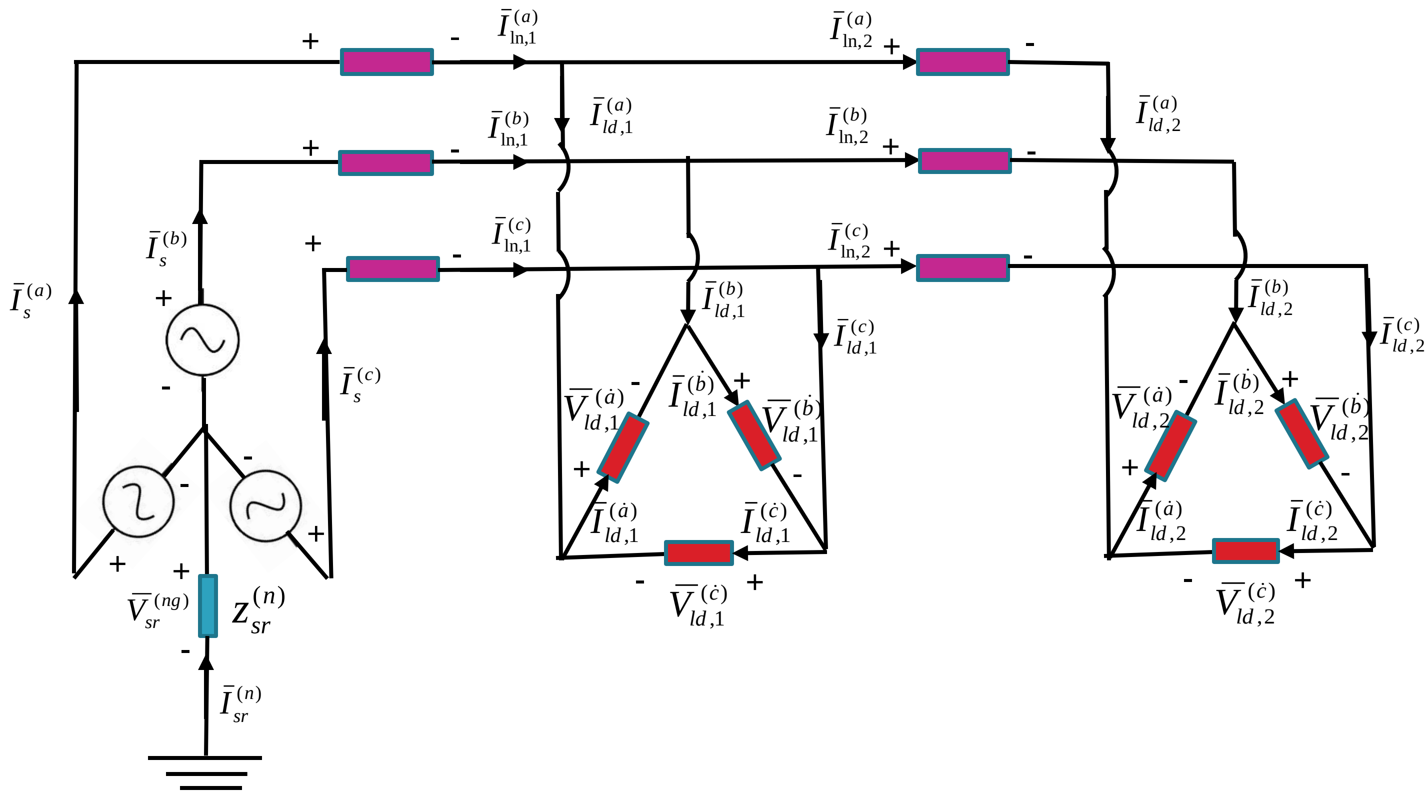
Structure of a Three-Phase Circuit (Con..)

Circuit 4:



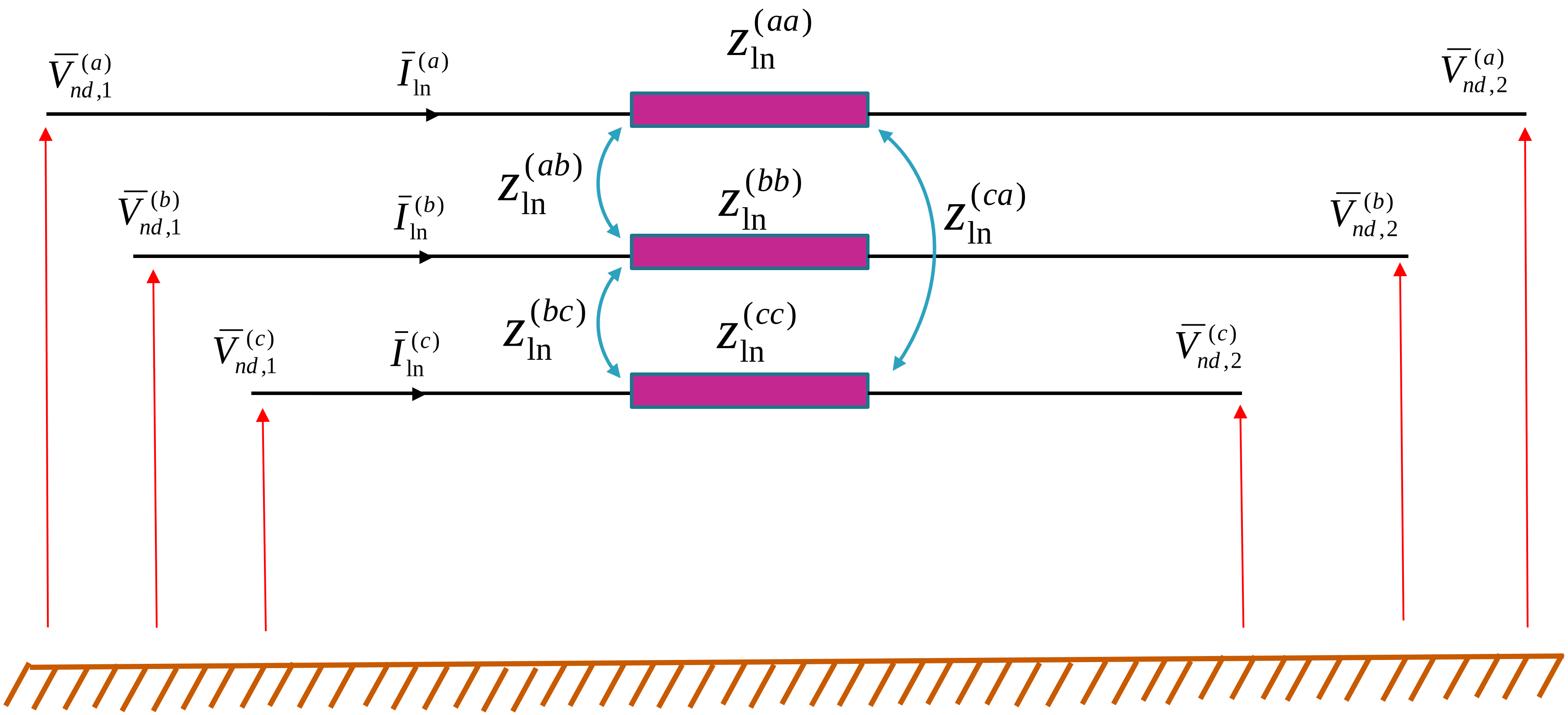
Structure of a Three-Phase Circuit (Con..)

Note:

1. The line voltage at a node (or, simply the node voltage) is computed with reference to the ground or the common neutral
2. The neutral lines connect the neutral points of star elements
3. The neutral lines are taken as zero impedance wires
4. The ground is taken as an equipotential surface
5. A physical neutral wire or the ground can be mathematically converted into an equipotential element by transferring its impedance to phase conductors

Matrix Representation of Line, Load and Source

Impedance/admittance matrix of three-phase line



Matrix Representation of Line, Load and Source (Con..)

By applying KVL,

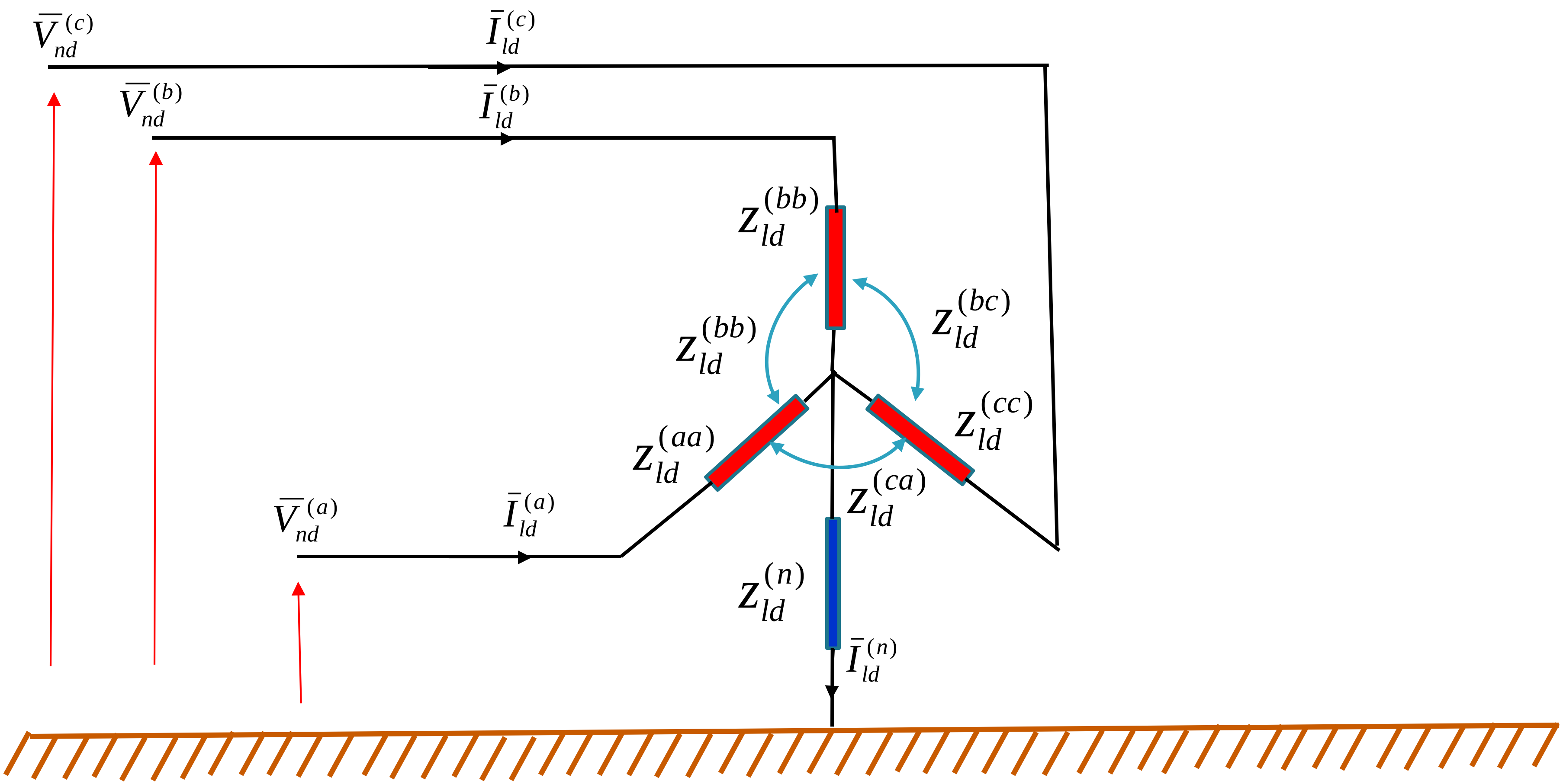
$$\overline{V}_{\ln}^{(a)} = \overline{V}_{nd,1}^{(a)} - \overline{V}_{nd,2}^{(a)} = z_{\ln}^{(aa)}\overline{I}_{\ln}^{(a)} + z_{\ln}^{(ab)}\overline{I}_{\ln}^{(b)} + z_{\ln}^{(ca)}\overline{I}_{\ln}^{(c)}$$
$$\overline{V}_{\ln}^{(b)} = \overline{V}_{nd,1}^{(b)} - \overline{V}_{nd,2}^{(b)} = z_{\ln}^{(ab)}\overline{I}_{\ln}^{(a)} + z_{\ln}^{(bb)}\overline{I}_{\ln}^{(b)} + z_{\ln}^{(bc)}\overline{I}_{\ln}^{(c)}$$
$$\overline{V}_{\ln}^{(c)} = \overline{V}_{nd,1}^{(c)} - \overline{V}_{nd,2}^{(c)} = z_{\ln}^{(ca)}\overline{I}_{\ln}^{(a)} + z_{\ln}^{(bc)}\overline{I}_{\ln}^{(b)} + z_{\ln}^{(cc)}\overline{I}_{\ln}^{(c)}$$

The above set of equations can be written down in the vector-matrix form as follows,

$$\begin{bmatrix} \overline{V}_{\ln}^{(a)} \\ \overline{V}_{\ln}^{(b)} \\ \overline{V}_{\ln}^{(c)} \end{bmatrix} = \begin{bmatrix} \overline{V}_{nd,1}^{(a)} \\ \overline{V}_{nd,1}^{(b)} \\ \overline{V}_{nd,1}^{(c)} \end{bmatrix} - \begin{bmatrix} \overline{V}_{nd,2}^{(a)} \\ \overline{V}_{nd,2}^{(b)} \\ \overline{V}_{nd,2}^{(c)} \end{bmatrix} = \begin{bmatrix} z_{\ln}^{(aa)} & z_{\ln}^{(ab)} & z_{\ln}^{(ca)} \\ z_{\ln}^{(ab)} & z_{\ln}^{(bb)} & z_{\ln}^{(bc)} \\ z_{\ln}^{(ca)} & z_{\ln}^{(bc)} & z_{\ln}^{(cc)} \end{bmatrix} \begin{bmatrix} \overline{I}_{\ln}^{(a)} \\ \overline{I}_{\ln}^{(b)} \\ \overline{I}_{\ln}^{(c)} \end{bmatrix}$$
$$\Rightarrow \overline{\mathbf{V}}_{\mathbf{ln}}^{\mathbf{(abc)}} = \hat{\mathbf{z}}_{\mathbf{ln}}^{\mathbf{(abc)}} \overline{\mathbf{I}}_{\mathbf{ln}}^{\mathbf{(abc)}}$$
$$\Rightarrow \overline{\mathbf{I}}_{\mathbf{ln}}^{\mathbf{(abc)}} = \left\{\hat{\mathbf{z}}_{\mathbf{ln}}^{\mathbf{(abc)}}\right\}^{-1} \overline{\mathbf{V}}_{\mathbf{ln}}^{\mathbf{(abc)}} = \hat{\mathbf{y}}_{\mathbf{ln}}^{\mathbf{(abc)}} \overline{\mathbf{V}}_{\mathbf{ln}}^{\mathbf{(abc)}}$$

Matrix Representation of Line, Load and Source (Con..)

Impedance/admittance matrix of a grounded star-connected load



Matrix Representation of Line, Load and Source (Con..)

According to the KCL,

$$\bar{I}_{ld}^{(n)} = \bar{I}_{ld}^{(a)} + \bar{I}_{ld}^{(b)} + \bar{I}_{ld}^{(c)}$$

Subsequently, by applying the KVL, we obtain,

$$\begin{aligned}
\bar{V}_{nd}^{(a)} &= z_{ld}^{(aa)}\bar{I}_{ld}^{(a)} + z_{ld}^{(ab)}\bar{I}_{ld}^{(b)} + z_{ld}^{(ca)}\bar{I}_{ld}^{(c)} + z_{ld}^{(n)}\bar{I}_{ld}^{(n)} \\
&= z_{ld}^{(aa)}\bar{I}_{ld}^{(a)} + z_{ld}^{(ab)}\bar{I}_{ld}^{(b)} + z_{ld}^{(ca)}\bar{I}_{ld}^{(c)} + z_{ld}^{(n)}\left\{\bar{I}_{ld}^{(a)} + \bar{I}_{ld}^{(b)} + \bar{I}_{ld}^{(c)}\right\} \\
&= \left\{z_{ld}^{(aa)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(a)} + \left\{z_{ld}^{(ab)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(b)} + \left\{z_{ld}^{(ca)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(c)}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\bar{V}_{nd}^{(b)} &= \left\{z_{ld}^{(ab)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(a)} + \left\{z_{ld}^{(bb)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(b)} + \left\{z_{ld}^{(bc)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(c)} \\
\bar{V}_{nd}^{(c)} &= \left\{z_{ld}^{(ca)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(a)} + \left\{z_{ld}^{(bc)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(b)} + \left\{z_{ld}^{(cc)} + z_{ld}^{(n)}\right\}\bar{I}_{ld}^{(c)}
\end{aligned}$$

Matrix Representation of Line, Load and Source (Con..)

The above set of KVL equations can be written down in the vector-matrix form as follows,

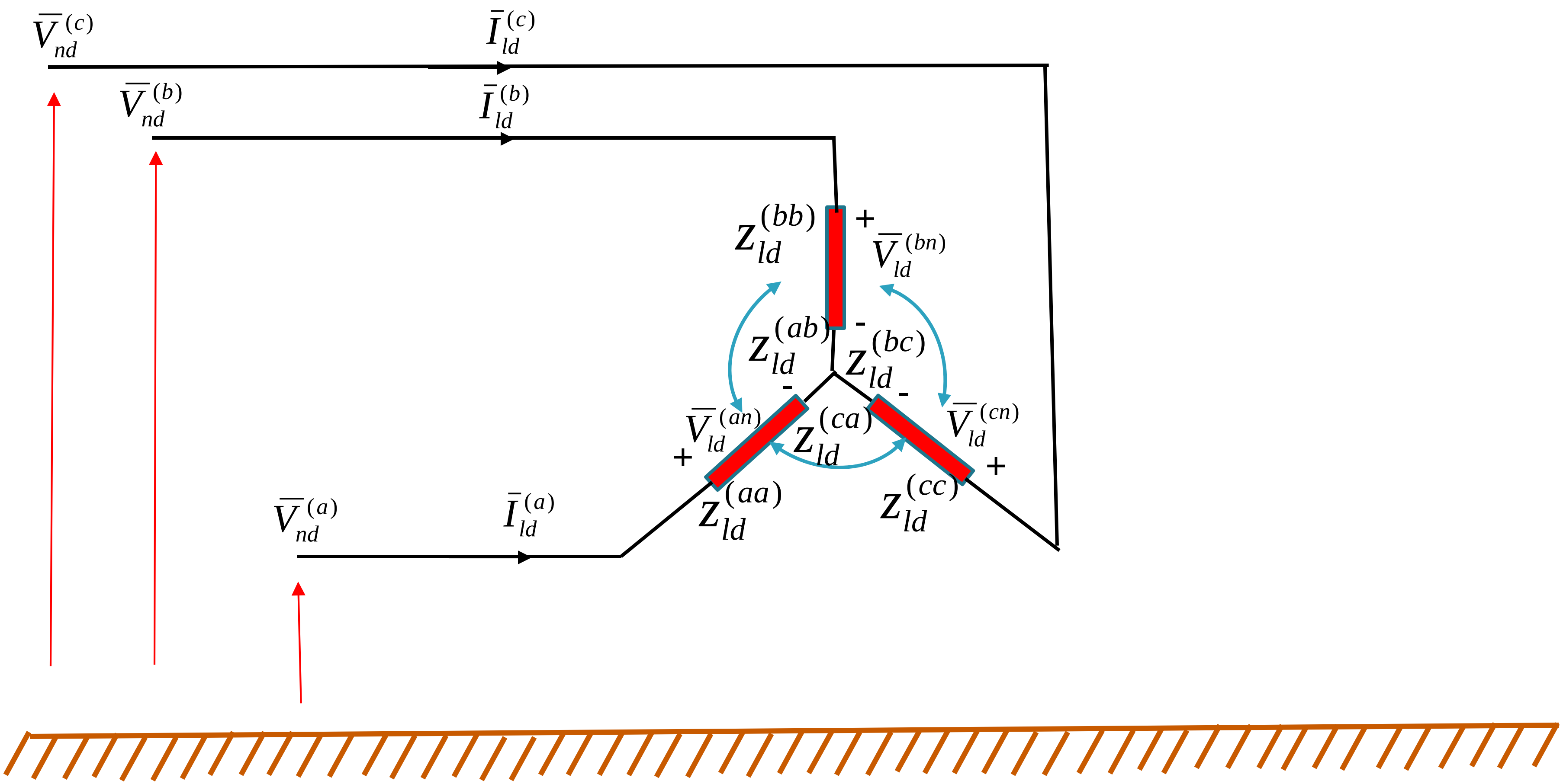
$$\begin{bmatrix} \overline{V}_{nd}^{(a)} \\ \overline{V}_{nd}^{(b)} \\ \overline{V}_{nd}^{(c)} \end{bmatrix} = \begin{bmatrix} z_{ld}^{(aa)} + z_{ld}^{(n)} & z_{ld}^{(ab)} + z_{ld}^{(n)} & z_{ld}^{(ca)} + z_{ld}^{(n)} \\ z_{ld}^{(ab)} + z_{ld}^{(n)} & z_{ld}^{(bb)} + z_{ld}^{(n)} & z_{ld}^{(bc)} + z_{ld}^{(n)} \\ z_{ld}^{(ca)} + z_{ld}^{(n)} & z_{ld}^{(bc)} + z_{ld}^{(n)} & z_{ld}^{(cc)} + z_{ld}^{(n)} \end{bmatrix} \begin{bmatrix} \overline{I}_{ld}^{(a)} \\ \overline{I}_{ld}^{(b)} \\ \overline{I}_{ld}^{(c)} \end{bmatrix}$$

$$\Rightarrow \overline{\mathbf{V}}_{\mathbf{nd}}^{\mathbf{(abc)}} = \hat{\mathbf{z}}_{\mathbf{ld}}^{\mathbf{(abc)}} \overline{\mathbf{I}}_{\mathbf{ld}}^{\mathbf{(abc)}}$$

$$\Rightarrow \overline{\mathbf{I}}_{\mathbf{ld}}^{\mathbf{(abc)}} = \left\{ \hat{\mathbf{z}}_{\mathbf{ld}}^{\mathbf{(abc)}} \right\}^{-1} \overline{\mathbf{V}}_{\mathbf{nd}}^{\mathbf{(abc)}} = \hat{\mathbf{y}}_{\mathbf{ld}}^{\mathbf{(abc)}} \overline{\mathbf{V}}_{\mathbf{nd}}^{\mathbf{(abc)}}$$

Matrix Representation of Line, Load and Source (Con..)

Admittance matrix of an ungrounded star-connected load



Matrix Representation of Line, Load and Source (Con..)

According to the KCL,

$$\bar{I}_{ld}^{(a)} + \bar{I}_{ld}^{(b)} + \bar{I}_{ld}^{(c)} = 0$$

Next, the following equations are obtained by applying the KVL

$$\begin{aligned}
\bar{V}_{nd}^{(a)} &= \bar{V}_{ld}^{(an)} - \bar{V}_{ld}^{(bn)} + \bar{V}_{nd}^{(b)} \\
&= z_{ld}^{(aa)}\bar{I}_{ld}^{(a)} + z_{ld}^{(ab)}\bar{I}_{ld}^{(b)} + z_{ld}^{(ca)}\bar{I}_{ld}^{(c)} - z_{ld}^{(ab)}\bar{I}_{ld}^{(a)} - z_{ld}^{(bb)}\bar{I}_{ld}^{(b)} - z_{ld}^{(bc)}\bar{I}_{ld}^{(c)} + \bar{V}_{nd}^{(b)} \\
&= \left\{z_{ld}^{(aa)} - z_{ld}^{(ab)}\right\}\bar{I}_{ld}^{(a)} + \left\{z_{ld}^{(ab)} - z_{ld}^{(bb)}\right\}\bar{I}_{ld}^{(b)} + \left\{z_{ld}^{(ca)} - z_{ld}^{(bc)}\right\}\bar{I}_{ld}^{(c)} + \bar{V}_{nd}^{(b)}
\end{aligned}$$

$$\begin{aligned}
\bar{V}_{nd}^{(a)} &= \bar{V}_{ld}^{(an)} - \bar{V}_{ld}^{(cn)} + \bar{V}_{nd}^{(c)} \\
&= z_{ld}^{(aa)}\bar{I}_{ld}^{(a)} + z_{ld}^{(ab)}\bar{I}_{ld}^{(b)} + z_{ld}^{(ca)}\bar{I}_{ld}^{(c)} - z_{ld}^{(ca)}\bar{I}_{ld}^{(a)} - z_{ld}^{(bc)}\bar{I}_{ld}^{(b)} - z_{ld}^{(cc)}\bar{I}_{ld}^{(c)} + \bar{V}_{nd}^{(c)} \\
&= \left\{z_{ld}^{(aa)} - z_{ld}^{(ca)}\right\}\bar{I}_{ld}^{(a)} + \left\{z_{ld}^{(ab)} - z_{ld}^{(bc)}\right\}\bar{I}_{ld}^{(b)} + \left\{z_{ld}^{(ca)} - z_{ld}^{(cc)}\right\}\bar{I}_{ld}^{(c)} + \bar{V}_{nd}^{(c)}
\end{aligned}$$

Matrix Representation of Line, Load and Source (Con..)

The KCL and KVL equations can be put together in the matrix-vector form as follows

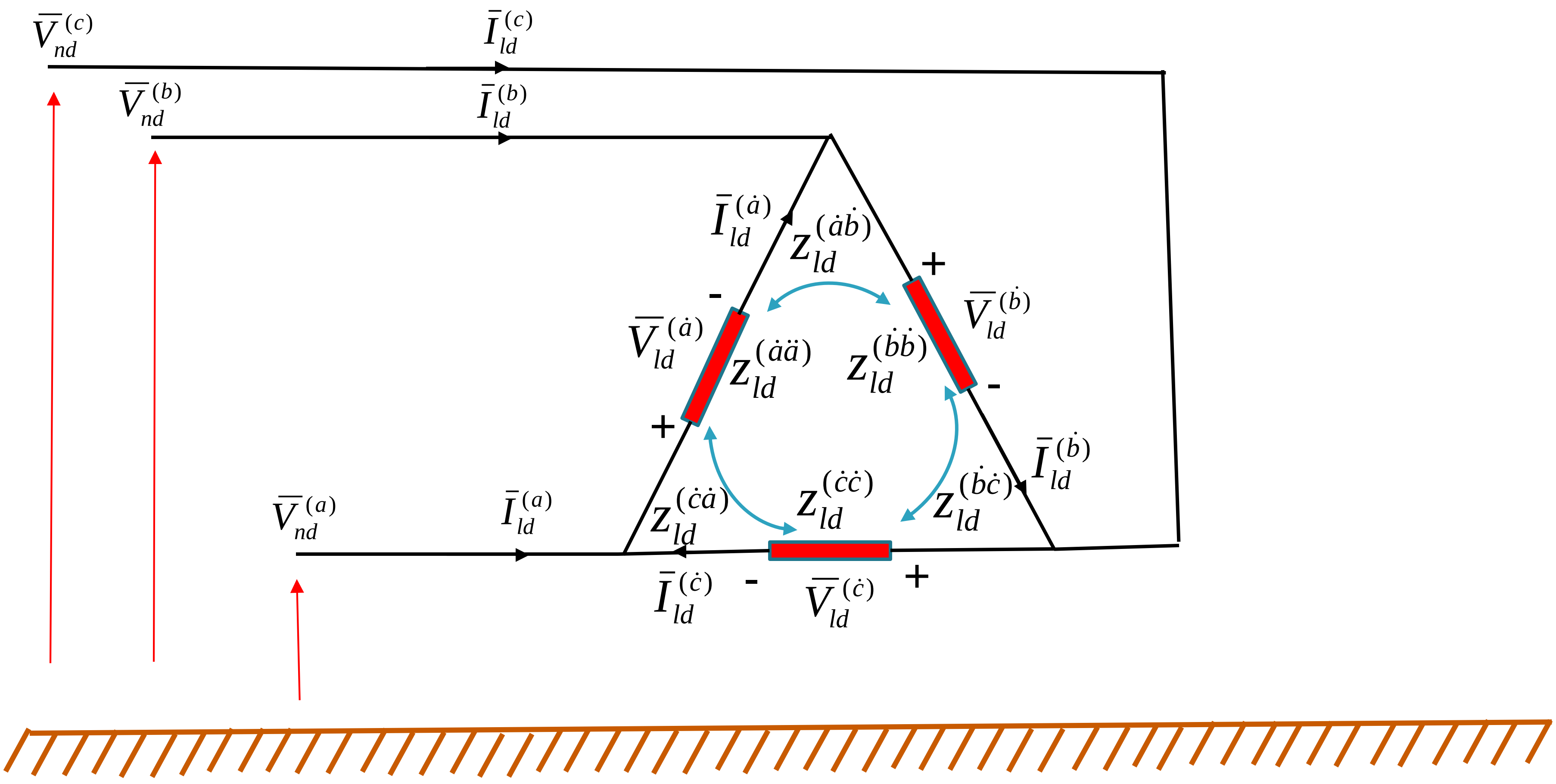
$$\begin{bmatrix} 1 & 1 & 1 \\ z_{ld}^{(aa)} - z_{ld}^{(ab)} & z_{ld}^{(ab)} - z_{ld}^{(bb)} & z_{ld}^{(ca)} - z_{ld}^{(bc)} \\ z_{ld}^{(aa)} - z_{ld}^{(ca)} & z_{ld}^{(ab)} - z_{ld}^{(bc)} & z_{ld}^{(ca)} - z_{ld}^{(cc)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ z_{ld}^{(aa)} - z_{ld}^{(ab)} & z_{ld}^{(ab)} - z_{ld}^{(bb)} & z_{ld}^{(ca)} - z_{ld}^{(bc)} \\ z_{ld}^{(aa)} - z_{ld}^{(ca)} & z_{ld}^{(ab)} - z_{ld}^{(bc)} & z_{ld}^{(ca)} - z_{ld}^{(cc)} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}}_{\hat{\mathbf{y}}_{\mathbf{ld}}^{(\mathbf{abc})}} \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix}$$

For the ungrounded star connection, the load impedance matrix does not exist

Matrix Representation of Line, Load and Source (Con..)

Admittance matrix of an delta-connected load



Matrix Representation of Line, Load and Source (Con..)

By applying the KVL,

$$\bar{V}_{nd}^{(a)} = \bar{V}_{ld}^{(\dot{a})} + \bar{V}_{nd}^{(b)} = z_{ld}^{(\dot{a}\dot{a})}\bar{I}_{ld}^{(\dot{a})} + z_{ld}^{(\dot{a}\dot{b})}\bar{I}_{ld}^{(\dot{b})} + z_{ld}^{(\dot{c}\dot{a})}\bar{I}_{ld}^{(\dot{c})} + \bar{V}_{nd}^{(b)}$$

$$\bar{V}_{nd}^{(b)} = \bar{V}_{ld}^{(\dot{b})} + \bar{V}_{nd}^{(c)} = z_{ld}^{(\dot{a}\dot{b})}\bar{I}_{ld}^{(\dot{a})} + z_{ld}^{(\dot{b}\dot{b})}\bar{I}_{ld}^{(\dot{b})} + z_{ld}^{(\dot{b}\dot{c})}\bar{I}_{ld}^{(\dot{c})} + \bar{V}_{nd}^{(c)}$$

$$\bar{V}_{nd}^{(c)} = \bar{V}_{ld}^{(\dot{c})} + \bar{V}_{nd}^{(a)} = z_{ld}^{(\dot{c}\dot{a})}\bar{I}_{ld}^{(\dot{a})} + z_{ld}^{(\dot{b}\dot{c})}\bar{I}_{ld}^{(\dot{b})} + z_{ld}^{(\dot{c}\dot{c})}\bar{I}_{ld}^{(\dot{c})} + \bar{V}_{nd}^{(a)}$$

The KVL equations can be put down in the vector/matrix form as follows,

$$\begin{bmatrix} z_{ld}^{(\dot{a}\dot{a})} & z_{ld}^{(\dot{a}\dot{b})} & z_{ld}^{(\dot{c}\dot{a})} \\ z_{ld}^{(\dot{a}\dot{b})} & z_{ld}^{(\dot{b}\dot{b})} & z_{ld}^{(\dot{b}\dot{c})} \\ z_{ld}^{(\dot{c}\dot{a})} & z_{ld}^{(\dot{b}\dot{c})} & z_{ld}^{(\dot{c}\dot{c})} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(\dot{a})} \\ \bar{I}_{ld}^{(\dot{b})} \\ \bar{I}_{ld}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix}$$

Matrix Representation of Line, Load and Source (Con..)

Therefore,

$$\begin{bmatrix} \bar{I}_{ld}^{(\dot{a})} \\ \bar{I}_{ld}^{(\dot{b})} \\ \bar{I}_{ld}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} z_{ld}^{(\dot{a}\dot{a})} & z_{ld}^{(\dot{a}\dot{b})} & z_{ld}^{(\dot{c}\dot{a})} \\ z_{ld}^{(\dot{a}\dot{b})} & z_{ld}^{(\dot{b}\dot{b})} & z_{ld}^{(\dot{b}\dot{c})} \\ z_{ld}^{(\dot{c}\dot{a})} & z_{ld}^{(\dot{b}\dot{c})} & z_{ld}^{(\dot{c}\dot{c})} \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix}$$

Next, we get the following equations by applying the KCL

$$\left.\begin{aligned} \bar{I}_{ld}^{(a)} &= \bar{I}_{ld}^{(\dot{a})} - \bar{I}_{ld}^{(\dot{c})} \\ \bar{I}_{ld}^{(b)} &= \bar{I}_{ld}^{(\dot{b})} - \bar{I}_{ld}^{(\dot{a})} \\ \bar{I}_{ld}^{(c)} &= \bar{I}_{ld}^{(\dot{c})} - \bar{I}_{ld}^{(\dot{b})} \end{aligned}\right\} \Rightarrow \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(\dot{a})} \\ \bar{I}_{ld}^{(\dot{b})} \\ \bar{I}_{ld}^{(\dot{c})} \end{bmatrix}$$

Matrix Representation of Line, Load and Source (Con..)

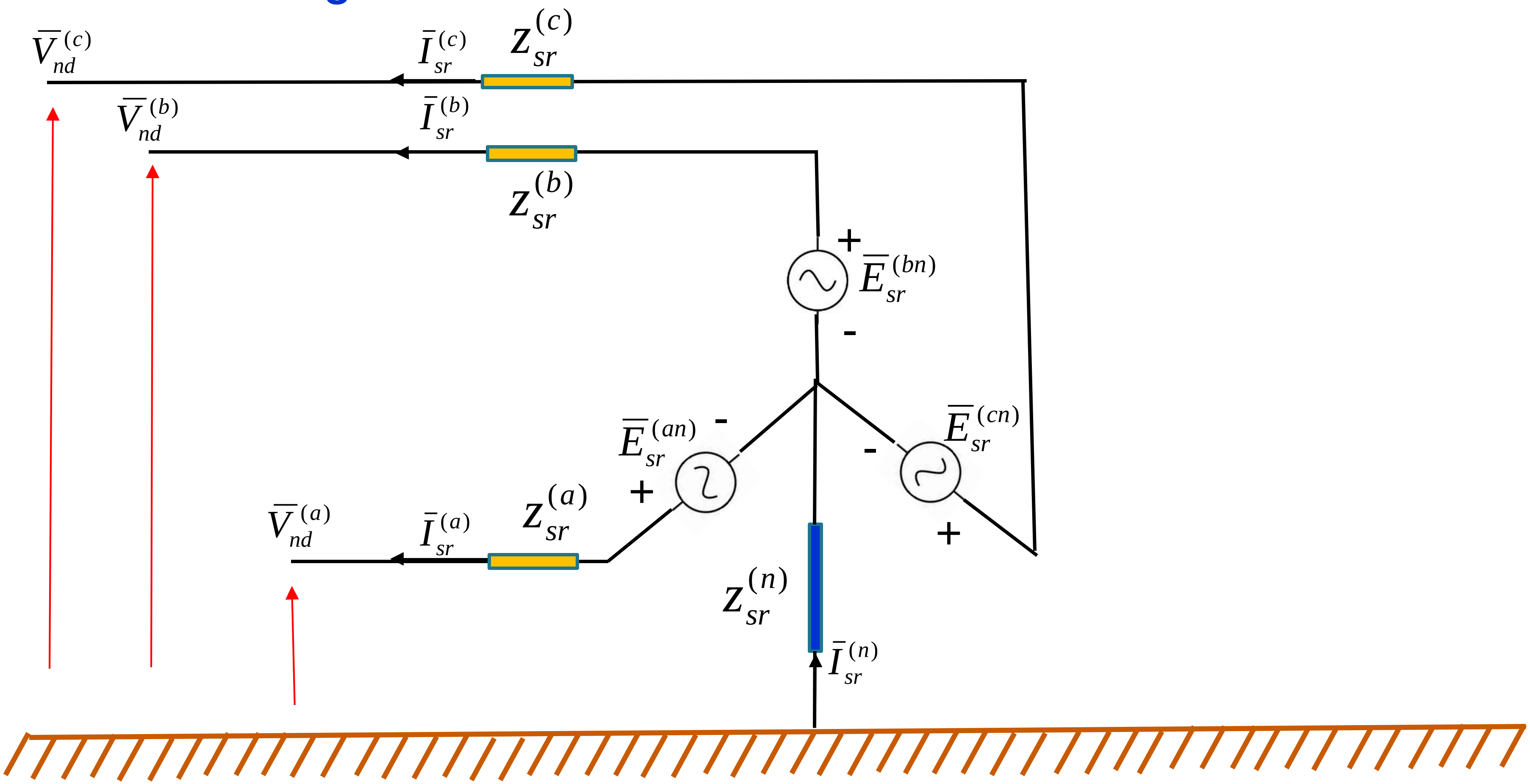
By replacing the delta phase currents in terms of the line voltage at the terminal node, we finally obtain the following equations

$$\begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} z_{ld}^{(\dot{a}\dot{a})} & z_{ld}^{(\dot{a}\dot{b})} & z_{ld}^{(\dot{c}\dot{a})} \\ z_{ld}^{(\dot{a}\dot{b})} & z_{ld}^{(\dot{b}\dot{b})} & z_{ld}^{(\dot{b}\dot{c})} \\ z_{ld}^{(\dot{c}\dot{a})} & z_{ld}^{(\dot{b}\dot{c})} & z_{ld}^{(\dot{c}\dot{c})} \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}}_{\hat{\mathbf{y}}_{\mathbf{ld}}^{\mathbf{(abc)}}} \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix}$$

For the delta connection, the load impedance matrix does not exist

Matrix Representation of Line, Load and Source (Con..)

Source modeling



Matrix Representation of Line, Load and Source (Con..)

According to the KCL,

$$\bar{I}_{sr}^{(n)} = \bar{I}_{sr}^{(a)} + \bar{I}_{sr}^{(b)} + \bar{I}_{sr}^{(c)}$$

The KVL, together with the KCL, gives the following equations,

$$\begin{aligned}
\bar{E}_{sr}^{(an)} &= \bar{V}_{nd}^{(a)} + z_{sr}^{(a)}\bar{I}_{sr}^{(a)} + z_{sr}^{(n)}\bar{I}_{sr}^{(n)} \\
&= \bar{V}_{nd}^{(a)} + \left\{z_{sr}^{(a)} + z_{sr}^{(n)}\right\}\bar{I}_{sr}^{(a)} + z_{sr}^{(n)}\bar{I}_{sr}^{(b)} + z_{sr}^{(n)}\bar{I}_{sr}^{(c)} \\
\bar{E}_{sr}^{(bn)} &= \bar{V}_{nd}^{(b)} + z_{sr}^{(b)}\bar{I}_{sr}^{(b)} + z_{sr}^{(n)}\bar{I}_{sr}^{(n)} \\
&= \bar{V}_{nd}^{(b)} + z_{sr}^{(n)}\bar{I}_{sr}^{(a)} + \left\{z_{sr}^{(b)} + z_{sr}^{(n)}\right\}\bar{I}_{sr}^{(b)} + z_{sr}^{(n)}\bar{I}_{sr}^{(c)} \\
\bar{E}_{sr}^{(cn)} &= \bar{V}_{nd}^{(c)} + z_{sr}^{(c)}\bar{I}_{sr}^{(c)} + z_{sr}^{(n)}\bar{I}_{sr}^{(n)} \\
&= \bar{V}_{nd}^{(c)} + z_{sr}^{(n)}\bar{I}_{sr}^{(a)} + z_{sr}^{(n)}\bar{I}_{sr}^{(b)} + \left\{z_{sr}^{(c)} + z_{sr}^{(n)}\right\}\bar{I}_{sr}^{(b)}
\end{aligned}$$

Matrix Representation of Line, Load and Source (Con..)

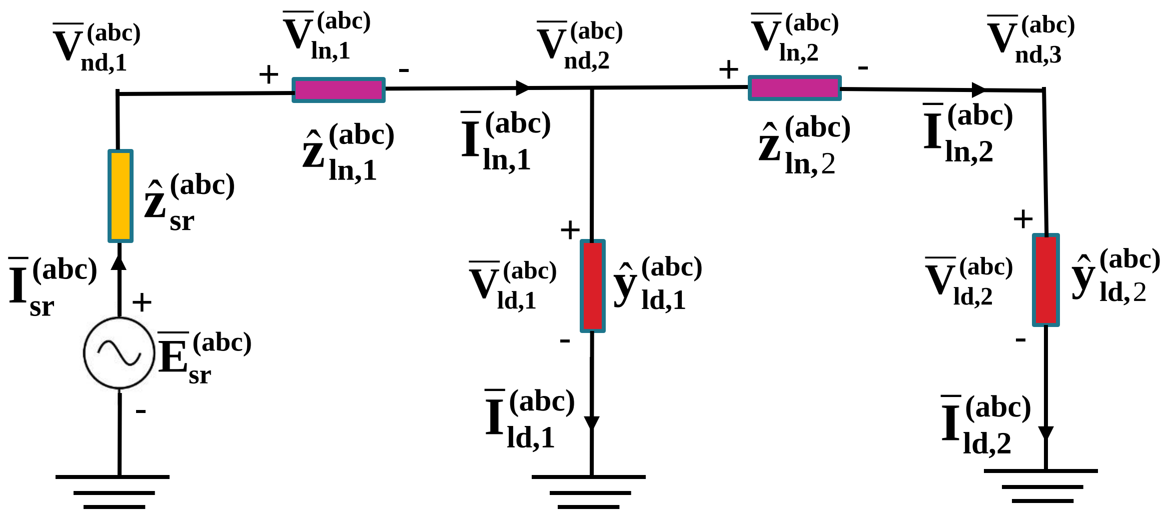
In the vector-matrix form,

$$\begin{bmatrix} \overline{E}_{sr}^{(an)} \\ \overline{E}_{sr}^{(bn)} \\ \overline{E}_{sr}^{(cn)} \end{bmatrix} = \begin{bmatrix} z_{sr}^{(a)} + z_{sr}^{(n)} & z_{sr}^{(n)} & z_{sr}^{(n)} \\ z_{sr}^{(n)} & z_{sr}^{(b)} + z_{sr}^{(n)} & z_{sr}^{(n)} \\ z_{sr}^{(n)} & z_{sr}^{(n)} & z_{sr}^{(c)} + z_{sr}^{(n)} \end{bmatrix} \begin{bmatrix} \overline{I}_{sr}^{(a)} \\ \overline{I}_{sr}^{(b)} \\ \overline{I}_{sr}^{(b)} \end{bmatrix} + \begin{bmatrix} \overline{V}_{nd}^{(a)} \\ \overline{V}_{nd}^{(b)} \\ \overline{V}_{nd}^{(c)} \end{bmatrix}$$

$$\Rightarrow \overline{\mathbf{E}}_{\mathbf{sr}}^{(\mathbf{abc})} = \hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})} \overline{\mathbf{I}}_{\mathbf{sr}}^{(\mathbf{abc})} + \overline{\mathbf{V}}_{\mathbf{nd}}^{(\mathbf{abc})}$$

Matrix Analysis of a Three-Phase Circuit

The three-phase circuits shown in Slides 6, 7, 8 and 9 can be compactly presented as follows



According to the KCL,

$$\bar{\mathbf{I}}^{(abc)}_{ln,2} = \bar{\mathbf{I}}^{(abc)}_{ld,2} = \hat{\mathbf{y}}^{(abc)}_{ld,2}\bar{\mathbf{V}}^{(abc)}_{nd,3}$$

$$\bar{\mathbf{I}}^{(abc)}_{sr} = \bar{\mathbf{I}}^{(abc)}_{ln,1} = \bar{\mathbf{I}}^{(abc)}_{ld,1} + \bar{\mathbf{I}}^{(abc)}_{ln,2} = \hat{\mathbf{y}}^{(abc)}_{ld,1}\bar{\mathbf{V}}^{(abc)}_{nd,2} + \hat{\mathbf{y}}^{(abc)}_{ld,2}\bar{\mathbf{V}}^{(abc)}_{nd,3}$$

Matrix Analysis of a Three-Phase Circuit (Con..)

According to the KVL,

$$\overline{\mathbf{E}}_{\mathbf{sr}}^{\mathbf{(abc)}} = \hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}}\overline{\mathbf{I}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\overline{\mathbf{I}}_{\mathbf{ln,1}}^{\mathbf{(abc)}} + \overline{\mathbf{V}}_{\mathbf{nd,2}}^{\mathbf{(abc)}}$$

$$\overline{\mathbf{E}}_{\mathbf{sr}}^{\mathbf{(abc)}} = \hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}}\overline{\mathbf{I}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\overline{\mathbf{I}}_{\mathbf{ln,1}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,2}}^{\mathbf{(abc)}}\overline{\mathbf{I}}_{\mathbf{ln,2}}^{\mathbf{(abc)}} + \overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}}$$

After replacing KCL relations into KVL equations, we obtain,

$$\begin{aligned}\overline{\mathbf{E}}_{\mathbf{sr}}^{\mathbf{(abc)}} &= \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\right)\left(\hat{\mathbf{y}}_{\mathbf{ld,1}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,2}}^{\mathbf{(abc)}} + \hat{\mathbf{y}}_{\mathbf{ld,2}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}}\right) + \overline{\mathbf{V}}_{\mathbf{nd,2}}^{\mathbf{(abc)}} \\ &= \left\{\mathbf{U} + \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\right)\hat{\mathbf{y}}_{\mathbf{ld,1}}^{\mathbf{(abc)}}\right\}\overline{\mathbf{V}}_{\mathbf{nd,2}}^{\mathbf{(abc)}} + \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\right)\hat{\mathbf{y}}_{\mathbf{ld,2}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}}\end{aligned}$$

$$\begin{aligned}\overline{\mathbf{E}}_{\mathbf{sr}}^{\mathbf{(abc)}} &= \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\right)\left(\hat{\mathbf{y}}_{\mathbf{ld,1}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,2}}^{\mathbf{(abc)}} + \hat{\mathbf{y}}_{\mathbf{ld,2}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}}\right) + \hat{\mathbf{z}}_{\mathbf{ln,2}}^{\mathbf{(abc)}}\hat{\mathbf{y}}_{\mathbf{ld,2}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}} + \overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}} \\ &= \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\right)\hat{\mathbf{y}}_{\mathbf{ld,1}}^{\mathbf{(abc)}}\overline{\mathbf{V}}_{\mathbf{nd,2}}^{\mathbf{(abc)}} + \left\{\mathbf{U} + \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,1}}^{\mathbf{(abc)}}\right)\hat{\mathbf{y}}_{\mathbf{ld,2}}^{\mathbf{(abc)}} + \hat{\mathbf{z}}_{\mathbf{ln,2}}^{\mathbf{(abc)}}\hat{\mathbf{y}}_{\mathbf{ld,2}}^{\mathbf{(abc)}}\right\}\overline{\mathbf{V}}_{\mathbf{nd,3}}^{\mathbf{(abc)}}\end{aligned}$$

$$\boxed{\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Identity matrix}}$$

Matrix Analysis of a Three-Phase Circuit (Con..)

Therefore,

$$\begin{bmatrix} \left\{\mathbf{U}+\left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,1}}^{(\mathbf{abc})}\right\} & \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,2}}^{(\mathbf{abc})} \\ \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,1}}^{(\mathbf{abc})} & \left\{\mathbf{U}+\left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,2}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,2}}^{(\mathbf{abc})}\hat{\mathbf{y}}_{\mathbf{ld,2}}^{(\mathbf{abc})}\right\} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{V}}_{\mathbf{nd,2}}^{(\mathbf{abc})} \\ \overline{\mathbf{V}}_{\mathbf{nd,3}}^{(\mathbf{abc})} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{E}}_{\mathbf{sr}}^{(\mathbf{abc})} \\ \overline{\mathbf{E}}_{\mathbf{sr}}^{(\mathbf{abc})} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \overline{\mathbf{V}}_{\mathbf{nd,2}}^{(\mathbf{abc})} \\ \overline{\mathbf{V}}_{\mathbf{nd,3}}^{(\mathbf{abc})} \end{bmatrix} = \begin{bmatrix} \left\{\mathbf{U}+\left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,1}}^{(\mathbf{abc})}\right\} & \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,2}}^{(\mathbf{abc})} \\ \left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,1}}^{(\mathbf{abc})} & \left\{\mathbf{U}+\left(\hat{\mathbf{z}}_{\mathbf{sr}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,1}}^{(\mathbf{abc})}\right)\hat{\mathbf{y}}_{\mathbf{ld,2}}^{(\mathbf{abc})}+\hat{\mathbf{z}}_{\mathbf{ln,2}}^{(\mathbf{abc})}\hat{\mathbf{y}}_{\mathbf{ld,2}}^{(\mathbf{abc})}\right\} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{E}}_{\mathbf{sr}}^{(\mathbf{abc})} \\ \overline{\mathbf{E}}_{\mathbf{sr}}^{(\mathbf{abc})} \end{bmatrix}$$

Once the voltage vectors at Node 2 and Node 3 are known, all other quantities can be easily determined

<u>Procedural summary:</u>

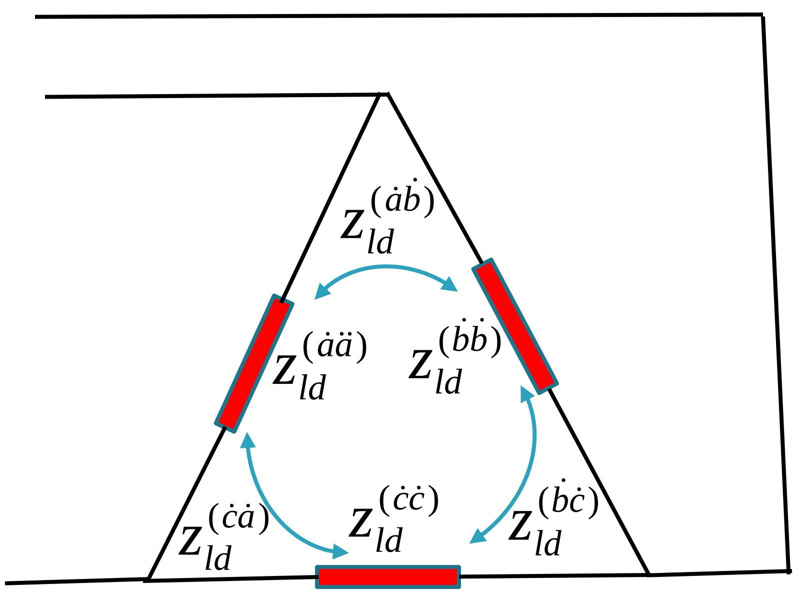
1. Find out the matrix model of each three-phase component
2. Represent the circuit like an ordinary single-phase circuit
3. Replace scalar quantities (i.e., voltages or currents) with vector quantities and scalar parameters (i.e., impedances or admittances) with matrix parameters
4. Instead of scalar KVL and KCL equations, write matrix KVL and KCL equations
5. You may solve those matrix KVL and KCL equations by using any software such as the MATLAB

Properties of a Balanced Three-Phase Circuit

Balanced element

1. The same self impedance for each phase
2. The same mutual impedance for each phase pair
3. The same source current or voltage magnitude for each phase
4. A phase angle separation of 120 degree between the source voltage and current phasors any two phases

Example:

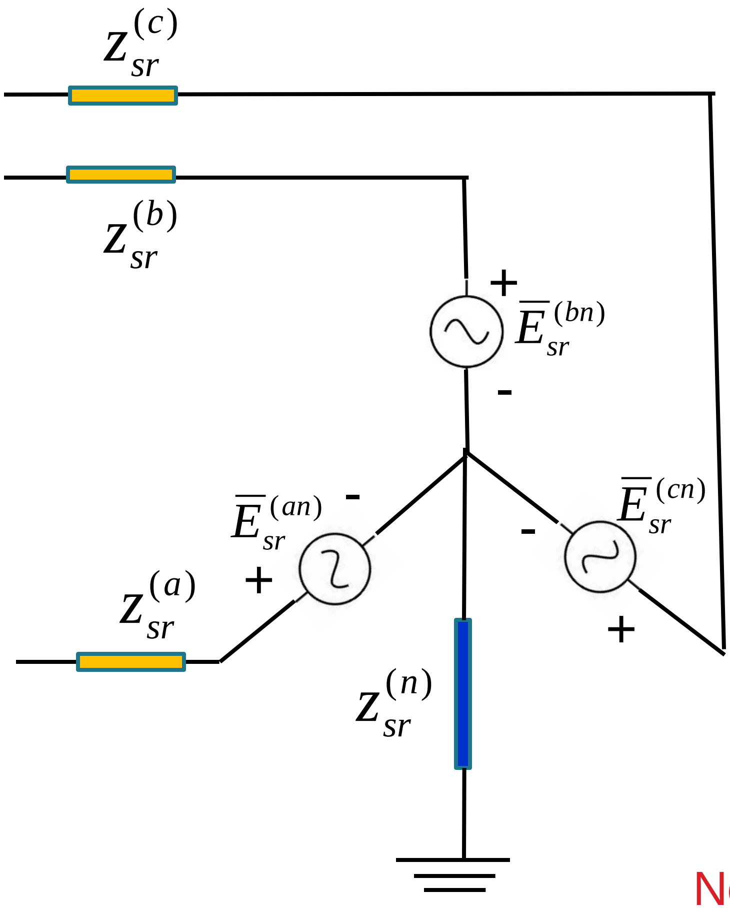


Balancing condition

$$Z_{ld}^{(\dot{a}\dot{a})} = Z_{ld}^{(\dot{b}\dot{b})} = Z_{ld}^{(\dot{c}\dot{c})} = Z_{ld}^{(ps)}$$

$$Z_{ld}^{(\dot{a}\dot{b})} = Z_{ld}^{(\dot{b}\dot{c})} = Z_{ld}^{(\dot{c}\dot{a})} = Z_{ld}^{(pm)}$$

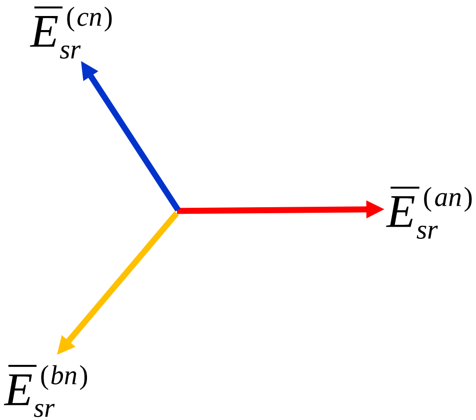
Properties of a Balanced Three-Phase Circuit (Con..)



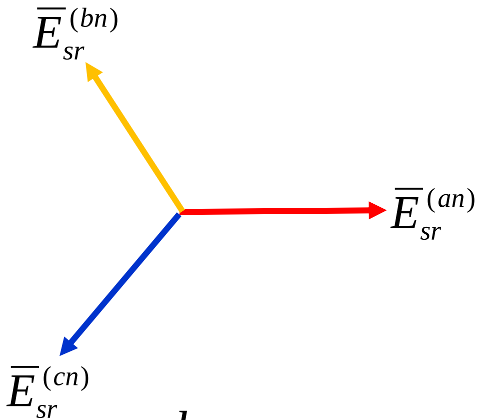
Balancing condition

$$z_{sr}^{(a)} = z_{sr}^{(b)} = z_{sr}^{(c)} = z_{sr}^{(ps)}$$

$$\overline{E}_{sr}^{(cn)} = \overline{E}_{sr}^{(bn)} e^{\mp j\frac{2\pi}{3}} = \overline{E}_{sr}^{(an)} e^{\mp j\frac{4\pi}{3}}$$



abc sequence



acb sequence

Normally, phase voltages/currents follow the abc sequence

Properties of a Balanced Three-Phase Circuit (Con..)

Balanced circuit

1. A three-phase balanced circuit comprises on balanced elements
2. Any three-phase voltage or current vector is found to follow the same phasor pattern as in the previous slide if the circuit is balanced
3. There always exists a single-phase equivalent of a balanced three-phase circuit; therefore, a balanced three-phase circuit can be solved just by solving ordinary scalar equations

Some simple derivations

Consider the following balanced three-phase signal vector

$$\overline{\mathbf{F}}^{(\mathbf{abc})} = \begin{bmatrix} \overline{F}^{(a)} \\ \overline{F}^{(b)} \\ \overline{F}^{(c)} \end{bmatrix}$$

Properties of a Balanced Three-Phase Circuit (Con..)

Sum of the phase quantities:

$$\overline{F}^{(a)} + \overline{F}^{(b)} + \overline{F}^{(c)} = \overline{F}^{(a)}\left\{1 + e^{\mp j\frac{2\pi}{3}} + e^{\mp j\frac{4\pi}{3}}\right\}$$

$$= \overline{F}^{(a)}\left[\left\{1 + \cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{4\pi}{3}\right)\right\} \mp j\left\{\sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right)\right\}\right]$$

$$= 0$$

Difference between two phase quantities:

$$\overline{F}^{(ab)} = \overline{F}^{(a)} - \overline{F}^{(b)}$$

$$= \overline{F}^{(a)}\left(1 - e^{\mp j\frac{2\pi}{3}}\right)$$

$$= \overline{F}^{(a)}\left[\left\{1 - \cos\left(\frac{2\pi}{3}\right)\right\} \pm j\sin\left(\frac{2\pi}{3}\right)\right]$$

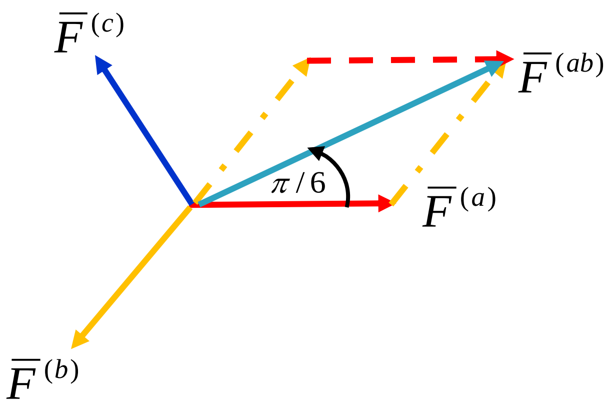
$$= \sqrt{3}\overline{F}^{(a)} e^{\pm j\frac{\pi}{6}}$$

Similarly,

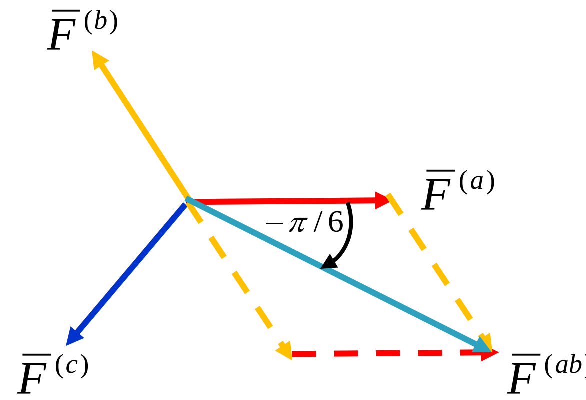
$$\overline{F}^{(bc)} = \sqrt{3}\overline{F}^{(b)} e^{\pm j\frac{\pi}{6}}$$

$$\overline{F}^{(ca)} = \sqrt{3}\overline{F}^{(c)} e^{\pm j\frac{\pi}{6}}$$

Properties of a Balanced Three-Phase Circuit (Con..)



abc sequence



acb sequence

Conclusion:

1. The neutral current of a star connection is zero
2. The neutral-to-ground voltage of a star connection is zero
3. The line-to-line voltage phasor at a node leads the corresponding line voltage phasor by 30 degree in the case of the abc sequence
4. The line-to-line voltage phasor at a node lags the corresponding line voltage phasor by 30 degree in the case of the acb sequence

Properties of a Balanced Three-Phase Circuit (Con..)

5. The line-to-line voltage magnitude at a node is $\sqrt{3}$ times of the line voltage magnitude at that node
6. The phase current of a delta connection leads the corresponding line current by 30 degree in the case of the abc sequence
7. The phase current of a delta connection lags the corresponding line current by 30 degree in the case of the acb sequence
8. The phase current magnitude of delta connection is $1/\sqrt{3}$ times of the corresponding line current magnitude
9. The phase voltage of a star connection is the same as the line voltage at the terminal node
10. The phase current of a star connection is the same as the corresponding line current
11. The phase voltage of a delta connection is the same as the line-to-line voltage at the terminal node

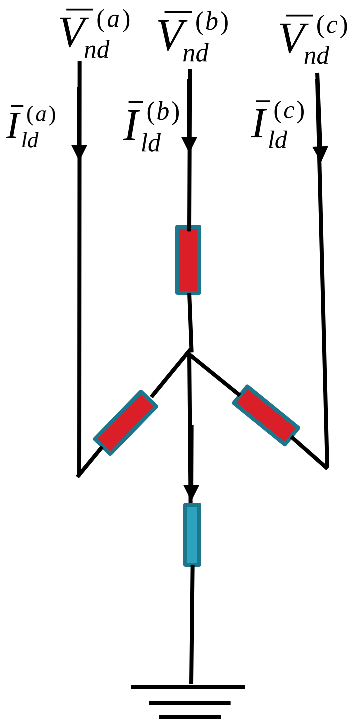
The green highlighted statements are also applicable to an unbalanced circuit

Properties of a Balanced Three-Phase Circuit (Con..)

Three-phase instantaneous power input/output of an element

For the sake of simplicity, we will consider only the abc sequence here

Star connection



Let,

$$v_{nd}^{(a)} = \sqrt{2}V_{nd}\cos(\omega t) \qquad i_{ld}^{(a)} = \sqrt{2}I_{ld}\cos(\omega t + \delta)$$

$$v_{nd}^{(b)} = \sqrt{2}V_{nd}\cos\left(\omega t - \frac{2\pi}{3}\right) \qquad i_{ld}^{(b)} = \sqrt{2}I_{ld}\cos\left(\omega t + \delta - \frac{2\pi}{3}\right)$$

$$v_{nd}^{(c)} = \sqrt{2}V_{nd}\cos\left(\omega t + \frac{2\pi}{3}\right) \qquad i_{ld}^{(c)} = \sqrt{2}I_{ld}\cos\left(\omega t + \delta + \frac{2\pi}{3}\right)$$

Properties of a Balanced Three-Phase Circuit (Con..)

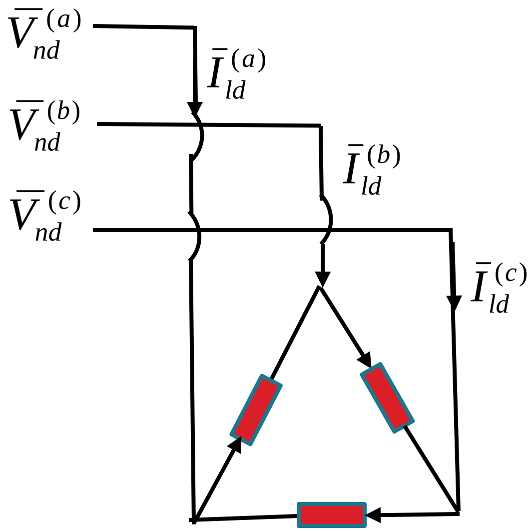
The instantaneous power consumed by the load is given by,

$$
\begin{aligned}
p_{ins} &= v_{nd}^{(a)} i_{ld}^{(a)} + v_{nd}^{(b)} i_{ld}^{(b)} + v_{nd}^{(c)} i_{ld}^{(c)} \\
&= 2V_{nd} I_{ld} \left\{ \cos(\omega t)\cos(\omega t + \delta) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cos\left(\omega t + \delta - \frac{2\pi}{3}\right) + \cos\left(\omega t + \frac{2\pi}{3}\right)\cos\left(\omega t + \delta + \frac{2\pi}{3}\right) \right\} \\
&= 2V_{nd} I_{ld} \left[\frac{1}{2}\{\cos\delta + \cos(2\omega t + \delta)\} + \frac{1}{2}\left\{\cos\delta + \cos\left(2\omega t + \delta - \frac{4\pi}{3}\right)\right\} + \frac{1}{2}\left\{\cos\delta + \cos\left(2\omega t + \delta + \frac{4\pi}{3}\right)\right\} \right] \\
&= 3V_{nd} I_{ld} \cos\delta + V_{nd} I_{ld} \left\{ \cos(2\omega t + \delta) + \cos\left(2\omega t + \delta - \frac{4\pi}{3}\right) + \cos\left(2\omega t + \delta + \frac{4\pi}{3}\right) \right\} \\
&= 3V_{nd} I_{ld} \cos\delta + V_{nd} I_{ld} \left\{ \cos(2\omega t + \delta) + \cos\left(2\omega t + \delta + \frac{2\pi}{3}\right) + \cos\left(2\omega t + \delta - \frac{2\pi}{3}\right) \right\} \\
&= 3V_{nd} I_{ld} \cos\delta
\end{aligned}
$$

Therefore, the instantaneous power delivered to a star connected three-phase load is a constant number if the circuit is balanced

Properties of a Balanced Three-Phase Circuit (Con..)

Delta connection



$$p_{ins} = \left(v_{nd}^{(a)} - v_{nd}^{(c)}\right) i_{ld}^{(a)} + \left(v_{nd}^{(b)} - v_{nd}^{(c)}\right) i_{ld}^{(b)}$$
$$= v_{nd}^{(a)} i_{ld}^{(a)} + v_{nd}^{(b)} i_{ld}^{(b)} - v_{nd}^{(c)} \left(i_{ld}^{(a)} + i_{ld}^{(b)}\right)$$

According to the KCL,

$$i_{ld}^{(a)} + i_{ld}^{(b)} + i_{ld}^{(c)} = 0$$
$$\Rightarrow i_{ld}^{(a)} + i_{ld}^{(b)} = -i_{ld}^{(c)}$$

Thus,

$$p_{ins} = v_{nd}^{(a)} i_{ld}^{(a)} + v_{nd}^{(b)} i_{ld}^{(b)} - v_{nd}^{(c)} \left(-i_{ld}^{(c)}\right)$$
$$= v_{nd}^{(a)} i_{ld}^{(a)} + v_{nd}^{(b)} i_{ld}^{(b)} + v_{nd}^{(c)} i_{ld}^{(c)}$$
$$= 3 V_{nd} I_{ld} \cos\delta$$

So we observe the same result as for the star connection

Three-phase Transformer Connections

Code name of a three-phase transformer connection

HV side winding connection	LV side winding connection	Phase shift caused by the transformer

HV winding connection code:

D: The HV winding is delta connected
Y: The HV winding is star connected with a floating neutral
YN: The HV winding is star connected with a grounded neutral
Z: The HV winding is zig-zag star connected with a floating neutral
ZN: The HV winding is zig-zag star connected with a grounded neutral

LV winding connection code:

d: The LV winding is delta connected
y: The LV winding is star connected with a floating neutral
yn: The LV winding is star connected with a grounded neutral
z: The LV winding is zig-zag star connected with a floating neutral
zn: The LV winding is zig-zag star connected with a grounded neutral

Three-phase Transformer Connections (Con..)

Phase shift code:

Let the LV line-to-line voltage lags the HV line-to-line voltage by ϕ degree in the ideal case

$$\text{Phase shift code} = \frac{\phi}{30}$$

Example:

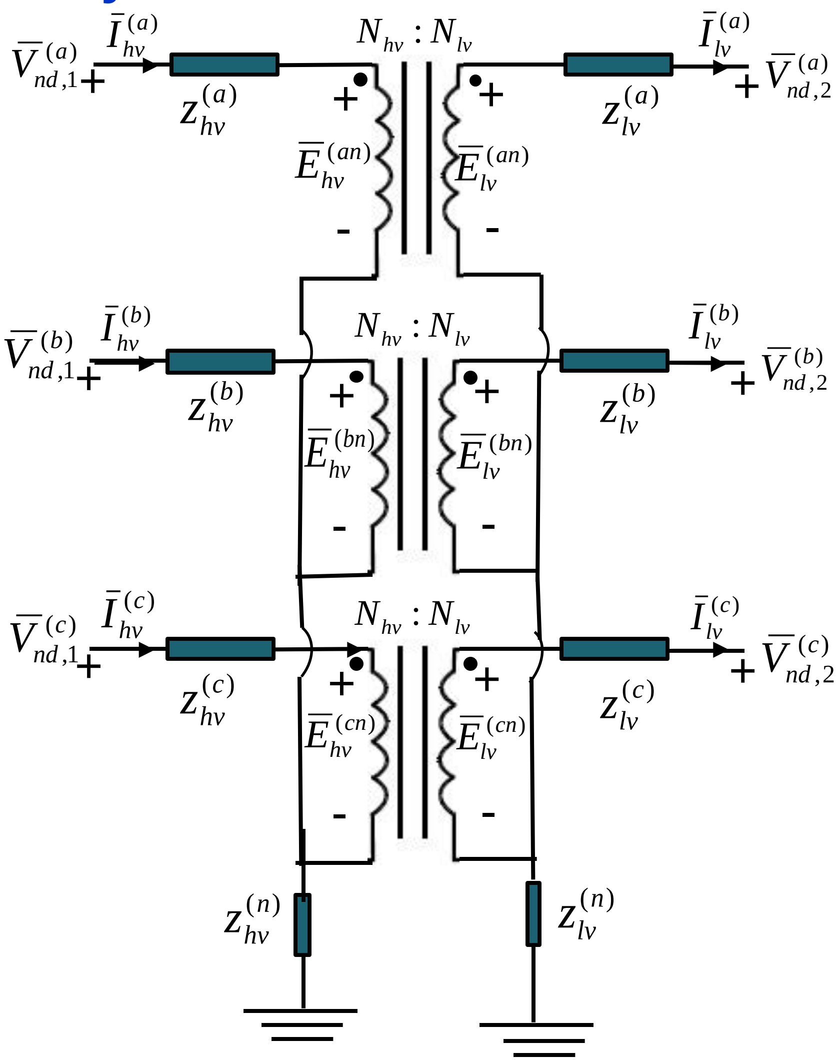
$$\boxed{\text{YN-d-11}}$$

1. The HV winding is star connected
2. The HV winding neutral is grounded
3. The LV winding is delta connected
4. The LV line-to-line voltage lags the HV line-to-line voltage by 330 degree (i.e., leads by 30 degree)

If there is a tertiary winding, two more code blocks are required at the end for indicating the type of the tertiary winding connection and the phase shift from the HV winding to the tertiary winding

Three-phase Transformer Connections (Con..)

YN-yn-0 connection



If the circuit is balanced and transformer impedances are zero, we get the following relations

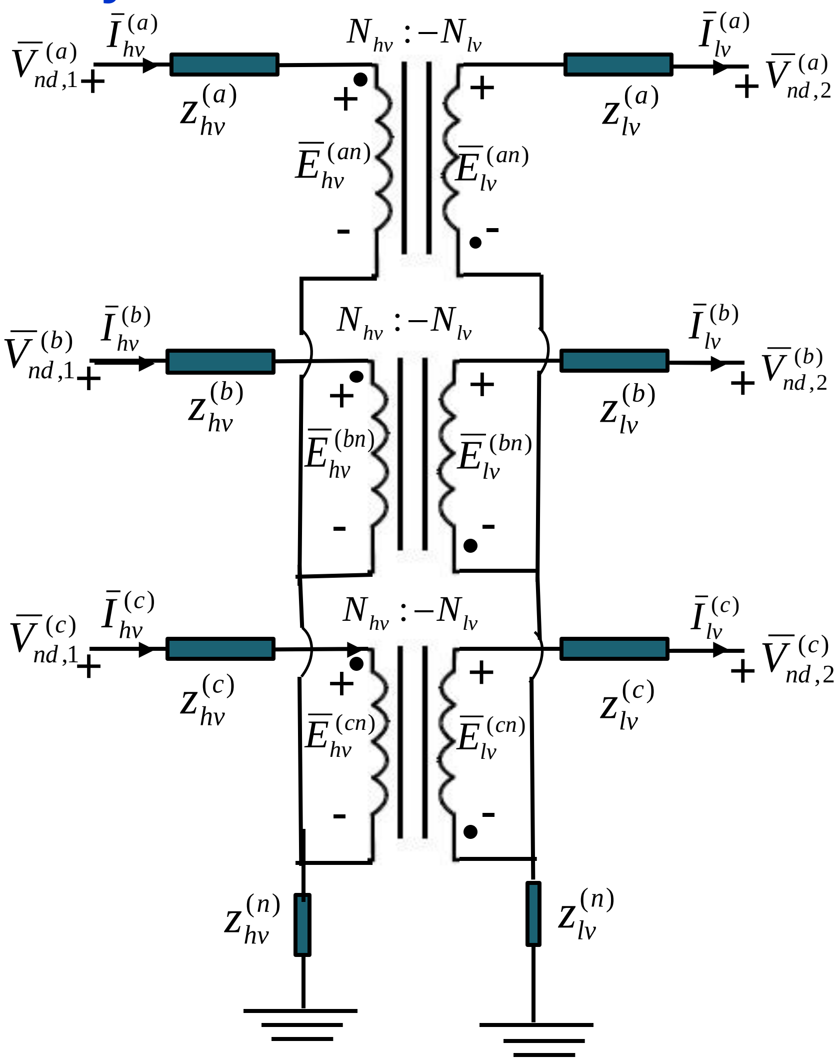
$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = \frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

YN-yn-6 connection



If the circuit is balanced and transformer is ideal,

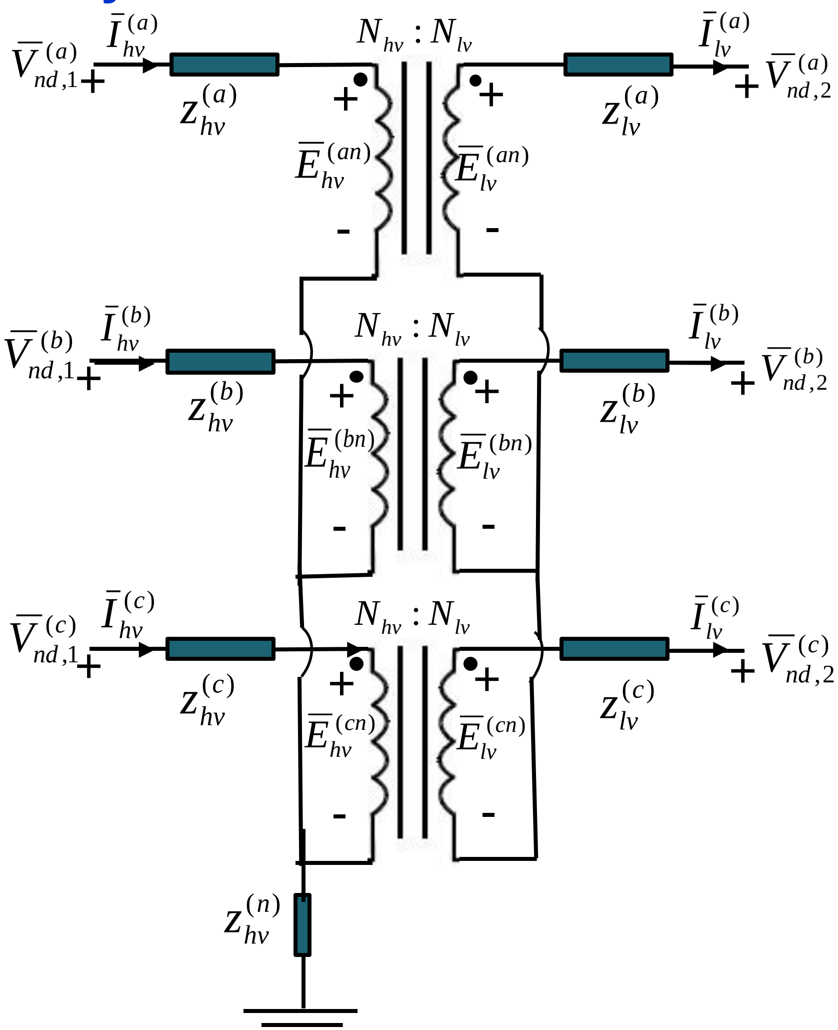
$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = -\frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

YN-y-0 connection



If the circuit is balanced and transformer is ideal,

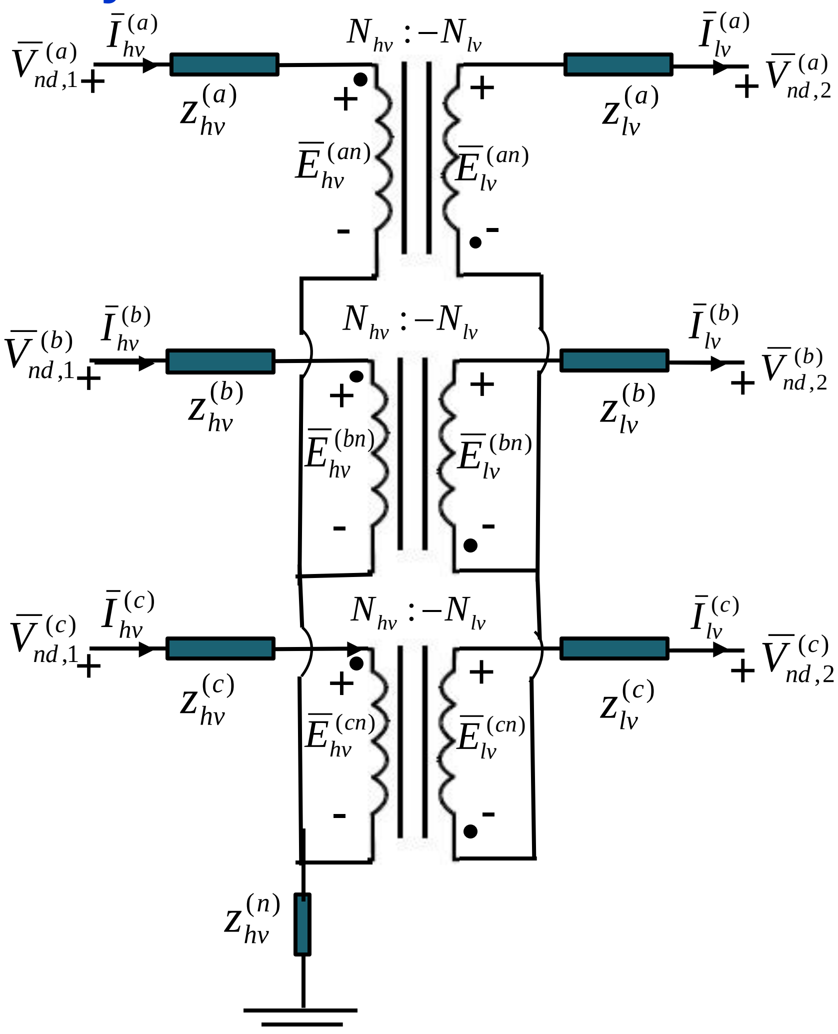
$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = \frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

YN-y-6 connection



If the circuit is balanced and transformer is ideal,

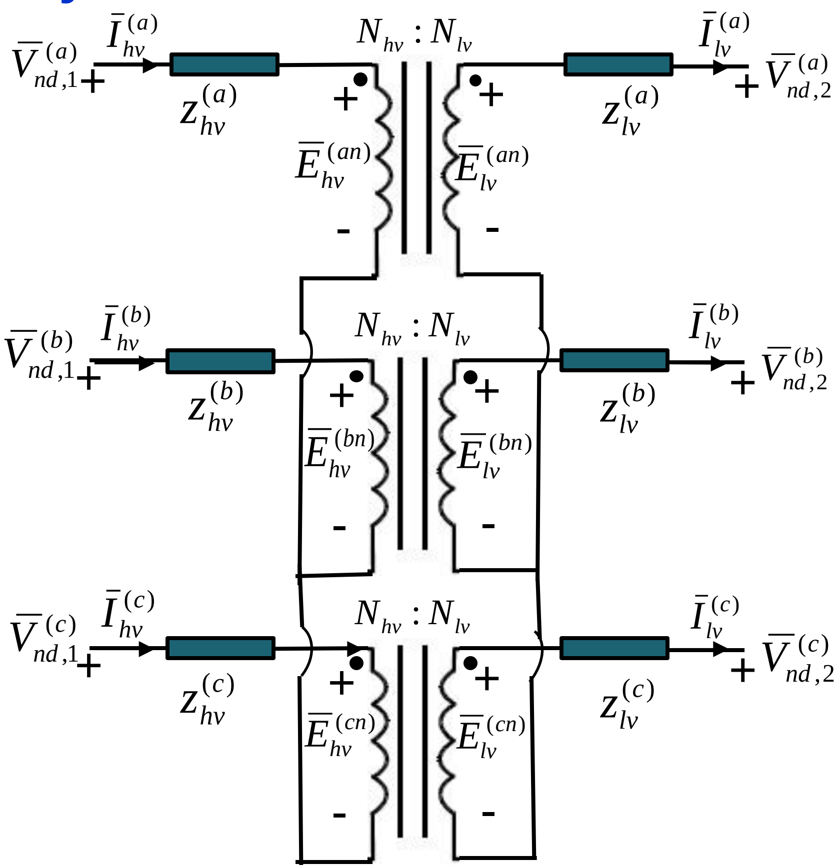
$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = -\frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

Y-y-0 connection



If the circuit is balanced and transformer is ideal,

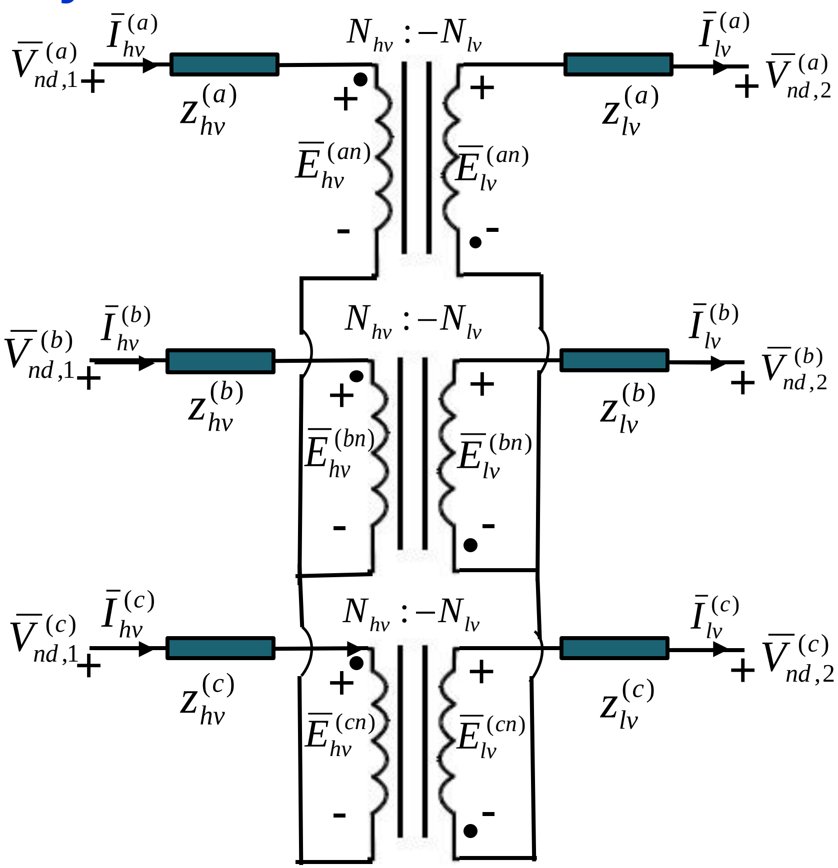
$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = \frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

Y-y-6 connection



If the circuit is balanced and transformer is ideal,

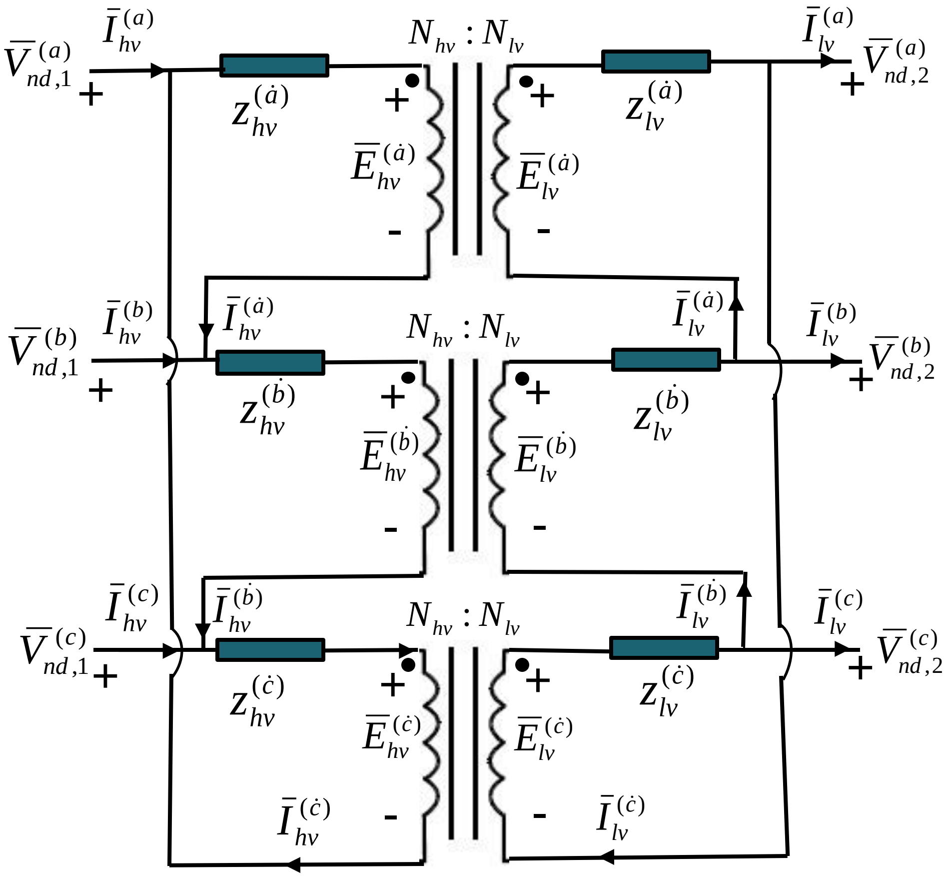
$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = -\frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

D-d-0 connection



If the circuit is balanced and transformer is ideal,

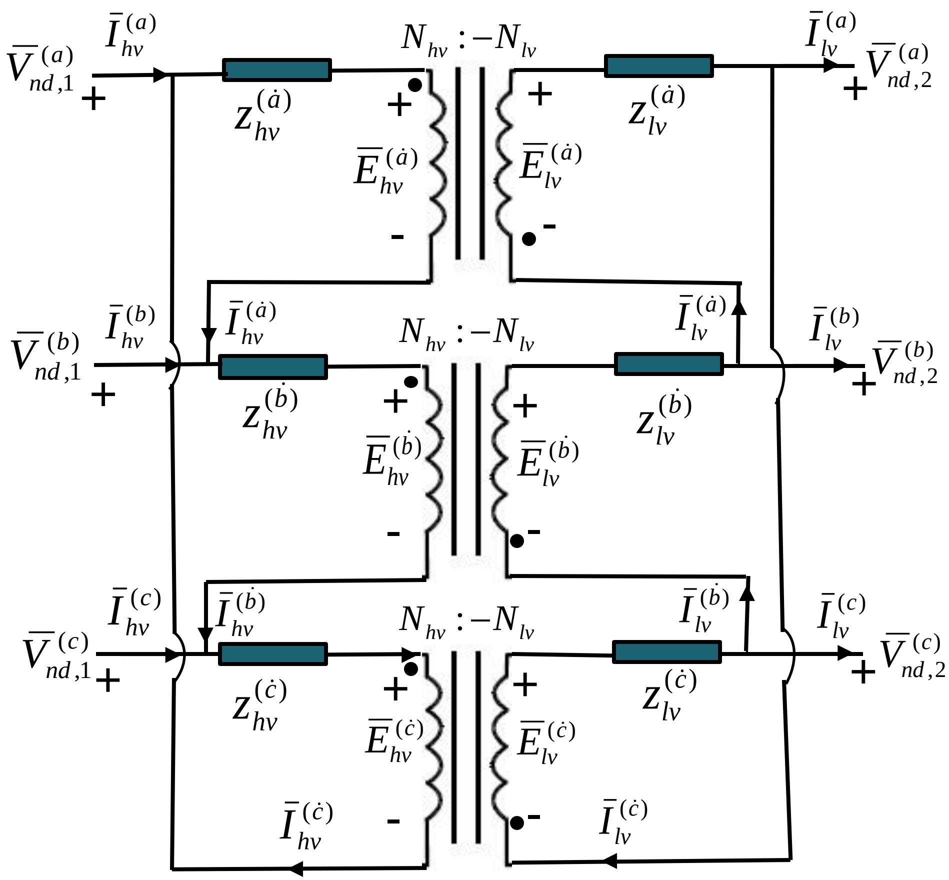
$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = \frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

D-d-6 connection



If the circuit is balanced and transformer is ideal,

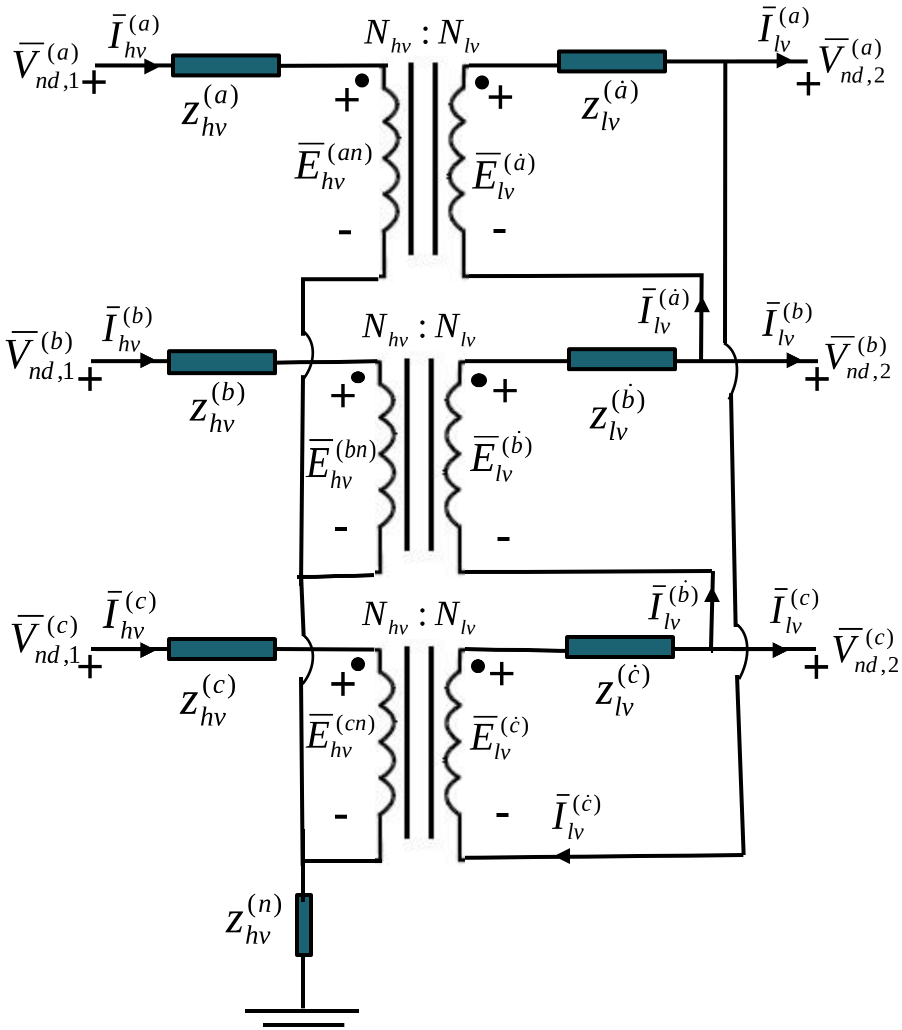
$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = -\frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = -\frac{N_{hv}}{N_{lv}}$$

Three-phase Transformer Connections (Con..)

YN-d-1 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

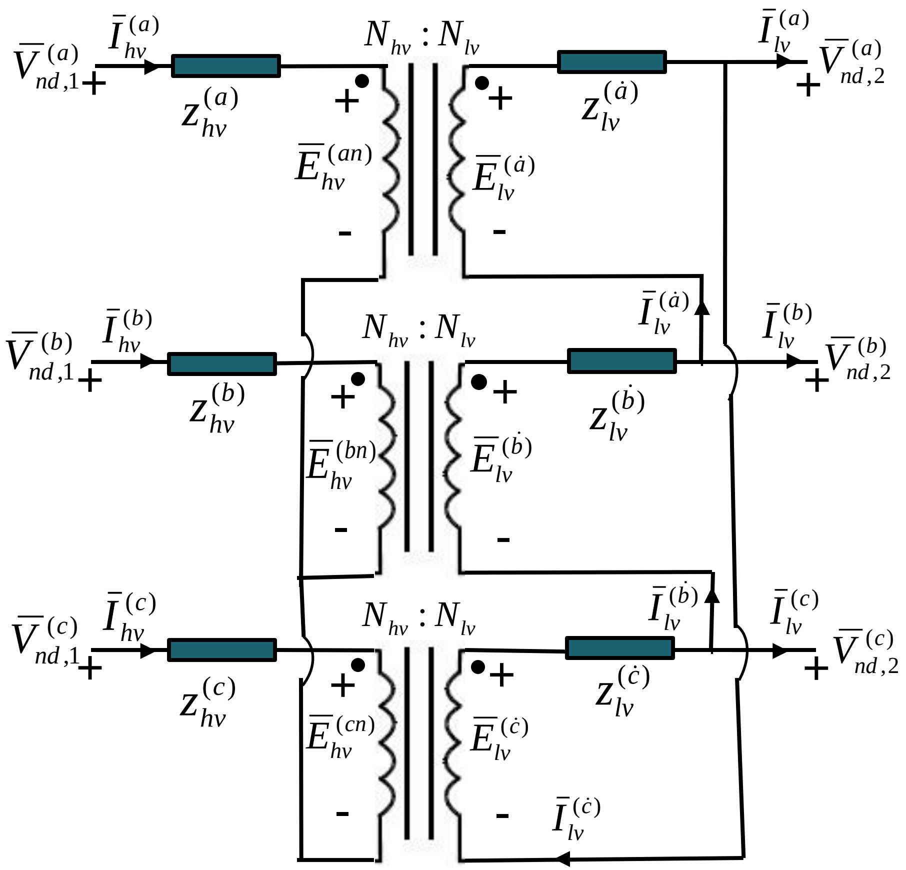
$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{6}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{6}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{-j\frac{\pi}{6}}$$

Three-phase Transformer Connections (Con..)

Y-d-1 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\overline{V}^{(ab)}_{nd,2}}{\overline{V}^{(a)}_{nd,1}} = \frac{\overline{V}^{(bc)}_{nd,2}}{\overline{V}^{(b)}_{nd,1}} = \frac{\overline{V}^{(ca)}_{nd,2}}{\overline{V}^{(c)}_{nd,1}} = \frac{N_{lv}}{N_{hv}}$$

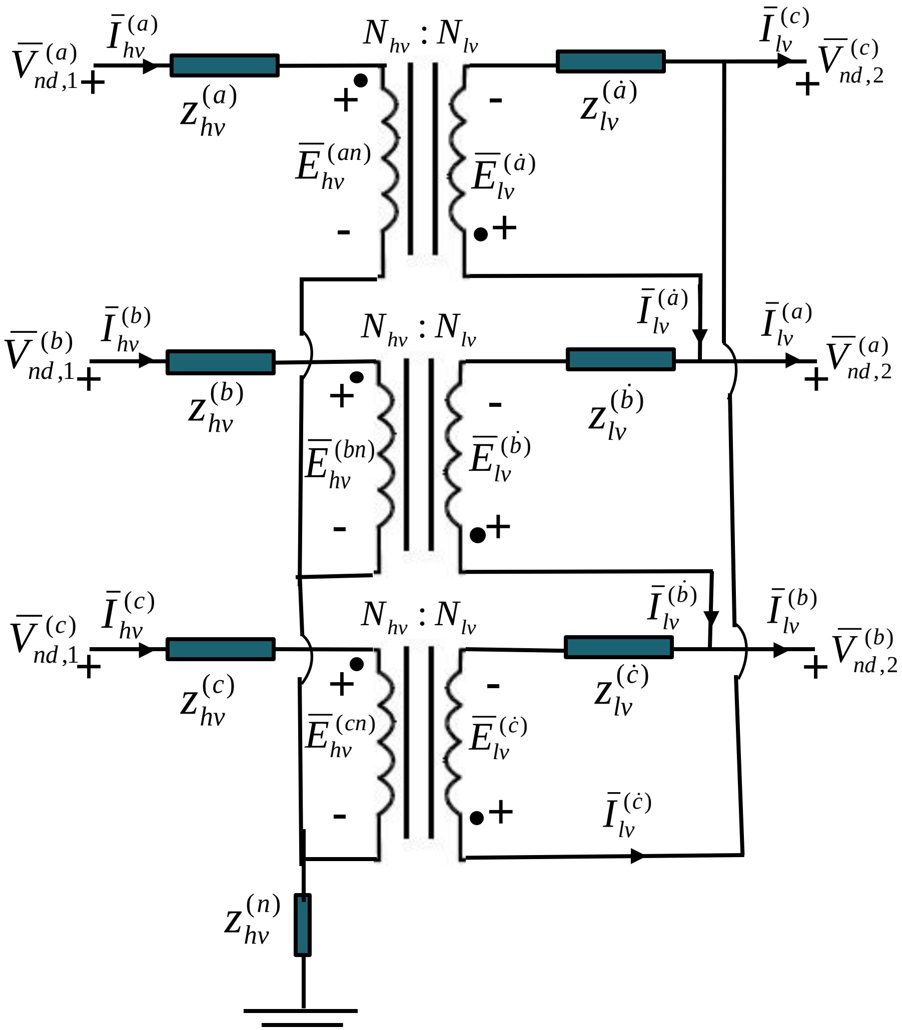
$$\frac{\overline{V}^{(a)}_{nd,2}}{\overline{V}^{(a)}_{nd,1}} = \frac{\overline{V}^{(b)}_{nd,2}}{\overline{V}^{(b)}_{nd,1}} = \frac{\overline{V}^{(c)}_{nd,2}}{\overline{V}^{(c)}_{nd,1}} = \frac{1}{\sqrt{3}}\frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{6}}$$

$$\frac{\overline{V}^{(ab)}_{nd,2}}{\overline{V}^{(ab)}_{nd,1}} = \frac{\overline{V}^{(bc)}_{nd,2}}{\overline{V}^{(bc)}_{nd,1}} = \frac{\overline{V}^{(ca)}_{nd,2}}{\overline{V}^{(ca)}_{nd,1}} = \frac{1}{\sqrt{3}}\frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{6}}$$

$$\frac{\overline{I}^{(a)}_{lv}}{\overline{I}^{(a)}_{hv}} = \frac{\overline{I}^{(b)}_{lv}}{\overline{I}^{(b)}_{hv}} = \frac{\overline{I}^{(c)}_{lv}}{\overline{I}^{(c)}_{hv}} = \sqrt{3}\frac{N_{hv}}{N_{lv}} e^{-j\frac{\pi}{6}}$$

Three-phase Transformer Connections (Con..)

YN-d-11 connection



If the circuit is balanced and transformer is ideal,

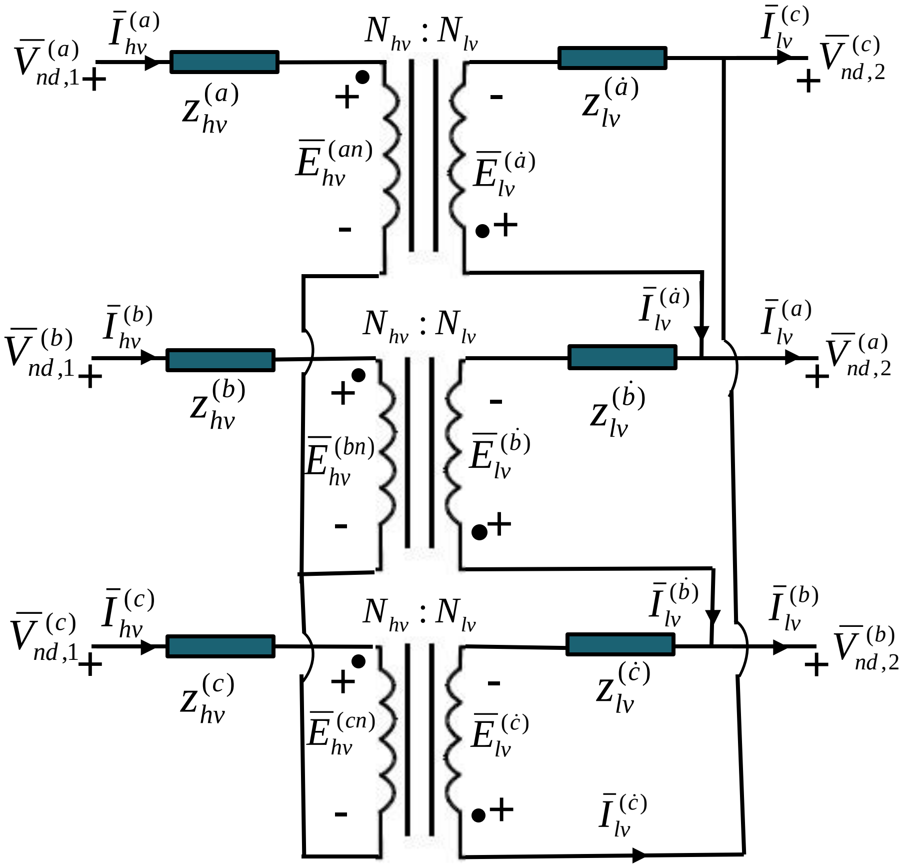
$$\frac{\overline{V}_{nd,2}^{(ac)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(ba)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(cb)}}{\overline{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

$$\frac{\overline{V}_{nd,2}^{(a)}}{\overline{V}_{nd,1}^{(a)}} = \frac{\overline{V}_{nd,2}^{(b)}}{\overline{V}_{nd,1}^{(b)}} = \frac{\overline{V}_{nd,2}^{(c)}}{\overline{V}_{nd,1}^{(c)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{6}}$$

$$\frac{\overline{V}_{nd,2}^{(ab)}}{\overline{V}_{nd,1}^{(ab)}} = \frac{\overline{V}_{nd,2}^{(bc)}}{\overline{V}_{nd,1}^{(bc)}} = \frac{\overline{V}_{nd,2}^{(ca)}}{\overline{V}_{nd,1}^{(ca)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{6}}$$

$$\frac{\overline{I}_{lv}^{(a)}}{\overline{I}_{hv}^{(a)}} = \frac{\overline{I}_{lv}^{(b)}}{\overline{I}_{hv}^{(b)}} = \frac{\overline{I}_{lv}^{(c)}}{\overline{I}_{hv}^{(c)}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{j\frac{\pi}{6}}$$

Y-d-11 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\bar{V}_{nd,2}^{(ac)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(ba)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(cb)}}{\bar{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}}$$

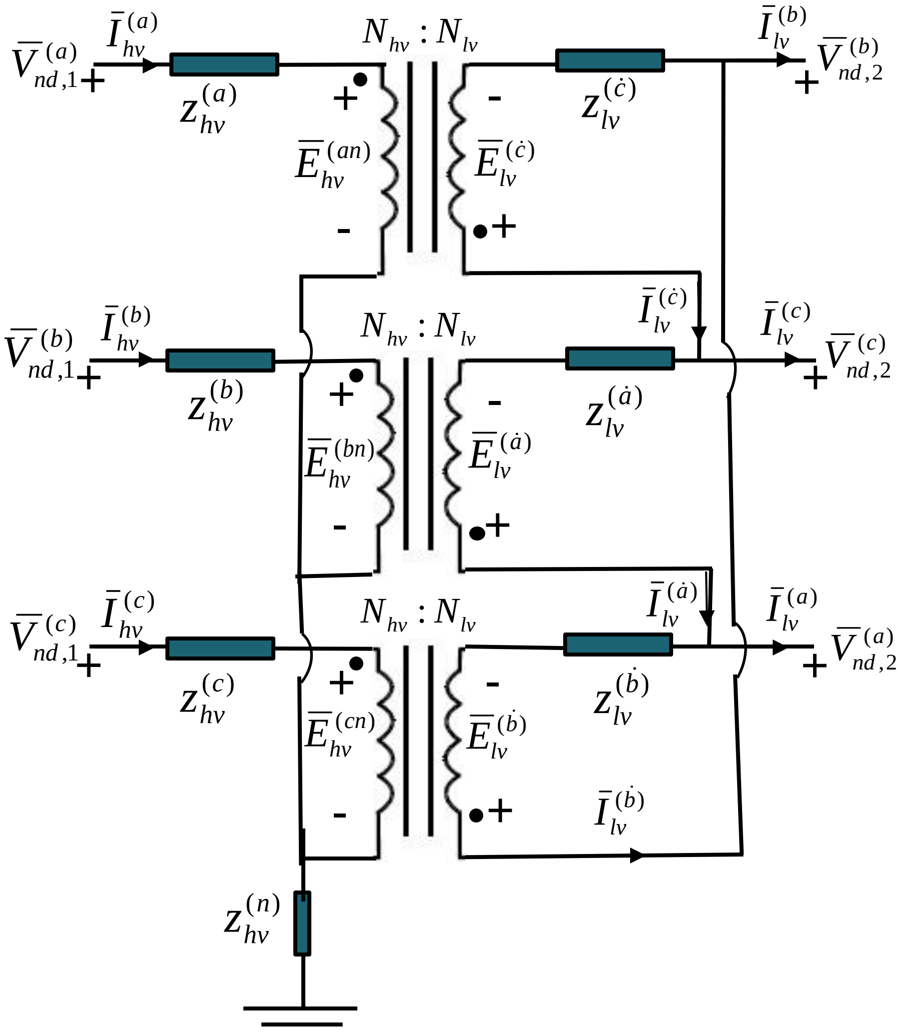
$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{6}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{6}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{j\frac{\pi}{6}}$$

Three-phase Transformer Connections (Con..)

YN-d-3 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\bar{V}_{nd,2}^{(ac)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(ba)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(cb)}}{\bar{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}} e^{-j\frac{2\pi}{3}}$$

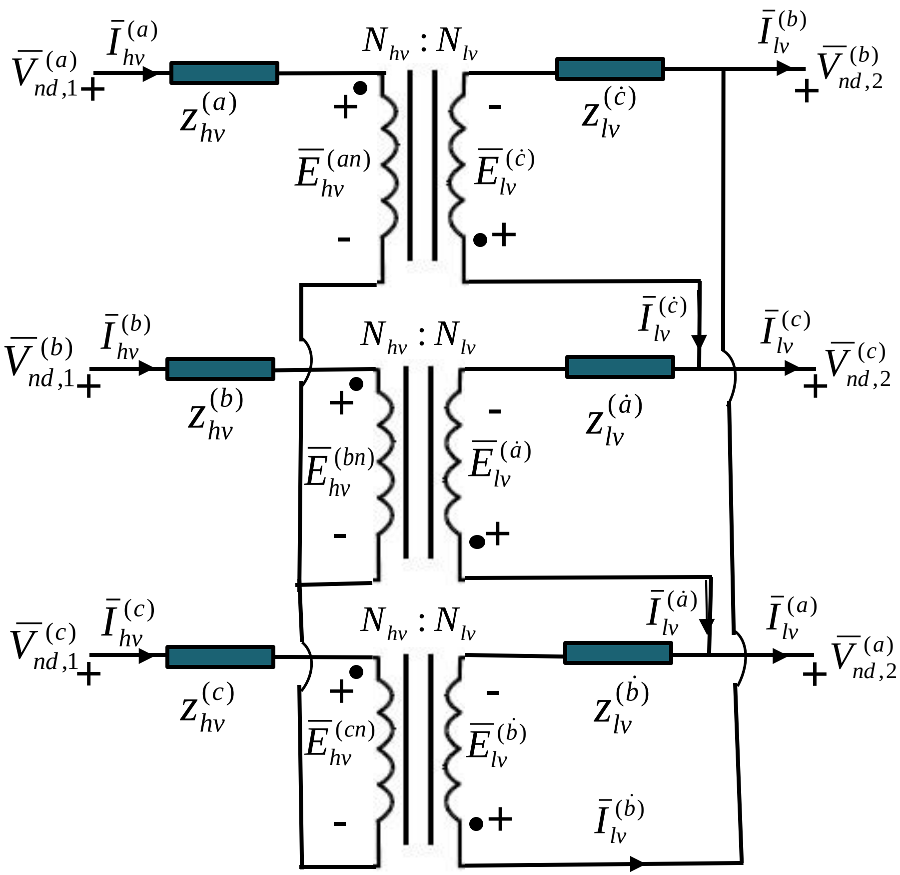
$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{2}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{2}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{-j\frac{\pi}{2}}$$

Three-phase Transformer Connections (Con..)

Y-d-3 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\overline{V}^{(ac)}_{nd,2}}{\overline{V}^{(a)}_{nd,1}} = \frac{\overline{V}^{(ba)}_{nd,2}}{\overline{V}^{(b)}_{nd,1}} = \frac{\overline{V}^{(cb)}_{nd,2}}{\overline{V}^{(c)}_{nd,1}} = \frac{N_{lv}}{N_{hv}} e^{-j\frac{2\pi}{3}}$$

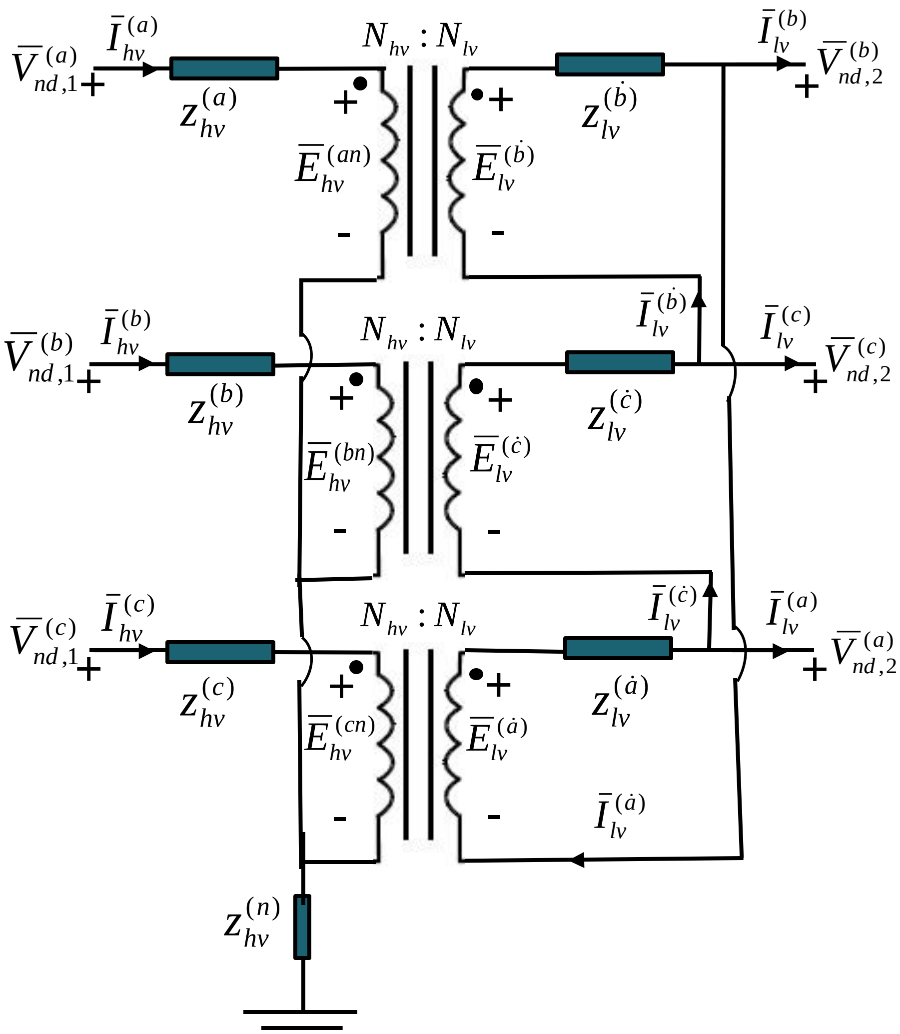
$$\frac{\overline{V}^{(a)}_{nd,2}}{\overline{V}^{(a)}_{nd,1}} = \frac{\overline{V}^{(b)}_{nd,2}}{\overline{V}^{(b)}_{nd,1}} = \frac{\overline{V}^{(c)}_{nd,2}}{\overline{V}^{(c)}_{nd,1}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{2}}$$

$$\frac{\overline{V}^{(ab)}_{nd,2}}{\overline{V}^{(ab)}_{nd,1}} = \frac{\overline{V}^{(bc)}_{nd,2}}{\overline{V}^{(bc)}_{nd,1}} = \frac{\overline{V}^{(ca)}_{nd,2}}{\overline{V}^{(ca)}_{nd,1}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{-j\frac{\pi}{2}}$$

$$\frac{\overline{I}^{(a)}_{lv}}{\overline{I}^{(a)}_{hv}} = \frac{\overline{I}^{(b)}_{lv}}{\overline{I}^{(b)}_{hv}} = \frac{\overline{I}^{(c)}_{lv}}{\overline{I}^{(c)}_{hv}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{-j\frac{\pi}{2}}$$

Three-phase Transformer Connections (Con..)

YN-d-9 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}} e^{j\frac{2\pi}{3}}$$

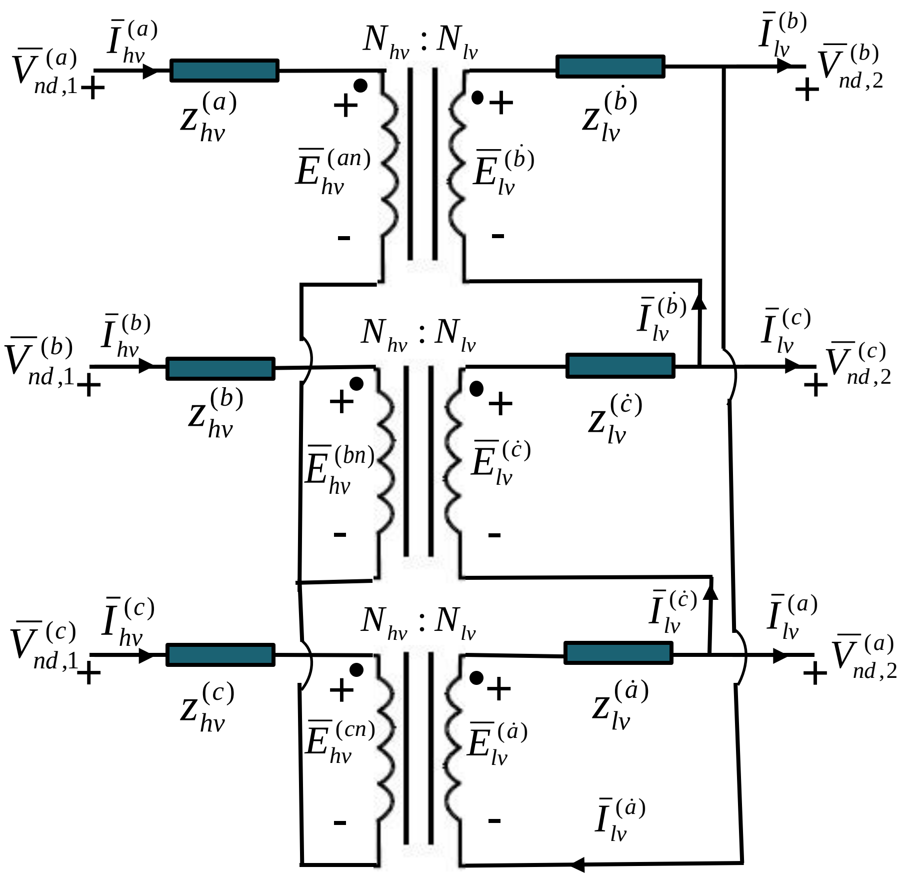
$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{2}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{2}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{j\frac{\pi}{2}}$$

Three-phase Transformer Connections (Con..)

Y-d-9 connection



If the circuit is balanced and transformer is ideal,

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(c)}} = \frac{N_{lv}}{N_{hv}} e^{j\frac{2\pi}{3}}$$

$$\frac{\bar{V}_{nd,2}^{(a)}}{\bar{V}_{nd,1}^{(a)}} = \frac{\bar{V}_{nd,2}^{(b)}}{\bar{V}_{nd,1}^{(b)}} = \frac{\bar{V}_{nd,2}^{(c)}}{\bar{V}_{nd,1}^{(c)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{2}}$$

$$\frac{\bar{V}_{nd,2}^{(ab)}}{\bar{V}_{nd,1}^{(ab)}} = \frac{\bar{V}_{nd,2}^{(bc)}}{\bar{V}_{nd,1}^{(bc)}} = \frac{\bar{V}_{nd,2}^{(ca)}}{\bar{V}_{nd,1}^{(ca)}} = \frac{1}{\sqrt{3}} \frac{N_{lv}}{N_{hv}} e^{j\frac{\pi}{2}}$$

$$\frac{\bar{I}_{lv}^{(a)}}{\bar{I}_{hv}^{(a)}} = \frac{\bar{I}_{lv}^{(b)}}{\bar{I}_{hv}^{(b)}} = \frac{\bar{I}_{lv}^{(c)}}{\bar{I}_{hv}^{(c)}} = \sqrt{3} \frac{N_{hv}}{N_{lv}} e^{j\frac{\pi}{2}}$$

Principle of Symmetrical Transformation

Objective

1. To derive the single-phase equivalent of a balanced three-phase circuit
2. To simplify the modeling and analysis of an unbalanced system with the unbalance being restricted only to shunt elements
3. To develop the control techniques for an unbalanced system

Concept of symmetrical components

It is possible to decompose any three-phase signal vector in the following form

$$\overline{\mathbf{F}}^{(\mathbf{abc})} = \overline{\mathbf{F}}_{\mathbf{0}}^{(\mathbf{abc})} + \overline{\mathbf{F}}_{1}^{(\mathbf{abc})} + \overline{\mathbf{F}}_{2}^{(\mathbf{abc})}$$

where,

$$\left.\overline{F}_0^{(c)} = \overline{F}_0^{(b)} = \overline{F}_0^{(a)}\right\} \text{Zero sequence componenets}$$

$$\left.\overline{F}_1^{(c)} = \overline{F}_1^{(b)} e^{-j\frac{2\pi}{3}} = \overline{F}_1^{(a)} e^{-j\frac{4\pi}{3}}\right\} \text{Positive sequence componenets}$$

$$\left.\overline{F}_1^{(c)} = \overline{F}_1^{(b)} e^{j\frac{2\pi}{3}} = \overline{F}_1^{(a)} e^{j\frac{4\pi}{3}}\right\} \text{Negative sequence componenets}$$

Principle of Symmetrical Transformation (Con..)

Example:

$$\underbrace{\begin{bmatrix} 150e^{-j\left(\frac{\pi}{12}\right)} \\ 55e^{-j\left(\frac{\pi}{2}\right)} \\ 180e^{j\left(\frac{2\pi}{3}\right)} \end{bmatrix}}_{\overline{\mathbf{F}}^{(abc)}} = \underbrace{27.62e^{j0.847}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\overline{\mathbf{F}}_0^{(abc)}} \underbrace{+124.23e^{-j0.03}\begin{bmatrix} 1 \\ e^{-j\left(\frac{2\pi}{3}\right)} \\ e^{j\left(\frac{2\pi}{3}\right)} \end{bmatrix}}_{\overline{\mathbf{F}}_1^{(abc)}} \underbrace{+55.79e^{-j1.527}\begin{bmatrix} 1 \\ e^{j\left(\frac{2\pi}{3}\right)} \\ e^{-j\left(\frac{2\pi}{3}\right)} \end{bmatrix}}_{\overline{\mathbf{F}}_2^{(abc)}}$$

In the time domain,

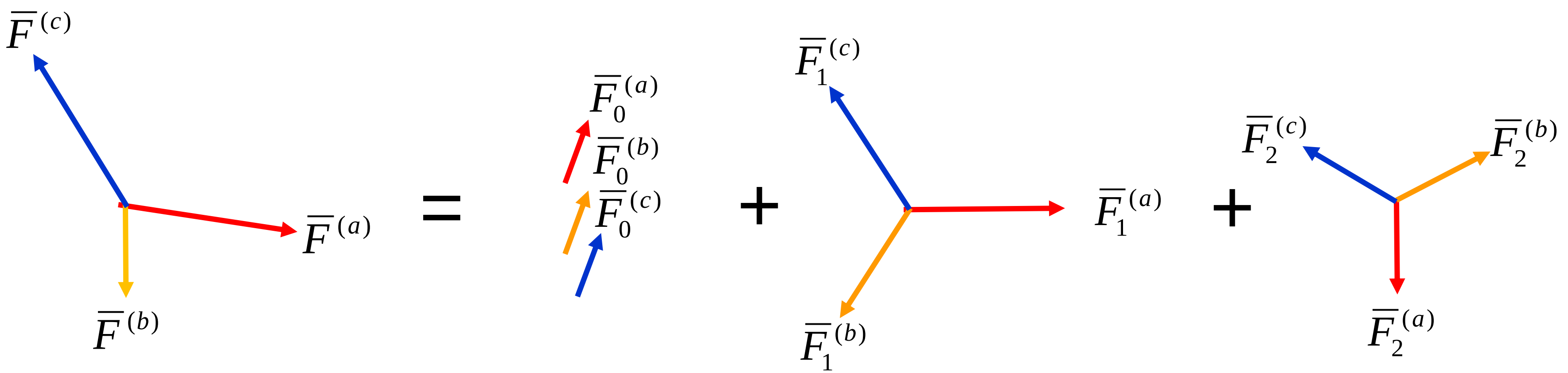
$$150\sqrt{2}\cos\left(\omega t - \frac{\pi}{12}\right) = 27.62\sqrt{2}\cos(\omega t + 0.847) + 124.23\sqrt{2}\cos(\omega t - 0.03) + 55.79\sqrt{2}\cos(\omega t - 1.527)$$

$$55\sqrt{2}\cos\left(\omega t - \frac{\pi}{2}\right) = 27.62\sqrt{2}\cos(\omega t + 0.847) + 124.23\sqrt{2}\cos\left(\omega t - 0.03 - \frac{2\pi}{3}\right) + 55.79\sqrt{2}\cos\left(\omega t - 1.527 + \frac{2\pi}{3}\right)$$

Principle of Symmetrical Transformation (Con..)

$$180\sqrt{2}\cos\left(\omega t+\frac{2\pi}{3}\right)=27.62\sqrt{2}\cos\left(\omega t+0.847\right)+124.23\sqrt{2}\cos\left(\omega t-0.03+\frac{2\pi}{3}\right)$$
$$+55.79\sqrt{2}\cos\left(\omega t-1.527-\frac{2\pi}{3}\right)$$

The phasor diagram representation of the symmetrical component decomposition is presented below,



Principle of Symmetrical Transformation (Con.)

Derivation of symmetrical components

Let us first define a complex number α as follows

$$\alpha = e^{j\frac{2\pi}{3}} = e^{-j\frac{4\pi}{3}}$$

$$\Rightarrow \alpha^2 = e^{j\frac{4\pi}{3}} = e^{-j\frac{2\pi}{3}}$$

From the definition of symmetrical components

$$\overline{F}_0^{(b)} = \overline{F}_0^{(a)} \qquad \overline{F}_1^{(b)} = \alpha^2 \overline{F}_1^{(a)} \qquad \overline{F}_2^{(b)} = \alpha \overline{F}_2^{(a)}$$

$$\overline{F}_0^{(c)} = \overline{F}_0^{(a)} \qquad \overline{F}_1^{(c)} = \alpha \overline{F}_1^{(a)} \qquad \overline{F}_2^{(c)} = \alpha^2 \overline{F}_2^{(a)}$$

Therefore,

$$F^{(a)} = \overline{F}_0^{(a)} + \overline{F}_1^{(a)} + \overline{F}_2^{(a)}$$

$$F^{(b)} = \overline{F}_0^{(b)} + \overline{F}_1^{(b)} + \overline{F}_2^{(b)} = \overline{F}_0^{(a)} + \alpha^2 \overline{F}_1^{(a)} + \alpha \overline{F}_2^{(a)}$$

$$F^{(c)} = \overline{F}_0^{(c)} + \overline{F}_1^{(c)} + \overline{F}_2^{(c)} = \overline{F}_0^{(a)} + \alpha \overline{F}_1^{(a)} + \alpha^2 \overline{F}_2^{(a)}$$

Principle of Symmetrical Transformation (Con.)

In the matrix-vector form,

$$\begin{bmatrix} F^{(a)} \\ F^{(b)} \\ F^{(c)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} \overline{F}_0^{(a)} \\ \overline{F}_1^{(a)} \\ \overline{F}_2^{(a)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \overline{F}_0^{(a)} \\ \overline{F}_1^{(a)} \\ \overline{F}_2^{(a)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^{-1} \begin{bmatrix} F^{(a)} \\ F^{(b)} \\ F^{(c)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} F^{(a)} \\ F^{(b)} \\ F^{(c)} \end{bmatrix}$$

Once the symmetrical/sequence components of the a-phase quantity are obtained, the symmetrical/sequence components of other phase quantities can be obtained simply by using the relations in Slide 59

From now onwards, we will concentrate only on the symmetrical/sequence components of the a-phase quantity with the following revised notations

$$\boxed{\overline{F}^{(0)} \Rightarrow \overline{F}_0^{(a)} \qquad \overline{F}^{(1)} \Rightarrow \overline{F}_1^{(a)} \qquad \overline{F}^{(2)} \Rightarrow \overline{F}_2^{(a)}}$$

Principle of Symmetrical Transformation (Con.)

The power relationship

3-phase active power

3-phase reactive power

$$P_{3\varphi} + jQ_{3\varphi} = \overline{V}^{(a)}\overline{I}^{(a)*} + \overline{V}^{(b)}\overline{I}^{(b)*} + \overline{V}^{(c)}\overline{I}^{(c)*}$$

$$= \left\{\overline{\mathbf{V}}^{(\mathbf{abc})}\right\}^T \overline{\mathbf{I}}^{(\mathbf{abc})*}$$

$$= \left\{\mathbf{C}\overline{\mathbf{V}}^{(012)}\right\}^T \mathbf{C}^*\overline{\mathbf{I}}^{(012)*}$$

$$= \left\{\overline{\mathbf{V}}^{(012)}\right\}^T \mathbf{C}^T\mathbf{C}^*\overline{\mathbf{I}}^{(012)*}$$

The complex number α shows the following property since it is essentially a cube root of 1

$$\alpha^3 = e^{j\frac{6\pi}{3}} = e^{j2\pi} = 1$$

Therefore,

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \Rightarrow \mathbf{C}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} = 3\mathbf{C}^{-1} \Rightarrow \mathbf{C}\mathbf{C}^* = 3\mathbf{U}$$

Principle of Symmetrical Transformation (Con.)

Finally, we get the following relationships between the phase-domain and sequence-domain power quantities

$$P_{3\varphi} + jQ_{3\varphi} = 3\left\{\overline{\mathbf{V}}^{(012)}\right\}^{T} \overline{\mathbf{I}}^{(012)*} = 3\left(P_{seq} + jQ_{seq}\right)$$

Sequence domain active power

Sequence domain reactive power

Note:

1. Here, the sequence domain power is defined in terms of the sequence components of the a-phase since only those sequence components will be used in the subsequent analyses
2. With the above definition of the sequence domain power, the symmetrical transformation performed here is power variant
3. If the sequence domain power is redefined by considering the contributions of the sequence components of all the phases, the symmetrical transformation appears to be power invariant
4. The modified expression of the sequence domain power by considering the contributions of the sequence components of all the phases appears as is shown in the next slide

Principle of Symmetrical Transformation (Con.)

$$P'_{seq} + jQ'_{seq} = \left\{\overline{\mathbf{V}}^{(\mathbf{a})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{a})*}_{\mathbf{012}} + \left\{\overline{\mathbf{V}}^{(\mathbf{b})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{b})*}_{\mathbf{012}} + \left\{\overline{\mathbf{V}}^{(\mathbf{c})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{c})*}_{\mathbf{012}}$$

Modified sequence domain active power

Modified sequence domain reactive power

It can be easily shown that

$$\left\{\overline{\mathbf{V}}^{(\mathbf{a})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{a})*}_{\mathbf{012}} = \left\{\overline{\mathbf{V}}^{(\mathbf{b})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{b})*}_{\mathbf{012}} = \left\{\overline{\mathbf{V}}^{(\mathbf{c})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{c})*}_{\mathbf{012}}$$

Therefore,

$$P'_{seq} + jQ'_{seq} = 3\left\{\overline{\mathbf{V}}^{(\mathbf{a})}_{\mathbf{012}}\right\}^T \overline{\mathbf{I}}^{(\mathbf{a})*}_{\mathbf{012}} = 3\left\{\overline{\mathbf{V}}^{(\mathbf{012})}\right\}^T \overline{\mathbf{I}}^{(\mathbf{012})*} = P_{3\varphi} + jQ_{3\varphi}$$

Some basic derivations

Derivation 1:

Given that

$$\mathbf{\Gamma^{(abc)}} = \begin{bmatrix} \Gamma^{(s)} & \Gamma^{(m)} & \Gamma^{(m)} \\ \Gamma^{(m)} & \Gamma^{(s)} & \Gamma^{(m)} \\ \Gamma^{(m)} & \Gamma^{(m)} & \Gamma^{(s)} \end{bmatrix}$$

We need to evaluate $\mathbf{C}^{-1}\mathbf{\Gamma^{(abc)}}\mathbf{C}$

Solution:

$$\mathbf{\Gamma^{(abc)}C} = \begin{bmatrix} \Gamma^{(s)} & \Gamma^{(m)} & \Gamma^{(m)} \\ \Gamma^{(m)} & \Gamma^{(s)} & \Gamma^{(m)} \\ \Gamma^{(m)} & \Gamma^{(m)} & \Gamma^{(s)} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} = \begin{bmatrix} \Gamma^{(s)} + 2\Gamma^{(m)} & \Gamma^{(s)} + \left(\alpha^2 + \alpha\right)\Gamma^{(m)} & \Gamma^{(s)} + \left(\alpha + \alpha^2\right)\Gamma^{(m)} \\ \Gamma^{(s)} + 2\Gamma^{(m)} & \alpha^2\Gamma^{(s)} + \left(1 + \alpha\right)\Gamma^{(m)} & \alpha\Gamma^{(s)} + \left(1 + \alpha^2\right)\Gamma^{(m)} \\ \Gamma^{(s)} + 2\Gamma^{(m)} & \alpha\Gamma^{(s)} + \left(1 + \alpha^2\right)\Gamma^{(m)} & \alpha^2\Gamma^{(s)} + \left(1 + \alpha\right)\Gamma^{(m)} \end{bmatrix}$$

We know that

$$1+\alpha+\alpha^2=0 \Rightarrow \begin{cases} \alpha+\alpha^2=-1 \\ 1+\alpha=-\alpha^2 \\ 1+\alpha^2=-\alpha \end{cases}$$

Therefore, the expression of $\boldsymbol{\Gamma}^{(\mathbf{abc})}\mathbf{C}$ can be simplified as follows

$$\boldsymbol{\Gamma}^{(\mathbf{abc})}\mathbf{C}=\begin{bmatrix} \Gamma^{(s)}+2\Gamma^{(m)} & \Gamma^{(s)}-\Gamma^{(m)} & \Gamma^{(s)}-\Gamma^{(m)} \\ \Gamma^{(s)}+2\Gamma^{(m)} & \alpha^2\left(\Gamma^{(s)}-\Gamma^{(m)}\right) & \alpha\left(\Gamma^{(s)}-\Gamma^{(m)}\right) \\ \Gamma^{(s)}+2\Gamma^{(m)} & \alpha\left(\Gamma^{(s)}-\Gamma^{(m)}\right) & \alpha^2\left(\Gamma^{(s)}-\Gamma^{(m)}\right) \end{bmatrix}=\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}}_{\mathbf{C}}\begin{bmatrix} \Gamma^{(s)}+2\Gamma^{(m)} & 0 & 0 \\ 0 & \Gamma^{(s)}-\Gamma^{(m)} & 0 \\ 0 & 0 & \Gamma^{(s)}-\Gamma^{(m)} \end{bmatrix}$$

Finally,

$$\mathbf{C}^{-1}\boldsymbol{\Gamma}^{(\mathbf{abc})}\mathbf{C}=\begin{bmatrix} \Gamma^{(s)}+2\Gamma^{(m)} & 0 & 0 \\ 0 & \Gamma^{(s)}-\Gamma^{(m)} & 0 \\ 0 & 0 & \Gamma^{(s)}-\Gamma^{(m)} \end{bmatrix}$$

Derivation 2:

Given that

$$\mathbf{A}_\Delta = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

We need to evaluate $\mathbf{C}^{-1}\mathbf{A}_\Delta\mathbf{C}$

Solution:

$$\mathbf{C}^{-1}\mathbf{A}_\Delta\mathbf{C} = \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 & 1-\alpha^2 & 1-\alpha \\ 0 & \alpha^2-\alpha & \alpha-\alpha^2 \\ 0 & \alpha-1 & \alpha^2-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha^2 & 0 \\ 0 & 0 & 1-\alpha \end{bmatrix}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Derivation 3:

Given that

$$\mathbf{A}_{\Delta} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

We need to evaluate $\mathbf{C}^{-1}\{\mathbf{A}_{\Delta}\}^{T}\mathbf{C}$

Solution:

$$\mathbf{C}^{-1}\{\mathbf{A}_{\Delta}\}^{T}\mathbf{C} = \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 & 1-\alpha & 1-\alpha^2 \\ 0 & \alpha^2-1 & \alpha-1 \\ 0 & \alpha-\alpha^2 & \alpha^2-\alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & 1-\alpha^2 \end{bmatrix}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Derivation 4:

Given that

$$\mathbf{\Theta} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

We need to evaluate $\mathbf{C}^{-1}\mathbf{\Theta}\mathbf{C}$

Solution:

$$\mathbf{C}^{-1}\mathbf{\Theta}\mathbf{C} = \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha^2 \end{bmatrix}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Derivation 5:

Given that

$$\mathbf{\Theta} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

We need to evaluate $\mathbf{C}^{-1}\mathbf{\Theta}^{T}\mathbf{C}$

Solution:

$$\mathbf{C}^{-1}\mathbf{\Theta}^{T}\mathbf{C} = \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

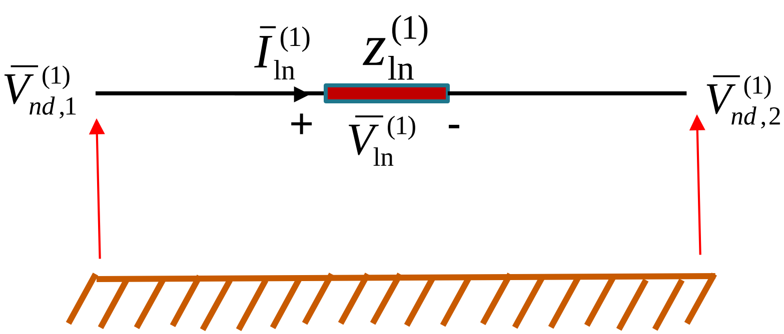
Balanced line

Refer to Slides 11 and 12, and enforce the impedance balance condition as per Slide 29

$$\begin{bmatrix}\overline{V}_{\ln}^{(a)}\\ \overline{V}_{\ln}^{(b)}\\ \overline{V}_{\ln}^{(c)}\end{bmatrix}=\begin{bmatrix}\overline{V}_{nd,1}^{(a)}\\ \overline{V}_{nd,1}^{(b)}\\ \overline{V}_{nd,1}^{(c)}\end{bmatrix}-\begin{bmatrix}\overline{V}_{nd,2}^{(a)}\\ \overline{V}_{nd,2}^{(b)}\\ \overline{V}_{nd,2}^{(c)}\end{bmatrix}=\begin{bmatrix}z_{\ln}^{(ps)} & z_{\ln}^{(pm)} & z_{\ln}^{(pm)}\\ z_{\ln}^{(pm)} & z_{\ln}^{(ps)} & z_{\ln}^{(pm)}\\ z_{\ln}^{(m)} & z_{\ln}^{(m)} & z_{\ln}^{(s)}\end{bmatrix}\begin{bmatrix}\overline{I}_{\ln}^{(a)}\\ \overline{I}_{\ln}^{(b)}\\ \overline{I}_{\ln}^{(c)}\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\overline{V}_{\ln}^{(0)}\\ \overline{V}_{\ln}^{(1)}\\ \overline{V}_{\ln}^{(2)}\end{bmatrix}=\begin{bmatrix}\overline{V}_{nd,1}^{(0)}\\ \overline{V}_{nd,1}^{(1)}\\ \overline{V}_{nd,1}^{(2)}\end{bmatrix}-\begin{bmatrix}\overline{V}_{nd,2}^{(0)}\\ \overline{V}_{nd,2}^{(1)}\\ \overline{V}_{nd,2}^{(2)}\end{bmatrix}=\mathbf{C}^{-1}\begin{bmatrix}z_{\ln}^{(ps)} & z_{\ln}^{(pm)} & z_{\ln}^{(pm)}\\ z_{\ln}^{(pm)} & z_{\ln}^{(ps)} & z_{\ln}^{(pm)}\\ z_{\ln}^{(pm)} & z_{\ln}^{(pm)} & z_{\ln}^{(ps)}\end{bmatrix}\mathbf{C}\begin{bmatrix}\overline{I}_{\ln}^{(0)}\\ \overline{I}_{\ln}^{(1)}\\ \overline{I}_{\ln}^{(2)}\end{bmatrix}=\begin{bmatrix}z_{\ln}^{(ps)}+2z_{\ln}^{(pm)} & 0 & 0\\ 0 & z_{\ln}^{(ps)}-z_{\ln}^{(pm)} & 0\\ 0 & 0 & z_{\ln}^{(ps)}-z_{\ln}^{(pm)}\end{bmatrix}\begin{bmatrix}\overline{I}_{\ln}^{(0)}\\ \overline{I}_{\ln}^{(1)}\\ \overline{I}_{\ln}^{(2)}\end{bmatrix}$$

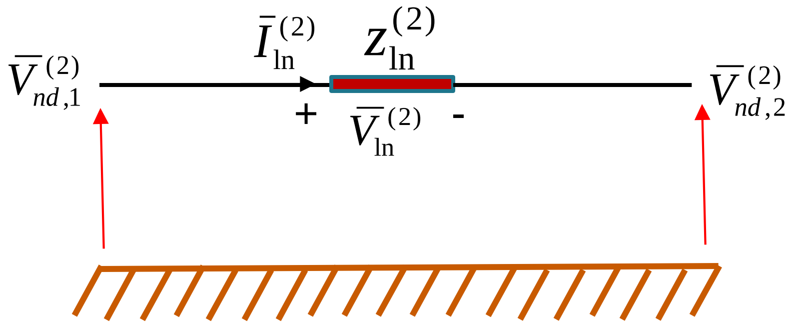
<u>Positive sequence network</u>



$$z_{\ln}^{(1)}=z_{\ln}^{(ps)}-z_{\ln}^{(pm)}\ :+\text{ve sequence impedance}$$

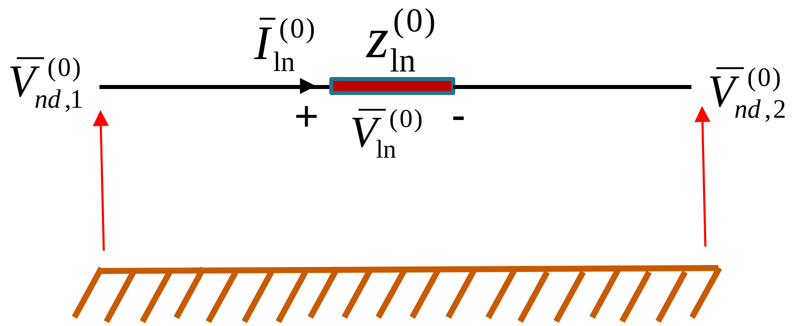
Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Negative sequence network



$$z_{\mathrm{ln}}^{(2)} = z_{\mathrm{ln}}^{(ps)} - z_{\mathrm{ln}}^{(pm)} : -\text{ve sequence impedance}$$

Zero sequence network



$$z_{\mathrm{ln}}^{(0)} = z_{\mathrm{ln}}^{(ps)} + 2z_{\mathrm{ln}}^{(pm)} : \text{zero sequence impedance}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Balanced star-connected load with grounded neutral

Refer to Slides 13 and 15 along with the impedance balance condition

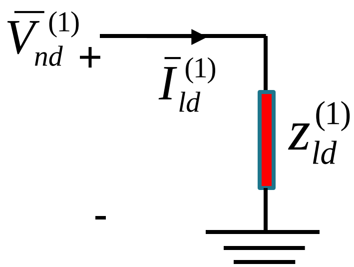
$$\begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix} = \begin{bmatrix} z_{ld}^{(ps)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} \\ z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(ps)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} \\ z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(ps)} + z_{ld}^{(n)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{V}_{nd}^{(0)} \\ \bar{V}_{nd}^{(1)} \\ \bar{V}_{nd}^{(2)} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} z_{ld}^{(ps)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} \\ z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(ps)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} \\ z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(pm)} + z_{ld}^{(n)} & z_{ld}^{(ps)} + z_{ld}^{(n)} \end{bmatrix} \mathbf{C} \begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{V}_{nd}^{(0)} \\ \bar{V}_{nd}^{(1)} \\ \bar{V}_{nd}^{(2)} \end{bmatrix} = \begin{bmatrix} z_{ld}^{(ps)} + 2z_{ld}^{(pm)} + 3z_{ld}^{(n)} & 0 & 0 \\ 0 & z_{ld}^{(ps)} - z_{ld}^{(pm)} & 0 \\ 0 & & z_{ld}^{(ps)} - z_{ld}^{(pm)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix}$$

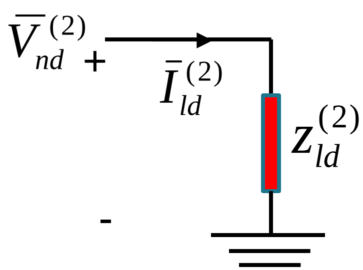
Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Positive sequence network



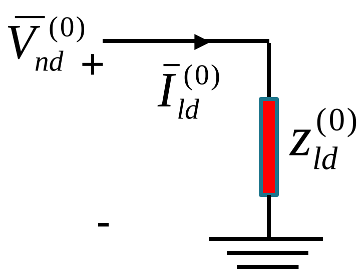
$$z_{ld}^{(1)} = z_{ld}^{(ps)} - z_{ld}^{(pm)}$$

Negative sequence network



$$z_{ld}^{(2)} = z_{ld}^{(ps)} - z_{ld}^{(pm)}$$

Zero sequence network



$$z_{ld}^{(0)} = z_{ld}^{(ps)} + 2z_{ld}^{(pm)} + 3z_{ld}^{(n)}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Balanced star-connected load with ungrounded neutral

Refer to Slides 16 along with the impedance balance condition

$$\begin{bmatrix} \bar{V}_{ld}^{(an)} \\ \bar{V}_{ld}^{(bn)} \\ \bar{V}_{ld}^{(cn)} \end{bmatrix} = \begin{bmatrix} z_{ld}^{(ps)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(ps)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(pm)} & z_{ld}^{(ps)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \bar{V}_{ld}^{(ng)} = \begin{bmatrix} z_{ld}^{(ps)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(ps)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(pm)} & z_{ld}^{(ps)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix} = \begin{bmatrix} z_{ld}^{(ps)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(ps)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(pm)} & z_{ld}^{(ps)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \bar{V}_{ld}^{(ng)}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

After performing the symmetrical transformation,

$$\begin{bmatrix} \bar{V}_{nd}^{(0)} \\ \bar{V}_{nd}^{(1)} \\ \bar{V}_{nd}^{(2)} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} z_{ld}^{(ps)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(ps)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(pm)} & z_{ld}^{(ps)} \end{bmatrix} \mathbf{C} \begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix} + \mathbf{C}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \bar{V}_{ld}^{(ng)}$$

$$= \begin{bmatrix} z_{ld}^{(ps)} + 2z_{ld}^{(pm)} & 0 & 0 \\ 0 & z_{ld}^{(ps)} - z_{ld}^{(pm)} & 0 \\ 0 & 0 & z_{ld}^{(ps)} - z_{ld}^{(pm)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \bar{V}_{ld}^{(ng)}$$

Therefore,

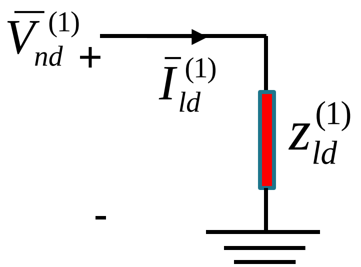
$$\bar{V}_{nd}^{(1)} = \left(z_{ld}^{(ps)} - z_{ld}^{(pm)} \right) \bar{I}_{ld}^{(1)}$$
$$\bar{V}_{nd}^{(2)} = \left(z_{ld}^{(ps)} - z_{ld}^{(pm)} \right) \bar{I}_{ld}^{(2)}$$

As per the KCL,

$$I_{ld}^{(a)} + I_{ld}^{(b)} + I_{ld}^{(c)} = 0 \Rightarrow I_{ld}^{(0)} = 0$$

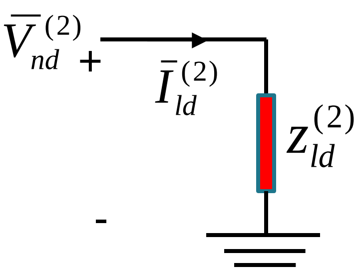
Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Positive sequence network



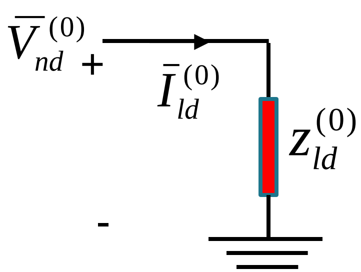
$$z_{ld}^{(1)} = z_{ld}^{(ps)} - z_{ld}^{(pm)}$$

Negative sequence network



$$z_{ld}^{(2)} = z_{ld}^{(ps)} - z_{ld}^{(pm)}$$

Zero sequence network



$$z_{ld}^{(0)} = \infty$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Balanced delta-connected load

Refer to Slides 19, 20 and 21 along with the impedance balance condition

$$\begin{bmatrix} z_{ld}^{(ps)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(ps)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(\dot{a})} \\ \bar{I}_{ld}^{(\dot{b})} \\ \bar{I}_{ld}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{V}_{nd}^{(a)} \\ \bar{V}_{nd}^{(b)} \\ \bar{V}_{nd}^{(c)} \end{bmatrix}$$

$$\Rightarrow \mathbf{C}^{-1} \begin{bmatrix} z_{ld}^{(ps)} & z_{ld}^{(pm)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(ps)} & z_{ld}^{(pm)} \\ z_{ld}^{(pm)} & z_{ld}^{(pm)} & z_{ld}^{(ps)} \end{bmatrix} \mathbf{C} \begin{bmatrix} \bar{I}_{ld}^{(\dot{0})} \\ \bar{I}_{ld}^{(\dot{1})} \\ \bar{I}_{ld}^{(\dot{2})} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \mathbf{C} \begin{bmatrix} \bar{V}_{nd}^{(0)} \\ \bar{V}_{nd}^{(1)} \\ \bar{V}_{nd}^{(2)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} z_{ld}^{(ps)} + 2z_{ld}^{(pm)} & 0 & 0 \\ 0 & z_{ld}^{(ps)} - z_{ld}^{(pm)} & 0 \\ 0 & 0 & z_{ld}^{(ps)} - z_{ld}^{(pm)} \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(\dot{0})} \\ \bar{I}_{ld}^{(\dot{1})} \\ \bar{I}_{ld}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha^{2} & 0 \\ 0 & 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} \bar{V}_{nd}^{(0)} \\ \bar{V}_{nd}^{(1)} \\ \bar{V}_{nd}^{(2)} \end{bmatrix}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Therefore,

$$\bar{I}_{ld}^{(\dot{0})} = 0$$

$$\bar{I}_{ld}^{(\dot{1})} = \frac{1}{z_{ld}^{(ps)} - z_{ld}^{(pm)}} \left(1-\alpha^2\right) \bar{V}_{nd}^{(1)}$$

$$\bar{I}_{ld}^{(\dot{2})} = \frac{1}{z_{ld}^{(ps)} - z_{ld}^{(pm)}} \left(1-\alpha\right) \bar{V}_{nd}^{(2)}$$

Further, from the KCL

$$\begin{bmatrix} \bar{I}_{ld}^{(a)} \\ \bar{I}_{ld}^{(b)} \\ \bar{I}_{ld}^{(c)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(\dot{a})} \\ \bar{I}_{ld}^{(\dot{b})} \\ \bar{I}_{ld}^{(\dot{c})} \end{bmatrix} \Rightarrow \mathbf{C} \begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \mathbf{C} \begin{bmatrix} \bar{I}_{ld}^{(\dot{0})} \\ \bar{I}_{ld}^{(\dot{1})} \\ \bar{I}_{ld}^{(\dot{2})} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \mathbf{C} \begin{bmatrix} \bar{I}_{ld}^{(\dot{0})} \\ \bar{I}_{ld}^{(\dot{1})} \\ \bar{I}_{ld}^{(\dot{2})} \end{bmatrix}$$

Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Therefore,

$$\begin{bmatrix} \bar{I}_{ld}^{(0)} \\ \bar{I}_{ld}^{(1)} \\ \bar{I}_{ld}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} \bar{I}_{ld}^{(\dot{0})} \\ \bar{I}_{ld}^{(\dot{1})} \\ \bar{I}_{ld}^{(\dot{2})} \end{bmatrix}$$

Finally, we obtain

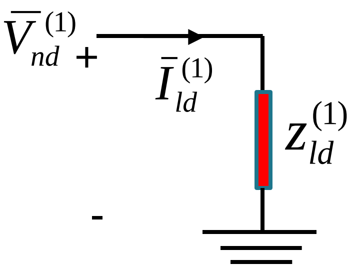
$$\bar{I}_{ld}^{(0)} = 0$$

$$\bar{I}_{ld}^{(1)} = \frac{(1-\alpha)}{z_{ld}^{(ps)} - z_{ld}^{(pm)}}(1-\alpha^2)\bar{V}_{nd}^{(1)} = \frac{3}{z_{ld}^{(ps)} - z_{ld}^{(pm)}}\bar{V}_{nd}^{(1)}$$

$$\bar{I}_{ld}^{(2)} = \frac{(1-\alpha^2)}{z_{ld}^{(ps)} - z_{ld}^{(pm)}}(1-\alpha)\bar{V}_{nd}^{(2)} = \frac{3}{z_{ld}^{(ps)} - z_{ld}^{(pm)}}\bar{V}_{nd}^{(2)}$$

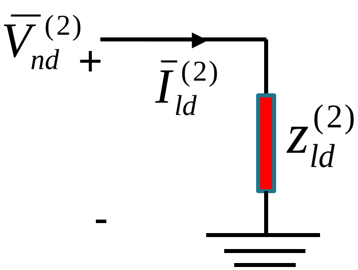
Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Positive sequence network



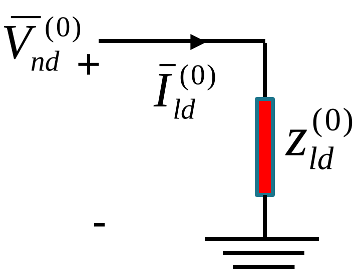
$$z_{ld}^{(1)} = \frac{z_{ld}^{(ps)} - z_{ld}^{(pm)}}{3}$$

Negative sequence network



$$z_{ld}^{(2)} = \frac{z_{ld}^{(ps)} - z_{ld}^{(pm)}}{3}$$

Zero sequence network



$$z_{ld}^{(0)} = \infty$$

Source with balanced internal impedance

Refer to Slides 23 and 25 along with the impedance balance condition

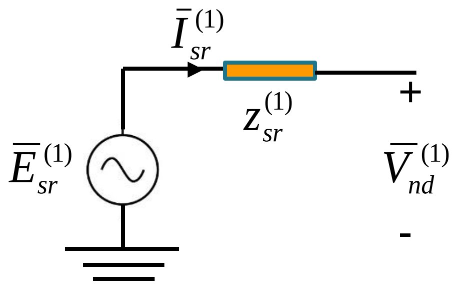
$$\begin{bmatrix} \overline{E}_{sr}^{(an)} \\ \overline{E}_{sr}^{(bn)} \\ \overline{E}_{sr}^{(cn)} \end{bmatrix} = \begin{bmatrix} z_{sr}^{(ps)} + z_{sr}^{(n)} & z_{sr}^{(n)} & z_{sr}^{(n)} \\ z_{sr}^{(n)} & z_{sr}^{(ps)} + z_{sr}^{(n)} & z_{sr}^{(n)} \\ z_{sr}^{(n)} & z_{sr}^{(n)} & z_{sr}^{(ps)} + z_{sr}^{(n)} \end{bmatrix} \begin{bmatrix} \overline{I}_{sr}^{(a)} \\ \overline{I}_{sr}^{(b)} \\ \overline{I}_{sr}^{(b)} \end{bmatrix} + \begin{bmatrix} \overline{V}_{nd}^{(a)} \\ \overline{V}_{nd}^{(b)} \\ \overline{V}_{nd}^{(c)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \overline{E}_{sr}^{(0)} \\ \overline{E}_{sr}^{(1)} \\ \overline{E}_{sr}^{(2)} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} z_{sr}^{(ps)} + z_{sr}^{(n)} & z_{sr}^{(n)} & z_{sr}^{(n)} \\ z_{sr}^{(n)} & z_{sr}^{(ps)} + z_{sr}^{(n)} & z_{sr}^{(n)} \\ z_{sr}^{(n)} & z_{sr}^{(n)} & z_{sr}^{(ps)} + z_{sr}^{(n)} \end{bmatrix} \mathbf{C} \begin{bmatrix} \overline{I}_{sr}^{(0)} \\ \overline{I}_{sr}^{(1)} \\ \overline{I}_{sr}^{(2)} \end{bmatrix} + \mathbf{C}^{-1}\mathbf{C} \begin{bmatrix} \overline{V}_{nd}^{(0)} \\ \overline{V}_{nd}^{(1)} \\ \overline{V}_{nd}^{(2)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \overline{E}_{sr}^{(0)} \\ \overline{E}_{sr}^{(1)} \\ \overline{E}_{sr}^{(2)} \end{bmatrix} = \begin{bmatrix} z_{sr}^{(ps)} + 3z_{sr}^{(n)} & 0 & 0 \\ 0 & z_{sr}^{(ps)} & 0 \\ 0 & 0 & z_{sr}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{sr}^{(0)} \\ \overline{I}_{sr}^{(1)} \\ \overline{I}_{sr}^{(2)} \end{bmatrix} + \begin{bmatrix} \overline{V}_{nd}^{(0)} \\ \overline{V}_{nd}^{(1)} \\ \overline{V}_{nd}^{(2)} \end{bmatrix}$$

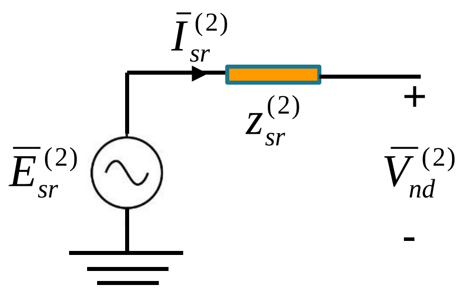
Sequence-domain Line, Load and Source Modeling Under Impedance Balance (Con..)

Positive sequence network



$$z_{sr}^{(1)} = z_{sr}^{(ps)}$$

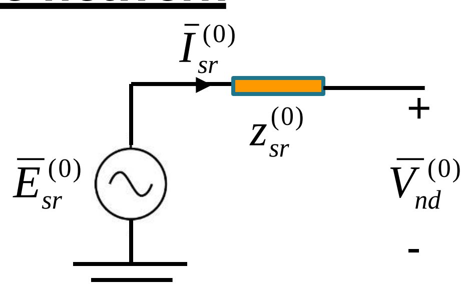
Negative sequence network



$$z_{sr}^{(2)} = z_{sr}^{(ps)}$$

If mutual impedances are present, those can be taken into account in the same way as was done in the case of the load and the line

Zero sequence network



$$z_{sr}^{(0)} = z_{sr}^{(ps)} + 3z_{sr}^{(n)}$$

Sequence-domain Modeling of a Balanced Transformer

Balanced YN-yn-xx transformer

Refer to Slides 40 and 41 along with the impedance balance condition

$$\begin{bmatrix} \overline{E}_{lv}^{(an)} \\ \overline{E}_{lv}^{(bn)} \\ \overline{E}_{lv}^{(cn)} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix} \left(\pm \frac{N_{lv}}{N_{hv}} \right) \Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(0)} \\ \overline{E}_{lv}^{(1)} \\ \overline{E}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\pm \frac{N_{lv}}{N_{hv}} \right)$$

$$\begin{bmatrix} \overline{I}_{lv}^{(a)} \\ \overline{I}_{lv}^{(b)} \\ \overline{I}_{lv}^{(c)} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(a)} \\ \overline{I}_{hv}^{(b)} \\ \overline{I}_{hv}^{(c)} \end{bmatrix} \left(\pm \frac{N_{hv}}{N_{lv}} \right) \Rightarrow \begin{bmatrix} \overline{I}_{lv}^{(0)} \\ \overline{I}_{lv}^{(1)} \\ \overline{I}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(0)} \\ \overline{I}_{hv}^{(1)} \\ \overline{I}_{hv}^{(2)} \end{bmatrix} \left(\pm \frac{N_{hv}}{N_{lv}} \right)$$

According to the KVL and KCL on the HV side

$$\begin{bmatrix} \overline{V}_{nd,1}^{(a)} \\ \overline{V}_{nd,1}^{(b)} \\ \overline{V}_{nd,1}^{(c)} \end{bmatrix} = \begin{bmatrix} z_{hv}^{(ps)} + z_{hv}^{(n)} & z_{hv}^{(n)} & z_{hv}^{(n)} \\ z_{hv}^{(n)} & z_{hv}^{(ps)} + z_{hv}^{(n)} & z_{hv}^{(n)} \\ z_{hv}^{(n)} & z_{hv}^{(n)} & z_{hv}^{(ps)} + z_{hv}^{(n)} \end{bmatrix} \begin{bmatrix} \overline{I}_{hv}^{(a)} \\ \overline{I}_{hv}^{(b)} \\ \overline{I}_{hv}^{(c)} \end{bmatrix} + \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

The subsequent symmetrical transformation leads to the following equation

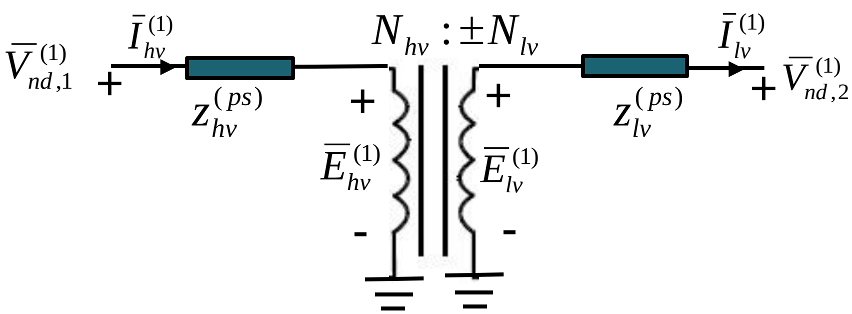
$$\begin{bmatrix} \bar{V}_{nd,1}^{(0)} \\ \bar{V}_{nd,1}^{(1)} \\ \bar{V}_{nd,1}^{(2)} \end{bmatrix} = \begin{bmatrix} z_{hv}^{(ps)} + 3z_{hv}^{(n)} & 0 & 0 \\ 0 & z_{hv}^{(ps)} & 0 \\ 0 & 0 & z_{hv}^{(ps)} \end{bmatrix} \begin{bmatrix} \bar{I}_{hv}^{(0)} \\ \bar{I}_{hv}^{(1)} \\ \bar{I}_{hv}^{(2)} \end{bmatrix} + \begin{bmatrix} \bar{E}_{hv}^{(0)} \\ \bar{E}_{hv}^{(1)} \\ \bar{E}_{hv}^{(2)} \end{bmatrix}$$

In the same way, the following equation can be derived for the LV side

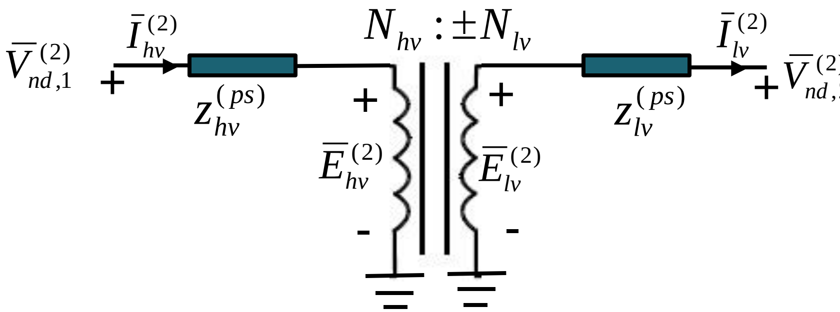
$$\begin{bmatrix} \bar{E}_{lv}^{(0)} \\ \bar{E}_{lv}^{(1)} \\ \bar{E}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} \bar{V}_{nd,2}^{(0)} \\ \bar{V}_{nd,2}^{(1)} \\ \bar{V}_{nd,2}^{(2)} \end{bmatrix} + \begin{bmatrix} z_{lv}^{(ps)} + 3z_{lv}^{(n)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \bar{I}_{lv}^{(0)} \\ \bar{I}_{lv}^{(1)} \\ \bar{I}_{lv}^{(2)} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

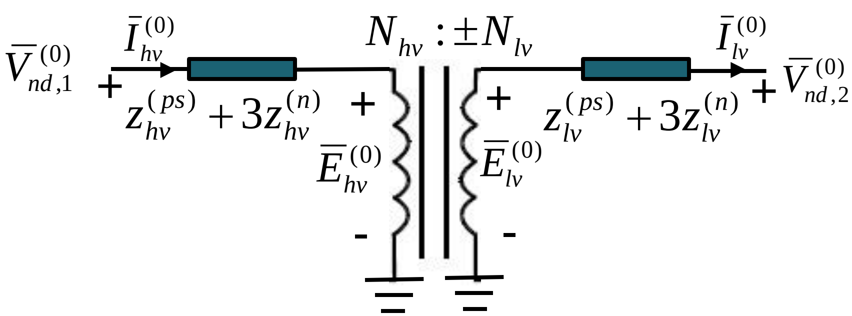
Positive sequence network



Negative sequence network



Zero sequence network



Sequence-domain Modeling of a Balanced Transformer (Con..)

Balanced Y-y-xx transformer

Refer to Slides 44 and 45 along with the impedance balance condition

$$\begin{bmatrix} \overline{E}_{lv}^{(an)} \\ \overline{E}_{lv}^{(bn)} \\ \overline{E}_{lv}^{(cn)} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix} \left(\pm \frac{N_{lv}}{N_{hv}} \right) \Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(0)} \\ \overline{E}_{lv}^{(1)} \\ \overline{E}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\pm \frac{N_{lv}}{N_{hv}} \right)$$

$$\begin{bmatrix} \overline{I}_{lv}^{(a)} \\ \overline{I}_{lv}^{(b)} \\ \overline{I}_{lv}^{(c)} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(a)} \\ \overline{I}_{hv}^{(b)} \\ \overline{I}_{hv}^{(c)} \end{bmatrix} \left(\pm \frac{N_{hv}}{N_{lv}} \right) \Rightarrow \begin{bmatrix} \overline{I}_{lv}^{(0)} \\ \overline{I}_{lv}^{(1)} \\ \overline{I}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(0)} \\ \overline{I}_{hv}^{(1)} \\ \overline{I}_{hv}^{(2)} \end{bmatrix} \left(\pm \frac{N_{hv}}{N_{lv}} \right)$$

According to the KVL on the HV side

$$\begin{bmatrix} \overline{V}_{nd,1}^{(a)} \\ \overline{V}_{nd,1}^{(b)} \\ \overline{V}_{nd,1}^{(c)} \end{bmatrix} = \begin{bmatrix} z_{hv}^{(ps)} & 0 & 0 \\ 0 & z_{hv}^{(ps)} & 0 \\ 0 & 0 & z_{hv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{hv}^{(a)} \\ \overline{I}_{hv}^{(b)} \\ \overline{I}_{hv}^{(c)} \end{bmatrix} + \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \overline{V}_{hv}^{(ng)}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

The subsequent symmetrical transformation leads to the following equation

$$\begin{bmatrix} \overline{V}_{nd,1}^{(0)} \\ \overline{V}_{nd,1}^{(1)} \\ \overline{V}_{nd,1}^{(2)} \end{bmatrix} = \begin{bmatrix} z_{hv}^{(ps)} & 0 & 0 \\ 0 & z_{hv}^{(ps)} & 0 \\ 0 & 0 & z_{hv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{hv}^{(0)} \\ \overline{I}_{hv}^{(1)} \\ \overline{I}_{hv}^{(2)} \end{bmatrix} + \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} + \begin{bmatrix} \overline{V}_{hv}^{(ng)} \\ 0 \\ 0 \end{bmatrix}$$

In the same way, the following KVL equation can be derived for the LV side

$$\begin{bmatrix} \overline{E}_{lv}^{(0)} \\ \overline{E}_{lv}^{(1)} \\ \overline{E}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} z_{lv}^{(ps)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(0)} \\ \overline{I}_{lv}^{(1)} \\ \overline{I}_{lv}^{(2)} \end{bmatrix} + \begin{bmatrix} \overline{V}_{nd,2}^{(0)} \\ \overline{V}_{nd,2}^{(1)} \\ \overline{V}_{nd,2}^{(2)} \end{bmatrix} - \begin{bmatrix} \overline{V}_{lv}^{(ng)} \\ 0 \\ 0 \end{bmatrix}$$

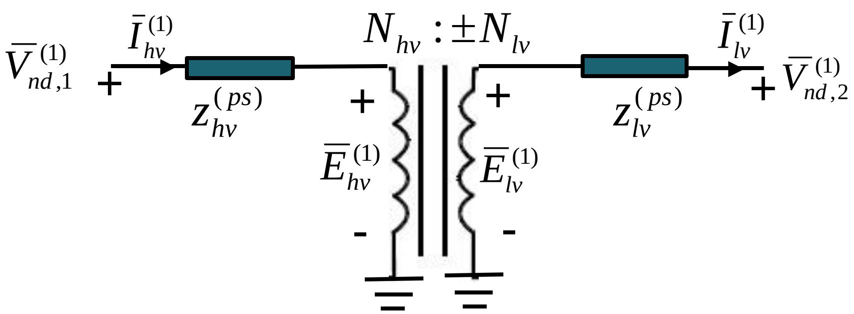
According to the KCL

$$\overline{I}_{hv}^{(a)} + \overline{I}_{hv}^{(b)} + \overline{I}_{hv}^{(c)} = 0 \Rightarrow \overline{I}_{hv}^{(0)} = 0$$

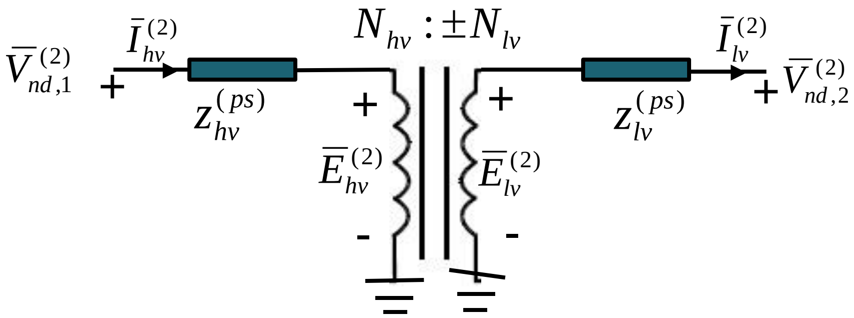
$$\overline{I}_{lv}^{(a)} + \overline{I}_{lv}^{(b)} + \overline{I}_{lv}^{(c)} = 0 \Rightarrow \overline{I}_{lv}^{(0)} = 0$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

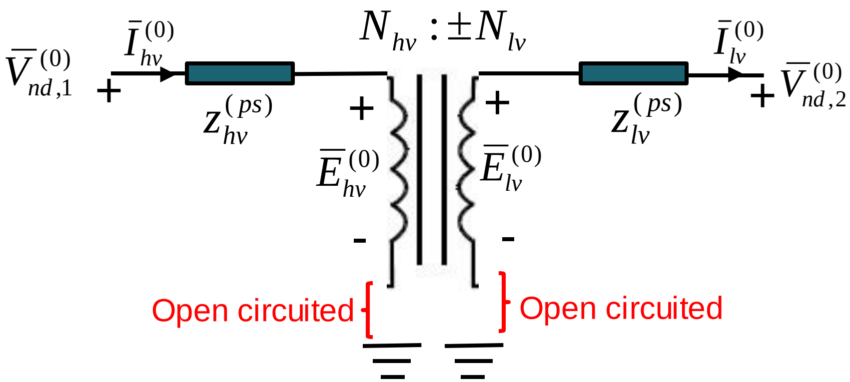
Positive sequence network



Negative sequence network



Zero sequence network

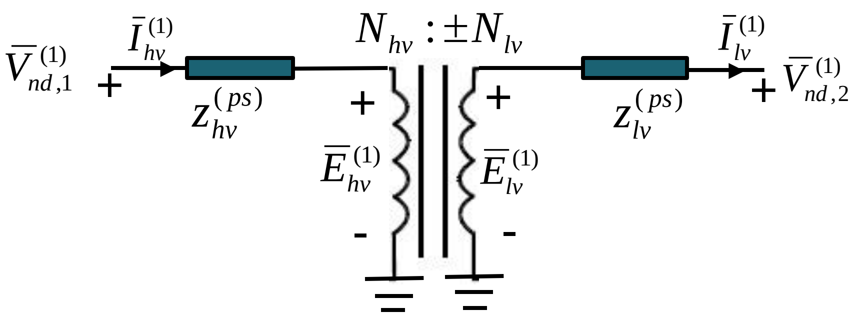


Sequence-domain Modeling of a Balanced Transformer (Con..)

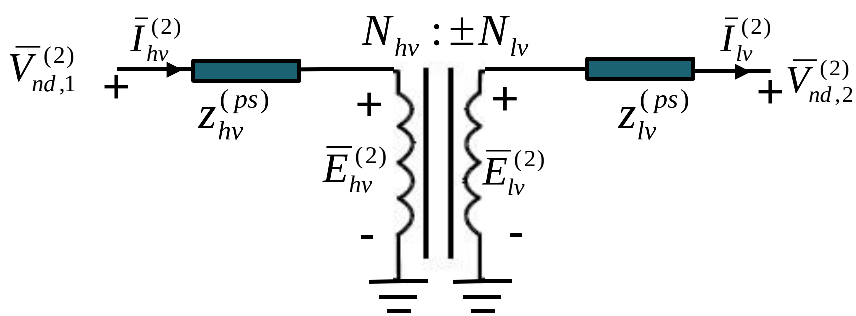
Balanced YN-y-xx transformer

We need to combine the derivations for YN-yn-xx and Y-y-xx connections to get the following sequence networks

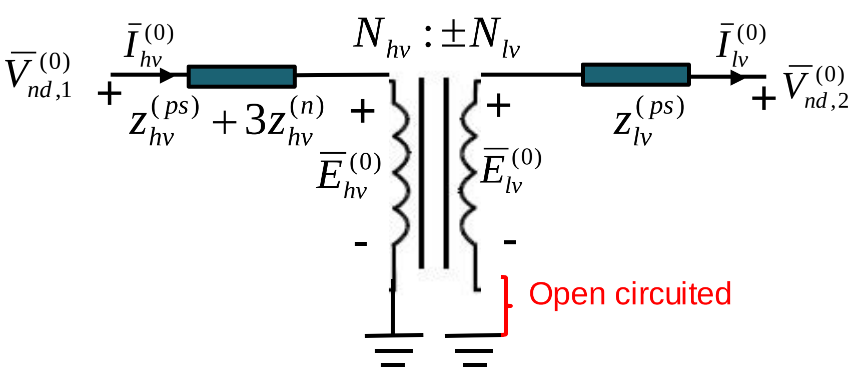
Positive sequence network



Negative sequence network



Zero sequence network



Note that, according to the current transformation principle, the HV side zero-sequence current also becomes zero

Balanced D-d-xx transformer

Refer to Slides 46 and 47 along with the impedance balance condition

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(\dot{a})} \\ \overline{E}_{hv}^{(\dot{b})} \\ \overline{E}_{hv}^{(\dot{c})} \end{bmatrix} \left(\pm \frac{N_{lv}}{N_{hv}} \right) \Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(\dot{0})} \\ \overline{E}_{hv}^{(\dot{1})} \\ \overline{E}_{hv}^{(\dot{2})} \end{bmatrix} \left(\pm \frac{N_{lv}}{N_{hv}} \right)$$

$$\begin{bmatrix} \overline{I}_{lv}^{(\dot{a})} \\ \overline{I}_{lv}^{(\dot{b})} \\ \overline{I}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(\dot{a})} \\ \overline{I}_{hv}^{(\dot{b})} \\ \overline{I}_{hv}^{(\dot{c})} \end{bmatrix} \left(\pm \frac{N_{hv}}{N_{lv}} \right) \Rightarrow \begin{bmatrix} \overline{I}_{lv}^{(\dot{0})} \\ \overline{I}_{lv}^{(\dot{1})} \\ \overline{I}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(\dot{0})} \\ \overline{I}_{hv}^{(\dot{1})} \\ \overline{I}_{hv}^{(\dot{2})} \end{bmatrix} \left(\pm \frac{N_{hv}}{N_{lv}} \right)$$

According to the KVL on the HV side

$$\begin{bmatrix} \overline{E}_{hv}^{(\dot{a})} \\ \overline{E}_{hv}^{(\dot{b})} \\ \overline{E}_{hv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,1}^{(a)} \\ \overline{V}_{nd,1}^{(b)} \\ \overline{V}_{nd,1}^{(c)} \end{bmatrix} - \begin{bmatrix} z_{hv}^{(ps)} & 0 & 0 \\ 0 & z_{hv}^{(ps)} & 0 \\ 0 & 0 & z_{hv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{hv}^{(\dot{a})} \\ \overline{I}_{hv}^{(\dot{b})} \\ \overline{I}_{hv}^{(\dot{c})} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

The subsequent symmetrical transformation leads to the following equation

$$\begin{bmatrix} \overline{E}_{hv}^{(\dot{0})} \\ \overline{E}_{hv}^{(\dot{1})} \\ \overline{E}_{hv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha^2 & 0 \\ 0 & 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,1}^{(0)} \\ \overline{V}_{nd,1}^{(1)} \\ \overline{V}_{nd,1}^{(2)} \end{bmatrix} - \begin{bmatrix} z_{hv}^{(ps)} & 0 & 0 \\ 0 & z_{hv}^{(ps)} & 0 \\ 0 & 0 & z_{hv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{hv}^{(\dot{0})} \\ \overline{I}_{hv}^{(\dot{1})} \\ \overline{I}_{hv}^{(\dot{2})} \end{bmatrix}$$

In the same way, the following KVL equation can be derived for the LV side

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha^2 & 0 \\ 0 & 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,2}^{(0)} \\ \overline{V}_{nd,2}^{(1)} \\ \overline{V}_{nd,2}^{(2)} \end{bmatrix} + \begin{bmatrix} z_{lv}^{(ps)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{0})} \\ \overline{I}_{lv}^{(\dot{1})} \\ \overline{I}_{lv}^{(\dot{2})} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

As per the derivation in Slide 79

$$\begin{bmatrix} \bar{I}_{hv}^{(0)} \\ \bar{I}_{hv}^{(1)} \\ \bar{I}_{hv}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} \bar{I}_{hv}^{(\dot{0})} \\ \bar{I}_{hv}^{(\dot{1})} \\ \bar{I}_{hv}^{(\dot{2})} \end{bmatrix} \qquad \begin{bmatrix} \bar{I}_{lv}^{(0)} \\ \bar{I}_{lv}^{(1)} \\ \bar{I}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} \bar{I}_{lv}^{(\dot{0})} \\ \bar{I}_{lv}^{(\dot{1})} \\ \bar{I}_{lv}^{(\dot{2})} \end{bmatrix}$$

Finally, the HV side and LV side equations can be put down as follows

$$\bar{I}_{hv}^{(0)} = 0$$

$$\bar{E}_{hv}^{(\dot{0})} = -z_{hv}^{(ps)} \bar{I}_{hv}^{(\dot{0})}$$

$$\bar{V}_{nd,1}^{(1)} = \frac{1}{1-\alpha^2} \bar{E}_{hv}^{(\dot{1})} + \frac{z_{hv}^{(ps)}}{(1-\alpha^2)(1-\alpha)} \bar{I}_{hv}^{(1)}$$

$$= \frac{\bar{E}_{hv}^{(\dot{1})}}{\sqrt{3}} e^{-j\frac{\pi}{6}} + \frac{z_{hv}^{(ps)}}{3} \bar{I}_{hv}^{(1)}$$

$$\bar{V}_{nd,1}^{(2)} = \frac{1}{1-\alpha} \bar{E}_{hv}^{(\dot{2})} + \frac{z_{hv}^{(ps)}}{(1-\alpha^2)(1-\alpha)} \bar{I}_{hv}^{(2)}$$

$$= \frac{\bar{E}_{hv}^{(\dot{2})}}{\sqrt{3}} e^{j\frac{\pi}{6}} + \frac{z_{hv}^{(ps)}}{3} \bar{I}_{hv}^{(2)}$$

$$\bar{I}_{lv}^{(0)} = 0$$

$$\bar{E}_{lv}^{(\dot{0})} = z_{lv}^{(ps)} \bar{I}_{lv}^{(\dot{0})}$$

$$\bar{V}_{nd,2}^{(1)} = \frac{1}{1-\alpha^2} \bar{E}_{lv}^{(\dot{1})} - \frac{z_{lv}^{(ps)}}{(1-\alpha^2)(1-\alpha)} \bar{I}_{lv}^{(1)}$$

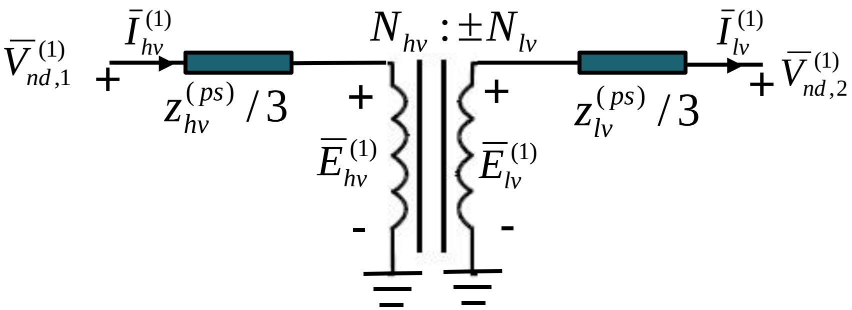
$$= \frac{\bar{E}_{lv}^{(\dot{1})}}{\sqrt{3}} e^{-j\frac{\pi}{6}} - \frac{z_{lv}^{(ps)}}{3} \bar{I}_{lv}^{(1)}$$

$$\bar{V}_{nd,2}^{(2)} = \frac{1}{1-\alpha} \bar{E}_{lv}^{(\dot{2})} - \frac{z_{lv}^{(ps)}}{(1-\alpha^2)(1-\alpha)} \bar{I}_{lv}^{(2)}$$

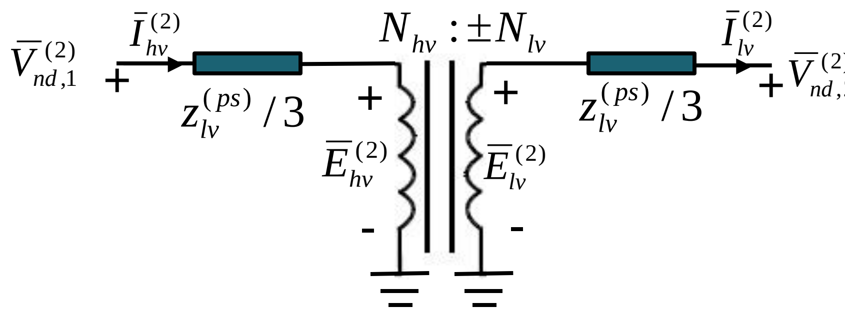
$$= \frac{\bar{E}_{lv}^{(\dot{2})}}{\sqrt{3}} e^{j\frac{\pi}{6}} - \frac{z_{lv}^{(ps)}}{3} \bar{I}_{lv}^{(2)}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

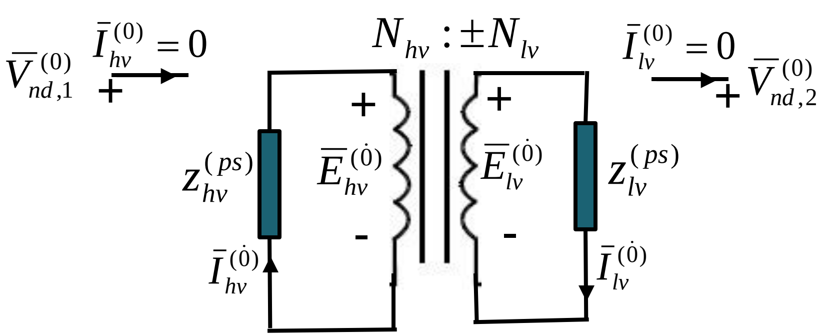
Positive sequence network



Negative sequence network



Zero sequence network



$$\bar{E}_{hv}^{(1)}=\frac{\bar{E}_{hv}^{(\dot{1})}}{\sqrt{3}}e^{-j\left(\frac{\pi}{6}\right)}$$

$$\bar{E}_{lv}^{(1)}=\frac{\bar{E}_{lv}^{(\dot{1})}}{\sqrt{3}}e^{-j\left(\frac{\pi}{6}\right)}$$

$$\bar{E}_{hv}^{(2)}=\frac{\bar{E}_{hv}^{(\dot{2})}}{\sqrt{3}}e^{j\left(\frac{\pi}{6}\right)}$$

$$\bar{E}_{lv}^{(2)}=\frac{\bar{E}_{lv}^{(\dot{2})}}{\sqrt{3}}e^{j\left(\frac{\pi}{6}\right)}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

Balanced YN-d-1 transformer

Refer to Slides 48 with the impedance balance condition

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right) \Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

$$\begin{bmatrix} \overline{I}_{lv}^{(\dot{a})} \\ \overline{I}_{lv}^{(\dot{b})} \\ \overline{I}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(a)} \\ \overline{I}_{hv}^{(b)} \\ \overline{I}_{hv}^{(c)} \end{bmatrix} \left(\frac{N_{hv}}{N_{lv}} \right) \Rightarrow \begin{bmatrix} \overline{I}_{lv}^{(\dot{0})} \\ \overline{I}_{lv}^{(\dot{1})} \\ \overline{I}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{I}_{hv}^{(0)} \\ \overline{I}_{hv}^{(1)} \\ \overline{I}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{hv}}{N_{lv}} \right)$$

According to the KVL and KCL equations on the HV side (*vide* Slide 84)

$$\begin{bmatrix} \overline{V}_{nd,1}^{(0)} \\ \overline{V}_{nd,1}^{(1)} \\ \overline{V}_{nd,1}^{(2)} \end{bmatrix} = \begin{bmatrix} z_{hv}^{(ps)} + 3z_{hv}^{(n)} & 0 & 0 \\ 0 & z_{hv}^{(ps)} & 0 \\ 0 & 0 & z_{hv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{hv}^{(0)} \\ \overline{I}_{hv}^{(1)} \\ \overline{I}_{hv}^{(2)} \end{bmatrix} + \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

According to the KVL on the LV side

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,2}^{(a)} \\ \overline{V}_{nd,2}^{(b)} \\ \overline{V}_{nd,2}^{(c)} \end{bmatrix} + \begin{bmatrix} z_{lv}^{(ps)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{a})} \\ \overline{I}_{lv}^{(\dot{b})} \\ \overline{I}_{lv}^{(\dot{c})} \end{bmatrix}$$

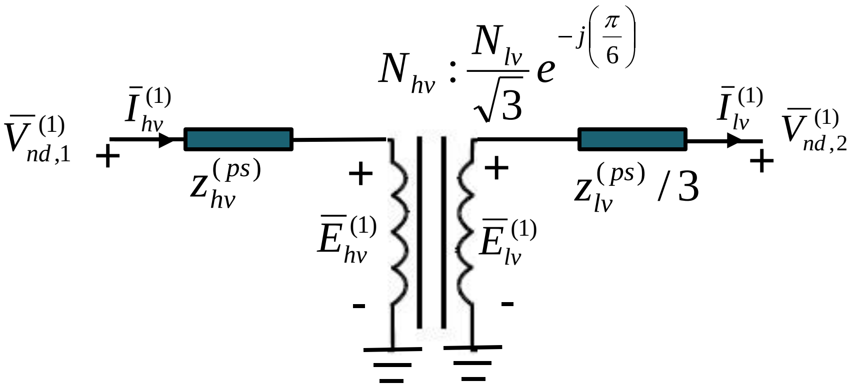
$$\Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha^2 & 0 \\ 0 & 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,2}^{(0)} \\ \overline{V}_{nd,2}^{(1)} \\ \overline{V}_{nd,2}^{(2)} \end{bmatrix} + \begin{bmatrix} z_{lv}^{(ps)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{0})} \\ \overline{I}_{lv}^{(\dot{1})} \\ \overline{I}_{lv}^{(\dot{2})} \end{bmatrix}$$

The KCL equations on the LV side are presented below

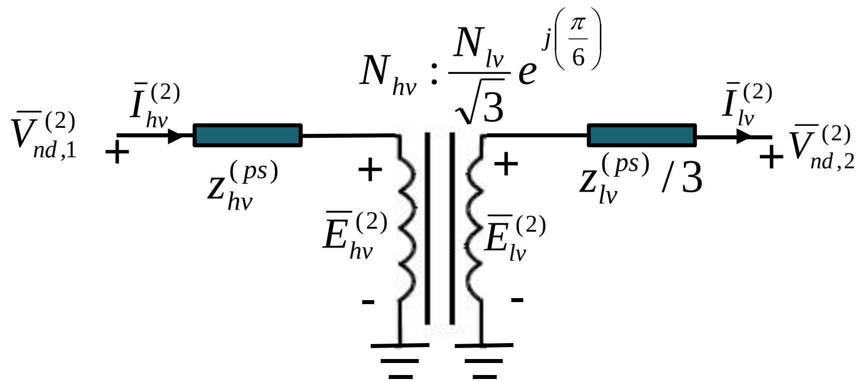
$$\begin{bmatrix} \overline{I}_{lv}^{(a)} \\ \overline{I}_{lv}^{(b)} \\ \overline{I}_{lv}^{(c)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{a})} \\ \overline{I}_{lv}^{(\dot{b})} \\ \overline{I}_{lv}^{(\dot{c})} \end{bmatrix} \Rightarrow \begin{bmatrix} \overline{I}_{lv}^{(0)} \\ \overline{I}_{lv}^{(1)} \\ \overline{I}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{0})} \\ \overline{I}_{lv}^{(\dot{1})} \\ \overline{I}_{lv}^{(\dot{2})} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

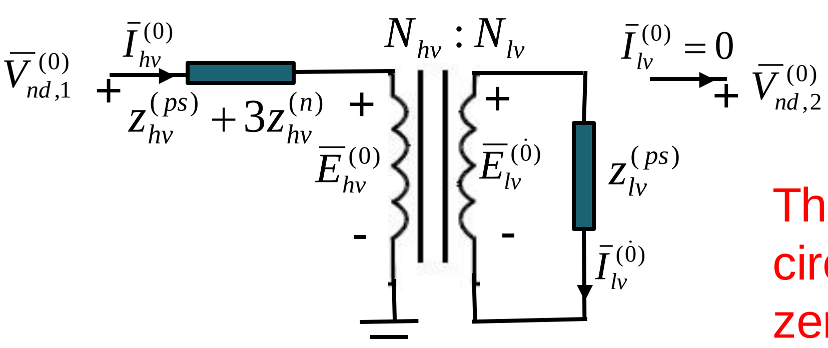
Positive sequence network



Negative sequence network



Zero sequence network



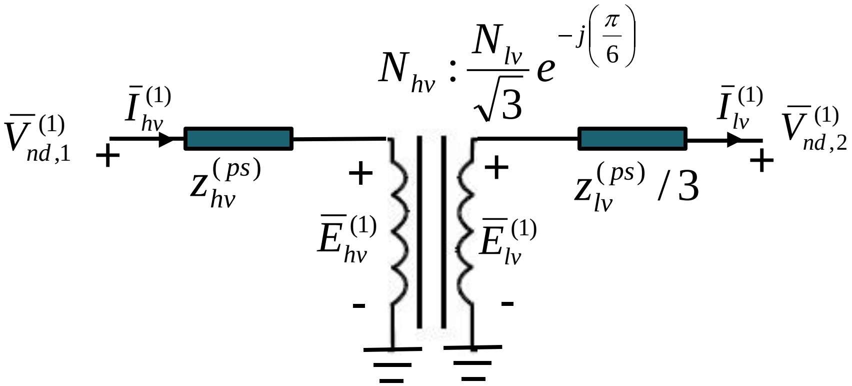
$$\overline{E}^{(1)}_{lv} = \frac{\overline{E}^{(1)}_{lv}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)} \qquad \overline{E}^{(2)}_{lv} = \frac{\overline{E}^{(2)}_{lv}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)}$$

There can be zero-sequence phase current (or circulating current) on the LV side although the zero-sequence line current output on the LV side is zero

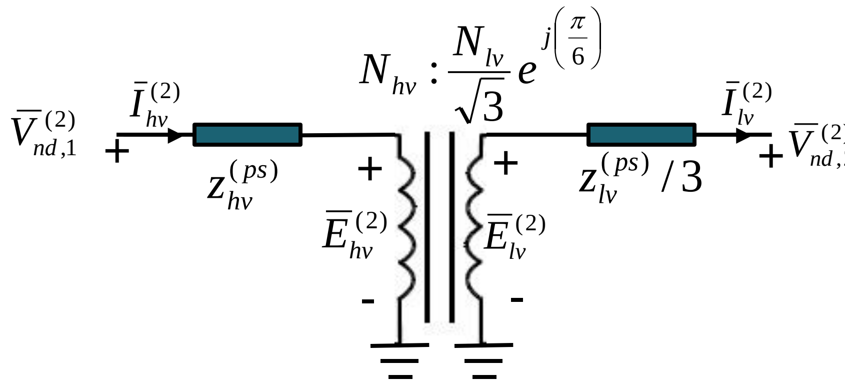
Sequence-domain Modeling of a Balanced Transformer (Con..)

Balanced Y-d-1 transformer

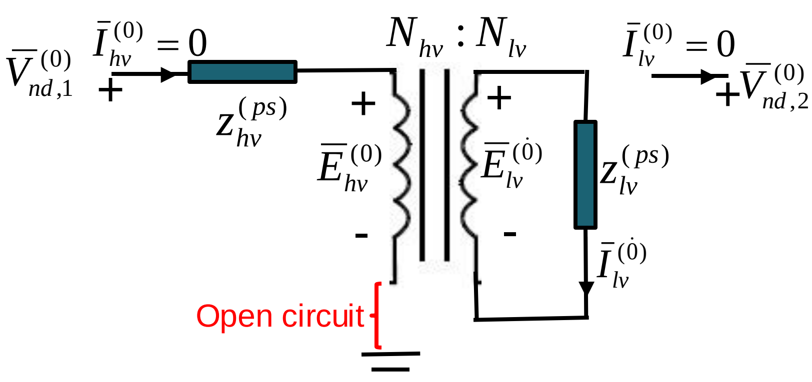
Positive sequence network



Negative sequence network



Zero sequence network



$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(1)}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)} \qquad \bar{E}_{lv}^{(2)} = \frac{\bar{E}_{lv}^{(2)}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)}$$

There is no circulating current on the LV side

Sequence-domain Modeling of a Balanced Transformer (Con..)

YN-d-11 connection

1. Refer to Slides 50 along with the impedance balance condition
2. The winding voltage and winding current transformation equations remain the same as in Slide 94
3. The KVL/KCL equations on the HV side also remain the same as in Slide 94
4. Only the KVL and KCL equations on the LV side need to be changed

KVL equations:

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,2}^{(a)} \\ \overline{V}_{nd,2}^{(b)} \\ \overline{V}_{nd,2}^{(c)} \end{bmatrix} + \begin{bmatrix} z_{lv}^{(ps)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{a})} \\ \overline{I}_{lv}^{(\dot{b})} \\ \overline{I}_{lv}^{(\dot{c})} \end{bmatrix}$$

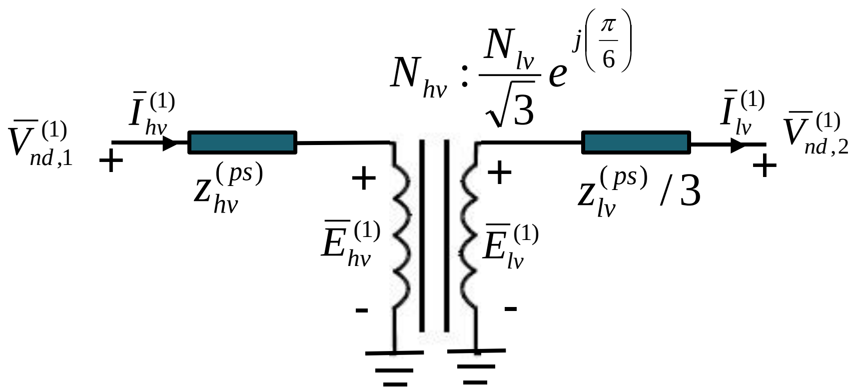
$$\Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} \overline{V}_{nd,2}^{(0)} \\ \overline{V}_{nd,2}^{(1)} \\ \overline{V}_{nd,2}^{(2)} \end{bmatrix} + \begin{bmatrix} z_{lv}^{(ps)} & 0 & 0 \\ 0 & z_{lv}^{(ps)} & 0 \\ 0 & 0 & z_{lv}^{(ps)} \end{bmatrix} \begin{bmatrix} \overline{I}_{lv}^{(\dot{0})} \\ \overline{I}_{lv}^{(\dot{1})} \\ \overline{I}_{lv}^{(\dot{2})} \end{bmatrix}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

KCL equations:

$$\begin{bmatrix} \bar{I}_{lv}^{(a)} \\ \bar{I}_{lv}^{(b)} \\ \bar{I}_{lv}^{(c)} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_{lv}^{(\dot{a})} \\ \bar{I}_{lv}^{(\dot{b})} \\ \bar{I}_{lv}^{(\dot{c})} \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{I}_{lv}^{(0)} \\ \bar{I}_{lv}^{(1)} \\ \bar{I}_{lv}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1-\alpha^2 & 0 \\ 0 & 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} \bar{I}_{lv}^{(\dot{0})} \\ \bar{I}_{lv}^{(\dot{1})} \\ \bar{I}_{lv}^{(\dot{2})} \end{bmatrix}$$

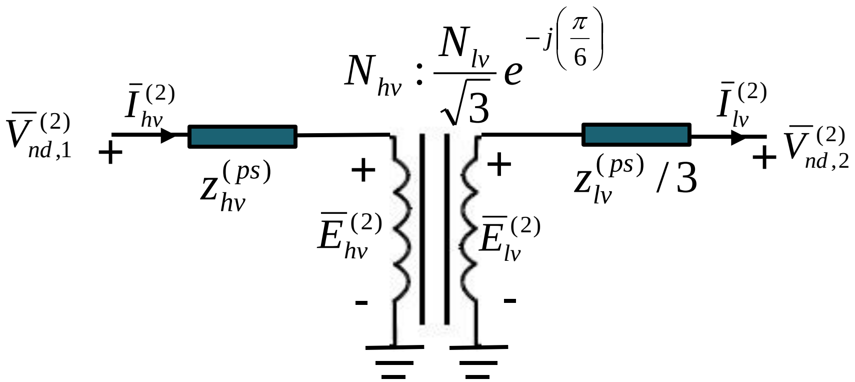
Positive sequence network



$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(\dot{1})}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)}$$

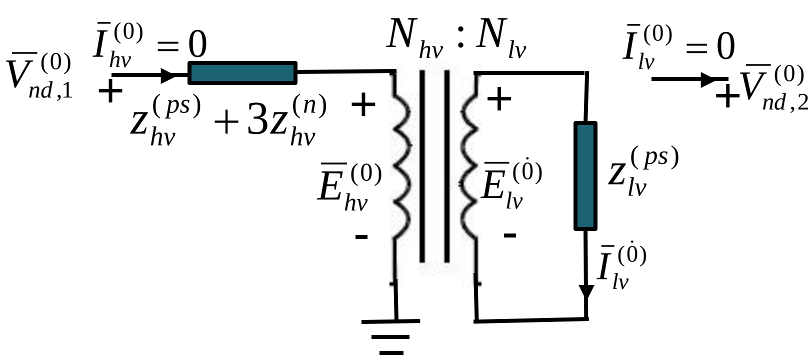
Sequence-domain Modeling of a Balanced Transformer (Con..)

Negative sequence network



$$\bar{E}_{lv}^{(2)} = \frac{\bar{E}_{lv}^{(2)}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)}$$

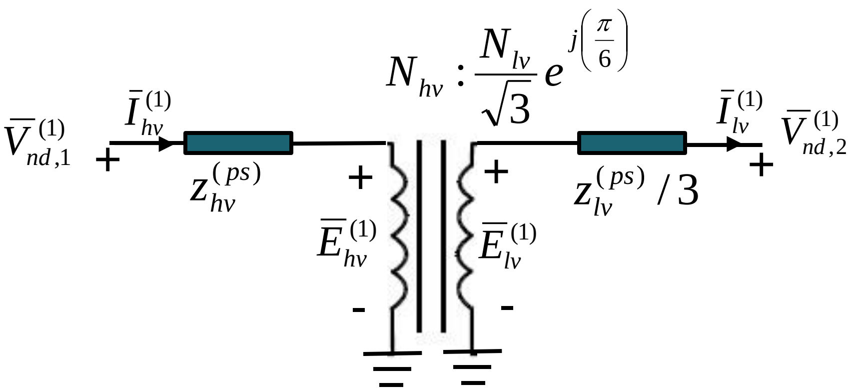
Zero sequence network



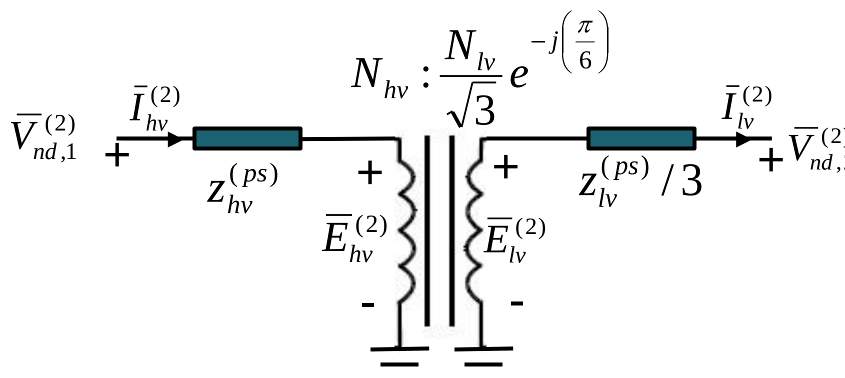
Sequence-domain Modeling of a Balanced Transformer (Con..)

Y-d-11 connection

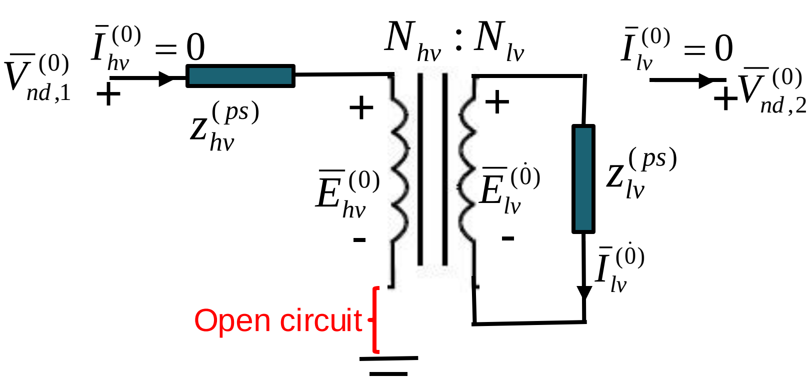
Positive sequence network



Negative sequence network



Zero sequence network



$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(1)}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)} \qquad \bar{E}_{lv}^{(2)} = \frac{\bar{E}_{lv}^{(2)}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

YN-d-3 connection

1. Refer to Slides 52 along with the impedance balance condition
2. The KVL/KCL equations on the HV side remain the same as in Slide 94
3. The KVL/KCL equations on the LV side remain the same as in Slides 98 and 99
4. Only the winding voltage and winding current transformation equations need to be changed

Winding voltage transformation equation:

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{c})} \\ \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right) \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(a)} \\ \overline{E}_{hv}^{(b)} \\ \overline{E}_{hv}^{(c)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

$$\Rightarrow \mathbf{C}^{-1} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{C} \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

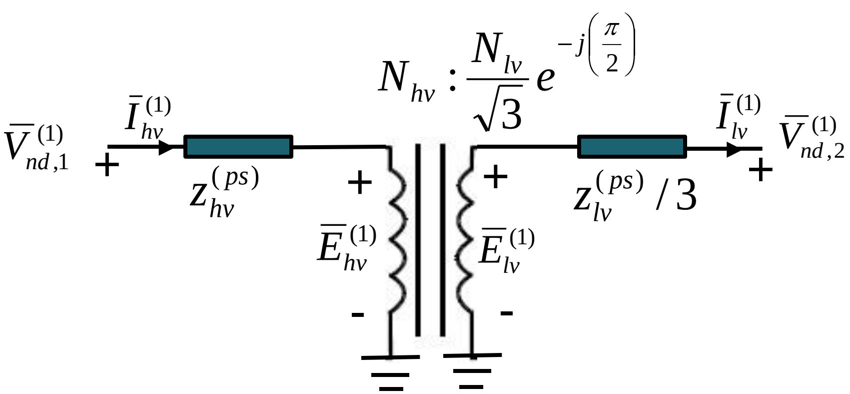
$$\Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \alpha \overline{E}_{lv}^{(\dot{1})} \\ \alpha^2 \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

Winding current transformation equation:

$$\begin{bmatrix} \bar{I}_{lv}^{(\dot{c})} \\ \bar{I}_{lv}^{(\dot{a})} \\ \bar{I}_{lv}^{(\dot{b})} \end{bmatrix} = \begin{bmatrix} \bar{I}_{hv}^{(a)} \\ \bar{I}_{hv}^{(b)} \\ \bar{I}_{hv}^{(c)} \end{bmatrix} \left(\frac{N_{hv}}{N_{lv}} \right) \Rightarrow \begin{bmatrix} \bar{I}_{lv}^{(\dot{0})} \\ \alpha \bar{I}_{lv}^{(\dot{1})} \\ \alpha^2 \bar{I}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \bar{I}_{hv}^{(0)} \\ \bar{I}_{hv}^{(1)} \\ \bar{I}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{hv}}{N_{lv}} \right)$$

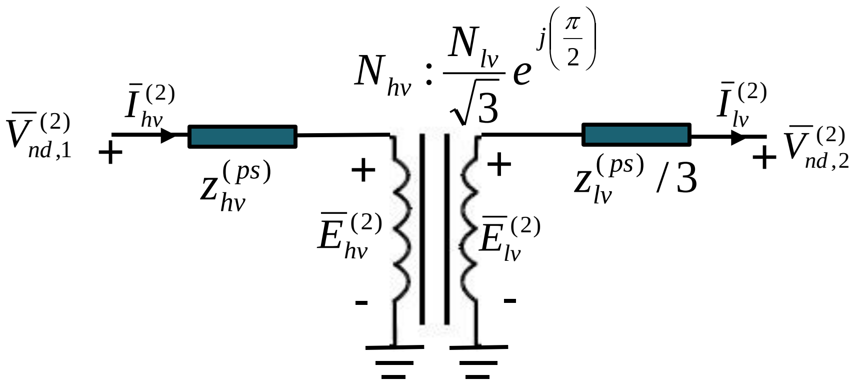
Positive sequence network



$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(\dot{1})}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)}$$

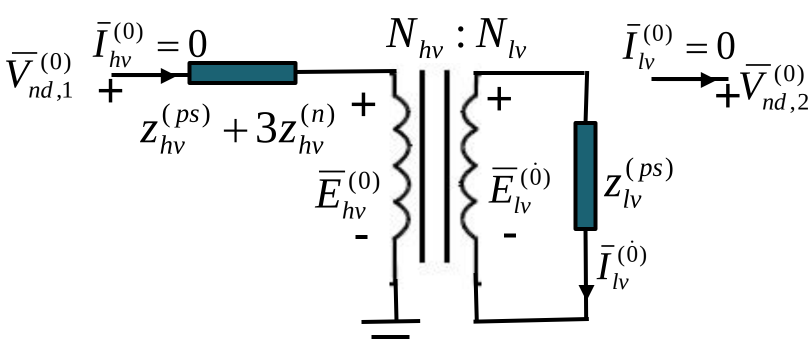
Sequence-domain Modeling of a Balanced Transformer (Con..)

Negative sequence network



$$\bar{E}_{lv}^{(2)} = \frac{\bar{E}_{lv}^{(\dot{2})}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)}$$

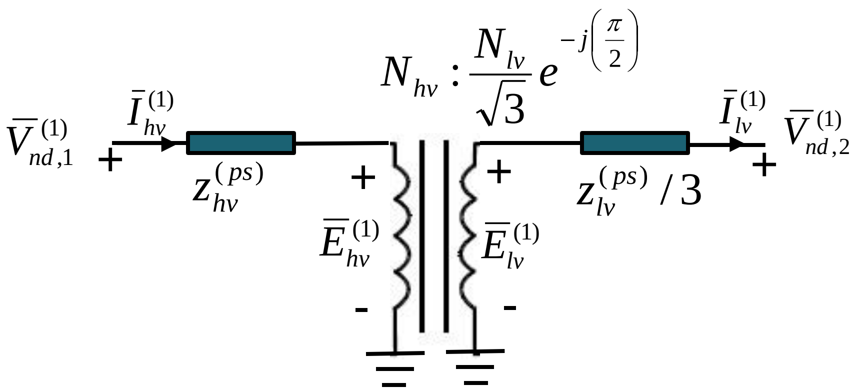
Zero sequence network



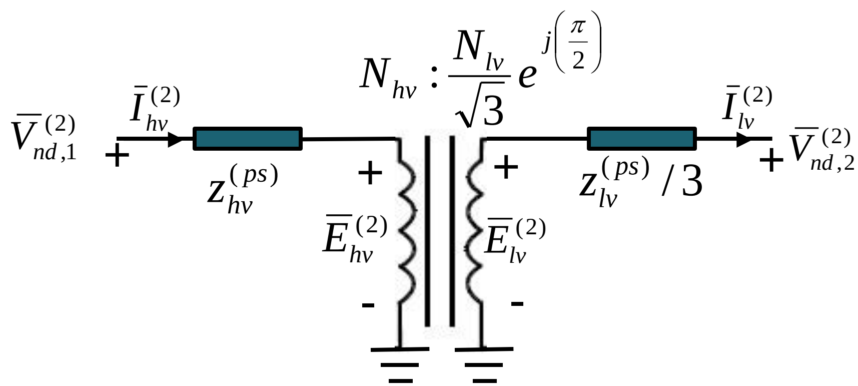
Sequence-domain Modeling of a Balanced Transformer (Con..)

Y-d-3 connection

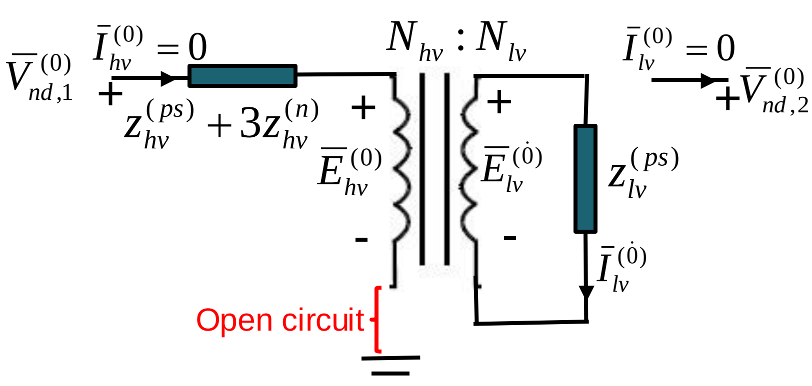
Positive sequence network



Negative sequence network



Zero sequence network



$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(\dot{1})}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)} \qquad \bar{E}_{lv}^{(2)} = \frac{\bar{E}_{lv}^{(\dot{2})}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)}$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

YN-d-9 connection

1. Refer to Slides 54 along with the impedance balance condition
2. The KVL/KCL equations on the HV side remain the same as in Slide 94
3. The KVL/KCL equations on the LV side remain the same as in Slides 95
4. Only the winding voltage and winding current transformation equations need to be changed

Winding voltage transformation equation:

$$\begin{bmatrix} \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \\ \overline{E}_{lv}^{(\dot{a})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(an)} \\ \overline{E}_{hv}^{(bn)} \\ \overline{E}_{hv}^{(cn)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right) \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overline{E}_{lv}^{(\dot{a})} \\ \overline{E}_{lv}^{(\dot{b})} \\ \overline{E}_{lv}^{(\dot{c})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(a)} \\ \overline{E}_{hv}^{(b)} \\ \overline{E}_{hv}^{(c)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

$$\Rightarrow \mathbf{C}^{-1} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \mathbf{C} \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \overline{E}_{lv}^{(\dot{1})} \\ \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

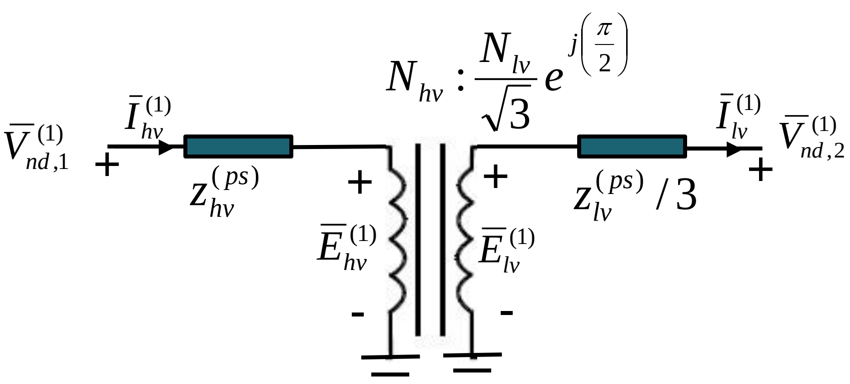
$$\Rightarrow \begin{bmatrix} \overline{E}_{lv}^{(\dot{0})} \\ \alpha^2 \overline{E}_{lv}^{(\dot{1})} \\ \alpha \overline{E}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \overline{E}_{hv}^{(0)} \\ \overline{E}_{hv}^{(1)} \\ \overline{E}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{lv}}{N_{hv}} \right)$$

Sequence-domain Modeling of a Balanced Transformer (Con..)

Winding current transformation equation:

$$\begin{bmatrix} \bar{I}_{lv}^{(\dot{b})} \\ \bar{I}_{lv}^{(\dot{c})} \\ \bar{I}_{lv}^{(\dot{a})} \end{bmatrix} = \begin{bmatrix} \bar{I}_{hv}^{(a)} \\ \bar{I}_{hv}^{(b)} \\ \bar{I}_{hv}^{(c)} \end{bmatrix} \left(\frac{N_{hv}}{N_{lv}} \right) \Rightarrow \begin{bmatrix} \bar{I}_{lv}^{(\dot{0})} \\ \alpha^2 \bar{I}_{lv}^{(\dot{1})} \\ \alpha \bar{I}_{lv}^{(\dot{2})} \end{bmatrix} = \begin{bmatrix} \bar{I}_{hv}^{(0)} \\ \bar{I}_{hv}^{(1)} \\ \bar{I}_{hv}^{(2)} \end{bmatrix} \left(\frac{N_{hv}}{N_{lv}} \right)$$

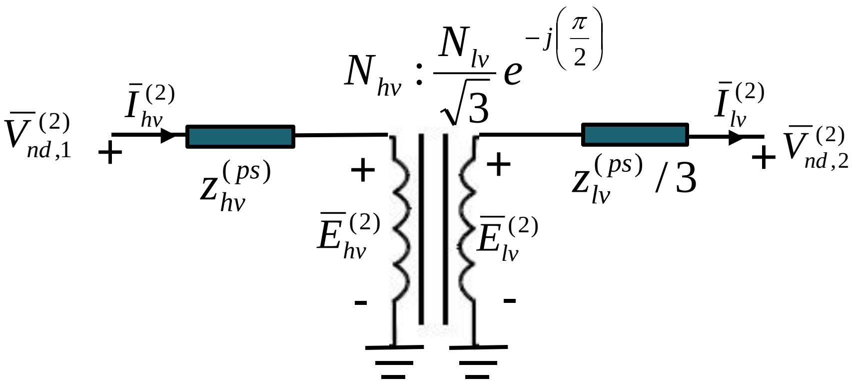
Positive sequence network



$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(\dot{1})}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)}$$

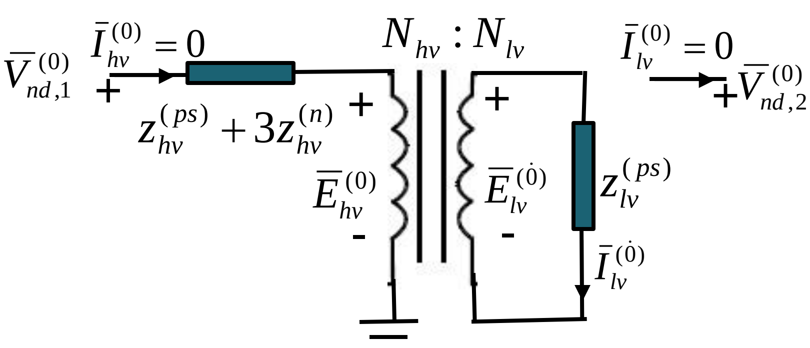
Sequence-domain Modeling of a Balanced Transformer (Con..)

Negative sequence network



$$\overline{E}_{lv}^{(2)} = \frac{\overline{E}_{lv}^{(\dot{2})}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)}$$

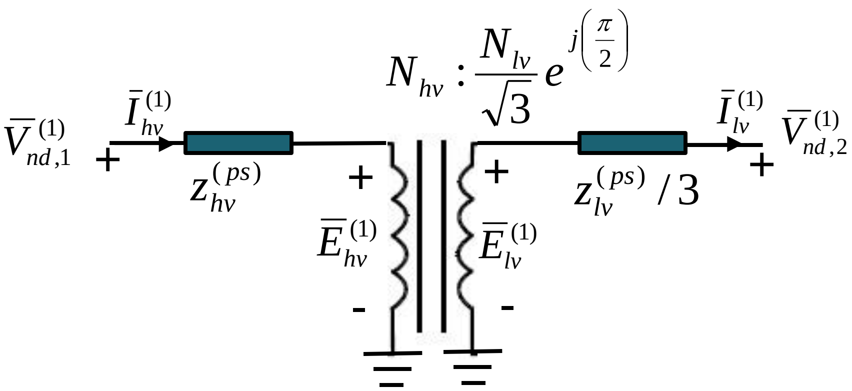
Zero sequence network



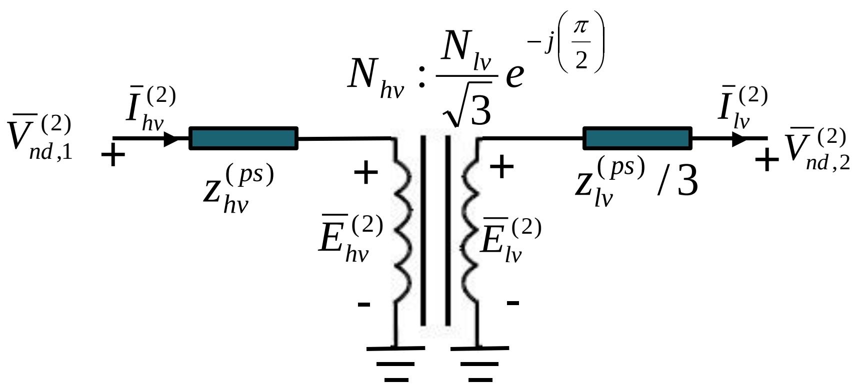
Sequence-domain Modeling of a Balanced Transformer (Con..)

Y-d-9 connection

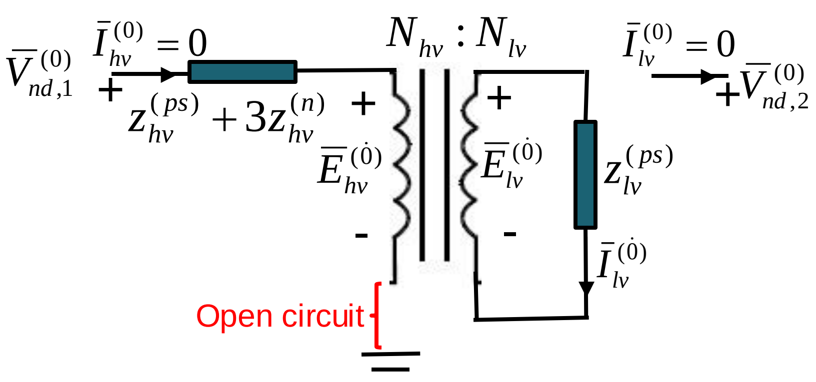
Positive sequence network



Negative sequence network



Zero sequence network

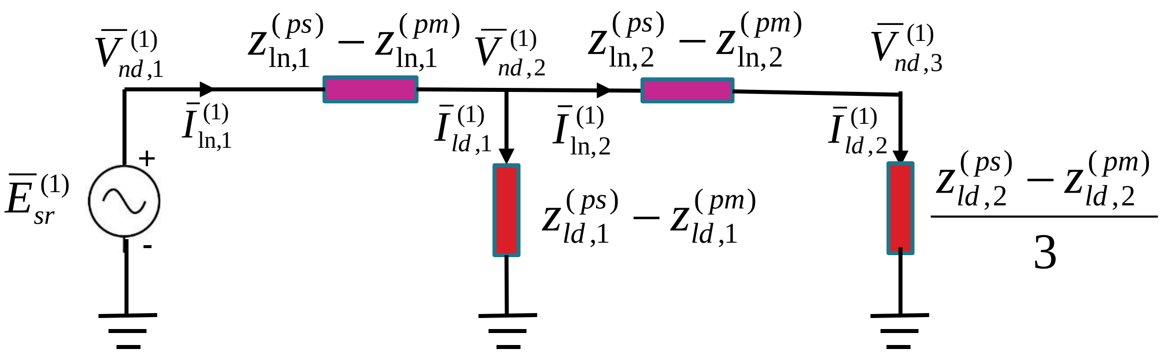


$$\bar{E}_{lv}^{(1)} = \frac{\bar{E}_{lv}^{(1)}}{\sqrt{3}} e^{-j\left(\frac{\pi}{6}\right)} \qquad \bar{E}_{lv}^{(2)} = \frac{\bar{E}_{lv}^{(2)}}{\sqrt{3}} e^{j\left(\frac{\pi}{6}\right)}$$

Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation

The 3-phase circuit of Slide 6:

Equivalent positive sequence circuit



Equivalent negative sequence circuit

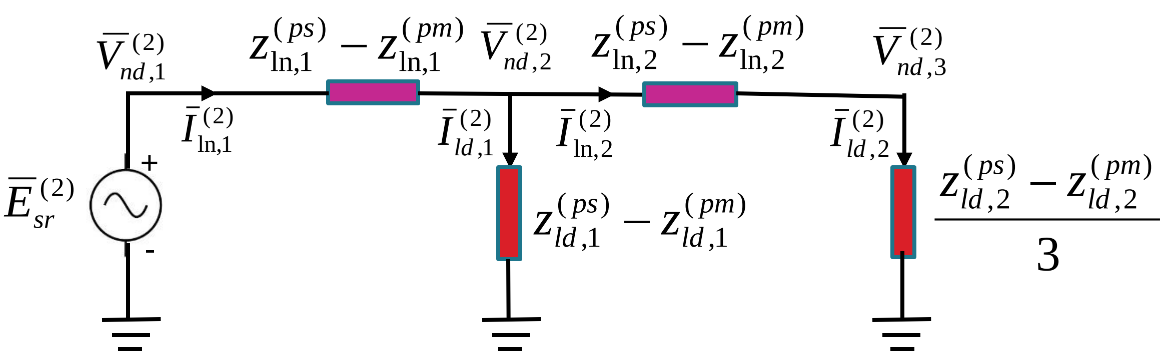
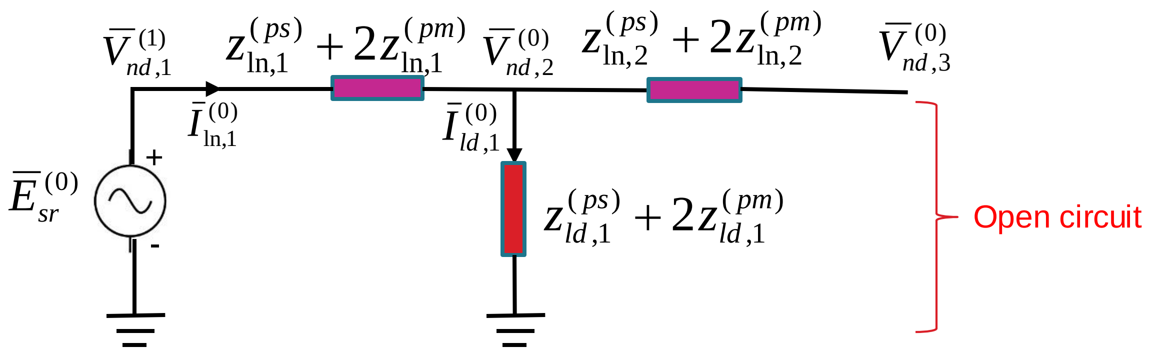


Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation (Con..)

Equivalent zero sequence circuit



The 3-phase circuit of Slide 7:

Equivalent positive sequence circuit

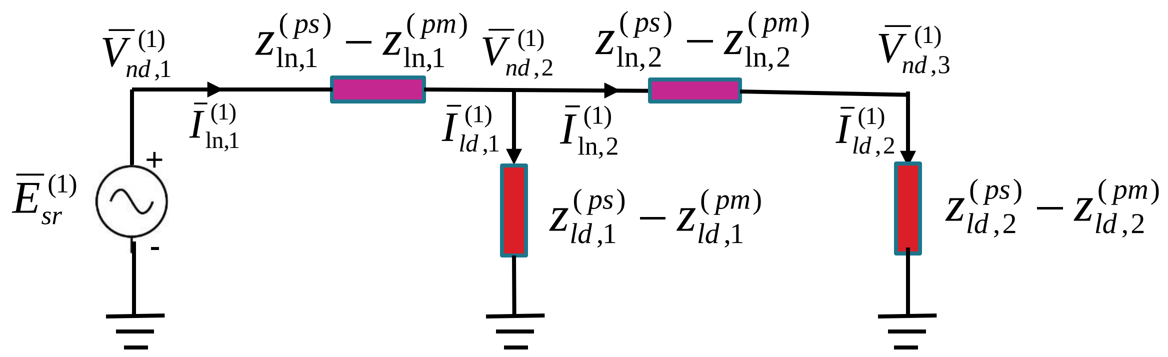


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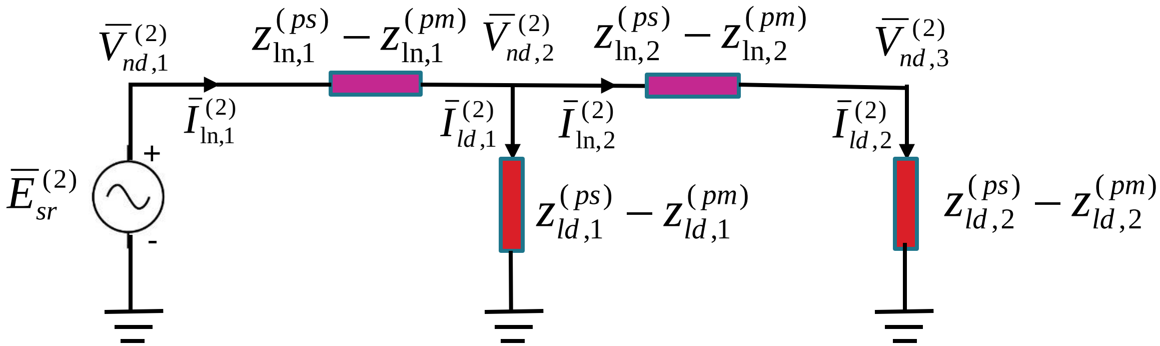
Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation (Con..)

Equivalent negative sequence circuit



Equivalent zero sequence circuit

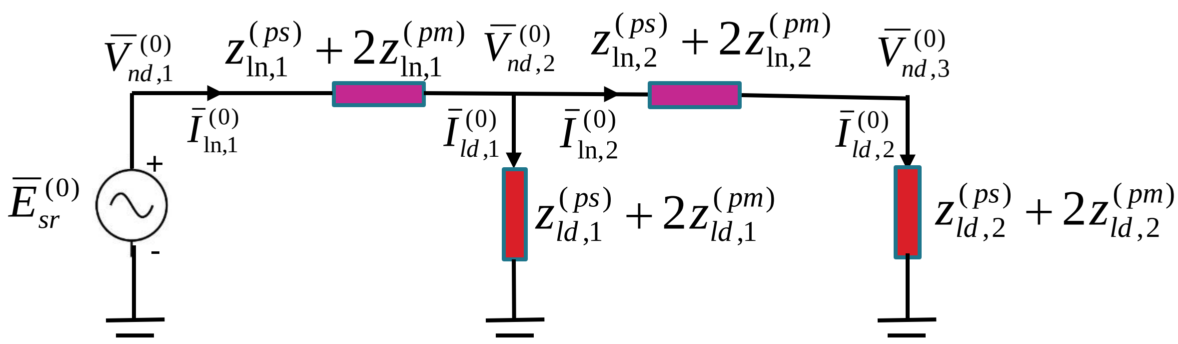
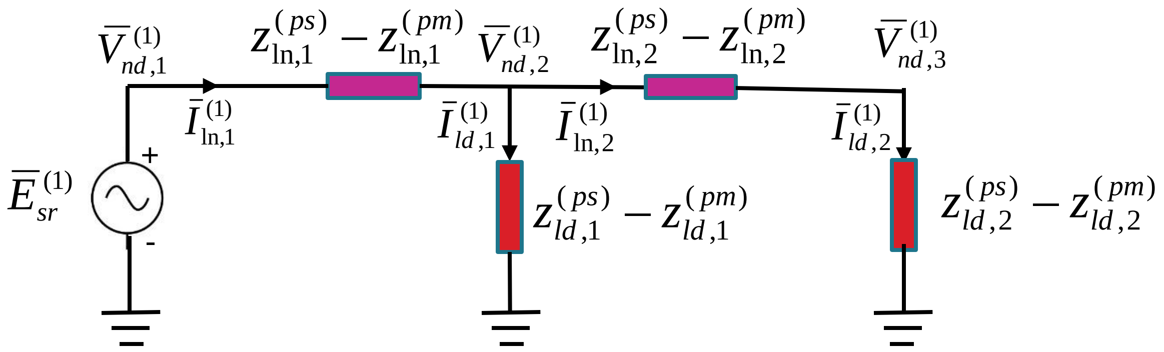


Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation (Con..)

The 3-phase circuit of Slide 8:

Equivalent positive sequence circuit



Equivalent negative sequence circuit

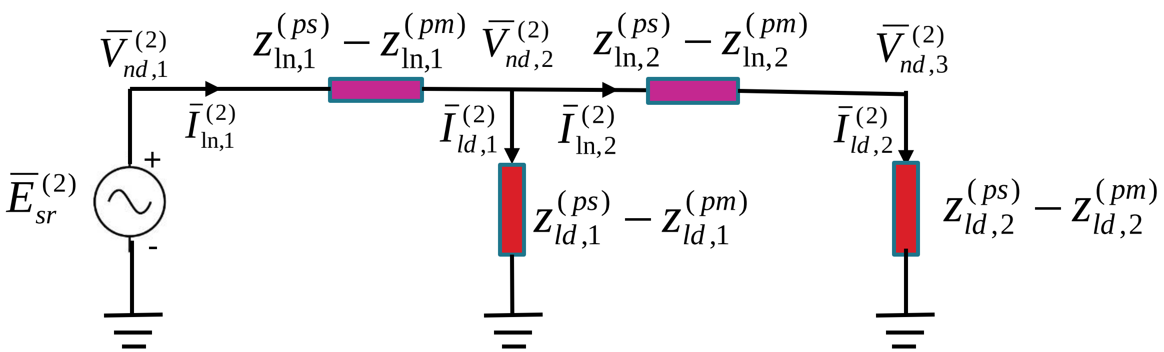
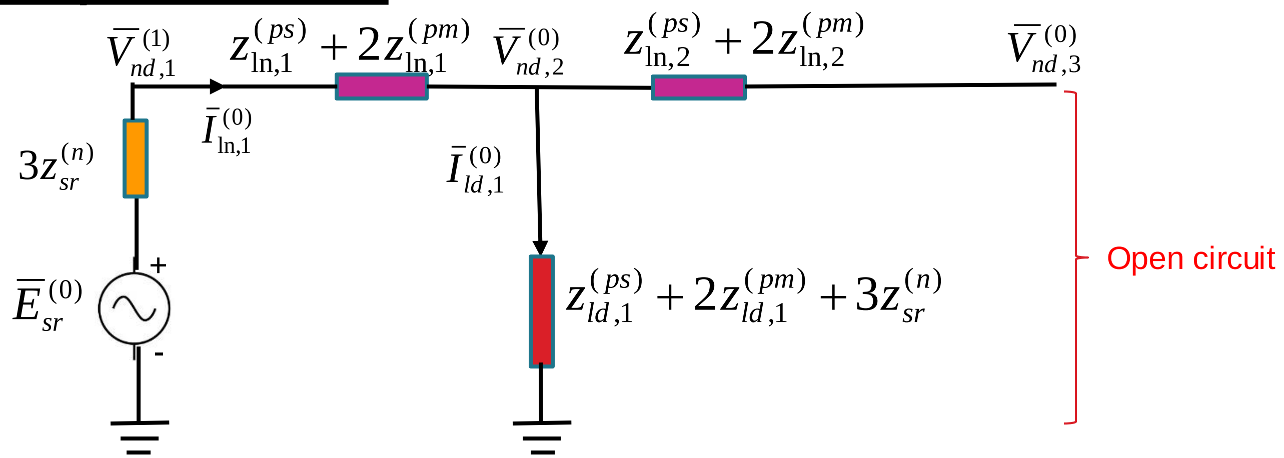


Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation (Con..)

Equivalent zero sequence circuit



The 3-phase circuit of Slide 9:

Equivalent positive sequence circuit

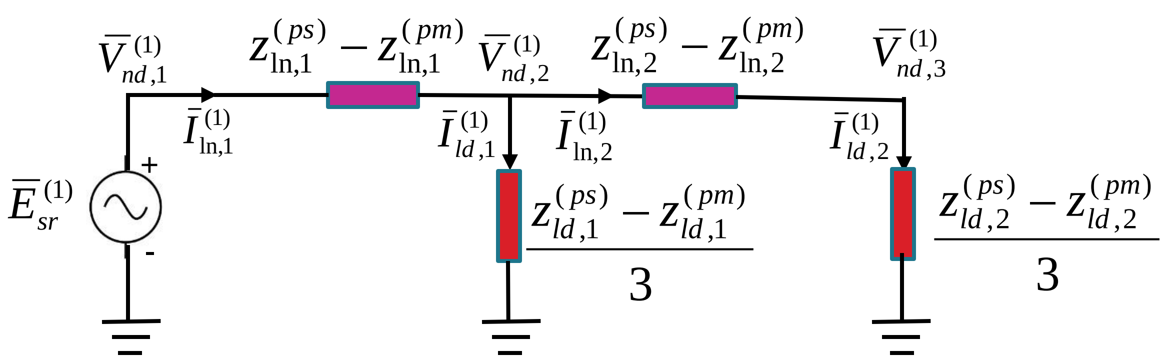
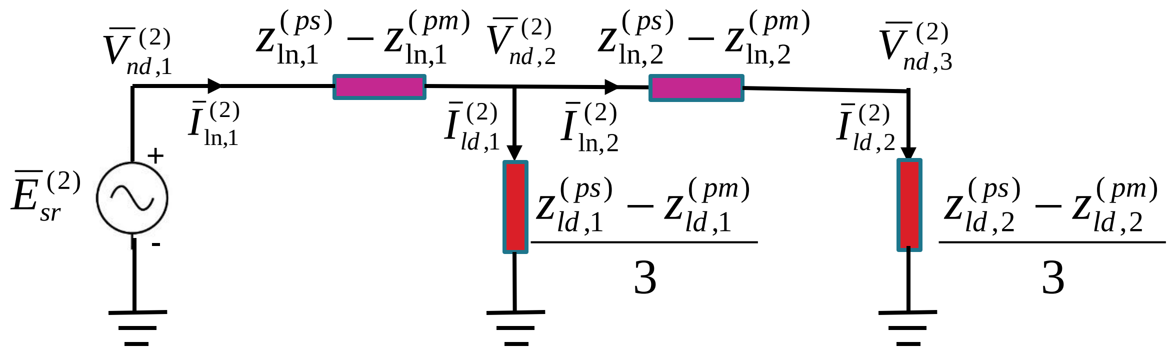


Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation (Con..)

Equivalent negative sequence circuit



Equivalent zero sequence circuit

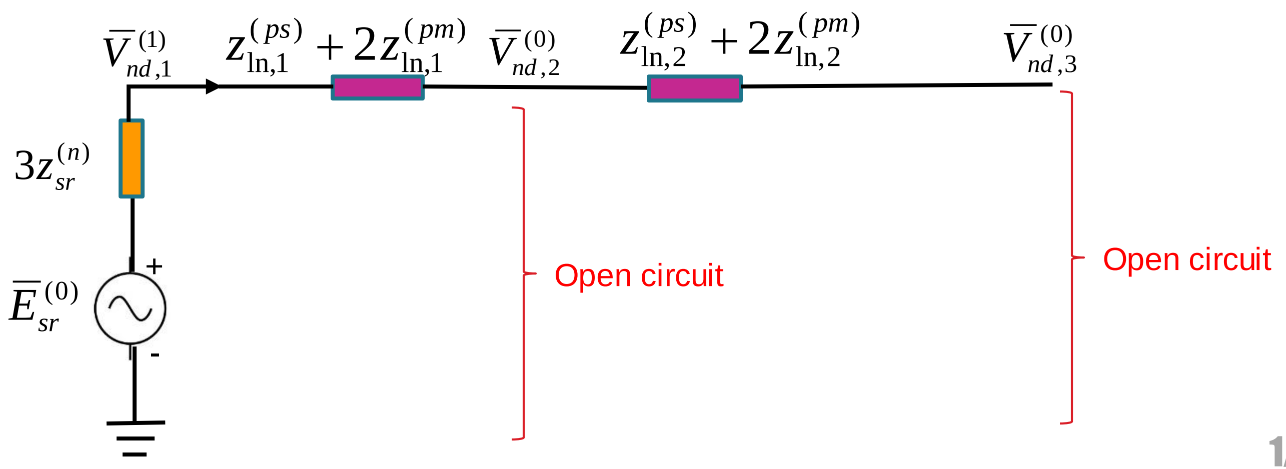


Illustration of 3-Phase Circuit Analysis via Symmetrical Transformation (Con..)

Note:

1. Here the phase impedances of each three-phase element are assumed to be balanced
2. In the case the phase EMFs of the source are also balanced, the negative and zero sequence circuits become immaterial
3. In general, a three-phase network of lines and loads can be divided into two parts
   - The balanced subnetwork
   - The unbalanced subnetwork
   - The decoupled sequence networks are derived only for the balanced portion of the given three-phase network
   - For the unbalanced portion, the mutual coupling may exist among difference sequences
   - The success of the symmetrical transformation technique for analyzing an unbalanced 3-phase network depends upon relative sizes of its balanced and unbalanced parts

THANK YOU