Chapter-13 Probability
Excercise-3

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I. QUESTION 10.13.3.38

In a game, the entry fee is Rs 5.The game consists of a tossing a coin 3 times.If one or two heads show, Sweta gets her entry fee back.If she throws 3 heads, she receives double the entry fees.Otherwise she will lose. For tossing a coin three times, find the probability that she

1) loses the entry fee.
2) gets double entry fee.
3) just gets her entry fee.

II. SOLUTION

Let, X_i be the sequence of independent Bernoulli random varibles.

$$X = \begin{cases} 0, & \text{Tails} \\ 1, & \text{Heads} \end{cases} \tag{1}$$

which means

$$p_X(Z) = \begin{cases} \frac{1}{2} = p, Z = 0 \\ \frac{1}{2} = q, Z = 1 \end{cases} \tag{2}$$

Let, the total number of trials be n and Z be the random variable that represents the number of tails in n trials which is given by:

$$p_X(Z = k) = {}^nC_k p^{n-k} q^k \tag{3}$$

For only tails in 3 trials,

$$p_X(Z = 0) = {}^3C_0 \left(\frac{1}{2}\right)^{3-0} \left(\frac{1}{2}\right)^0 \tag{4}$$

$$= (1)\left(\frac{1}{2}\right)^3 (1) \tag{5}$$

$$= \left(\frac{1}{2}\right)^3 \tag{6}$$

$$= 0.125 \tag{7}$$

Probability that she loses the entry fees = 0.125
For only heads in 3 trials,

$$p_X(Z = 1) = {}^3C_3 \left(\frac{1}{2}\right)^{3-3} \left(\frac{1}{2}\right)^3 \tag{8}$$

$$= (1)(1)\left(\frac{1}{2}\right)^3 \tag{9}$$

$$= \left(\frac{1}{2}\right)^3 \tag{10}$$

$$= 0.125 \tag{11}$$

Probability that she gets double entry fees = 0.125
Then probability for no heads or no tails in 3 trials,

$$\Pr(Z \neq 0, 1) = 1 - p_X(Z = 0) - p_X(Z = 1) \tag{12}$$

$$= 1 - 0.125 - 0.125 \tag{13}$$

$$= 0.750 \tag{14}$$

Probability that she just gets the entry fees = 0.750