

Chapter 13 Probability

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I. QUESTION 10.13.38

In a game, the entry fee is Rs 5. The game consists of a tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she

- 1) loses the entry fee.
- 2) gets double entry fee.
- 3) just gets her entry fee.

Probability that she just gets the entry fees,

$$\Pr(Z \neq 0, 1) = 1 - p_X(Z = 0) - p_X(Z = 1) \quad (9)$$

$$= 1 - 0.125 - 0.125 \quad (10)$$

$$= 0.750 \quad (11)$$

II. SOLUTION

Let, X_i be the sequence of independent Bernoulli random variables.

$$X = \begin{cases} 0, & \text{Tails} \\ 1, & \text{Heads} \end{cases} \quad (1)$$

Let, the total number of trials be n and Z be the random variable that represents the number of tails in n trials which is given by:

$$p_X(Z = k) = {}^nC_k p^{n-k} q^k \quad (2)$$

Probability	Value	Description
$\Pr(TTT)$	$p_X(0)$	loses the entry fees
$\Pr(HHH)$	$p_X(1)$	gets double entry fee
$\Pr(HHT + \dots + TTH)$	$1 - p_X(0) - p_X(1)$	just gets her entry fee

Probability that she loses the entry fees,

$$p_X(Z = 0) = {}^3C_0 \left(\frac{1}{2}\right)^{3-0} \left(\frac{1}{2}\right)^0 \quad (3)$$

$$= \left(\frac{1}{2}\right)^3 \quad (4)$$

$$= 0.125 \quad (5)$$

Probability that she gets double entry fees,

$$p_X(Z = 1) = {}^3C_3 \left(\frac{1}{2}\right)^{3-3} \left(\frac{1}{2}\right)^3 \quad (6)$$

$$= \left(\frac{1}{2}\right)^3 \quad (7)$$

$$= 0.125 \quad (8)$$