Chapter 13 Probability

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Q10.13.3.38: In a game, the entry fee is Rs 5.The game consists of a tossing a coin 3 times.If one or two heads show, Sweta gets her entry fee back.If she throws 3 heads, she receives double the entry fees.Otherwise she will lose. For tossing a coin three times, find the probability that she

1) loses the entry fee.
2) gets double entry fee.
3) just gets her entry fee.

Solution: Let, X_i and Y_i be the sequence of independent Bernoulli random variables and the total number of trials be n and Z be the random variable that represents the number of tails in n trials which is given by:

$$p_X(Z=k) = {}^nC_k p^{n-k} q^k \tag{1}$$

Random variable	denoted	Events
X	0	not loses the entry fees
	1	loses the entry fees
Y	0	not gets double entry fees
	1	gets double entry fees

TABLE 3
Random Variables

Probability that she loses the entry fees,

$$p_X(Z=1) = {}^3C_3 \left(\frac{1}{2}\right)^{3-3} \left(\frac{1}{2}\right)^3 \tag{2}$$

$$= \left(\frac{1}{2}\right)^3 \tag{3}$$

$$= 0.125 \tag{4}$$

Probability that she gets double entry fees,

$$p_Y(Z=1) = {}^3C_3 \left(\frac{1}{2}\right)^{3-3} \left(\frac{1}{2}\right)^3 \tag{5}$$

$$= \left(\frac{1}{2}\right)^3 \tag{6}$$

$$= 0.125 \tag{7}$$

Probability that she just gets the entry fees,

$$\Pr(Y=0|X=0) = 1 - p_X(Z=0) - p_X(Z=1) \tag{8}$$

$$= 1 - 0.125 - 0.125 \tag{9}$$

$$= 0.750 \tag{10}$$