Chapter 9 Gaussian

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Q9.3.6: The probability that a student is not a swimmer is $\frac{1}{5}$ .Then the probability that out of five students, four are swimmers

1) ${}^5C_4 \left(\frac{4}{5}\right)^4 \frac{1}{5}$

2) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$

3) ${}^5C_1 \frac{1}{5} \left(\frac{4}{5}\right)^4$

4) None of these

Solution: The pmf of X is,

Parameter	Value	Description
n	5	number of students
q	$\frac{1}{5}$	probability for not a swimmer
p	$\frac{4}{5}$	probability for a swimmer
k	4	number of swimmers

TABLE 4
GIVEN INFORMATION

$$p_X(k) = {}^nC_k p^k q^{n-k} \tag{1}$$

and the desired probability is

$$p_X(4) = {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^{5-4} \tag{2}$$

$$= 0.4096 \tag{3}$$

Let Y be gaussian variable

$$\mu = np \tag{4}$$

$$= 4 \tag{5}$$

$$\sigma^2 = npq \tag{6}$$

$$= \frac{4}{5} \tag{7}$$

Using Normal distribution at $X = 4$.

$$Z = \frac{X - \mu}{\sigma} \tag{8}$$

$$= \frac{4 - 4}{\sqrt{\frac{4}{5}}} \tag{9}$$

$$= 0 \tag{10}$$

For pdf calculation

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{11}$$

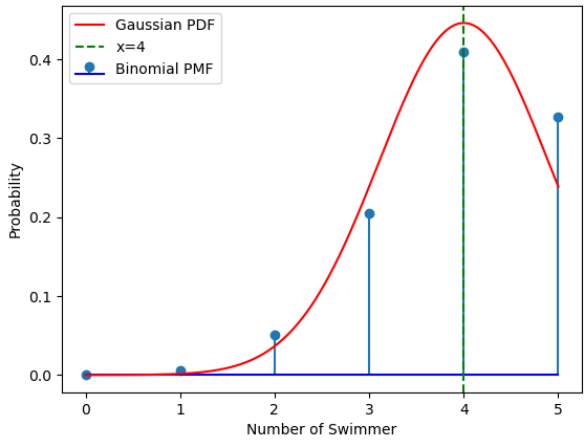

Fig. 4. Binomial pmf vs Gaussian pdf

From the plot, pmf is close to normal distribution pdf.

$$p_Y(4) = p_Z(0) \tag{12}$$

$$= 0.44603 \tag{13}$$

From (3) and (13),

$$p_X(4) \approx p_Y(4) \tag{14}$$

Hence, option (3) is correct