Chapter 13 Probability

Jay Vikrant EE22BTECH11025

I. QUESTION 10.13.3.38

In a game, the entry fee is Rs 5.The game consists of a tossing a coin 3 times.If one or two heads show, Sweta gets her entry fee back.If she throws 3 heads, she receives double the entry fees.Otherwise she will lose. For tossing a coin three times, find the probability that she
1) loses the entry fee.
2) gets double entry fee.
3) just gets her entry fee.

II. SOLUTION

Let, X_i be the sequence of independent Bernoulli random varibles.

$$X = \begin{cases} 0, & \text{Tails} \\ 1, & \text{Heads} \end{cases} \tag{1}$$

Let, the total number of trials be n and Z be the random variable that represents the number of tails in n trials which is given by:

$$p_X (Z = k) = {}^nC_k p^{n-k} q^k \tag{2}$$

Probabilty	Value	Description
$\Pr(TTT)$	$p_X (Z = 0)$	loses the entry fees
$\Pr(HHH)$	$p_X (Z = 1)$	gets double entry fee.
$\Pr(HHT + HTH + ... + TTH)$	$1 - p_X (Z = 0) - p_X (Z = 1)$	just gets her entry fee.

Probability that she loses the entry fees,

$$p_X(Z = 0) = {}^3C_0 \left(\frac{1}{2}\right)^{3-0} \left(\frac{1}{2}\right)^0 \tag{3}$$

$$= \left(\frac{1}{2}\right)^3 \tag{4}$$

$$= 0.125 \tag{5}$$

Probability that she gets double entry fees,

$$p_X(Z = 1) = {}^3C_3 \left(\frac{1}{2}\right)^{3-3} \left(\frac{1}{2}\right)^3 \tag{6}$$

$$= \left(\frac{1}{2}\right)^3 \tag{7}$$

$$= 0.125 \tag{8}$$

Probability that she just gets the entry fees,

$$\Pr(Z \neq 0, 1) = 1 - p_X(Z = 0) - p_X(Z = 1) \tag{9}$$

$$= 1 - 0.125 - 0.125 \tag{10}$$

$$= 0.750 \tag{11}$$