

GATE-EC2023

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Q9.3.6: The frequency of occurrence of 8 symbols (a-h) is shown in the table below. A symbol is chosen and it is determined by asking a series of “yes/no” questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is

Symbols	Frequency of occurrence
a	$\frac{1}{2}$
b	$\frac{1}{4}$
c	$\frac{1}{8}$
d	$\frac{1}{16}$
e	$\frac{1}{32}$
f	$\frac{1}{64}$
g	$\frac{1}{128}$
h	$\frac{1}{128}$

Solution:

Parameter	Value	Description
X	$1 \leq X \leq 8$	number of symbols
b	2	base of algorithm
$H(X)$	$\sum_i p_X(i) \log_b \left(\frac{1}{p_X(i)} \right)$	average number of question

$$H(X) = \sum_i p_X(i) \log_b \left(\frac{1}{p_X(i)} \right) \quad (1)$$

$$= \frac{1}{2} \log_2 (2) + \frac{1}{4} \log_2 (4) + \dots + \frac{1}{128} \log_2 (128) \quad (2)$$

$$= 0.5 + 0.5 + 0.375 + \dots + 0.0078125 \quad (3)$$

$$= 1.984375 \quad (4)$$