Chapter 9 Gaussian

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Q9.3.6: The probability that a student is not a swimmer is $\frac{1}{5}$ .Then the probability that out of five students, four are swimmers

1) ${}^5C_4 \left(\frac{4}{5}\right)^4 \frac{1}{5}$

2) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$

3) ${}^5C_1 \frac{1}{5} \left(\frac{4}{5}\right)^4$

4) None of these

Solution:

Parameter	Value	Description
n	5	number of students
q	$\frac{1}{5}$	not a swimmer
p	$\frac{4}{5}$	swimmer
k	4	number of swimmers
X	$0 \le X \le 5$	X swimmer out of 5
Y	$0 \le Y \le 5$	Gaussian variable
μ	$np = 4$	mean
σ^2	$npq = \frac{4}{5}$	variance
$\Pr(Y)$	$\approx \Pr(X)$	by central limit theorem

TABLE 4
GIVEN INFORMATION

The X is the random variable,We require pmf at $X = 4$,

$$p_X(4) = {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^{5-4} = 0.4096 \tag{1}$$

Using central limit theorem, we can use the gaussian distribution function:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (x \in Y) \tag{2}$$

Now, using Normal distribution at Y=4

$$p_Y(4) = \frac{1}{\sqrt{2\pi\left(\frac{4}{5}\right)}} e^{-\frac{(4-4)^2}{2\left(\frac{4}{5}\right)}} \tag{3}$$

$$= \frac{1}{\sqrt{2\pi\left(\frac{4}{5}\right)}} e^{0} \tag{4}$$

$$= 0.4463 \tag{5}$$

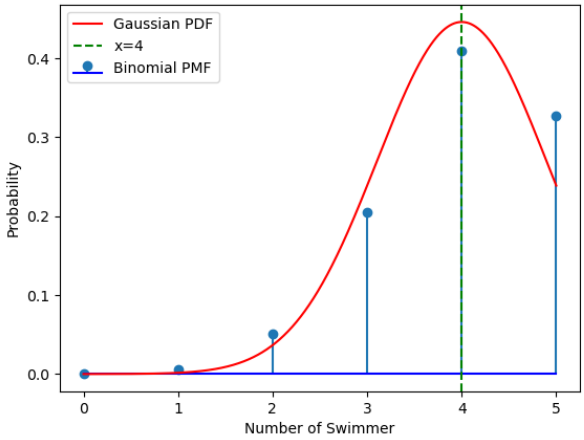

Fig. 4. Binomial pmf vs Gaussian pdf

From the plot also the pmf is close to normal distribution pdf.
Hence, $p_Y(4) \approx p_X(4)$
so, option (3) is correct