Chapter 9 Gaussian

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Q9.3.6: The probability that a student is not a swimmer is $\frac{1}{5}$ .Then the probability that out of five students, four are swimmers

1) ${}^5C_4 \left(\frac{4}{5}\right)^4 \frac{1}{5}$

2) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$

3) ${}^5C_1 \frac{1}{5} \left(\frac{4}{5}\right)^4$

4) None of these

Solution: The pmf of X is,

Parameter	Value	Description
n	5	number of students
q	$\frac{1}{5}$	probability for not a swimmer
p	$\frac{4}{5}$	probability for a swimmer
k	4	number of swimmers

TABLE 4
GIVEN INFORMATION

$$p_X(k) = {}^nC_k p^k q^{n-k} \tag{1}$$

and the desired probability is

$$p_X(4) = {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^{5-4} = 0.4096 \tag{2}$$

Let Y be gaussian variable

$$\mu = np = 4 \tag{3}$$

$$\sigma^2 = npq = \frac{4}{5} \tag{4}$$

by the central limit theorem we can take a random variable Z such that,

$$Z \approx \frac{X - \mu}{\sigma} \tag{5}$$

Using Normal distribution at $X = 4$.

$$Z = \frac{X - \mu}{\sigma} = \frac{4 - 4}{\sqrt{\frac{4}{5}}} = 0 \tag{6}$$

For pdf calculation

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{7}$$

$$\implies p_Z(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2}} \tag{8}$$

and the desired probability is,

$$p_Z(4) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{0}{2}} = \frac{1}{\sqrt{2\pi\sigma^2}} = 0.44603 \tag{9}$$

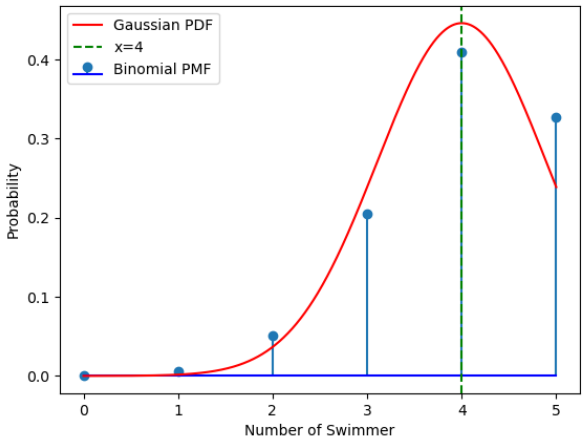

Fig. 4. Binomial pmf vs Gaussian pdf

From the plot also, pmf is close to normal distribution pdf.

$$p_Y(4) = p_Z(0) = 0.44603 \tag{10}$$

From (2) and (10),

$$p_X(4) \approx p_Y(4) \tag{11}$$

Hence, option (3) is correct