Chapter-13 Probability

Excercise-3

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I. Question 10.13.3.38

In a game, the entry fee is Rs 5.The game consists of a tossing a coin 3 times.If one or two heads show, Sweta gets her entry fee back.If she throws 3 heads, she receives double the entry fees.Otherwise she will lose. For tossing a coin three times, find the probability that she

1) loses the entry fee.
2) gets double entry fee.
3) just gets her entry fee.

II. Solution

Let, p_X be the sequence of independent Bernoulli random varibles for which Sweta gets no money back.

$$X = \begin{cases} 0, & \text{no money back} \\ 1, & \text{money back} \end{cases} \tag{1}$$

which means

$$p_X(0) = \frac{1}{2} \tag{2}$$
$$p_X(1) = \frac{1}{2} \tag{3}$$
$$\tag{4}$$

Let, the total number of trials be n and Z_1 be the random variable that represents the number of tails in n trials which is given by:

$$p_X(Z_1 = k) = {}^{n}C_k p^{n-k} q^k \tag{5}$$

where,

$$Z_1 = X_1 + X_2 + ... + X_n \tag{6}$$

For no money back in 3 trials,

$$p_X(Z_1 = 0) = p_X(X_1 = 0, X_2 = 0..., X_4 = 0) \tag{7}$$
$$= {}^{3}C_0 \left(\frac{1}{2}\right)^{3-0} \left(\frac{1}{2}\right)^{0} \tag{8}$$
$$= (1)\left(\frac{1}{2}\right)^{3} (1) \tag{9}$$
$$= \left(\frac{1}{2}\right)^{3} \tag{10}$$
$$= 0.125 \tag{11}$$

Now, Let p_Y be the sequence of independent Bernoulli random varibles for which sweta gets double money back. Then,

$$X = \begin{cases} 0, & \text{no double money back} \\ 1, & \text{double money back} \end{cases} \tag{12}$$

which means

$$p_Y(0) = \frac{1}{2} \tag{13}$$
$$p_Y(1) = \frac{1}{2} \tag{14}$$
$$\tag{15}$$

Let, the total number of trials be n and Z_2 be the random variable that represents the number of heads in n trials which is given by:

$$p_Y(Z_2 = k) = {}^{n}C_k p^{n-k} q^k \tag{16}$$

where,

$$Z_2 = Y_1 + Y_2 + ... + Y_n \tag{17}$$

For double money back in 3 trials,

$$
\begin{aligned}
p_Y(Z_2 = 1) &= p_Y(Y_1 = 1, Y_2 = 1, Y_3 = 1) && (18)\\
&= {}^3C_3 \left(\frac{1}{2}\right)^{3-3} \left(\frac{1}{2}\right)^3 && (19)\\
&= (1)\,(1) \left(\frac{1}{2}\right)^3 && (20)\\
&= \left(\frac{1}{2}\right)^3 && (21)\\
&= 0.125 && (22)
\end{aligned}
$$

From (11) and (22), The probability of only money back is

$$
\begin{aligned}
&1 - p_X(Z_1 = 0) - p_Y(Z_2 = 1) && (23)\\
&= 1 - 0.125 - 0.125 && (24)\\
&= 0.750 && (25)
\end{aligned}
$$