Solution to 1.5.10

Question: Verify that:

$$AE_3 = AF_3 = m, BD_3 = BF_3 = n, CD_3 = CE_3 = p. \tag{1}$$

Solution:

The coordinates of the points of contact of the circle and the triangle are:

$$\mathbf{D}_3 = \begin{pmatrix} \frac{-366\sqrt{74}-406\sqrt{122}-488\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \\ \frac{-610\sqrt{74}-170\sqrt{122}+732\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \end{pmatrix} \text{ from 1.5.8} \tag{2}$$

$$\mathbf{E}_3 = \begin{pmatrix} \frac{-111-20\sqrt{37}+5\sqrt{2257}}{74} \\ \frac{185+28\sqrt{37}-7\sqrt{2257}}{74} \end{pmatrix} \text{ from 1.5.9} \tag{3}$$

$$\mathbf{F}_3 = \begin{pmatrix} \frac{-2-\sqrt{37}+\sqrt{61}}{2} \\ \frac{-6-\sqrt{37}+\sqrt{61}}{2} \end{pmatrix} \text{ from 1.5.9} \tag{4}$$

Length of line segment between two points is given by:

1)

$$AE_3 = \sqrt{(\mathbf{E_3}-\mathbf{A})^\top(\mathbf{E_3}-\mathbf{A})} \tag{5}$$

$$\mathbf{E_3}-\mathbf{A} = \begin{pmatrix} -0.136-1 \\ -2.136+1 \end{pmatrix} \tag{6}$$

$$\implies AE_3 = \sqrt{\begin{pmatrix} -1.136 & -1.136 \end{pmatrix}\begin{pmatrix} -1.136 \\ -1.136 \end{pmatrix}} \tag{7}$$

$$= 1.607 \tag{8}$$

$$AF_3 = \sqrt{(\mathbf{F_3}-\mathbf{A})^\top(\mathbf{F_3}-\mathbf{A})} \tag{9}$$

$$\mathbf{F_3}-\mathbf{A} = \begin{pmatrix} 0.066-1 \\ 0.308+1 \end{pmatrix} \tag{10}$$

$$\implies AF_3 = \sqrt{\begin{pmatrix} -0.934 & 1.308 \end{pmatrix}\begin{pmatrix} -0.934 \\ 1.308 \end{pmatrix}} \tag{11}$$

$$= 1.607 \tag{12}$$

$\therefore AE_3 = AF_3 = m$ is verified.

2)

$$BD_3 = \sqrt{(\mathbf{D_3}-\mathbf{B})^\top(\mathbf{D_3}-\mathbf{B})} \tag{13}$$

$$\mathbf{D_3}-\mathbf{B} = \begin{pmatrix} -3.367+4 \\ -0.967-6 \end{pmatrix} \tag{14}$$

$$\implies BD_3 = \sqrt{\begin{pmatrix} 0.633 & 6.967 \end{pmatrix}\begin{pmatrix} 0.633 \\ 6.967 \end{pmatrix}} \tag{15}$$

$$= 6.995 \tag{16}$$

$$BF_3 = \sqrt{(\mathbf{F_3}-\mathbf{B})^\top(\mathbf{F_3}-\mathbf{B})} \tag{17}$$

$$\mathbf{F_3}-\mathbf{B} = \begin{pmatrix} 0.066+4 \\ 0.308-6 \end{pmatrix} \tag{18}$$

$$\implies BF_3 = \sqrt{\begin{pmatrix} 4.066 & -5.692 \end{pmatrix}\begin{pmatrix} 4.066 \\ -5.692 \end{pmatrix}} \tag{19}$$

$$= 6.995 \tag{20}$$

$\therefore BD_3 = BF_3 = n$ is verified.

3)

$$CD_3 = \sqrt{(\mathbf{D_3}-\mathbf{C})^\top(\mathbf{D_3}-\mathbf{C})} \tag{21}$$

$$\mathbf{D_3}-\mathbf{C} = \begin{pmatrix} -3.367+3 \\ -0.967+5 \end{pmatrix} \tag{22}$$

$$\implies CD_3 = \sqrt{\begin{pmatrix} -0.367 & 4.033 \end{pmatrix}\begin{pmatrix} -0.367 \\ 4.033 \end{pmatrix}} \tag{23}$$

$$= 4.0499 \tag{24}$$

$$CE_3 = \sqrt{(\mathbf{E_3}-\mathbf{C})^\top(\mathbf{E_3}-\mathbf{C})} \tag{25}$$

$$\mathbf{E_3}-\mathbf{C} = \begin{pmatrix} -0.136+3 \\ -2.136+5 \end{pmatrix} \tag{26}$$

$$\implies CE_3 = \sqrt{\begin{pmatrix} 2.864 & 2.864 \end{pmatrix}\begin{pmatrix} 2.864 \\ 2.864 \end{pmatrix}} \tag{27}$$

$$= 4.0499 \tag{28}$$

$\therefore CD_3 = CE_3 = p$ is verified.