Assignment 1
Probability and Random Processes

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EE22BTECH11038

Question 1.4.2
Find the intersection $\mathbf{O}$ of the perpendicular bisectors of $\mathbf{AB}$ and $\mathbf{AC}$.
Solution:
Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Vector equation of perpendicular bisector of $\mathbf{A} - \mathbf{B}$ is

$$(\mathbf{A} - \mathbf{B})^\top \left(\mathbf{x} - \frac{\mathbf{A} + \mathbf{B}}{2} \right) = 0 \tag{1}$$

where,

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} -3 \\ 5 \end{pmatrix} \tag{3}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{5}$$

$$\implies (\mathbf{A} - \mathbf{B})^\top = \begin{pmatrix} 5 & -7 \end{pmatrix} \tag{6}$$

$\therefore$ The vector equation of $\mathbf{O} - \mathbf{F}$ is

$$\begin{pmatrix} 5 & -7 \end{pmatrix} \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -3 \\ 5 \end{pmatrix} \right) = 0 \tag{7}$$

$$\implies \begin{pmatrix} 5 & -7 \end{pmatrix} \mathbf{x} = \frac{1}{2} \begin{pmatrix} 5 & -7 \end{pmatrix} \begin{pmatrix} -3 \\ 5 \end{pmatrix} \tag{8}$$

Performing matrix multiplication yields

$$\begin{pmatrix} 5 & -7 \end{pmatrix} \mathbf{x} = -25 \tag{9}$$

Vector equation of perpendicular bisector of $\mathbf{A} - \mathbf{C}$ is

$$(\mathbf{A} - \mathbf{C})^\top \left(\mathbf{x} - \frac{\mathbf{A} + \mathbf{C}}{2} \right) = 0 \tag{10}$$

where,

$$\mathbf{A} + \mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} -2 \\ -6 \end{pmatrix} \tag{12}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{14}$$

$$\implies (\mathbf{A} - \mathbf{C})^\top = \begin{pmatrix} 4 & 4 \end{pmatrix} \tag{15}$$

$\therefore$ The vector equatio of $\mathbf{O} - \mathbf{E}$ is

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -2 \\ -6 \end{pmatrix} \right) = 0 \tag{16}$$

$$\implies \begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = \frac{1}{2} \begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} -2 \\ -6 \end{pmatrix} \tag{17}$$

Performing matrix multiplication yields

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = -16 \tag{18}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = -4 \tag{19}$$

Thus,

$$\begin{pmatrix} 5 & -7 & -25 \\ 1 & 1 & -4 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 5R_2 - R_1} \begin{pmatrix} 5 & -7 & -25 \\ 0 & 12 & 5 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} 5 & -7 & -25 \\ 0 & 12 & 5 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{12}{7} R_1 + R_2} \begin{pmatrix} \frac{60}{7} & 0 & \frac{-265}{7} \\ 0 & 12 & 5 \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} \frac{60}{7} & 0 & \frac{-265}{7} \\ 0 & 12 & 5 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow \frac{7}{60} R_1]{R_2 \leftarrow \frac{1}{12} R_2} \begin{pmatrix} 1 & 0 & \frac{-53}{12} \\ 0 & 1 & \frac{5}{12} \end{pmatrix} \tag{22}$$

$$\therefore \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{23}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{24}$$

Therefore, the point of intersection of perpendicular bisectors of $\mathbf{A} - \mathbf{B}$ and $\mathbf{A} - \mathbf{C}$ is $\mathbf{O} = \frac{1}{12} \begin{pmatrix} -53 \\ 5 \end{pmatrix}$

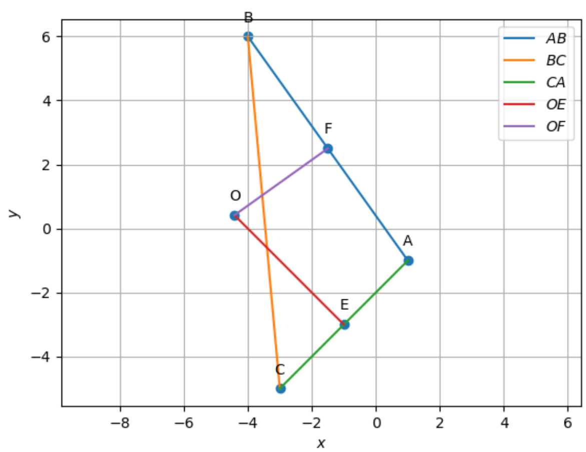

Fig. 0. $\mathbf{O} - \mathbf{E}$ and $\mathbf{O} - \mathbf{F}$ are perpendicular bisectors of $\mathbf{A} - \mathbf{C}$ and $\mathbf{A} - \mathbf{B}$ respectively