Question 1.5.1

Suppose the equations AB, BC and CA are respectively given by

$$\mathbf{n}_i^\top \mathbf{x} = c_i \quad i = 1, 2, 3 \tag{1}$$

The equations of the respective angle bisectors are then given by

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \quad i \neq j \tag{2}$$

Substitute numerical values and find the equations of the angle bisectors of A, B and C.

Solution: The internal angle bisector is obtained from the set of two bisectors by using:

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \quad i \neq j \tag{3}$$

This can be transformed to the normal equation of angle bisectors as follows

$$\left(\frac{\mathbf{n}_i^\top}{\|\mathbf{n}_i\|} - \frac{\mathbf{n}_j^\top}{\|\mathbf{n}_j\|}\right)\mathbf{x} = \frac{c_i}{\|\mathbf{n}_i\|} - \frac{c_j}{\|\mathbf{n}_j\|} \tag{4}$$

i and j values correspond to the sides including the angle

1) Angle Bisector of A

$$\left(\frac{\mathbf{n}_3^\top}{\|\mathbf{n}_3\|} - \frac{\mathbf{n}_1^\top}{\|\mathbf{n}_1\|}\right)\mathbf{x} = \frac{c_3}{\|\mathbf{n}_3\|} - \frac{c_1}{\|\mathbf{n}_1\|} \tag{5}$$

on substitution we obtain

$$\left(\frac{7}{\sqrt{74}} - \frac{1}{\sqrt{2}} \quad \frac{5}{\sqrt{74}} + \frac{1}{\sqrt{2}}\right)\mathbf{x} = \frac{2}{\sqrt{74}} - \frac{2}{\sqrt{2}} \tag{6}$$

2) Angle Bisector of B

$$\left(\frac{\mathbf{n}_2^\top}{\|\mathbf{n}_2\|} - \frac{\mathbf{n}_1^\top}{\|\mathbf{n}_1\|}\right)\mathbf{x} = \frac{c_2}{\|\mathbf{n}_2\|} - \frac{c_1}{\|\mathbf{n}_1\|} \tag{7}$$

on substitution we obtain

$$\left(\frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} \quad \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}}\right)\mathbf{x} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \tag{8}$$

3) Angle Bisector of C

$$\left(\frac{\mathbf{n}_2^\top}{\|\mathbf{n}_2\|} - \frac{\mathbf{n}_3^\top}{\|\mathbf{n}_3\|}\right)\mathbf{x} = \frac{c_2}{\|\mathbf{n}_2\|} - \frac{c_3}{\|\mathbf{n}_3\|} \tag{9}$$

on substitution we obtain

$$\left(\frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}}\right)\mathbf{x} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \tag{10}$$

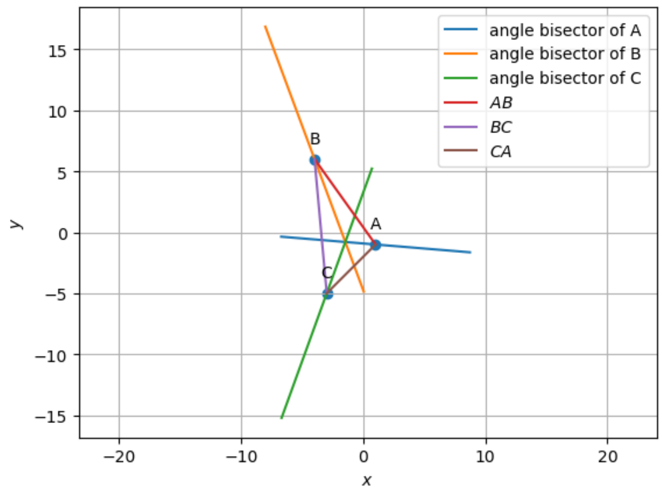

Fig. 1. Angle bisectors plotted using python