Solution of 1.2.2

Dhruv Parashar*

Question:- We are given the three vertices of a triangle(A, B, C) and the midpoints $\mathbf{D}, \mathbf{E}, \mathbf{F}$ of the sides AB, BC, AC respectively. We have to find the equations of the sides AD, BE, CF.

Solution:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The mid points $\mathbf{D}, \mathbf{E}, \mathbf{F}$ of sides AB, BC, AC are :-

$$\mathbf{D} = \frac{1}{2}\begin{pmatrix} -7 \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{5}$$

$$\mathbf{F} = \frac{1}{2}\begin{pmatrix} -3 \\ 5 \end{pmatrix} \tag{6}$$

Now, the direction vector of line $FC(\mathbf{m})$ is :-

$$\mathbf{m} = \mathbf{F} - \mathbf{C} \tag{7}$$

$$\implies \mathbf{m} = \frac{1}{2}\begin{pmatrix} 3 \\ 15 \end{pmatrix} \tag{8}$$

Now, we have to find $\mathbf{n}$,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{m} \tag{9}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 15 \end{pmatrix} \tag{10}$$

$$= \frac{1}{2}\begin{pmatrix} 15 \\ -3 \end{pmatrix} \tag{11}$$

Normal form of line CF is :

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{C}) = 0 \tag{12}$$

$$\mathbf{n}^\top\mathbf{x} = \mathbf{n}^\top\mathbf{C} \tag{13}$$

$$\frac{1}{2}\begin{pmatrix} 15 & -3 \end{pmatrix}\mathbf{x} = \frac{1}{2}\begin{pmatrix} 15 & -3 \end{pmatrix}\begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} 15 & -3 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 15 & -3 \end{pmatrix}\begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} 15 & -3 \end{pmatrix}\mathbf{x} = -30 \tag{16}$$

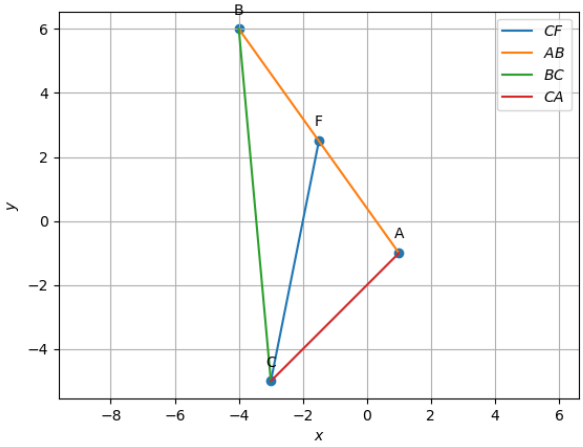

Fig. 0. Line CF