

1.5.5 Repeat the above exercise for the sides AB and AC .

Solution: We know the value of \mathbf{I} is

$$\mathbf{I} = \frac{1}{\sqrt{37} + 4 + \sqrt{61}} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \quad (1)$$

from the problem 1.5.2 .

1. The equation of AB is:

$$\begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{x} - 2 = 0 \quad (2)$$

Let r_1 be the distance between \mathbf{I} and AB , then

$$r_1 = \frac{\left| \begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{I} - 2 \right|}{\left\| \begin{pmatrix} 7 \\ 5 \end{pmatrix} \right\|} \quad (3)$$

$$= \frac{\left| \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} 7 & 5 \end{pmatrix} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} - 2 \right|}{\sqrt{7^2 + 5^2}} \quad (4)$$

$$= \frac{\frac{2\sqrt{61}-46\sqrt{37}+8}{\sqrt{37}+4+\sqrt{61}} - 2}{\sqrt{74}} \quad (5)$$

$$= \frac{48\sqrt{37}}{\sqrt{74}(\sqrt{37} + 4 + \sqrt{61})} \quad (6)$$

$$= \frac{48}{\sqrt{2}(\sqrt{37} + 4 + \sqrt{61})} \quad (7)$$

$$= \frac{24\sqrt{2}}{\sqrt{37} + 4 + \sqrt{61}} \quad (8)$$

$$= 1.8969 \quad (9)$$

2. Similarly, the equation of AC is

$$\begin{pmatrix} 4 & -4 \end{pmatrix} \mathbf{x} - 8 = 0 \quad (10)$$

Let r_2 be the distance between \mathbf{I} and AC , then

$$r_2 = \frac{\left| \begin{pmatrix} 4 & -4 \end{pmatrix} \mathbf{I} - 8 \right|}{\left\| \begin{pmatrix} 4 \\ -4 \end{pmatrix} \right\|} \quad (11)$$

$$= \frac{\left| \frac{1}{\sqrt{37+4+\sqrt{61}}} \begin{pmatrix} 4 & -4 \end{pmatrix} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix} - 8 \right|}{\sqrt{4^2 + (-4)^2}} \quad (12)$$

$$= \frac{\left| \frac{8\sqrt{61}+8\sqrt{37}-160}{\sqrt{37+4+\sqrt{61}}} - 8 \right|}{4\sqrt{2}} \quad (13)$$

$$= \frac{192}{4\sqrt{2}(\sqrt{37}+4+\sqrt{61})} \quad (14)$$

$$= \frac{48}{\sqrt{2}(\sqrt{37}+4+\sqrt{61})} \quad (15)$$

$$= \frac{24\sqrt{2}}{\sqrt{37}+4+\sqrt{61}} \quad (16)$$

$$= 1.8969 \quad (17)$$