Q. Find the equations of $AD$ , $BE$ and $CF$.
**Solution:** : $\mathbf{D}$,$\mathbf{E}$,$\mathbf{F}$ are the midpoints of $BC$,$CA$,$AB$ respectively, then

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{1}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{2}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \tag{3}$$

1) The normal equation for the median $AD$ is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{4}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \tag{5}$$

We have to find the $\mathbf{n}$ so that we can find $\mathbf{n}^\top$. Since,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{6}$$

Here $\mathbf{m} = \mathbf{D} - \mathbf{A}$ for median $AD$

$$\mathbf{m} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{8}$$

Since,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{9}$$

$$\implies \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \tag{11}$$

Hence the normal equation of median $AD$ is

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{12}$$

$$\implies \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \tag{13}$$

2) The normal equation for the median $BE$ is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{B}\right) = 0 \tag{14}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{B} \tag{15}$$

Here $\mathbf{m} = \mathbf{E} - \mathbf{B}$ for median $BE$

$$\mathbf{m} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{16}$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \tag{17}$$

Since,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{18}$$

$$\implies \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -9 \end{pmatrix} \tag{19}$$

$$= \begin{pmatrix} -9 \\ -3 \end{pmatrix} \tag{20}$$

Hence the normal equation of median $BE$ is

$$\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{21}$$

$$\implies \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18 \tag{22}$$

3) The normal equation for the median $CF$ is

$$\mathbf{n}^\top \left( \mathbf{x} - \mathbf{C} \right) = 0 \tag{23}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{C} \tag{24}$$

Here $\mathbf{m} = \mathbf{F} - \mathbf{C}$ for median $CF$

$$\mathbf{m} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{25}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \tag{26}$$

Since,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{27}$$

$$\implies \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \tag{28}$$

$$= \begin{pmatrix} \frac{15}{2} \\ \frac{-3}{2} \end{pmatrix} \tag{29}$$

Hence the normal equation of median $CF$ is

$$\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{30}$$

$$\implies \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15 \tag{31}$$

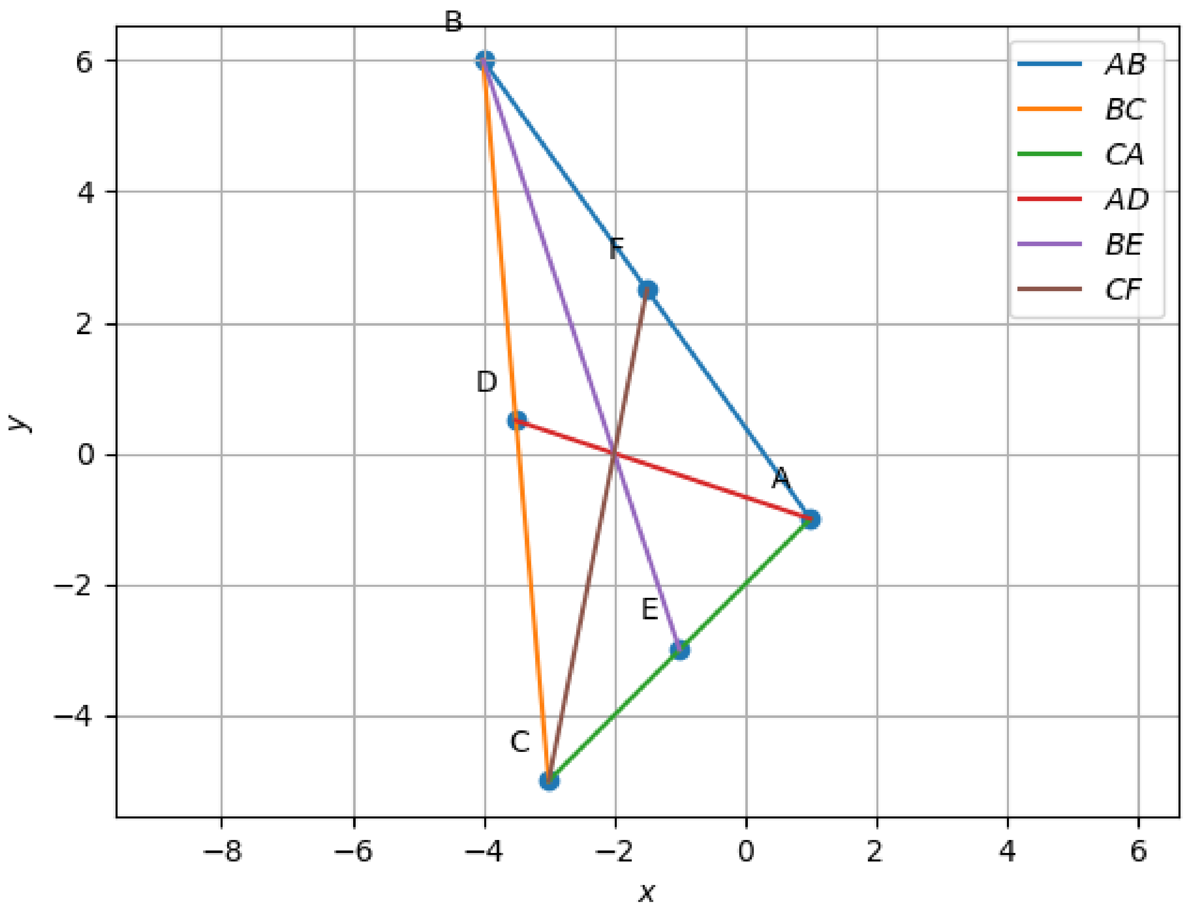


Fig. 3. Medians $AD$ , $BE$ and $CF$