Assignment 1
Probability And Random Processes

Barath Surya M
EE22BTECH11014

1 Question 1.1.7

find the angles $\mathbf{A}, \mathbf{B}, \mathbf{C}$, given that

$$\cos A \triangleq \frac{(\mathbf{B}-\mathbf{A})\top(\mathbf{C}-\mathbf{A})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|} \tag{1}$$

2 Solution

From the given values of $\mathbf{A}, \mathbf{B}, \mathbf{C}$,

1) Finding the value of angle A

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{2}$$

and

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{3}$$

also calculating the values of norms

$$\|\mathbf{B}-\mathbf{A}\| = \sqrt{74} \tag{4}$$

$$\|\mathbf{C}-\mathbf{A}\| = \sqrt{32} \tag{5}$$

and by doing matrix multiplication we get,

$$\begin{aligned}(\mathbf{B}-\mathbf{A})^{\top}(\mathbf{C}-\mathbf{A}) &= \begin{pmatrix} -5 & 7 \end{pmatrix}\begin{pmatrix} -4 \\ -4 \end{pmatrix} \\ &= -8\end{aligned} \tag{6}$$

so

$$\cos A = \frac{-8}{\sqrt{74}\sqrt{32}} \tag{7}$$

$$= \frac{-1}{\sqrt{37}} \tag{8}$$

$$\implies A = \cos^{-1}\frac{-1}{\sqrt{37}} \tag{9}$$

2) Finding the value of angle B

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{10}$$

and

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{11}$$

also calculating the values of norms

$$\|\mathbf{C}-\mathbf{B}\| = \sqrt{122} \tag{12}$$

$$\|\mathbf{A}-\mathbf{B}\| = \sqrt{74} \tag{13}$$

and by doing matrix multiplication we get,

$$\begin{aligned}(\mathbf{C}-\mathbf{B})^{\top}(\mathbf{A}-\mathbf{B}) &= \begin{pmatrix} 1 & -11 \end{pmatrix}\begin{pmatrix} 5 \\ -7 \end{pmatrix} \\ &= 82\end{aligned} \tag{14}$$

so

$$\cos B = \frac{82}{\sqrt{74}\sqrt{122}} \tag{15}$$

$$= \frac{41}{\sqrt{2257}} \tag{16}$$

$$\implies B = \cos^{-1}\frac{41}{\sqrt{2257}} \tag{17}$$

3) Finding the value of angle C

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{18}$$

and

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \tag{19}$$

also calculating the values of norms

$$\|\mathbf{A}-\mathbf{C}\| = \sqrt{32} \tag{20}$$

$$\|\mathbf{B}-\mathbf{C}\| = \sqrt{122} \tag{21}$$

and by doing matrix multiplication we get,

$$\begin{aligned}(\mathbf{A}-\mathbf{C})^{\top}(\mathbf{B}-\mathbf{C}) &= \begin{pmatrix} 4 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 11 \end{pmatrix} \\ &= 40\end{aligned} \tag{22}$$

so

$$\cos C = \frac{40}{\sqrt{32}\sqrt{122}} \tag{23}$$

$$= \frac{5}{\sqrt{61}} \tag{24}$$

$$\implies C = \cos^{-1}\frac{5}{\sqrt{61}} \tag{25}$$