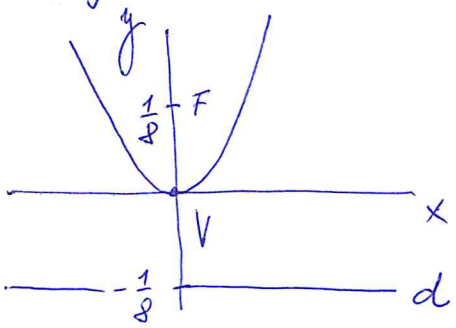
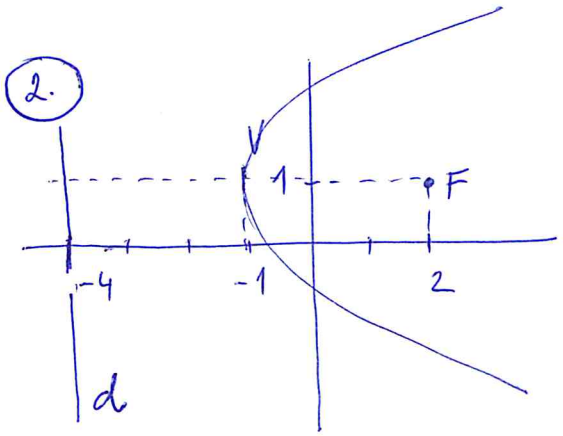
Cvičení 4

1. $y = 2x^2 \rightarrow x^2 = \frac{1}{2}y \rightarrow 2p = \frac{1}{2} \rightarrow p = \frac{1}{4}$, $V[0;0]$



$$F\left[0;\frac{1}{8}\right],\ d: y = -\frac{1}{8}$$

2.

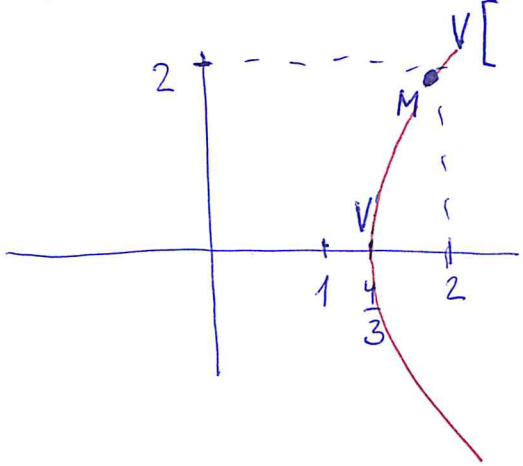


$\rightarrow V[-1;1]$; $p = 6$

$$(y-1)^2 = 2p(x+1)$$

$$(y-1)^2 = 12(x+1)$$

3. $y^2 - 6x + 8 = 0 \rightarrow y^2 = 6x - 8 \rightarrow y^2 = 6\left(x - \frac{4}{3}\right) \rightarrow V\left[\frac{4}{3};0\right]$



1) bodem M přímka $\parallel$ s osou x: $y = 2$

2) bodem M leží parabola

$$y^2 = 6\left(x - \frac{4}{3}\right),\ M[2;2]$$

$$t_M: \ y \cdot 2 = 3\left(x - \frac{4}{3}\right) + 3\left(2 - \frac{4}{3}\right)$$

$$2y = 3x - 4 + 6 - 4$$

$$2y = 3x - 2$$

$$y = \frac{3}{2}x - 1$$

4. $y^2 - 4y = -12x \rightarrow y^2 - 4x + 4 = -12x + 4$

$(y-2)^2 = -12\left(x - \frac{1}{3}\right)$ $T[0;0]$

$t: (y-2)(0-2) = -6\left(x - \frac{1}{3}\right) - 6\left(0 - \frac{1}{3}\right)$

$-2y + 4 = -6x + 2 + 2$

$-2y = -6x \rightarrow \boxed{y = 3x}$

5. $y = ax^2 + bx + c$

K ∈ parab. $\rightarrow$ $-3 = a \cdot 1 + b + c \rightarrow -3 = a + b - 1$

L ∈ parab. $\rightarrow$ $-1 = a \cdot 0 + b \cdot 0 + c \rightarrow \boxed{c = -1} \rightarrow$

M ∈ parab. $\rightarrow$ $-1 = a \cdot 4 + 2b + c \rightarrow -1 = 4a + 2b - 1$

$\rightarrow \begin{matrix} a+b=-2 \\ 4a+2b=0 \end{matrix} \rightarrow \begin{matrix} a+b=-2 \\ 2a+b=0 \end{matrix} \rightarrow \begin{matrix} a=-b-2 \\ 2(-b-2)+b=0 \rightarrow -2b-4+b=0 \rightarrow \\ \boxed{b=-4} \end{matrix} \rightarrow \boxed{a=2}$

6. $y^2 - 4y = 6x - 22 \rightarrow (y^2 - 4y + 4 = 6x - 22 + 4 \rightarrow$

$(y-2)^2 = 6x - 18 \rightarrow (y-2)^2 = 6(x-3)$ **není třeba**)

$p: y = x$; přímky s p posunuté : $t: y = x + q$

t má s parab. 1 spol. bod $\Rightarrow t \cap$ parab. = 1 řeš. $\rightarrow$

$(x+q)^2 - 4(x+q) - 6x + 22 = 0$

$x^2 + 2qx + q^2 - 4x - 4q - 6x + 22 = 0$

$x^2 + (2q - 10)x + q^2 - 4q + 22 = 0$ kvadr. rce $\rightarrow D \rightarrow D = 0$

$a = 1$; $b = 2q - 10$; $c = q^2 - 4q + 22$

$D = (2q-10)^2 - 4 \cdot 1 \cdot (q^2 - 4q + 22) = 4q^2 - 40q + 100 - 4q^2 + 16q - 88$

$= -24q - 88$

$D = 0 \Leftrightarrow -24q - 88 = 0 \Leftrightarrow q = -\frac{88}{24} = -\frac{11}{3}$

$\boxed{t: y = x - \frac{11}{3}}$

(7) $y^2 = 2px \cap y = \frac{x}{2} + 5$ = 1 bod; 1 řešení

$\left(\frac{x}{2} + 5\right)^2 = 2px \rightarrow \frac{x^2}{4} + 5x + 25 - 2px = 0 \rightarrow$

$x^2 + 20x + 100 - 8px = 0 \rightarrow x^2 + (20 - 8p)x + 100 = 0$

$a = 1;\ b = 20 - 8p;\ c = 100$

$D = (20 - 8p)^2 - 4 \cdot 1 \cdot 100 = 400 - 320p + 64p^2 - 400 =$

$= 64p^2 - 320p$

1 řešení $\Leftrightarrow D = 0 \Leftrightarrow 64p^2 - 320p = 0 \rightarrow p^2 - 5p = 0 \rightarrow$

$p(p - 5) = 0 \Leftrightarrow p = 0 \vee p = 5 \Rightarrow$

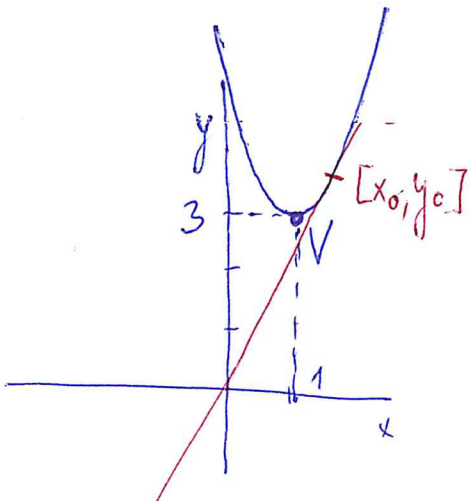
2 paraboly: $y^2 = 2 \cdot 0 \cdot x \rightarrow y^2 = 0$ → není parab.

$y^2 = 2 \cdot 5 \cdot x \rightarrow \boxed{y^2 = 10x}$

(8.) $x^2 - 2x + 1 = y - 4 + 1 \rightarrow (x - 1)^2 = y - 3 \rightarrow V[1; 3] \cup$

Počet společných přímek: $t: y = k \cdot x$

$t \cap$ parab. – 1 řešení:



$x^2 - 2x - y + 4 = 0$

$x^2 - 2x - kx + 4 = 0$

$x^2 + (-2 - k)x + 4 = 0$

$a = 1,\ b = -2 - k,\ c = 4$

$D = (-2 - k)^2 - 4 \cdot 1 \cdot 4 =$

$= 4 + 4k + k^2 - 16 = k^2 + 4k - 12$

1 řešení $\Leftrightarrow D = 0 \Leftrightarrow k^2 + 4k - 12 = 0 \Leftrightarrow$

$k_{1,2} = 2; -6 \rightarrow$ 2 tečny:

$\boxed{y = 2x}$

$\boxed{y = -6x}$