Czech Technical University in Prague
Faculty of Nuclear Sciences and Physical Engineering

Prediction of electron emission in the tokamak COMPASS using neural networks

Predikce emitovaných elektronů v tokamaku COMPASS pomocí neuronových sítí

Research project

Author: **Bc. Jazmína Kreanová**

Supervisor: **doc. Ing. Václav Šmídl, Ph.D.**

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- Zadání práce -

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Author's declaration:
I declare that this Research project is entirely my own work and I have listed all the used sources in the bibliography.

Prague, July 7, 2032 Jméno Autora

Název práce:

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Autor: Bc. Jazmína Kreanová

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Title:

Prediction of electron emission in the tokamak COMPASS using neural networks

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Abstract: Max. 10 lines of English abstract text.

Key words: keywords in alphabetical order separated by commas

Contents

Introduction

Describe COMPASS tokamak problem (emitted electrons could potentially damage metal walls of the vessel - or sth like it)

Describe contribution of machine learning/AI nowadays, in the field.

Our motivation

brief description of topics in this research project..

Chapter 1

Black-Box Modelling

Within a scientific field, the key problem is to sufficiently describe and interpret system or certain phenomena via mathematical model. Vital task is finding a mathematical formula of a relationship between past observation and future outputs in the system or a suitable parameter estimation (model structure) in complex phenomena.

However, determining models based on physical experiment includes noise, errors of measurements to the required outputs and thus resulting models are approximate.

Using the prior knowledge is crucial. We can distinguish between 3 levels of models according to the prior knowledge [1].

White Box Model: model is perfectly known with a possibility to construct it entirely from a prior knowledge and physical insight.

Grey Box Model: some physical insight is available, but there are several parameters which need to be estimate from measurements (observations). Two sub-cases are considered. Physical modelling, when model structure can be built on a physical grounds and a small amount of parameters that require to be determine from data. Secondly, Semi-Physical modelling which use physical insight to suggest a specific combinations of observation signal.

Black Box Model: model with no physical insight available, but the chosen model structure resides within a family known to dispose with good flexibility and have been *succesfully used in past.*

Doplniť popis časových radov - dynamické systémy, nájsť si literatúru

1.1 Linear Black-Box Models

Black-Box linear models represent a mapping from $\mathbb{R}$ to $\mathbb{R}^{pm}$, where p denotes number of outputs and m denotes number of inputs. In dynamical system, we can denote observations containing inputs $u(t)$ and outputs $y(t)$

$$\mathbf{u}^t = [u(1), u(2), \ldots, u(t)], \tag{1.1}$$

$$\mathbf{y}^t = [y(1), y(2), \ldots, y(t)]. \tag{1.2}$$

As we mentioned in text above, we seek s relationship between past observation $[\mathbf{u}^{t-1}, \mathbf{y}^{t-1}]$ and future outputs $y(t)$, written as

$$y(t) = g(\mathbf{u}^{t-1}, \mathbf{y}^{t-1}) + v(t), \tag{1.3}$$

where $v(t)$ is additive term, which is required to be small in order $g(\mathbf{u}^{t-1}, \mathbf{y}^{t-1})$ to be good prediction of $y(t)$ given by past data. The task is to find function $g(\cdot, \cdot)$ within a family of functions $\mathcal{G}$. Thus parametrization by

$theta \in \Theta$, within a finite parametric space $dim(\Theta) = k < +\infty$, can be used. With θ parametrization we might be able to rewrite function g as a concatenation of 2 mappings: $g(\mathbf{u}^{t-1}, \mathbf{y}^{t-1}, \theta)$. Firstly the increasing number of past observations $[\mathbf{u}^t, \mathbf{y}^t]$ is taken a and mapped into a vector of fixes and finite domension $\varphi(t)$. Secondly this vector is taken to the space of outputs:

$$g(\mathbf{u}^{t-1}, \mathbf{y}^{t-1}, \theta) = g(\varphi(t), \theta), \tag{1.4}$$

where $\varphi(t) = \varphi(\mathbf{u}^{t-1}, \mathbf{y}^{t-1})$ is called regression vector [1].

General Linear Black-box model, depicted on figure 1.1, has linear relationship between past observations and present output, with formula as follows

$$\mathbf{A}(q)y(t) = \frac{\mathbf{B}(q)}{\mathbf{F}(q)}u(t) + \frac{\mathbf{C}(q)}{\mathbf{D}(q)}e(t), \tag{1.5}$$

where q is a shift operator,which means

$$\begin{aligned} q^{-1}x(k) &= x(k-1) \\ q^{-2}x(k) &= x(k-2) \\ &\vdots \\ q^{-n}x(k) &= x(k-n), \end{aligned}$$

$\mathbf{A}(q), \mathbf{B}(q), \mathbf{C}(q), \mathbf{D}(q), \mathbf{F}(q)$ are polynomials in q^{-1}, $y(t)$ is system output, $u(t)$ is system input, $e(t)$ is system noise or disturbance. According to value of polynomials, there are several special cases of linear models: AR, ARX, ARMAX, BJ, OE. We will describe them briefly in following subsections.

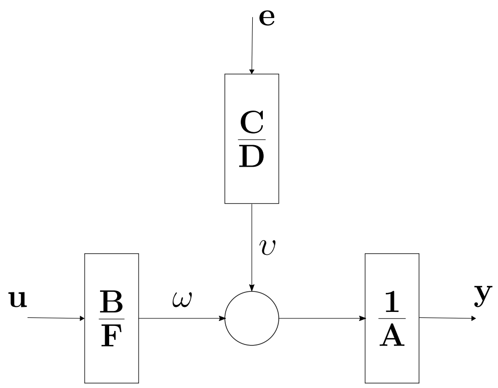

Figure 1.1: Scheme of a General Linear Black-box model.

1.1.1 AR model

Autoregressive model or simply AR model does not include relation between system input and output. This method is suitable for signal representation or as a prewhitening filter due to a dependency only on previous outputs and system noise (disturbance). Mathematical form is simple with $\mathbf{B} = 0, \mathbf{C} = \mathbf{D} = 1$

$$\mathbf{A}(q)y(t) = e(t), \tag{1.6}$$

q is shift operator, $e(t)$ system noise, $\mathbf{A}(q) = 1 + a_1 q^{-1} + \cdots + a_{k_a} q^{-k_a}$ is model polynomial in q^{-1} of order k_a. AR model is depicted via diagram 1.2.

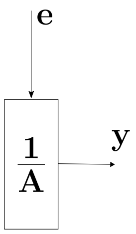

Figure 1.2: Scheme of a AR Black-box model.

1.1.2 ARX model

Autoregressive with exogenous terms model or simply ARX model integrate system input - stimulus signal. With special choice of model polynomials $\mathbf{C} = \mathbf{D} = \mathbf{F} = 1$ the formula can be written as

$$\mathbf{A}(q)y(t) = \mathbf{B}(q)u(t) + e(t), \tag{1.7}$$

q is shift operator, $e(t)$ system noise, $\mathbf{A}(q) = 1 + a_1 q^{-1} + \cdots + a_{k_a} q^{-k_a}$, $\mathbf{B}(q) = b_0 + b_1 q^{-1} + \cdots + b_{k_b-1} q^{-k_b+1}$ are model polynomials in q^{-1} of orders k_a and k_b respectively. ARX model is depicted via diagram 1.3.

Form (1.7) can be rewritten

$$y(t) + a_1 y(t-1) + \cdots + a_{k_a} = b_0 u(t) + b_1 u(t-1) + \cdots + b_{k_b-1} u(t - k_b + 1) + e(t). \tag{1.8}$$

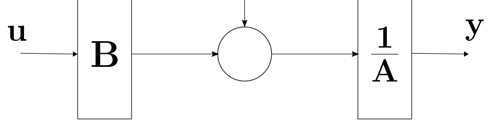

Figure 1.3: Scheme of a ARX Black-box model.

1.1.3 ARMAX

Autoregressive moving average with exogenous terms model or simply ARMAX model includes stochastic dynamics. This model can be useful if dominating disturbances enter in process with input. With special choice of model polynomials If model polynomials are chosen specifically $\mathbf{D} = \mathbf{F} = 1$, the formula can be written as

$$\mathbf{A}(q)y(t) = \mathbf{B}(q)u(t) + \mathbf{C}(q)e(t), \tag{1.9}$$

q is shift operator, $e(t)$ system noise, $\mathbf{A}(q) = 1+a_1q^{-1}+\cdots+a_{k_a}q^{-k_a}$, $\mathbf{B}(q) = b_0+b_1q^{-1}+\cdots+b_{k_b-1}q^{-k_b+1}$, $\mathbf{C}(q) = 1+c_1q^{-1}+\cdots+c_{k_c}q^{-k_c}$ are model polynomials in q^{-1} of orders k_a, k_b and k_c respectively. ARMAX model is depicted via diagram 1.4.

Form (1.9) can be rewritten

$$\begin{aligned} y(t) + a_1y(t-1) + \cdots + a_{k_a} &= b_0u(t) + b_1u(t-1) + \cdots + b_{k_b-1}u(t-k_b+1) && (1.10)\\ &+ e(t) + c_1e(t-1) + \cdots + c_{k_c}e(t-k_c). && (1.11) \end{aligned}$$

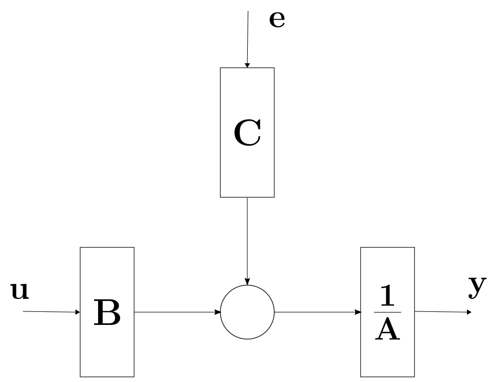

Figure 1.4: Scheme of a ARMAX Black-box model.

1.1.4 BJ

Box-Jenkins model or shortly BJ model incorporates system dynamicz and system disturbances. Model is useful, when disturbances are connected to measurements (outputs). BJ model is special case of linear black box model, when $\mathbf{A} = 1$. Form is as follows

$$y(t) = \frac{\mathbf{B}(q)}{\mathbf{F}(q)}u(t) + \frac{\mathbf{C}(q)}{\mathbf{D}(q)}e(t), \tag{1.12}$$

q is shift operator, $e(t)$ system noise, $\mathbf{B}(q) = b_0+b_1q^{-1}+\cdots+b_{k_b-1}q^{-k_b+1}$, $\mathbf{C}(q) = 1+c_1q^{-1}+\cdots+c_{k_c}q^{-k_c}$, $\mathbf{D}(q) = 1+d_1q^{-1}+\cdots+d_{k_d}q^{-k_d}$,$\mathbf{F}(q) = 1+f_1q^{-1}+\cdots+f_{k_f}q^{-k_f}$, are model polynomials in q^{-1} of orders k_b, k_c,k_d and k_f respectively. BJ model is depicted via diagram 1.5, where ω, υ are auxiliary variables (ni.com alebo to osdstaniť alebo ešte dohľadať).

Form 1.12 can be rewritten into time domain equations:

$$y(t) = w(t) + v(t), \tag{1.13}$$

$$w(t) + f_1w(t-1) + \cdots + f_{k_f}w(t-k_f) = b_0u(t) + b_1u(t-1) + \cdots + b_{k_b-1}u(t-k_b+1) \tag{1.14}$$

$$v(t) + d_1v(t-1) + \cdots + d_{k_d}v(t-k_d) = e(t) + c_1e(t-1) + \cdots + c_{k_c}e(t-k_c). \tag{1.15}$$

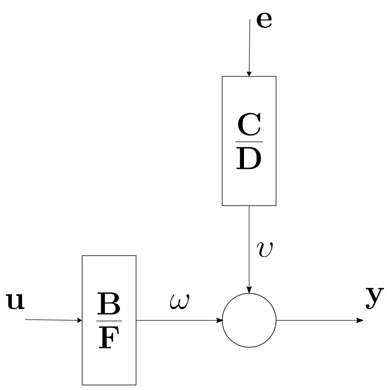

Figure 1.5: Scheme of a Box-Jenkins Black-box model.

1.1.5 OE

Output-Error model or OE model does not use any parameters for simulating disturbances. With a choice $\mathbf{A} = mathbfC = mathbfD = 1$ linear model of this form is OE:

$$y(t) = \frac{\mathbf{B}(q)}{\mathbf{F}(q)} u(t) + e(t), \tag{1.16}$$

q is shift operator, $e(t)$ system noise, $\mathbf{B}(q) = b_0 + b_1 q^{-1} + \cdots + b_{k_b-1} q^{-k_b+1}$, ,$\mathbf{F}(q) = 1 + f_1 q^{-1} + \cdots + f_{k_f} q^{-k_f}$, are model polynomials in q^{-1} of orders k_b, and k_f respectively. OE model is depicted via diagram 1.6

In this case, the model can be also rewritten to the time domain

$$y(t) = w(t) + e(t), \tag{1.17}$$

$$w(t) + f_1 w(t-1) + \cdots + f_{k_f} w(t-k_f) = b_0 u(t) + b_1 u(t-1) + \cdots + b_{k_b-1} u(t-k_b+1). \tag{1.18}$$

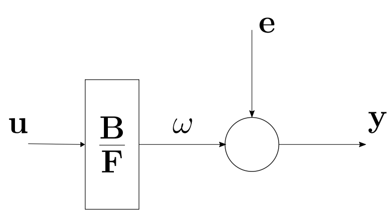

Figure 1.6: Scheme of a Output-Error Black-box model.

1.2 Non-linear Black-Box Models

Chapter 2

Neural Networks

Chapter 3

Artificial Data Experiments

Chapter 4

Evaluating Models of COMPASS Tokamak

4.1 Electron Problem

4.2 Data

4.3 Models

Conclusion

Text of the conclusion…

Bibliography

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