Adaptive Neural Network Control of a Robotic Manipulator With Time-Varying Output Constraints

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***Abstract*—The control problem of an uncertain *n*-degrees of freedom robotic manipulator subjected to time-varying output constraints is investigated in this paper. We describe the rigid robotic manipulator system as a multi-input and multi-output nonlinear system. We devise a disturbance observer to estimate the unknown disturbance from humans and environment. To solve the uncertain problem, a neural network which utilizes a radial basis function is used to estimate the unknown dynamics of the robotic manipulator. An asymmetric barrier Lyapunov function is employed in the process of control design to avert the contravention of the time-varying output constraints. Simulation results validate the validity of the presented control scheme.**

***Index Terms*—Adaptive neural network (NN) control, barrier Lyapunov function (BLF), disturbance observer (DO), robotic manipulator, time-varying constraints.**

I. Introduction

NOWADAYS, the control problem of constrained robots appeals to more and more scholars and researchers [1]–[3]. Universally, constraints exist both in the input and the output of the robotic system, such as saturation, dead-zone, and safety specifications [4]–[6]. As the robots are now required to have more physical interaction with humans and environment, the safety problems resulting from the contravention of these constraints cannot be ignored. Therefore, we need to design targeted controllers to deal with these problems.

The control of an uncertain robotic manipulator which is in interactions with humans and environment is challenging due to its unknown dynamics and unknown disturbance. To handle the problem of the unknown disturbance, the technique of disturbance observer (DO) is taken into consideration [7]–[9]. In [10], a sliding-mode control law is devised to handle the control problem of systems with mismatched uncertainties by using a DO. Li *et al.* [11] proposed an adaptive fuzzy control law to control a robotic exoskeleton with input saturation by using a DO. In [12], nonlinear DO is utilized to approximate a combined disturbance composed of two unknown parts. In [13], dynamic surface control based on DO is designed for a transport aircraft. Thus, it is believed that DO is a efficacious technique to deal with the unknown disturbance of the robotic manipulator.

As for the system uncertainties, adaptive control is a method to cope with it due to its ability of providing online estimations of unknown system parameters [14]–[19]. A novel adaptive predictor feedback control algorithm is proposed for a managed pressure drilling system in the presence of unknown parameters, disturbance, and time-delay in [20]. In [21], adaptive impedance control is proposed for a robot collaborating with a human partner, in the presence of unknown robot dynamics. Meanwhile, technique based on neural networks (NNs) has been confirmed to be an useful way in function approximation since the exact information of the system is nonrequired [22]–[25]. Recently, more and more scholars and researchers have utilized adaptive NNs to design appropriate control laws for nonlinear systems [26]–[29]. Zhang *et al.* [30] devised a control scheme based on NN for affine nonlinear systems which are discrete-time and subjected to constraints. Li *et al.* [31] put forward adaptive back-stepping control through applying radial basis function (RBF) NNs. In [32], the trajectory tracking problem of multi-input and multi-output (MIMO) nonlinear systems that are uncertain and subjected to input constraints is settled with adaptive NNs. In [33], fuzzy NN-based adaptive control is designed for a class of uncertain nonlinear stochastic systems. In [34], model predictive control based on NNs is devised for piezoelectric actuators. In [35] and [36], NN-based control is utilized for nonlinear string system. Two kinds of adaptive NN control schemes are, respectively, developed for coordinated multiple mobile manipulators and quadruped robots in [37] and [38]. Ren *et al.* [39] deal with the uncertain nonlinear systems subjected to hysteresis input by utilizing NNs. An adaptive control scheme based on NNs is designed for full state constrained nonlinear systems in [40]. Liu *et al.* [41] devise an adaptive NN control law for a humanoid robot subjected to unknown nonlinearities in output mechanism. In [42], an adaptive control is presented for a rehabilitation robot by using NNs.

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Yang *et al.* [43], [44] employed RBF NNs to compensate the effect caused by dynamics uncertainties in a teleoperation system. In [45], NN approximation technique is utilized to design a controller for Bimanual Robots, and experiments carried out on the Baxter Robot prove the effectiveness of the proposed control scheme.

Traditional Lyapunov function aims to achieve the guaranteed global or semi-global stability. While in this paper, we should not only guarantee the semi-global stability but also avoid the contravention of the time-varying constraints. The common constraints investigated are usually constant constraints which can be seen as a special kind of the time-varying constraints, and thus, it is more practical to investigate the time-varying constraints. Bechlioulis *et al.* [46], [47] utilized approximation based robust adaptive control via NNs to guarantee the prescribed performance of the MIMO nonlinear systems and single-input and single-output (SISO) strict feedback systems, respectively. This technique can be used even when the bounds of the error are time-varying, but there is a prerequisite that the performance function should be positive and converge to a constant. A good way to cope with the constant constraint problem is the technique of barrier Lyapunov functions (BLFs). In 2009, the technique of BLFs was first proposed to deal with the output constraint problem [48]. In [49], a BLF is devised for an uncertain robotic system subjected to state constraints. To improve the compensation performance for the output constraints, BLFs are considered to investigate the stability of the nonlinear system in [50]. A BLF is introduced in [51] to make sure that the unknown parameters of the function maintain within the designated superset. An asymmetric time-varying BLF is used for nonlinear systems to abstain from the transgression of the time-varying output constraints in [52]. Motivated by this, we use time-varying BLFs to deal with the output time-varying constraints. Bulk of existing papers consider the nonlinear systems without constraints [53]–[58], nonlinear systems with input constraints [59]–[63], nonlinear systems with constant output constraints [64], [65], or certain SISO nonlinear systems subjected to time-varying output constraints [52]. Therefore, the control problem of an MIMO unknown robotic system subjected to unknown disturbance and time-varying output constraints needs to be solved.

In this paper, we utilize adaptive NNs to compensate the uncertain dynamics of the robotic manipulator system. The DO is devised to compensate for the effect of unknown disturbance, and the asymmetric BLFs are utilized in the process of control design to avert the contravention of the time-varying output constraints. The diagram of output time-varying constraints is shown in Fig. 1. The major contributions of this paper are summarized as follows.

1) With the adaptive control law based on NNs, the influences of the system uncertainties have successfully been compensated and the robotic system robustness has also been improved.
2) The NN estimation errors and the unknown disturbance from humans and environment are integrated as a combined disturbance which is finally approximated by a DO.

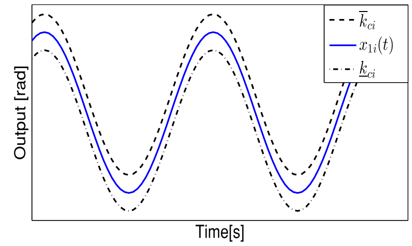


Fig. 1. Solid line represents the output $x_{1i}(t)$. Dashed lines represent constraints.

3) To avoid the contravention of the time-varying constraints, asymmetric BLFs are employed to design the control law and asymptotic tracking is achieved successfully.

In what follows, Section II presents some useful lemmas and definitions, and it formulates the time-varying output constraint problem of an MIMO uncertain robotic system. Section III describes the design of the model-based control first and then the adaptive NN control. Section IV provides a simulation example that proves the validity and rationality of the presented control laws. Finally, the conclusion is given in Section V.

II. Problem Formulation

The dynamics of a n-degrees of freedom (DOF) rigid robotic manipulator in joint space can be expressed as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = -f_{\mathrm{dis}}(t) + \tau(t) \tag{1}$$

where $q \in \mathbb{R}^n$ is a vector of joint variables, $\tau \in \mathbb{R}^n$ represents the control input torque, $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix which is symmetric and positive definite, $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ represents the Centripetal and Coriolis torques, and $G(q) \in \mathbb{R}^n$ denotes the gravitational force and $f_{\mathrm{dis}}(t) \in \mathbb{R}^n$ is the vector of disturbance from humans and environment.

Property 1 [66]: The matrix $(1/2)\dot{M}(q) - C(q,\dot{q})$ is skew-symmetric, i.e., $\forall r \in \mathbb{R}^n$, $r^T[(1/2)\dot{M}(q) - C(q,\dot{q})]r = 0$.

Lemma 1 [67]: For a continuous Lyapunov function $V(w,t)$, if it satisfies $\varpi_1(\|w\|) \le V(w,t) \le \varpi_2(\|w\|)$ and its derivative $\dot{V}(w,t) = dV(w,t)/dt$ satisfies $\dot{V}(w,t) \le -\kappa V(w,t) + C$, where κ and C are positive constants, then the solution $w(t)$ is bounded.

Lemma 2 [64]: Consider a positive constant $k_a \in \mathbb{R}$, if $x \in \mathbb{R}$ and $|x| < |k_a|$, the following inequality holds:

$$\ln\frac{k_a^2}{k_a^2 - x^2} \le \frac{x^2}{k_a^2 - x^2}. \tag{2}$$

Lemma 3 [66]: $\forall B \in \mathbb{R}^{n\times n}$, when B is symmetric and positive definite, the following inequality holds:

$$\forall y \in \mathbb{R}^n, \lambda_{\min}\|y\|^2 \le y^T B y \le \lambda_{\max}\|y\|^2 \tag{3}$$

where $\lambda_{\min}$ denotes the minimum eigenvalue of B, $\lambda_{\max}$ represents the maximum eigenvalue of B.

Lemma 4 [11]: For a continuously differentiable function $\Psi(t)$, if $\Psi(t)$ satisfies $|\Psi(t)| \le \psi$, $\forall t \in [0,\infty)$, where ψ is a positive constant, then $\dot{\Psi}(t)$ is bounded $\forall t \in [0,\infty)$.

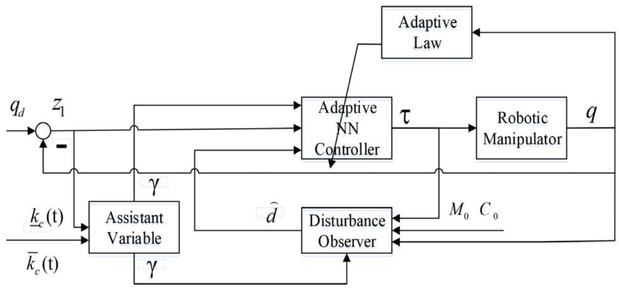


Fig. 2. Adaptive NN control strategy.

Assumption 1 [42]: Based on the actual application environment, the disturbance $f_{\rm dis}(t)$ exerted by humans and environment is assumed to be continuous and bounded.

We aim to devise a control law for the robotic manipulator system so that the output can follow a target trajectory $q_d(t) = [q_{d1}(t), q_{d2}(t), \dots q_{di}(t) \cdots q_{dn}(t)]^T$, while ensuring that the time-varying output constraints are never breached, i.e., if the time-varying bounds is given as $\underline{k}_c(t) = [\underline{k}_{c1}(t), \underline{k}_{c2}(t) \cdots \underline{k}_{ci}(t) \cdots \underline{k}_{cn}(t)]^T$ and $\overline{k}_c(t) = [\overline{k}_{c1}(t), \overline{k}_{c2}(t) \cdots \overline{k}_{ci}(t) \cdots \overline{k}_{cn}(t)]^T$, then the output $q(t)$ should remain within the region as

$$\Omega_q := \{q \in \mathbb{R}^n | \underline{k}_c(t) < q(t) < \overline{k}_c(t)\}, \ \forall t \geq 0. \tag{4}$$

III. Control Design

In this section, we utilize NNs to make up for the effect of the system uncertainties and use time-varying BLFs to avert the contravention of the time-varying constraints. We define a new compounded disturbance composed of the NN estimation error and the unknown external disturbance from humans and environment, which is estimated by a DO. Fig. 2 shows the strategy for the control.

We first consider that the parameters of the system $M(q)$, $C(q, \dot{q})$, and $G(q)$ are known forehand. The output tracking error variable is defined as $z_1(t) = q(t) - q_d(t)$ and we have $\dot{z}_1(t) = \dot{q}(t) - \dot{q}_d(t)$. Define a new error variable as $z_2(t) = \dot{q}(t) - \gamma(t)$ where $\gamma(t) = [\gamma_1(t), \gamma_2(t), \dots, \gamma_i(t), \dots, \gamma_n(t)]^T$ is a assistant variable to be designed. Consider (1), the time derivative of z_2 is as follows:

$$\dot{z}_2 = M^{-1}(q)\big[\tau - f_{\rm dis} - C(q, \dot{q})\dot{q} - G(q)\big] - \dot{\gamma}. \tag{5}$$

Time-varying bounds of z_1 are set to be

$$k_a(t) = q_d(t) - \underline{k}_c(t) \tag{6}$$
$$k_b(t) = \overline{k}_c(t) - q_d(t) \tag{7}$$

where $k_a(t) = [k_{a1}, k_{a2}, \dots, k_{ai}, \dots, k_{an}]^T$ and $k_b(t) = [k_{b1}, k_{b2}, \dots, k_{bi}, \dots, k_{bn}]^T$. Consider a Lyapunov function candidate as

$$V_1(t) = \sum_{i=1}^{n} \Bigg[\frac{h(i)}{2} \ln \frac{k_{bi}^2(t)}{k_{bi}^2(t) - z_{1i}^2(t)} + \frac{1 - h(i)}{2} \ln \frac{k_{ai}^2(t)}{k_{ai}^2(t) - z_{1i}^2(t)} \Bigg] \tag{8}$$

where $h(i)$ is defined as

$$h(i) = \begin{cases} 0, & z_{1i} \leq 0 \\ 1, & z_{1i} > 0 \quad i = 1, 2, 3, \dots, n. \end{cases} \tag{9}$$

By coordinates conversion of the error

$$\xi_{ai} = \frac{z_{1i}}{k_{ai}}, \xi_{bi} = \frac{z_{1i}}{k_{bi}}, \xi_i = h(i)\xi_{bi} + (1 - h(i))\xi_{ai} \tag{10}$$

we rewrite (8) into a new form

$$V_1(t) = \sum_{i=1}^{n} \frac{1}{2} \ln \frac{1}{1 - \xi_i^2}. \tag{11}$$

We can easily know that $V_1(t)$ holds positive definite and continuously differentiable as $|\xi_i| < 1$. Differentiating $V_1(t)$ yields

$$\dot{V}_1(t) = \sum_{i=1}^{n} \Bigg(\frac{\xi_{bi} h(i)}{\big(1 - \xi_{bi}^2\big) k_{bi}} \bigg(z_{2i} + \gamma_i - \dot{x}_{di} - z_{1i} \frac{\dot{k}_{bi}}{k_{bi}} \bigg) + \frac{\xi_{ai}(1 - h(i))}{\big(1 - \xi_{ai}^2\big) k_{ai}} \bigg(z_{2i} + \gamma_i - \dot{x}_{di} - z_{1i} \frac{\dot{k}_{ai}}{k_{ai}} \bigg) \Bigg). \tag{12}$$

We propose the virtual control γ as

$$\gamma = \begin{bmatrix} \dot{q}_{d1} - k_{11} z_{11} - \overline{k}_{11} z_{11} \\ \dot{q}_{d2} - k_{12} z_{12} - \overline{k}_{12} z_{12} \\ \vdots \\ \dot{q}_{di} - k_{1i} z_{1i} - \overline{k}_{1i} z_{1i} \\ \vdots \\ \dot{q}_{dn} - k_{1n} z_{1n} - \overline{k}_{1n} z_{1n} \end{bmatrix} \tag{13}$$

where $\overline{k}_{1i}$ is given by

$$\overline{k}_{1i} = \sqrt{\beta + \bigg(\frac{\dot{k}_{bi}(t)}{k_{bi}(t)} \bigg)^2 + \bigg(\frac{\dot{k}_{ai}(t)}{k_{ai}(t)} \bigg)^2} \tag{14}$$

β is a small positive constant ensuring that $\dot{\gamma}$ is bounded even if both $\dot{k}_a(t)$ and $\dot{k}_b(t)$ are zero, and the gain matrix $K_1 = \text{diag}[k_{11}, k_{12} \cdots k_{1i} \cdots k_{1n}] > 0$. Plugging (13) and (14) into (12), we have

$$\begin{aligned} V_1(t) = & \sum_{i=1}^{n} \frac{\xi_{bi} h(i)}{\big(1 - \xi_{bi}^2\big) k_{bi}} \bigg[z_{2i} - k_{1i} z_{1i} - \bigg(\overline{k}_{1i} + \frac{\dot{k}_{bi}}{k_{bi}} \bigg) z_{1i} \bigg] \\ & + \sum_{i=1}^{n} \frac{\xi_{ai}(1 - h(i))}{\big(1 - \xi_{ai}^2\big) k_{ai}} \bigg[z_{2i} - k_{1i} z_{1i} - \bigg(\overline{k}_{1i} + \frac{\dot{k}_{ai}}{k_{ai}} \bigg) z_{1i} \bigg] \\ \leq & \sum_{i=1}^{n} -k_{1i} \frac{\xi^2}{1 - \xi^2} + \sum_{i=1}^{n} \phi_i z_{1i} z_{2i}. \end{aligned} \tag{15}$$

In which ϕ_i is set as

$$\phi_i = \bigg(\frac{h(i)}{k_{bi}^2 - z_{1i}^2} + \frac{1 - h(i)}{k_{ai}^2 - z_{1i}^2} \bigg) i = 1, 2, \dots, n \tag{16}$$

and $\phi = \text{diag}[\phi_1, \dots, \phi_{\rm i}, \dots, \phi_{\rm n}]$.

To compensate for the disturbance, we use a nonlinear DO to estimate it. Define the following auxiliary variable as:

$$z_3 = f_{\rm dis} + \Theta(z_2) \tag{17}$$

where $\Theta(z_2) \in \mathbb{R}^n$ is a function vector of z_2 to be designed. Taking the time derivative of (17), we have

$$\begin{aligned}\dot{z}_3 &= \dot{f}_{\text{dis}} + D(z_2)\dot{z}_2 \\ &= \dot{f}_{\text{dis}} + D(z_2)M^{-1}(q)\big[(\tau - f_{\text{dis}} - C(q,\dot{q})\dot{q}) - G(q)\big] \\ &\quad - D(z_2)\dot{\gamma} \end{aligned} \tag{18}$$

where $D(z_2) = (\partial\Theta(z_2)/\partial(z_2^T)) \in \mathbb{R}^{n\times n}$. Generally, we design $\Theta(z_2)$ as a linear function of z_2 so that $D(z_2)$ is a constant matrix and the DO can be easily designed and implemented. Since z_2 can be obtained by the state feedback, we can get the approximate value of the disturbance f_{dis} if we have a appropriate way to estimate the z_3. To get the approximate value, the approximate $\hat{z}_3$ is presented as follows:

$$\dot{\hat{z}}_3 = D(z_2)M^{-1}(q)\Big[\Big(\tau - C(q,\dot{q})\dot{q} - G(q) - \hat{f}_{\text{dis}}\Big] - D(z_2)\dot{\gamma} \tag{19}$$

where $\hat{z}_3$ is the estimation of z_3. From (17), we have the estimation of the disturbance f_{dis}

$$\hat{f}_{\text{dis}} = \hat{z}_3 - \Theta(z_2). \tag{20}$$

It is easy to obtain that

$$\tilde{z}_3 = \hat{z}_3 - z_3 = \hat{f}_{\text{dis}} - f_{\text{dis}} = \tilde{f}_{\text{dis}}. \tag{21}$$

Taking its time derivative, we have

$$\dot{\tilde{f}}_{\text{dis}} = \dot{\tilde{z}}_3 = -\dot{f}_{\text{dis}} - D(z_2)M^{-1}(x_1)\tilde{f}_{\text{dis}}. \tag{22}$$

According to Lemma 4 and Assumption 1, we can obtain

$$\|\dot{f}_{\text{dis}}\| \le \varsigma_0 \tag{23}$$

where ς_0 is an unknown positive constant. Then we consider a BLF as follows:

$$V_2 = \frac{1}{2}z_2^T M z_2 + \frac{1}{2}\tilde{f}_{\text{dis}}^T\tilde{f}_{\text{dis}} + V_1. \tag{24}$$

Taking its time derivative results in

$$\begin{aligned}\dot{V}_2 &= z_2^T M\dot{z}_2 + \frac{1}{2}z_2^T\dot{M}z_2 + \tilde{f}_{\text{dis}}^T\dot{\tilde{f}}_{\text{dis}} + \dot{V}_1 \\ &\le \sum_{i=1}^n -k_{1i}\frac{\xi_i^2}{1-\xi_i^2} + \sum_{i=1}^n \phi_i z_{1i}z_{2i} + z_2^T(\tau - G(q) \\ &\quad - C(q,\dot{q})\gamma - M(q)\dot{\gamma} - f_{\text{dis}}) + \frac{1}{2}\varsigma_0^2 \\ &\quad + \frac{1}{2}\tilde{f}_{\text{dis}}^T\tilde{f}_{\text{dis}} - \tilde{f}_{\text{dis}}^T D(z_2)M^{-1}(q)\tilde{f}_{\text{dis}}. \end{aligned} \tag{25}$$

We devise the model-based control as

$$\begin{aligned}\tau &= \hat{f}_{\text{dis}} + G(q) + M\dot{\gamma} \\ &\quad - \phi z_1 - K_2 z_2 + C(q,\dot{q})\gamma \end{aligned} \tag{26}$$

where the gain matrix $K_2 = K_2^T > 0$. Substituting (26) to (25), we have

$$\begin{aligned}\dot{V}_2 &\le \sum_{i=1}^n -k_{1i}\frac{\xi_i^2}{1-\xi_i^2} - z_2^T\Big(K_2 - \frac{1}{2}I_{n\times n}\Big)z_2 \\ &\quad - \tilde{f}_{\text{dis}}^T\Big(D(z_2)M^{-1}(x_1) - I_{n\times n}\Big)\tilde{f}_{\text{dis}} + \frac{1}{2}\varsigma_0^2 \\ &\le -\kappa_1 V_2 + C_1 \end{aligned} \tag{27}$$

where $I_{n\times n}$ denotes identity matrix and

$$\kappa_1 = \min\left(\min_{i=1,2,\dots n}(2k_{1i}), \frac{2\lambda_{\min}\Big(K_2 - \frac{1}{2}I\Big)}{\lambda_{\max}(M)}, 2\lambda_{\min}\Big(D(z_2)M^{-1}(q) - I_{n\times n}\Big)\right) \tag{28}$$

$$C_1 = \frac{1}{2}\varsigma_0^2. \tag{29}$$

To ensure $\kappa_1 > 0$, the gain matrix K_2 and the linear function vector Θ_{z_2} are chosen to satisfy that

$$\lambda_{\min}\Big(K_2 - \frac{1}{2}I_{n\times n}\Big) > 0 \tag{30}$$

$$\lambda_{\min}\left(\frac{\partial\Theta(z_2)}{\partial\big(z_2^T\big)}M^{-1}(q) - I_{n\times n}\right) > 0. \tag{31}$$

Since $M^{-1}(q)$ is known, the $\Theta(z_2)$ is not hard to be designed as a linear function of z_2.

Theorem 1: For the robotic manipulator system dynamics (1), with bounded initial conditions, semi-global uniform boundedness stability is obtained under the control law (26). The output can asymptotically track the desired trajectory, i.e., as $t \to \infty$, $q(t) \to q_d(t)$. The time-varying output constraints are definitely not breached, namely $\forall t > 0$, $\underline{k}_{ci}(t) < q_i(t) < \overline{k}_{ci}(t)$. The error variables z_1, z_2, and $\tilde{f}_{\text{dis}}$ will be bounded over the compact sets Ω_{z_1}, Ω_{z_2}, and $\Omega_{\tilde{f}_{\text{dis}}}$, respectively, defined as follows:

$$\Omega_{z_1} := \Big\{z_1 \in \mathbb{R}^n | -\underline{D}_{z_{1i}} \le z_{1i} \le \overline{D}_{z_{1i}}\Big\} \tag{32}$$

$$\Omega_{z_2} := \left\{z_2 \in \mathbb{R}^n | \|z_2\| \le \sqrt{\frac{D}{\lambda_{\min}(M)}}\right\} \tag{33}$$

$$\Omega_{\tilde{f}_{\text{dis}}} := \Big\{\tilde{f}_{\text{dis}} \in \mathbb{R}^n | \|\tilde{f}_{\text{dis}}\| \le \sqrt{D}\Big\} \tag{34}$$

where $D = 2(V_2(0) + C_1/\kappa_1)$, κ_1 and C_1 are two positive constants shown in (28) and (29) and

$$\overline{D}_{z_{1i}} = \sqrt{k_{bi}^2(t)(1 - e^{-D})} \tag{35}$$

$$\underline{D}_{z_{1i}} = \sqrt{k_{ai}^2(t)(1 - e^{-D})}. \tag{36}$$

Proof: Multiplying (27) by $e^{\kappa_1 t}$ yields

$$\frac{d}{dt}\big(V_3 e^{\kappa_1 t}\big) \le C_1 e^{\kappa_1 t}. \tag{37}$$

Integrating (37) yields

$$\begin{aligned}V_2(t) &\le \Big(V_2(0) - \frac{C_1}{\kappa_1}\Big)e^{\kappa_1 t} + \frac{C_1}{\kappa_1} \\ &\le V_2(0) + \frac{C_1}{\kappa_1} = \frac{1}{2}D. \end{aligned} \tag{38}$$

Then, we obtain

$$\frac{1}{2}\ln\frac{1}{1-\xi_i^2} \le \frac{1}{2}D \tag{39}$$

$$\sqrt{k_{ai}^2(t)\big(1 - e^{-D}\big)} \le z_{1i} \le \sqrt{k_{bi}^2(t)\big(1 - e^{-D}\big)}. \tag{40}$$

Accordingly, z_1 converges to the compact set Ω_{z_1}. In the same way, we can demonstrate the bounds for z_2 and $\tilde{f}_{\text{dis}}$. ■

Since uncertainties exist in the parameters of the system, namely $M(q)$, $G(q)$, and $C(q, \dot{q})$, the control proposed above may not be appropriate. To deal with this problem, we assume that the actual value $M(q)$ can be divided into certain part denoted by $M_0(q)$ and uncertain part represented by $\triangle M(q)$, where $M(q) = M_0(q) + \triangle M(q)$ and $M_0(q)$ is a symmetric and positive definite matrix. Similarly, we design $C(q, \dot{q}) = C_0(q, \dot{q}) + \triangle C(q, \dot{q})$ and make sure that $(1/2)\dot{M}_0(q) - C_0(q, \dot{q})$ is skew symmetric. We employ NNs to estimate the uncertainties. The NN $\hat{W}^T X(Z)$ is an approximation of $W^{*T} X(Z)$ defined as follows:

$$\begin{aligned} W^{*T} X(Z) = & \triangle C(q, \dot{q})\dot{q} + G(q) - \omega(z) \\ & + \triangle M \dot{z}_2 + M(q)\dot{\gamma} \end{aligned} \tag{41}$$

where $Z = [q^T, \dot{q}^T, \gamma^T, \dot{\gamma}^T]$ are the input variables to the RBF, and $\omega(z)$ is the estimation error satisfying $|\omega(z)| \leq \overline{\omega}(z)$, where $\overline{\omega}(z) \in \mathbb{R}^n$ is the bound of $\omega(z)$. According to Lemma 4, $\dot{\omega}(z)$ is also bounded. The adaption law is

$$\dot{\hat{W}}_i = -\Gamma_i [X_i(Z) z_{2,i} + \sigma_i |z_{2i}| \hat{W}_i], \; i = 1, 2, 3, \dots, n \tag{42}$$

where $\hat{W}_i$ denote the actual weights which are utilized to estimate the ideal weight W_i^* of the NNs, Γ_i is a positive constant, $X(Z) = [X_1(Z), X_2(Z), \dots, X_l(Z)]^T$ are the basis Gaussian function, l is the node number of the NN and satisfies $l > 1$, σ_i is a small positive constant.

Property 2 [68]: For adaption law (42), $\hat{W}_i$ is bounded, i.e., $\Omega_{w1} = \{\hat{W}_i | \|\hat{W}_i\| \leq (x_i/\sigma_i)\}$ where $\|X_i(Z)\| \leq x_i$ with $x_i > 0$.

Define a compounded disturbance as

$$d = f_{\text{dis}} + \omega(Z). \tag{43}$$

It can be easily known that both d and $\dot{d}$ are bounded. Similar to (23), we can obtain $\|\dot{d}\| \leq \varsigma$ with an unknown constant ς. We design the new control law as

$$\tau = \hat{W}^T X(Z) - \phi z_1 - K_2 z_2 + C_0(q, \dot{q})\gamma + \hat{d}. \tag{44}$$

Combining (41), we can rewrite (5) as

$$\dot{z}_2 = M_0^{-1}(q)\big(\tau - d - C_0(q, \dot{q})\dot{q} - W^{*T} X(Z)\big). \tag{45}$$

Similarly, we devise a new DO to approximate the compounded disturbance d. Define an assistant variable as follows:

$$z_3 = \Theta(z_2) + d \tag{46}$$

where $\Theta(z_2) \in \mathbb{R}^n$ is a function vector of z_2 to be designed. Taking the time derivative of (46), we have

$$\begin{aligned} \dot{z}_3 &= \dot{d} + D(z_2)\dot{z}_2 \\ &= \dot{d} + D(z_2) M_0^{-1}(q)\big[\tau - d - C_0(q, \dot{q})\dot{q} - W^{*T} X(Z)\big] \end{aligned} \tag{47}$$

where $D(z_2) = (\partial\Theta(z_2)/\partial(z_2^T)) \in \mathbb{R}^{n\times n}$. The approximate $\hat{z}_3$ is presented as follows:

$$\dot{\hat{z}}_3 = D(z_2) M_0^{-1}(q)\Big(\tau - C_0(q, \dot{q})\dot{q} - \hat{d}\Big) \tag{48}$$

where $\hat{z}_3$ is the estimation of z_3. From (46), we can have the estimation of the disturbance d

$$\hat{d} = \hat{z}_3 - \Theta(z_2). \tag{49}$$

It is easy to know that

$$\tilde{z}_3 = \hat{z}_3 - z_3 = \hat{d} - d = \tilde{d}. \tag{50}$$

Taking its time derivative, we obtain

$$\dot{\tilde{d}} = \dot{\tilde{z}}_3 = -\dot{d} + D(z_2) M_0^{-1}(q)\Big(-\tilde{d} + W^{*T} X(Z)\Big). \tag{51}$$

Choosing another function V_3 as

$$V_3 = V_1 + \frac{1}{2} z_2^T M_0(q) z_2 + \frac{1}{2}\sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i + \frac{1}{2}\tilde{d}^T \tilde{d}. \tag{52}$$

Differentiating V_3 yields

$$\begin{aligned} \dot{V}_3 \leq & -\sum_{i=1}^{n} k_{1i} \frac{\xi_i^2}{1 - \xi_i^2} + \sum_{i=1}^{n} \phi_i z_{1i} z_{2i} - \tilde{d}^T \dot{d} \\ & + z_2^T\big(\tau - W^{*T} X(Z) - d - C_0(q, \dot{q})\gamma\big) \\ & - \tilde{d}^T D(z_2) M_0^{-1}(q)\tilde{d} + \tilde{d}^T D(z_2) M_0^{-1}(q) W^{*T} X(Z) \\ & - \sum_{i=1}^{n} \tilde{W}_i^T X_i(Z_i) z_{2i} - \sum_{i=1}^{n} \tilde{W}_i^T \sigma_i |z_{2i}| \hat{W}_i. \end{aligned} \tag{53}$$

Substituting (44) into (53), we have

$$\begin{aligned} \dot{V}_3 \leq & -\sum_{i=1}^{n} k_{1i} \frac{\xi_i^2}{1 - \xi_i^2} - z_2^T K_2 z_2 - \tilde{d}^T \dot{d} + z_2^T \tilde{d} \\ & - \tilde{d}^T D(z_2) M_0^{-1}(q)\tilde{d} + \tilde{d}^T D(z_2) M_0^{-1} W^{*T} X(Z) \\ & - \sum_{i=1}^{n} \tilde{W}_i^T \sigma_i |z_{2i}| \hat{W}_i. \end{aligned} \tag{54}$$

According to average value inequality, we obtain

$$z_2^T \tilde{d} \leq \frac{1}{2} z_2^T z_2 + \frac{1}{2}\tilde{d}^T \tilde{d} \tag{55}$$

$$\begin{aligned} \tilde{d}^T D(z_2) M_0^{-1} W^{*T} X(Z) \leq & \frac{\|D(z_2) M_0^{-1}\|^2}{2\theta}\|\tilde{d}\|^2 \\ & + \frac{\theta}{2}\sum_{i=1}^{n} \|W_i^*\|^2 \|X_i(Z)\|^2 \end{aligned} \tag{56}$$

$$-\tilde{d}^T \dot{d} \leq \frac{1}{2}\tilde{d}^T \tilde{d} + \frac{1}{2}\dot{d}^T \dot{d} \leq \frac{1}{2}\tilde{d}^T \tilde{d} + \frac{1}{2}\varsigma^2 \tag{57}$$

$$\begin{aligned} -\sum_{i=1}^{n} \tilde{W}_i{}^T \sigma_i |z_{2i}| \hat{W}_i \leq & \sum_{i=1}^{n} \frac{\sigma_i |z_{2i}|}{2}\Big(\|\tilde{W}_i\|^2 - \|W_i^*\|^2\Big) \\ \leq & \frac{1}{2} z_2^T z_2 + \frac{1}{8}\sum_{i=1}^{n} \sigma_i^2 \Big(\|\tilde{W}_i\|^2 - \|W_i^*\|^2\Big)^2 \end{aligned} \tag{58}$$

where θ is a positive number. Invoking Property 2 yields

$$\|\tilde{W}_i\| = \|\hat{W}_i - W_i^*\| \leq \frac{s_i}{\sigma_i} + \|W_i^*\|. \tag{59}$$

For simplification, we define

$$\|\tilde{W}_i\| \leq \frac{s_i}{\sigma_i} + \|W_i^*\| = \nu. \tag{60}$$

Considering the basis function of Gaussian RBF NN. we have $\|X_i(Z)\| \le l, i = 1, 2, \ldots, n$ with $l > 0$. Substituting (55)–(60) into (54)

$$\begin{aligned}\dot{V}_3 \le & -\sum_{i=1}^{n} k_{1i}\frac{\xi_i^2}{1-\xi_i^2} - z_2^T(K_2 - I)z_2 + \frac{1}{2}\|\varsigma\|^2 \\ & - \tilde{d}^T\left(-\frac{\|D(z_2)M_0^{-1}\|^2 + 2\theta}{2\theta}I_{n\times n} + D(z_2)M_0^{-1}\right)\tilde{d} \\ & - \sum_{i=1}^{n}\frac{\sigma_i^2}{4}\|W_i^*\|^2\|\tilde{W}_i\|^2 + \sum_{i=1}^{n}\frac{\sigma_i^2}{8}\left(\|W_i^*\|^4 + \nu^4\right) \\ \le & -\kappa_2 V_3 + C_2 \end{aligned} \tag{61}$$

where

$$\begin{aligned}\kappa_2 = \min\Bigg(&\min_{i=1,2,\ldots n}(2k_{1i}), \frac{2\lambda_{\min}(K_2 - I)}{\lambda_{\max}(M)} \\ &\min_{i=1,2,\ldots n}\left(\frac{\sigma_i^2\|W_i^*\|^2}{2\lambda_{\max}\left(\Gamma_i^{-1}\right)}\right), 2\lambda_{\min}\Bigg(D(z_2)M_0^{-1} \\ &- \left(1 + \frac{\|D(z_2)M_0^{-1}\|^2}{2\theta}\right)I_{n\times n}\Bigg)\Bigg)\end{aligned} \tag{62}$$

$$C_2 = \frac{1}{2}\varsigma^2 + \sum_{i=1}^{n}\frac{\sigma_i + \theta l^2}{2}\|W_i^*\|^2 + \sum_{i=1}^{n}\frac{\sigma_i^2}{8}\left(\nu^4 + \|W_i^*\|^4\right). \tag{63}$$

To make sure that $\kappa_2 > 0$, the constant θ, the gain matrix K_2 and the auxiliary variable $\Theta(z_2)$ are chosen to satisfy that

$$\theta > 0, \lambda_{\min}(K_2 - I) > 0 \tag{64}$$

$$\lambda_{\min}\left(-\frac{\|D(z_2)M_0^{-1}\|^2 + 2\theta}{2\theta}I_{n\times n} + D(z_2)M_0^{-1}\right) > 0. \tag{65}$$

Since $M_0^{-1}(q)$ and θ are known, the $\Theta(z_2)$ is not hard to be designed as a linear function of z_2.

Theorem 2: For the unknown robotic manipulator system dynamics (1), with bounded initial conditions, semi-global uniform boundedness stability is obtained under the adaptive NN control law (44). The output can asymptotically track the desired trajectory, i.e., as $t \to \infty$, $q(t) \to q_d(t)$. The time-varying output constraints are definitely not breached, namely $\forall t > 0$, $\underline{k}_c(t) < q(t) < \overline{k}_c(t)$. The error variables z_1, z_2, $\tilde{W}_i$, and $\tilde{d}$ will be bounded over the compact sets Ω_{z_1}, Ω_{z_2}, $\Omega_{\tilde{W}_i}$, and $\Omega_{\tilde{d}}$ respectively, defined as follows:

$$\Omega_{z_1} := \left\{z_1 \in \mathbb{R}^n | -\underline{D}_{z_{1i}} \le z_{1i} \le \overline{D}_{z_{1i}}\right\} \tag{66}$$

$$\Omega_{z_2} := \left\{z_2 \in \mathbb{R}^n | \|z_2\| \le \sqrt{\frac{D}{\lambda_{\min}(M)}}\right\} \tag{67}$$

$$\Omega_{\tilde{W}_i} := \left\{\tilde{W}_i \in \mathbb{R}^l | \|\tilde{W}_i\| \le \sqrt{\frac{D}{\lambda_{\min}\left(\Gamma_i^{-1}\right)}}\right\} \tag{68}$$

$$\Omega_{\tilde{d}} := \left\{\tilde{d} \in \mathbb{R}^n | \|\tilde{d}\| \le \sqrt{D}\right\} \tag{69}$$

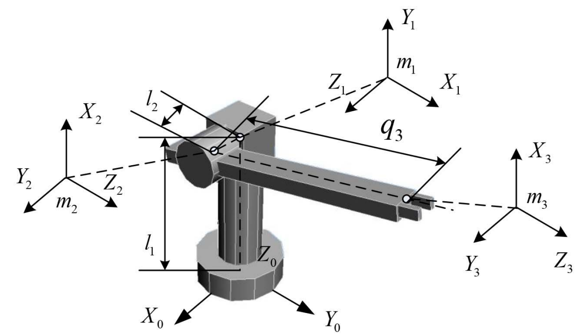


Fig. 3. Coordinates frame for each link using D-H method.

where $D = 2(V_3(0) + C_2/\kappa_2)$, κ_2 and C_2 are two positive constants shown in (62) and (63) and

$$\overline{D}_{z_{1i}} = \sqrt{k_{bi}^2(t)\left(1 - e^{-D}\right)} \tag{70}$$

$$\underline{D}_{z_{1i}} = \sqrt{k_{ai}^2(t)\left(1 - e^{-D}\right)}. \tag{71}$$

The proof is similar to the above proof of the Theorem 1.

IV. Simulation

We consider a 3-DOF robotic manipulator as shown in Fig. 3. The robot has two rotary joints and one prismatic joint. The position of the wrist is determined by two rotations and one translation through two rotary joints and a prismatic joint (RRP).

To verify the validity of the presented control, simulations of a 3-DOF robotic system are carried out. We define

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{72}$$

where q_1 and q_2 denote the two rotary joints' rotation angles and q_3 denotes the prismatic joint's translational displacement. Through the Denavit–Hartenberg representation and the Lagrange–Euler equations, $M(q)$, $C(q, \dot{q})$, and $G(q)$ can be expressed as

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix} \tag{73}$$

$$C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & 0 \end{bmatrix} \tag{74}$$

$$G(q) = \begin{bmatrix} 0 \\ G_{21} \\ G_{31} \end{bmatrix} \tag{75}$$

where $M_{11} = m_3q_3^2\sin q_2^2 + m_3l_1^2 + m_2l_1^2 + (1/4)m_1l_1^2$, $M_{12} = M_{21} = m_3q_3l_1\cos q_2$, $M_{13} = M_{31} = m_3l_1\sin q_2$, $M_{22} = m_3q_3^2 + (1/4)m_2l_2^2$, $M_{33} = m_3$, $C_{11} = m_3q_3^2\sin q_2\cos q_2\dot{q}_2 + m_3q_3^2\sin q_2^2\dot{q}_3$, $C_{12} = m_3q_3^2\sin q_2\cos q_2\dot{q}_1 - m_3l_1q_3\sin q_2\dot{q}_2 - m_3l_1q_3\sin q_2\dot{q}_3$, $C_{13} = m_3q_3^2\sin q_2^2\dot{q}_1 - m_3l_1q_3\sin q_2\dot{q}_2$, $C_{21} = -m_3q_3\sin q_2\cos q_2\dot{q}_1$, $C_{22} = m_3q_3\dot{q}_3$, $C_{23} = m_3q_3\dot{q}_2 + m_3l_1\cos q_2\dot{q}_3$, $C_{31} = -m_3q_3\sin q_2^2\dot{q}_1 + m_3l_1\cos q_2\dot{q}_2$, $C_{32} = m_3l_1\cos q_2\dot{q}_1 - m_3q_3\dot{q}_2$, $G_{21} = -m_3gq_3\cos q_2$,

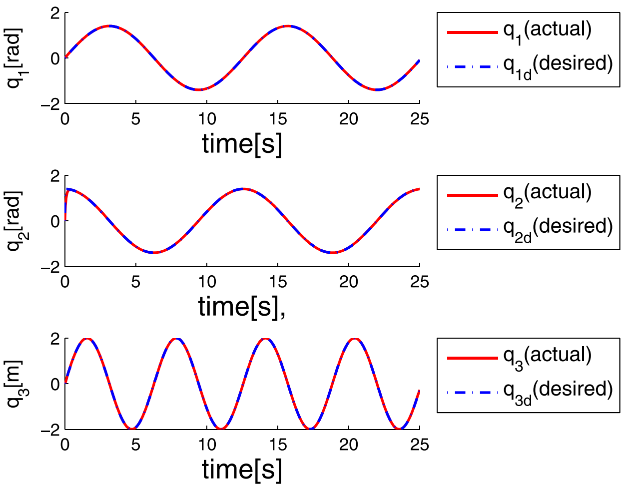


Fig. 4. Tracking performance of q under the MB control.

$G_{31} = -m_3 g \sin q_2$ and m_i denotes the equivalent lumped masses of links i, respectively. Parameters of the 3-DOF robotic manipulator are listed as follows:

$$l_1 = 0.3 \text{ m}, l_2 = 0.4 \text{ m} \tag{76}$$

$$m_1 = 2 \text{ kg}, m_2 = 2 \text{ kg}, m_3 = 1 \text{ kg}. \tag{77}$$

The initial states of the 3-DOF robot manipulator are set as 0. The control goal is to make the output q track a target trajectory

$$q_d = \begin{bmatrix} 1.4\sin 0.5t \\ 1.4\cos 0.5t \\ 2\sin t \end{bmatrix} \tag{78}$$

where $t \in (0, t_f)$ and $t_f = 25s$. We choose the external disturbance f_{dis} as $f_{\text{dis}}(t) = [\sin(t) + 1, 2\cos t + 0.5, 2\sin t + 1]^T$. The time-varying constraints for the output are designed as $\overline{k}_c(t) = [2 + 0.6\cos 0.5t, 2 + 0.6\sin 0.5t, 2.5 + 0.5\cos(t)]^T$, $\underline{k}_c(t) = [-1.1 + 0.4\sin 0.5t, -1.1 + 0.4\cos 0.5t, -1.6 + \sin t]^T$, so the bounds of tracking error $z_1 = [z_{11}, z_{12}, z_{13}]^T$ are set as $k_b(t) = [2 + 0.6\cos 0.5t - 1.4\sin 0.5t, 2 + 0.6\sin 0.5t - 1.4\cos 0.5t, 2.5 = 0.5\cos t - 2\sin t]^T$, $k_a(t) = [1.1 + \sin 0.5t, 1.1 + \cos 0.5t, 1.6 + 1.5\sin t]^T$. We can easily know that the joint constraints can be guaranteed by ensuring $-k_{ai}(t) < z_{1i}(t) < k_{bi}(t)$.

Three different kind of control laws are carried out for the simulation investigation. We first investigate the proposed model-based control (26). Then we validate the effectiveness of the presented adaptive NN control law (44). Finally, the traditional PD control designed as $\tau = -K_p z_1 - K_d \dot{z}_1$ is considered and the three controllers are compared to show the advantages of our NN control scheme.

A. Model-Based (MB) Control

For the MB control law (26), we design the gain matrix as $K_1 = \text{diag}[12, 12, 12]$, $K_2 = \text{diag}[8, 8, 8]$. The assistant quantity β is chosen as $\beta = 0.0001$. The $\Theta(z_2)$ is set as $3z_2$. Figs. 4–9 illustrate the simulation results. From Fig. 4, we can see that the output q can track the desired trajectories q_d exactly. From Figs. 5–7, we find that the tracking errors z_{11}, z_{12}, and z_{13} are convergent to a small value close to zero.

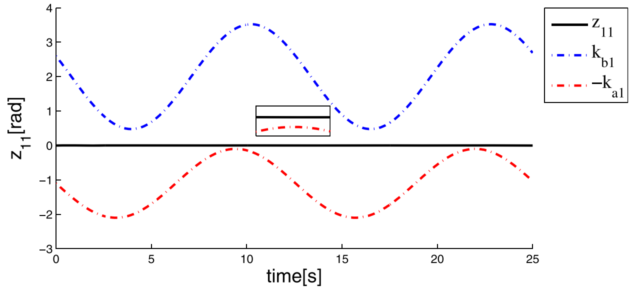


Fig. 5. Tracking performance of z_{11} under the MB control.

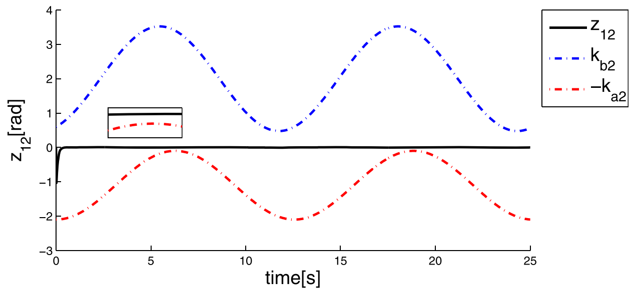


Fig. 6. Tracking performance of z_{12} under the MB control.

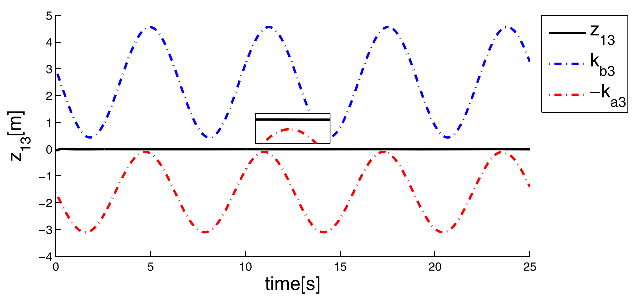


Fig. 7. Tracking performance of z_{13} under the MB control.

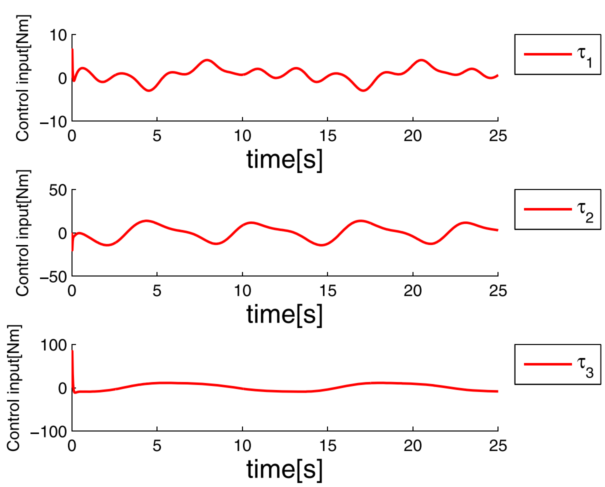


Fig. 8. Control input torque of the MB control.

Meanwhile, the constraints of them are never breached. Fig. 8 gives the control input torques τ_1, τ_2, and τ_3. The approximate errors of the DO are given in Fig. 9. We can see that the DO estimate errors are getting smaller and smaller and finally close to zero. Totally speaking, the proposed model-based control can obtain satisfying control effect. However, the model-based

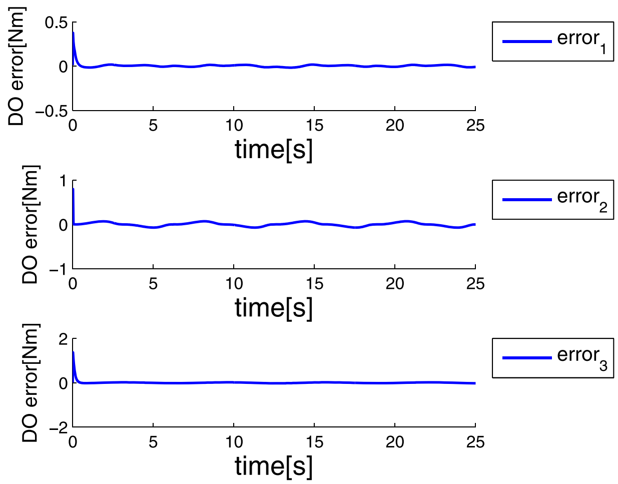


Fig. 9. DO approximate errors of the MB control.

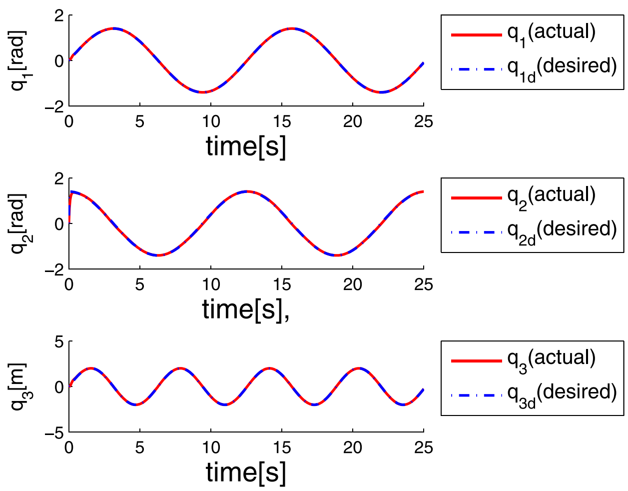


Fig. 10. Tracking performance of q under the NN control.

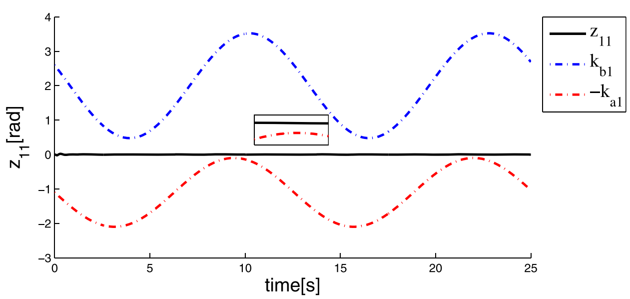


Fig. 11. Tracking performance of z_{11} under the NN control.

control law is based on that we know the exact parameters of the robotic manipulator system, which are generally unknown in practice.

B. Adaptive Neural Network Control

For the NN control law (44), 64 nodes are used for the NN. The RBF NN centers for $X_i(Z)$ are evenly distributed in domain of $[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]$. Variance for the NNs is set to be $\eta^2=1$.

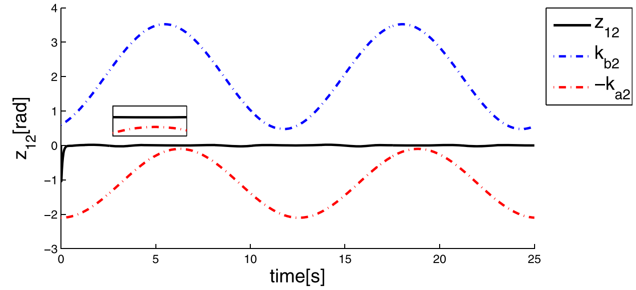


Fig. 12. Tracking performance of z_{12} under the NN control.

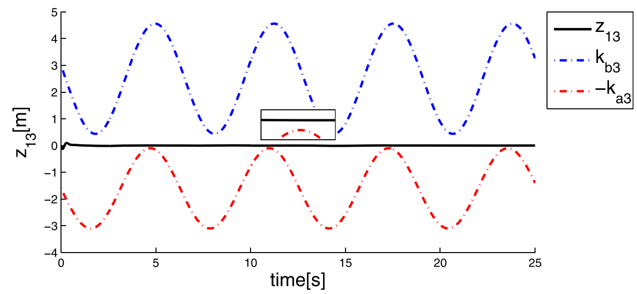


Fig. 13. Tracking performance of z_{13} under the NN control.

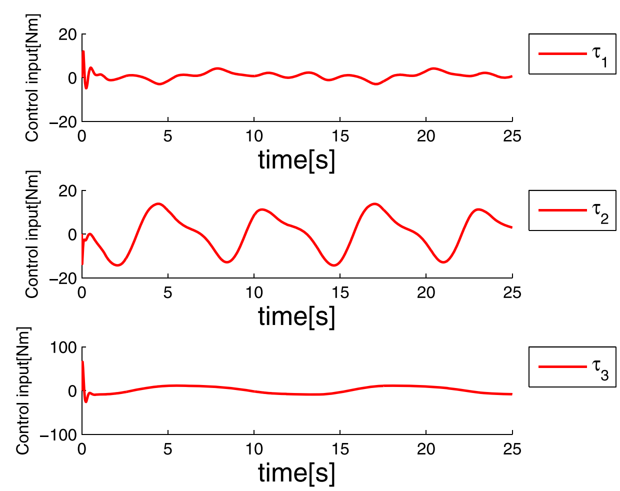


Fig. 14. Control input torque of the NN control.

The NNs weight are initialized from zero. The gain matrix K_1 and K_2 are chosen as $K_1=\mathrm{diag}[12,12,12]$, $K_2=\mathrm{diag}[8,8,8]$ which is the same as MB control, and $\sigma_1=\sigma_2=\sigma_3=0.02$, $\Gamma_1=\Gamma_2=\Gamma_3=10I_{64\times 64}$. We design $M_0=\mathrm{diag}[\sin(t)+2,\sin(t)+2,\sin(t)+2]$ and $C_0=\mathrm{diag}[0.5\cos(t)+1,0.5\cos(t)+1,0.5\cos(t)+1]$ to ensure that M_0 is positive definite and $\dot{M}_0-2C_0$ is symmetric. Figs. 10–17 illustrate simulation results. Fig. 10 indicates that the output q can follow the desired trajectories q_d with a tiny error. From Figs. 11–13, we find that the tracking errors z_{11}, z_{12}, and z_{13} are convergent to a small value close to zero. meanwhile, all of them are obviously repelled from the time-varying bounds $-k_a(t)$ and $k_b(t)$, which is equivalent to that the output constraints are not violated. Fig. 15 gives the adaption weights $\hat{W}_1(t)$, $\hat{W}_2(t)$, and $\hat{W}_3(t)$, from which we can see that the weights are also bounded. The DO estimate errors are shown in Fig. 16. Similarly as Fig. 9, we can see

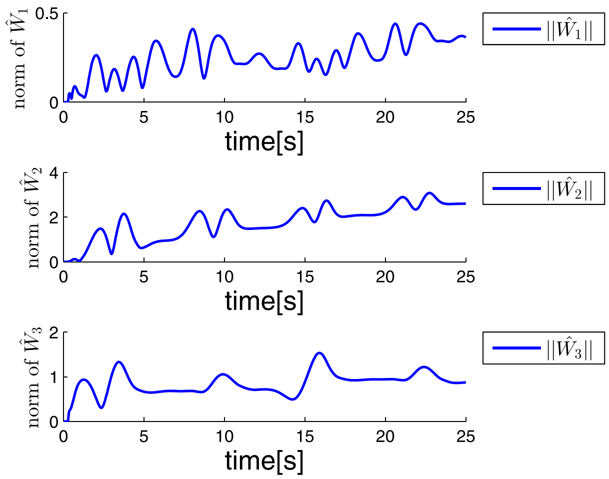


Fig. 15. Norms of the NN weights.

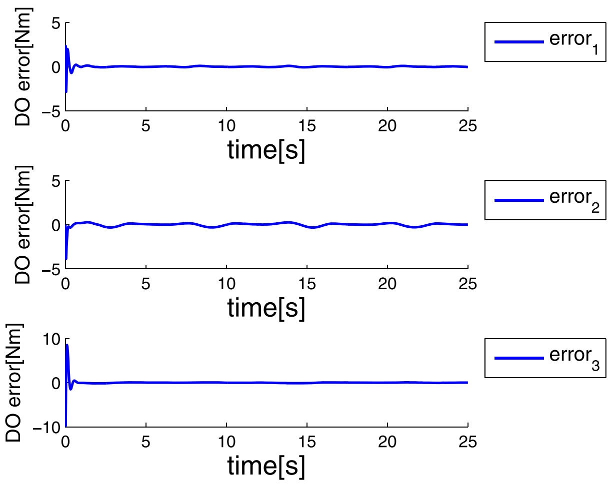


Fig. 16. DO approximate errors of the NN control.

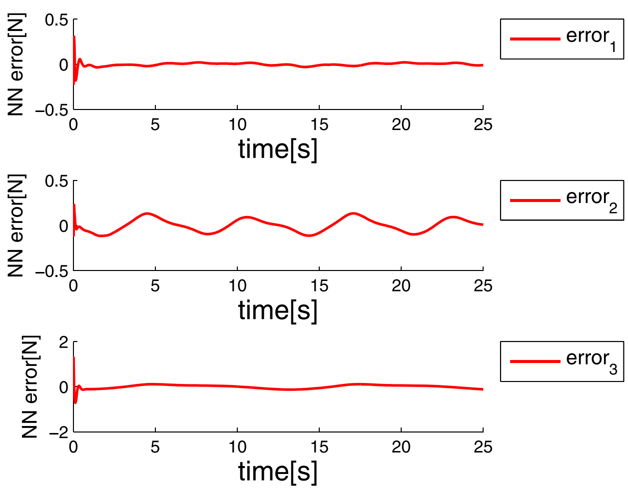


Fig. 17. NN approximate errors.

that the errors are getting smaller with the time and finally close to zero. The proposed adaptive NNs produce low norms of approximate errors as shown in Fig. 17. The good tracking performance indicates that NNs can ensure the control performance even when the exact information of the system

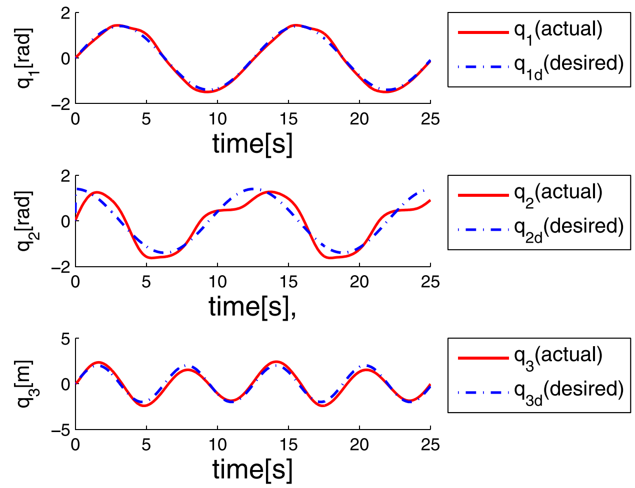


Fig. 18. Tracking performance of q under the PD control.

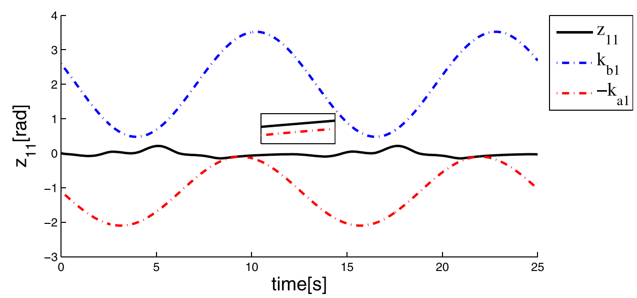


Fig. 19. Tracking performance of z_{11} under the PD control.

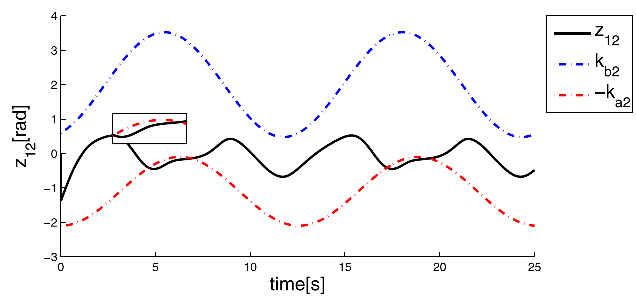


Fig. 20. Tracking performance of z_{12} under the PD control.

is unknown, and the DO is capable of compensating for the effect of the unknown external disturbance. From these figures, we know that desired performance can be guaranteed by the presented control law even when we have no knowledge of the robotic dynamics and the disturbance from humans and environment.

C. PD Control

To obtain more convincing simulation results, PD control simulation is conducted to compare with the proposed NN control. After many times of PD tuning, we choose $K_p = \text{diag}[10, 10, 10]$ and $K_d = \text{diag}[10, 10, 10]$. The tracking performance is given in Figs. 18–22. Fig. 18 indicates that the output q can follow the desired trajectories q_d roughly, but from Figs. 20 and 21, it is obvious that the constraints of q_2 and q_3 are breached. Fig. 22 gives the PD control input torques, from which we can see that the input is bounded.

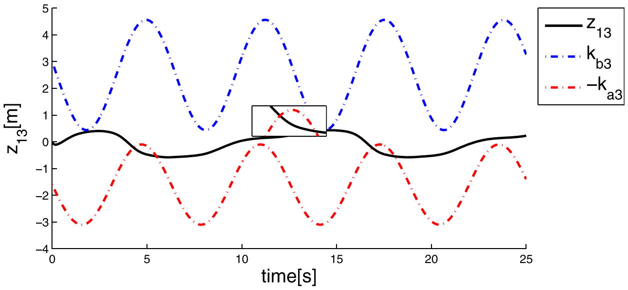


Fig. 21. Tracking performance of z_{13} under the PD control.

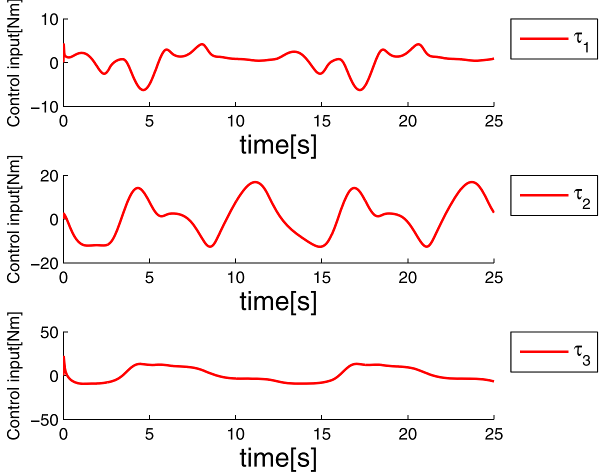


Fig. 22. Control input torque of the PD control.

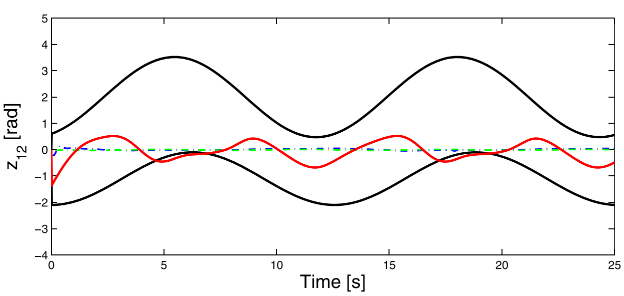


Fig. 23. Tracking performance of z_{12}. The black solid lines indicate the constraints. The red line indicates z_{12} under PD control. The green line indicates z_{12} under NN control. The blue dashed line indicates z_{12} under MB control.

To illustrate that the proposed adaptive NN control can achieve both guaranteed global stability and tailored transient performance, we make the tracking performance of z_{12} under the above three controllers in one figure as shown in Fig. 23. From this figure, we can clearly see that the NN control with our designed DO has a good tracking performance even the exact parameters of the robotic manipulator system are unknown.

V. Conclusion

An adaptive NN control scheme based on DOs has been presented for an uncertain n-DOF robot subjected to time-varying constraints and unknown disturbance. We have used the NNs to estimate the unknown dynamic model of the robotic manipulator and employed DOs to approximate the time-varying disturbance. An asymmetric BLF is used to avert the contravention of the output constraints. The simulation results have indicated that the presented control scheme can make the output well follow the target trajectory while ensuring the constraints satisfaction.

References

[1] R. Cui, J. Guo, and B. Gao, “Game theory-based negotiation for multiple robots task allocation,” *Robotica*, vol. 31, no. 6, pp. 923–934, 2013.

[2] L. Wang, Z. Liu, C. L. P. Chen, Y. Zhang, S. Lee, and X. Chen, “Energy-efficient SVM learning control system for biped walking robots,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 5, pp. 831–837, May 2013.

[3] S. P. Buerger and N. Hogan, “Complementary stability and loop shaping for improved human–robot interaction,” *IEEE Trans. Robot.*, vol. 23, no. 2, pp. 232–244, Apr. 2007.

[4] Z. Liu, F. Wang, and Y. Zhang, “Adaptive visual tracking control for manipulator with actuator fuzzy dead-zone constraint and unmodeled dynamic,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 45, no. 10, pp. 1301–1312, Oct. 2015.

[5] Z. Li, S. Xiao, S. S. Ge, and H. Su, “Constrained multilegged robot system modeling and fuzzy control with uncertain kinematics and dynamics incorporating foot force optimization,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 1, pp. 1–15, Jan. 2016.

[6] Z. Li, Z. Huang, W. He, and C.-Y. Su, “Adaptive impedance control for an upper limb robotic exoskeleton using biological signals,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 2, pp. 1664–1674, Feb. 2017.

[7] Z. Zhang and S. Xu, “Observer design for uncertain nonlinear systems with unmodeled dynamics,” *Automatica*, vol. 51, pp. 80–84, Jan. 2015.

[8] N. Wang, C. Qian, J.-C. Sun, and Y.-C. Liu, “Adaptive robust finite-time trajectory tracking control of fully actuated marine surface vehicles,” *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 4, pp. 1454–1462, Jul. 2016.

[9] Q. Guo, Y. Zhang, B. G. Celler, and S. W. Su, “Backstepping control of electro-hydraulic system based on extended-state-observer with plant dynamics largely unknown,” *IEEE Trans. Ind. Electron.*, vol. 63, no. 11, pp. 6909–6920, Nov. 2016.

[10] J. Yang, S. Li, and X. Yu, “Sliding-mode control for systems with mismatched uncertainties via a disturbance observer,” *IEEE Trans. Ind. Electron.*, vol. 60, no. 1, pp. 160–169, Jan. 2013.

[11] Z. Li, C.-Y. Su, L. Wang, Z. Chen, and T. Chai, “Nonlinear disturbance observer-based control design for a robotic exoskeleton incorporating fuzzy approximation,” *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5763–5775, Sep. 2015.

[12] M. Chen and S. S. Ge, “Adaptive neural output feedback control of uncertain nonlinear systems with unknown hysteresis using disturbance observer,” *IEEE Trans. Ind. Electron.*, vol. 62, no. 12, pp. 7706–7716, Dec. 2015.

[13] B. Xu, “Disturbance observer-based dynamic surface control of transport aircraft with continuous heavy cargo airdrop,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 1, pp. 161–170, Jan. 2017.

[14] J. Zhou, C. Wen, and G. Yang, “Adaptive backstepping stabilization of nonlinear uncertain systems with quantized input signal,” *IEEE Trans. Autom. Control*, vol. 59, no. 2, pp. 460–464, Feb. 2014.

[15] C. Mu, Z. Ni, C. Sun, and H. He, “Air-breathing hypersonic vehicle tracking control based on adaptive dynamic programming,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 3, pp. 584–598, Mar. 2017.

[16] J. Guo, B. Mu, L. Y. Wang, G. Yin, and L. Xu, “Decision-based system identification and adaptive resource allocation,” *IEEE Trans. Autom. Control*, vol. 62, no. 5, pp. 2166–2179, 2017.

[17] J. Zhou and C. Wen, *Adaptive Backstepping Control of Uncertain Systems: Nonsmooth Nonlinearities, Interactions or Time-Variations*. Berlin, Germany: Springer, 2008.

[18] B. Gao, W. L. Woo, Y. He, and G. Y. Tian, “Unsupervised sparse pattern diagnostic of defects with inductive thermography imaging system,” *IEEE Trans. Ind. Informat.*, vol. 12, no. 1, pp. 371–383, Feb. 2016.

[19] B. Gao *et al.*, “Multidimensional tensor-based inductive thermography with multiple physical fields for offshore wind turbine gear inspection,” *IEEE Trans. Ind. Electron.*, vol. 63, no. 10, pp. 6305–6315, Oct. 2016.

[20] J. Zhou and M. Krstic, “Adaptive predictor control for stabilizing pressure in a managed pressure drilling system under time-delay,” *J. Process Control*, vol. 40, pp. 106–118, Apr. 2016.

[21] Y. Li and S. S. Ge, “Human–robot collaboration based on motion intention estimation,” *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 3, pp. 1007–1014, Jun. 2014.

[22] C. Sun and Y. Xia, “An analysis of a neural dynamical approach to solving optimization problems,” *IEEE Trans. Autom. Control*, vol. 54, no. 8, pp. 1972–1977, Aug. 2009.

[23] B. Xu, Z. Shi, C. Yang, and F. Sun, "Composite neural dynamic surface control of a class of uncertain nonlinear systems in strict-feedback form," *IEEE Trans. Cybern.*, vol. 14, no. 12, pp. 2626–2634, Dec. 2014.

[24] C. L. P. Chen, G.-X. Wen, Y.-J. Liu, and Z. Liu, "Observer-based adaptive backstepping consensus tracking control for high-order nonlinear semi-strict-feedback multiagent systems," *IEEE Trans. Cybern.*, vol. 46, no. 7, pp. 1591–1601, Jul. 2016.

[25] C. L. P. Chen, G.-X. Wen, Y.-J. Liu, and F.-Y. Wang, "Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 6, pp. 1217–1226, Jun. 2014.

[26] B. Luo, T. Huang, H.-N. Wu, and X. Yang, "Data-driven h_∞ control for nonlinear distributed parameter systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 11, pp. 2949–2961, Nov. 2015.

[27] S.-L. Dai, C. Wang, and M. Wang, "Dynamic learning from adaptive neural network control of a class of nonaffine nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 1, pp. 111–123, Jan. 2014.

[28] S.-L. Dai, M. Wang, and C. Wang, "Neural learning control of marine surface vessels with guaranteed transient tracking performance," *IEEE Trans. Ind. Electron.*, vol. 63, no. 3, pp. 1717–1727, Mar. 2016.

[29] H. Li, L. Bai, L. Wang, Q. Zhou, and H. Wang, "Adaptive neural control of uncertain nonstrict-feedback stochastic nonlinear systems with output constraint and unknown dead zone," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2016.2605706.

[30] H. Zhang, Y. Luo, and D. Liu, "Neural-network-based near-optimal control for a class of discrete-time affine nonlinear systems with control constraints," *IEEE Trans. Neural Netw.*, vol. 20, no. 9, pp. 1490–1503, Sep. 2009.

[31] Y. Li, S. Qiang, X. Zhuang, and O. Kaynak, "Robust and adaptive backstepping control for nonlinear systems using RBF neural networks," *IEEE Trans. Neural Netw.*, vol. 15, no. 3, pp. 693–701, May 2004.

[32] M. Chen, S. S. Ge, and B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, 2011.

[33] C. L. P. Chen, Y.-J. Liu, and G.-X. Wen, "Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems," *IEEE Trans. Cybern.*, vol. 44, no. 5, pp. 583–593, May 2014.

[34] L. Cheng, W. Liu, Z.-G. Hou, J. Yu, and M. Tan, "Neural-network-based nonlinear model predictive control for piezoelectric actuators," *IEEE Trans. Ind. Electron.*, vol. 62, no. 12, pp. 7717–7727, Dec. 2015.

[35] W. He and S. S. Ge, "Robust adaptive boundary control of a vibrating string under unknown time-varying disturbance," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 1, pp. 48–58, Jan. 2012.

[36] W. He, S. Zhang, and S. S. Ge, "Adaptive boundary control of a nonlinear flexible string system," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 3, pp. 1088–1093, May 2014.

[37] Z. Li and C.-Y. Su, "Neural-adaptive control of single-master–multiple-slaves teleoperation for coordinated multiple mobile manipulators with time-varying communication delays and input uncertainties," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 9, pp. 1400–1413, Sep. 2013.

[38] Z. Li, S. S. Ge, and S. Liu, "Contact-force distribution optimization and control for quadruped robots using both gradient and adaptive neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 8, pp. 1460–1473, Aug. 2014.

[39] B. Ren, S. S. Ge, C.-Y. Su, and T. H. Lee, "Adaptive neural control for a class of uncertain nonlinear systems in pure-feedback form with hysteresis input," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 39, no. 2, pp. 431–443, Apr. 2009.

[40] Y.-J. Liu and S.-C. Tong, "Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems," *Automatica*, vol. 76, no. 2, pp. 143–152, Feb. 2017.

[41] Z. Liu, C. Chen, Y. Zhang, and C. L. P. Chen, "Adaptive neural control for dual-arm coordination of humanoid robot with unknown nonlinearities in output mechanism," *IEEE Trans. Cybern.*, vol. 45, no. 3, pp. 507–518, Mar. 2015.

[42] W. He, S. S. Ge, Y. Li, E. Chew, and Y. S. Ng, "Neural network control of a rehabilitation robot by state and output feedback," *J. Intell. Robot. Syst.*, vol. 80, no. 1, pp. 15–31, 2015.

[43] C. Yang, X. Wang, Z. Li, Y. Li, and C.-Y. Su, "Teleoperation control based on combination of wave variable and neural networks," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2016.2615061.

[44] C. Yang, J. Luo, Y. Pan, Z. Liu, and C.-Y. Su, "Personalized variable gain control with tremor attenuation for robot teleoperation," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2017.2694020.

[45] C. Yang, X. Wang, L. Cheng, and H. Ma, "Neural-learning-based telerobot control with guaranteed performance," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2016.2573837.

[46] C. P. Bechlioulis and G. A. Rovithakis, "Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance," *IEEE Trans. Autom. Control*, vol. 53, no. 9, pp. 2090–2099, Oct. 2008.

[47] C. P. Bechlioulis and G. A. Rovithakis, "Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems," *Automatica*, vol. 45, no. 2, pp. 532–538, 2009.

[48] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918–927, 2009.

[49] W. He, Y. Chen, and Z. Yin, "Adaptive neural network control of an uncertain robot with full-state constraints," *IEEE Trans. Cybern.*, vol. 46, no. 3, pp. 620–629, Mar. 2016.

[50] Y.-J. Liu and S. Tong, "Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints," *Automatica*, vol. 64, pp. 70–75, Feb. 2016.

[51] B. Ren, S. S. Ge, K. P. Tee, and T. H. Lee, "Adaptive neural control for output feedback nonlinear systems using a barrier Lyapunov function," *IEEE Trans. Neural Netw.*, vol. 21, no. 8, pp. 1339–1345, Aug. 2010.

[52] K. P. Tee, B. Ren, and S. S. Ge, "Control of nonlinear systems with time-varying output constraints," *Automatica*, vol. 47, no. 11, pp. 2511–2516, 2011.

[53] R. Cui, Y. Li, and W. Yan, "Mutual information-based multi-AUV path planning for scalar field sampling using multidimensional RRT*," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 7, pp. 993–1004, Jul. 2016.

[54] D. Wang, D. Liu, H. Li, B. Luo, and H. Ma, "An approximate optimal control approach for robust stabilization of a class of discrete-time nonlinear systems with uncertainties," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 5, pp. 713–717, May 2016.

[55] Y. Kang, D.-H. Zhai, G.-P. Liu, and Y.-B. Zhao, "On input-to-state stability of switched stochastic nonlinear systems under extended asynchronous switching," *IEEE Trans. Cybern.*, vol. 46, no. 5, pp. 1092–1105, May 2016.

[56] X. Wu and D. Gao, "Fault tolerance control of SOFC systems based on nonlinear model predictive control," *Int. J. Hydrogen Energy*, vol. 42, no. 4, pp. 2288–2308, 2017.

[57] Z. Liu, F. Wang, Y. Zhang, X. Chen, and C. L. P. Chen, "Adaptive fuzzy output-feedback controller design for nonlinear systems via backstepping and small-gain approach," *IEEE Trans. Cybern.*, vol. 44, no. 10, pp. 1714–1725, Oct. 2014.

[58] Y. Kang, D.-H. Zhai, G.-P. Liu, Y.-B. Zhao, and P. Zhao, "Stability analysis of a class of hybrid stochastic retarded systems under asynchronous switching," *IEEE Trans. Autom. Control*, vol. 59, no. 6, pp. 1511–1523, Jun. 2014.

[59] Y.-J. Liu, Y. Gao, S.-C. Tong, and Y.-M. Li, "Fuzzy approximation-based adaptive backstepping optimal control for a class of nonlinear discrete-time systems with dead-zone," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 1, pp. 16–28, Feb. 2016.

[60] Z. Zhang, S. Xu, and B. Zhang, "Exact tracking control of nonlinear systems with time delays and dead-zone input," *Automatica*, vol. 52, pp. 272–276, Feb. 2015.

[61] S. Zhang, W. He, and D. Huang, "Active vibration control for a flexible string system with input backlash," *IET Control Theory Appl.*, vol. 10, no. 7, pp. 800–805, Apr. 2016.

[62] Y.-J. Liu, Y. Gao, S.-C. Tong, and C. L. P. Chen, "A unified approach to adaptive neural control for nonlinear discrete-time systems with nonlinear dead-zone input," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 1, pp. 139–150, Jan. 2016.

[63] W. He, Y. Dong, and C. Sun, "Adaptive neural impedance control of a robotic manipulator with input saturation," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 3, pp. 334–344, Mar. 2016.

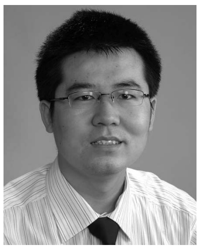
[64] Z. Zhao, W. He, and S. S. Ge, "Adaptive neural network control of a fully actuated marine surface vessel with multiple output constraints," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 4, pp. 1536–1543, Jul. 2014.

[65] Y. Li and S. Tong, "Adaptive fuzzy output constrained control design for multi-input multioutput stochastic nonstrict-feedback nonlinear systems," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2016.2600263.

[66] S. S. Ge, T. H. Lee, and C. J. Harris, *Adaptive Neural Network Control of Robotic Manipulators*. London, U.K.: World Sci., 1998.

[67] S. S. Ge and C. Wang, "Adaptive neural control of uncertain MIMO nonlinear systems," *IEEE Trans. Neural Netw.*, vol. 15, no. 3, pp. 674–692, May 2004.

[68] W. Meng, Q. Yang, J. Si, and Y. Sun, "Adaptive neural control of a class of output-constrained nonaffine systems," *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 85–95, Jan. 2016.



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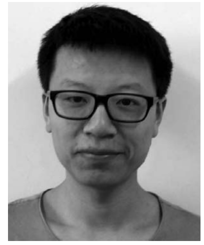

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