

# Lecture Notes in Control and Information Sciences 310

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Editors: M. Thoma · M. Morari

# Lecture Notes in Control and Information Sciences

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A. Janczak

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# **Identification of Nonlinear Systems Using Neural Networks and Polynomial Models**

**A Block-Oriented Approach**

With 79 Figures and 22 Tables



**Springer**

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## Preface

The identification of nonlinear systems using the block-oriented approach has been developed since the half of 1960s. A large amount of knowledge on this subject has been accumulated through literature. However, publications are scattered over many papers and there is no book which presents the subject in a unified framework. This has created an increasing need to systemize the existing identification methods and along with a presentation of some original results have been the main incentive to write this book. In writing the book, an attempt has been made at the presentation of some new ideas concerning the model parameter adjusting with gradient-based techniques.

Two types of models, considered in this book, use neural networks and polynomials as representations of Wiener and Hammerstein systems. The focus is placed on Wiener and Hammerstein models in which the nonlinear element is represented by a polynomial or a two-layer perceptron neural network with hyperbolic tangent hidden layer nodes and linear output nodes. Pulse transfer function models are common representations of system dynamics in both neural network and polynomial Wiener and Hammerstein models.

Neural network and polynomial models reveal different properties such as the approximation accuracy, computational complexity, available parameter and structure optimization methods, etc. All these differences make them complementary in solving many practical problems. For example, it is well known that the approximation of some nonlinear functions requires polynomials of a high order and this, in turn, results in a high parameter variance error. The approximation with neural network models is an interesting alternative in such cases.

The book results mainly from my research in the area of nonlinear system identification that have been performed since 1995. Two exceptions from this rule are Chapter 1, containing the introductory notes, and Chapter 5, which reviews the well-known Hammerstein system identification methods based on polynomial models of the nonlinearity. In writing the book, an emphasis has been put on presenting various identification methods, which are applicable to both neural network and polynomial models of Wiener and Hammerstein systems, in a unified framework.

The book starts with a survey of discrete-time models of time-invariant dynamic systems. Then the multilayer perceptron neural network is introduced and a brief review of the existing methods for the identification of Wiener and Hammerstein systems is presented. Two subsequent Chapters (2 and 3) introduce neural network models of Wiener and Hammerstein systems and present different algorithms for the calculation of the gradient or the approximate gradient of the model output w.r.t. model parameters. For both Wiener and Hammerstein models, the accuracy of gradient evaluation with the truncated backpropagation through time algorithm is analyzed. The discussion also includes advantages and disadvantages of the algorithms in terms of their approximation accuracy, computational requirements, and weight updating methods. Next, in Chapter 4, we present identification methods, which use polynomial models of Wiener systems. The parameters of the linear dynamic system and the inverse nonlinearity are estimated with the least squares method, and a combined least squares and instrumental variables approach. To estimate parameters of the noninverted nonlinearity, the recursive prediction error and the pseudolinear regression methods are proposed. Then the existing identification methods based on polynomial Hammerstein models are reviewed and presented in Chapter 5. Wiener and Hammerstein models are two examples of block-oriented models which have found numerous industrial applications. The most important of them, including nonlinear system modelling, control, and fault detection and isolation, are reviewed in Chapter 6. This chapters presents also two applications of Wiener and Hammerstein models – estimation of system parameter changes, and modelling vapor pressure dynamics in a five stage sugar evaporation station.

The book contains the results od research conducted by the author with the kind support of the State Committee for Scientific Research in Poland under the grant No 4T11A01425 and with the additional support of the European Union within the 5th Framework Programme under *DAMADICS* project No HPRN-CT-2000-00110.

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Zielona Góra,  
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*Andrzej Janczak*

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# Contents

<b>Symbols and notation</b> .....	XI
<b>1 Introduction</b> .....	1
1.1 Models of dynamic systems .....	5
1.1.1 Linear models .....	5
1.1.2 Nonlinear models .....	8
1.1.3 Series-parallel and parallel models .....	10
1.1.4 State space models .....	10
1.1.5 Nonlinear models composed of sub-models .....	11
1.1.6 State-space Wiener models .....	15
1.1.7 State-space Hammerstein models .....	15
1.2 Multilayer perceptron .....	16
1.2.1 MLP architecture .....	16
1.2.2 Learning algorithms .....	17
1.2.3 Optimizing the model architecture .....	18
1.3 Identification of Wiener systems .....	19
1.4 Identification of Hammerstein systems .....	25
1.5 Summary .....	30
<b>2 Neural network Wiener models</b> .....	31
2.1 Introduction .....	31
2.2 Problem formulation .....	32
2.3 Series-parallel and parallel neural network Wiener models ....	34
2.3.1 SISO Wiener models .....	34
2.3.2 MIMO Wiener models .....	37
2.4 Gradient calculation .....	40
2.4.1 Series-parallel SISO model. Backpropagation method ..	40
2.4.2 Parallel SISO model. Backpropagation method .....	42
2.4.3 Parallel SISO model. Sensitivity method .....	42
2.4.4 Parallel SISO model. Backpropagation through time method .....	43

2.4.5	Series-parallel MIMO model. Backpropagation method .	46
2.4.6	Parallel MIMO model. Backpropagation method . . . . .	48
2.4.7	Parallel MIMO model. Sensitivity method . . . . .	48
2.4.8	Parallel MIMO model. Backpropagation through time method . . . . .	49
2.4.9	Accuracy of gradient calculation with truncated BPTT.	49
2.4.10	Gradient calculation in the sequential mode. . . . .	51
2.4.11	Computational complexity . . . . .	52
2.5	Simulation example . . . . .	53
2.6	Two-tank system example . . . . .	61
2.7	Prediction error method . . . . .	65
2.7.1	Recursive prediction error learning algorithm . . . . .	65
2.7.2	Pneumatic valve simulation example . . . . .	66
2.8	Summary . . . . .	69
2.9	Appendix 2.1. Gradient derivation of the truncated BPTT. SISO Wiener models . . . . .	71
2.10	Appendix 2.2. Gradient derivation of truncated BPTT. MIMO Wiener models . . . . .	72
2.11	Appendix 2.3. Proof of Theorem 2.1 . . . . .	73
2.12	Appendix 2.4. Proof of Theorem 2.2 . . . . .	74
<b>3</b>	<b>Neural network Hammerstein models . . . . .</b>	<b>77</b>
3.1	Introduction . . . . .	77
3.2	Problem formulation . . . . .	78
3.3	Series-parallel and parallel neural network Hammerstein models	79
3.3.1	SISO Hammerstein models . . . . .	79
3.3.2	MIMO Hammerstein models . . . . .	82
3.4	Gradient calculation . . . . .	84
3.4.1	Series-parallel SISO model. Backpropagation method ..	84
3.4.2	Parallel SISO model. Backpropagation method . . . . .	85
3.4.3	Parallel SISO model. Sensitivity method . . . . .	85
3.4.4	Parallel SISO model. Backpropagation through time method . . . . .	87
3.4.5	Series-parallel MIMO model. Backpropagation method .	87
3.4.6	Parallel MIMO model. Backpropagation method . . . . .	90
3.4.7	Parallel MIMO model. Sensitivity method . . . . .	90
3.4.8	Parallel MIMO model. Backpropagation through time method . . . . .	91
3.4.9	Accuracy of gradient calculation with truncated BPTT.	92
3.4.10	Gradient calculation in the sequential mode. . . . .	96
3.4.11	Computational complexity . . . . .	97
3.5	Simulation example . . . . .	97
3.6	Combined steepest descent and least squares learning algorithms . . . . .	104
3.7	Summary . . . . .	106



3.8	Appendix 3.1. Gradient derivation of truncated BPTT. SISO Hammerstein models . . . . .	108
3.9	Appendix 3.2. Gradient derivation of truncated BPTT. MIMO Hammerstein models . . . . .	109
3.10	Appendix 3.3. Proof of Theorem 3.1 . . . . .	111
3.11	Appendix 3.4. Proof of Theorem 3.2 . . . . .	113
3.12	Appendix 3.5. Proof of Theorem 3.3 . . . . .	114
3.13	Appendix 3.6. Proof of Theorem 3.4 . . . . .	115
<b>4</b>	<b>Polynomial Wiener models . . . . .</b>	<b>117</b>
4.1	Least squares approach to the identification of Wiener systems	118
4.1.1	Identification error . . . . .	119
4.1.2	Nonlinear characteristic with the linear term . . . . .	121
4.1.3	Nonlinear characteristic without the linear term . . . . .	122
4.1.4	Asymptotic bias error of the LS estimator . . . . .	123
4.1.5	Instrumental variables method . . . . .	125
4.1.6	Simulation example. Nonlinear characteristic with the linear term . . . . .	126
4.1.7	Simulation example. Nonlinear characteristic without the linear term . . . . .	128
4.2	Identification of Wiener systems with the prediction error method . . . . .	130
4.2.1	Polynomial Wiener model . . . . .	130
4.2.2	Recursive prediction error method . . . . .	132
4.2.3	Gradient calculation . . . . .	132
4.2.4	Pneumatic valve simulation example . . . . .	133
4.3	Pseudolinear regression method . . . . .	137
4.3.1	Pseudolinear-in-parameters polynomial Wiener model . . . . .	137
4.3.2	Pseudolinear regression identification method . . . . .	138
4.3.3	Simulation example . . . . .	138
4.4	Summary . . . . .	141
<b>5</b>	<b>Polynomial Hammerstein models . . . . .</b>	<b>143</b>
5.1	Noniterative least squares identification of Hammerstein systems . . . . .	143
5.2	Iterative least squares identification of Hammerstein systems . . . . .	145
5.3	Identification of Hammerstein systems in the presence of correlated noise . . . . .	147
5.4	Identification of Hammerstein systems with the Laguerre function expansion . . . . .	149
5.5	Prediction error method . . . . .	151
5.6	Identification of MISO systems with the pseudolinear regression method . . . . .	153
5.7	Identification of systems with two-segment nonlinearities . . . . .	155
5.8	Summary . . . . .	157

- 6   Applications** ..... 159
  - 6.1   General review of applications..... 159
  - 6.2   Fault detection and isolation with Wiener and Hammerstein  
      models ..... 166
    - 6.2.1   Definitions of residuals ..... 167
    - 6.2.2   Hammerstein system. Parameter estimation of the  
          residual equation ..... 171
    - 6.2.3   Wiener system. Parameter estimation of the residual  
          equation ..... 175
  - 6.3   Sugar evaporator. Identification of the nominal model of  
      steam pressure dynamics ..... 180
    - 6.3.1   Theoretical model ..... 180
    - 6.3.2   Experimental models of steam pressure dynamics ..... 181
    - 6.3.3   Estimation results ..... 182
  - 6.4   Summary..... 185
- References** ..... 187
- Index** ..... 195

# Symbols and notation

$a_k$	$k$ th parameter of the polynomial $A(q^{-1})$
$\hat{a}_k$	$k$ th parameter of the polynomial $\hat{A}(q^{-1})$
$A(q^{-1})$	denominator polynomial of the system pulse transfer function
$\hat{A}(q^{-1})$	denominator polynomial of the model pulse transfer function
$\mathcal{A}(q^{-1})$	denominator polynomial of the faulty system pulse transfer function
$\mathbf{A}$	state space matrix, $(nx \times nx)$
$\hat{\mathbf{A}}^{(m)}$	$m$ th parameter matrix of the MIMO dynamic model. The MIMO Wiener model, $(ns \times ns)$ ; the MIMO Hammerstein model, $(ny \times ny)$
$b_k$	$k$ th parameter of the polynomial $B(q^{-1})$
$\hat{b}_k$	$k$ th parameter of the polynomial $\hat{B}(q^{-1})$
$B(q^{-1})$	nominator polynomial of the system pulse transfer function
$\hat{B}(q^{-1})$	nominator polynomial of the model pulse transfer function
$\mathcal{B}(q^{-1})$	nominator polynomial of the faulty system pulse transfer function
$\mathbf{B}$	control matrix, $(nx \times nu)$
$\hat{\mathbf{B}}^{(m)}$	$m$ th parameter matrix of the MIMO dynamic model. The MIMO Wiener model, $(ns \times nu)$ ; the MIMO Hammerstein model, $(ny \times nf)$
$\mathbf{C}$	observation matrix, $(ny \times nx)$
$\mathbf{D}$	matrix describing the effect of inputs on outputs, $(ny \times nu)$
$e(n)$	identification error (one-step-ahead prediction error)
$E$	mathematical expectation
$f(\cdot)$	nonlinear function of the system
$\hat{f}(\cdot)$	nonlinear function of the model
$\hat{f}_k(\cdot)$	$k$ th nonlinear function of the MIMO nonlinear element model
$\hat{\mathbf{f}}(\cdot)$	nonlinear function of the MIMO nonlinear element model
$f^{-1}(\cdot)$	inverse nonlinear function of the system
$\hat{f}^{-1}(\cdot)$	inverse nonlinear function of the model
$\hat{g}_k(\cdot)$	$k$ th nonlinear function of the MIMO inverse nonlinear element model
$\hat{\mathbf{g}}(\cdot)$	nonlinear function of the MIMO inverse nonlinear element model
$h(n)$	system impulse response
$h_1(n)$	impulse response of the sensitivity model
$h_2(n)$	impulse response of the linear dynamic model
$H_1(q^{-1})$	pulse transfer function of sensitivity models
$H_2(q^{-1})$	pulse transfer function of the linear dynamic model
$J$	global error function
$J(n)$	local error function
$K$	number of unfolded time steps
$m_f$	expected value of $\hat{f}(u(n), \mathbf{w})$

$m_{w_c}$	expected value of $\partial \hat{f}(u(n), \mathbf{w}) / \partial w_c$
$M$	number of nonlinear nodes
$n$	discrete time
$na$	order of the polynomials $A(q^{-1})$ and $\hat{A}(q^{-1})$
$nb$	order of the polynomials $B(q^{-1})$ and $\hat{B}(q^{-1})$
$nf$	number of outputs of MIMO Hammerstein nonlinear element model
$ns$	number of outputs of MIMO Wiener linear dynamic model;
$nu$	number of system (model) inputs
$ny$	number of system (model) outputs
$N$	number of input-output measurements
$q^{-1}$	backward shift operator
$\mathbb{R}^d$	Euclidean $d$ -dimensional space
$s(n)$	output of the linear dynamic part of Wiener system
$\hat{s}_k(n)$	$k$ th output of the linear dynamic part of MIMO Wiener model
$\hat{s}(n)$	output of the linear dynamic part of Wiener model
$\hat{\mathbf{s}}(n)$	output of the linear dynamic part of MIMO Wiener model
$u(n)$	system input
$\mathbf{u}(n)$	MIMO system input
$\text{var}$	variance
$v_{ji}^{(1)}$	$i$ th weight of the $j$ th hidden layer node of the inverse nonlinear element model
$v_{kj}^{(2)}$	$j$ th weight of the $k$ th output node of the inverse nonlinear element model
$\hat{v}(n)$	output of the nonlinear element part of Hammerstein model
$\mathbf{v}$	weight vector of the inverse nonlinear element model
$\mathbf{v}_k$	$k$ th path weight vector of the MIMO inverse nonlinear element model
$\mathbf{V}$	weight vector of the MIMO inverse nonlinear element model
$w_{ji}^{(1)}$	$i$ th weight of the $j$ th hidden layer node of the nonlinear element model
$w_{kj}^{(2)}$	$j$ th weight of the $k$ th output node of the nonlinear element model
$\mathbf{w}$	weight vector of the nonlinear element model
$\mathbf{w}_k$	$k$ th path weight vector of the MIMO nonlinear element model,
$\mathbf{W}$	weight vector of the MIMO nonlinear element model
$x_j(n)$	activation of the $j$ th nonlinear node
$\mathbf{x}(n)$	regression vector; system state
$\hat{\mathbf{x}}(n)$	model state
$y(n)$	system output
$\hat{y}(n)$	model output
$y_k(n)$	$k$ th output of the MIMO Wiener (Hammerstein) system
$\hat{y}_k(n)$	$k$ th output of the MIMO Wiener (Hammerstein) model
$\hat{y}(n n-1)$	one-step-ahead predictor of $y(n)$
$\mathbf{y}(n)$	MIMO system output
$\hat{\mathbf{y}}(n)$	MIMO model output

$z_j(n)$	activation of the $j$ th nonlinear node of the inverse nonlinear element model
$\mathbf{z}(n)$	instrumental variables vector
$\hat{\gamma}_k$	$k$ th parameter of the polynomial of the inverse nonlinear element model
$\gamma_k$	$k$ th parameter of the polynomial of the inverse nonlinear element
$\Delta A(q^{-1})$	change in the pulse transfer function denominator of the linear dynamic system
$\Delta f(\cdot)$	change in the nonlinear function of the nonlinear element
$\Delta f^{-1}(\cdot)$	change in the nonlinear function of the inverse nonlinear element
$\Delta \hat{s}_{\hat{a}_k}(n)$	computation error of $\partial \hat{s}(n)/\partial \hat{a}_k$
$\Delta \hat{s}_{\hat{b}_k}(n)$	computation error of $\partial \hat{s}(n)/\partial \hat{b}_k$
$\Delta \hat{y}_{\hat{a}_k}(n)$	computation error of $\partial \hat{y}(n)/\partial \hat{a}_k$
$\Delta \hat{y}_{\hat{b}_k}(n)$	computation error of $\partial \hat{y}(n)/\partial \hat{b}_k$
$\Delta \hat{y}_{w_c}(n)$	computation error of $\partial \hat{y}(n)/\partial w_c$
$\epsilon(n)$	discrete white noise disturbance
$\varepsilon(n)$	additive system output disturbance
$\eta$	learning rate
$\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}$	parameter vector
$\lambda$	exponential forgetting factor
$\mu_k$	$k$ th parameter of the polynomial of the nonlinear element,
$\hat{\mu}_k$	$k$ th parameter of the polynomial of the nonlinear element model
$\xi_{a_k}(n)$	calculation accuracy degree of $\partial \hat{s}(n)/\partial \hat{a}_k$ – the Wiener model; $\partial \hat{y}(n)/\partial \hat{a}_k$ – the Hammerstein model
$\xi_{b_k}(n)$	calculation accuracy degree of $\partial \hat{s}(n)/\partial \hat{b}_k$ – the Wiener model; $\partial \hat{y}(n)/\partial \hat{b}_k$ – the Hammerstein model
$\xi_{w_c}(n)$	calculation accuracy degree of $\partial \hat{y}(n)/\partial w_c$
$\sigma^2$	variance of $u(n)$
$\sigma_f^2$	variance of $\hat{f}(u(n), \mathbf{w})$
$\sigma_{w_c}^2$	variance of $\partial \hat{f}(u(n), \mathbf{w})/\partial w_c$
$\varphi(\cdot)$	nonlinear activation function
$\psi(n)$	gradient of the Wiener model output

## List of abbreviations

i.i.d.	independent and identically distributed
r.h.s	right hand side
w.r.t	with respect to
AR	autoregressive
ARMA	autoregressive moving average
ARMAX	autoregressive moving average with exogenous input

ARX	autoregressive with exogenous input
BJ	Box-Jenkins
BP	back propagation
BPP	back propagation for parallel models
BPPT	back propagation through time
BPS	back propagation for series-parallel models
CSTR	continuous stirred tank reactor
DLOP	discrete Legendre orthogonal polynomial
FDI	fault detection and isolation
ELS	extended least squares
FIR	finite impulse response model
IMC	internal model control
IV	instrumental variables
MA	moving average model
MIMO	multiple-input multiple-output
MISO	multiple-input single-output
MLP	multilayer perceptron
MPC	model-based predictive control
MSE	mean square error
NAR	nonlinear autoregressive
NARMA	nonlinear autoregressive moving average
NARMAX	nonlinear autoregressive moving average with exogenous input
NARX	nonlinear autoregressive with exogenous input
NBJ	nonlinear Box-Jenkins
NFIR	nonlinear finite impulse response
NOBF	nonlinear orthonormal basis function
NOE	nonlinear output error
NMA	nonlinear moving average
OBFP	orthogonal basis with fixed poles
OE	output error
PE	prediction error
PI	proportional plus integral
PID	proportional plus integral plus derivative
PRBS	pseudorandom binary sequence
RIV	recursive instrumental variables
RLS	recursive least squares
RELS	recursive extended least squares
RMS	root mean square
RPE	recursive prediction error
RPLR	recursive pseudolinear regression
SIMO	single-input multiple-output
SISO	single-input single-output
SM	sensitivity method
WMPC	Wiener model-based predictive control