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CALCULUS 2: Exercises MI1121E

Chapter 2: Multiple integrals

References: The list of suggested exercises in MI1121.

1.1 Double integrals

1. Change the order of integration

(a) $\int\limits_{-1}^{1} dx \int\limits_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)dy$

(b) $\int\limits_{0}^{1} dy \int\limits_{2-y}^{1+\sqrt{1-y^2}} f(x,y)dx$

(c) $\int\limits_{0}^{2} dx \int\limits_{\sqrt{2x-x^2}}^{\sqrt{2x}} f(x,y)dy$

(d) $\int\limits_{0}^{\pi/2} dy \int\limits_{\sin y}^{1+y^2} f(x,y)dx$

(e) $\int\limits_{0}^{\sqrt{2}} dy \int\limits_{0}^{y} f(x,y)dx + \int\limits_{\sqrt{2}}^{2} dy \int\limits_{0}^{\sqrt{4-y^2}} f(x,y)dx$

2. Evaluate the integral.

(a) $\iint\limits_D \frac{y}{1+xy}dxdy$, $D = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le 1; 0 \le y \le 2\}$.

(b) $\iint\limits_D x^2(y-x)dxdy$, D is bounded by $y = x^2$ and $x = y^2$.

(c) $\iint\limits_D 2xydxdy$, D is enclosed by $x = y^2$, $x = -1$, $y = 0$ and $y = 1$.

(d) $\iint\limits_D (x+y)dxdy$, D is defined by $x^2 + y^2 \le 1$ and $\sqrt{x} + \sqrt{y} \le 1$.

(e) $\iint\limits_D |x+y|dxdy$, $D = \{(x,y) \in \mathbb{R}^2 \mid |x| \le 1, |y| \le 1)\}$.

(f) $\iint\limits_{|x|+|y|\le 1} (|x| + |y|)\, dxdy$.

(g) $\int\limits_{0}^{1} dx \int\limits_{0}^{1-x^2} \frac{xe^{3y}}{1-y}dy$.

3. Find the limits of integration in polar coordinates of $\iint\limits_D f(x,y)dxdy$, where D is described by

(a) $a^2 \le x^2 + y^2 \le b^2$

(b) $x^2 + y^2 \ge 4x$, $x^2 + y^2 \le 8x$, $y \ge x$, $y \le \sqrt{3}x$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2}$, $y \ge 0$ $(a, b > 0)$

(d) $x^2 + y^2 \le 2x$, $x^2 + y^2 \le 2y$

4. Evaluate the given integral by changing to polar coordinates.

(a) $\int\limits_{0}^{R} dx \int\limits_{0}^{\sqrt{R^2-x^2}} \ln(1 + x^2 + y^2)dy$ $(R > 0)$.

(b) $\iint\limits_D xydxdy$, where $D\colon (x-2)^2+y^2 \le 1, y \ge 0$.

(c) $\iint\limits_D (\sin y + 3x)dxdy$, where $D\colon (x-2)^2+y^2 \le 1$.

(d) $\iint\limits_D |x+y|dxdy$, where $D\colon x^2+y^2 \le 1$.

5. Converting the following integral to an integral in variables u and v

(a) $I = \int\limits_0^1 dx \int\limits_{-x}^{x} f(x,y)dy$, where $u = x+y$ and $v = x-y$.

(b) Using the above change of variables to compute I where $f(x,y) = (2-x-y)^2$.

6. Evaluate the double integral.

(a) $\iint\limits_D \frac{2xy+1}{\sqrt{1+x^2+y^2}}dxdy$, where $D\colon x^2+y^2 \le 1$

(b) $\iint\limits_D \frac{dxdy}{(x^2+y^2)^2}$, where $D\colon y \le x^2+y^2 \le 2y;\, x \le y \le \sqrt{3}x$

(c) $\iint\limits_D \frac{xy}{x^2+y^2}dxdy$, where $D\colon 2x \le x^2+y^2 \le 12;\, x^2+y^2 \ge 2\sqrt{3}y;\, x \ge 0;\, y \ge 0$

(d) $\iint\limits_D |9x^2-4y^2|dxdy$, where $D\colon \frac{x^2}{4}+\frac{y^2}{9} \le 1$

(e) $\iint\limits_D (3x+2xy)dxdy$, where $D\colon 1 \le xy \le 9;\, y \le x \le 4y$

1.2 Triple integrals

Evaluate the following triple integrals.

7. $\iiint\limits_V zdxdydz$, where $V\colon \begin{cases} 0 \le x \le 1 \\ x \le y \le 2x \\ 0 \le z \le \sqrt{5-x^2-y^2} \end{cases}$

8. $\iiint\limits_V (3xy^2-4xyz)dxdydz$, where $V\colon \begin{cases} 1 \le y \le 2 \\ 0 \le xy \le 2 \\ 0 \le z \le 2 \end{cases}$

9. $\iiint\limits_V xye^{yz^2}dxdydz$, where $V\colon \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 \\ x^2 \le z \le 1 \end{cases}$

10. $\iiint\limits_V (x^2+y^2)dxdydz$, where $V\colon \begin{cases} x^2+y^2+z^2 \le 1 \\ x^2+y^2-z^2 \le 0 \end{cases}$

11. $\iiint\limits_V z\sqrt{x^2+y^2}dxdydz$, where

   (a) V is bounded by the cylinder $x^2+y^2=2x$ and the planes $z=0, z=a$ $(y\geq 0, a>0)$;
   (b) V is a half of the sphere $x^2+y^2+z^2\leq a^2, z\geq 0$ $(a>0)$;
   (c) V is a half of the ellipsoid $\dfrac{x^2+y^2}{a^2}+\dfrac{z^2}{b^2}\leq 1, z\geq 0$ $(a,b>0)$.

12. $\iiint\limits_V ydxdydz$, where V is bounded by $y=\sqrt{x^2+z^2}$ and $y=h$ $(h\geq 0)$.

13. $\iiint\limits_V \dfrac{x^2}{a^2}dxdydz$, where $V\colon \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}\leq 1$ $(a,b,c>0)$.

14. $\iiint\limits_V (x^2+y^2+z^2)dxdydz$, where $V\colon \begin{cases}1\leq x^2+y^2+z^2\leq 4\\ x^2+y^2\leq z^2.\end{cases}$

15. $\iiint\limits_V \sqrt{x^2+y^2}dxdydz$, where V is bounded by $x^2+y^2=z^2$ and $z=-1$.

16. $\iiint\limits_V \dfrac{dxdydz}{(x^2+y^2+(z-2)^2)^2}$, where $V\colon \begin{cases}x^2+y^2\leq 1\\ |z|\leq 1.\end{cases}$

17. $\iiint\limits_V \sqrt{x^2+y^2+z^2}dxdydz$, where $V\colon x^2+y^2+z^2\leq z$.

1.3 Applications of multiple integrals

18. Find the area of the domain D bounded by $\begin{cases}y^2=x, y^2=2x\\ x^2=y, x^2=2y\end{cases}$.

19. Find the area of the domain D bounded by $\begin{cases}y=0, y^2=4ax\\ x+y=3a, y\leq 0\end{cases}$ $(a\geq 0)$.

20. Find the area of the domain D defined by $\begin{cases}2x\leq x^2+y^2\leq 4x\\ 0\leq y\leq x\end{cases}$.

21. Find the area of the domain D defined by $r\geq 1, r\leq \dfrac{2}{\sqrt{3}}\cos\varphi$.

22. Find the area of the domain D bounded by $(a>0)$

   a) $(x^2+y^2)^2=2a^2xy$ b) $r=a(1+\cos\varphi)$

23. Show that the area of the domain D defined by $x^2+(ax-y)^2\leq 4$ is unchanged when a runs over the set of real numbers.

24. Find the volume of the object defined by $\begin{cases} x+y \geq 1 \\ x+2y \leq 2 \\ y \geq 0, 0 \leq z \leq 2 \leq 2-x-y \end{cases}$.

25. Find the volume of the object bounded by $\begin{cases} z = 4 - x^2 - y^2 \\ 2z = 2 + x^2 + y^2 \end{cases}$.

26. Find the volume of the object defined by $|x-y| + |x+3y| + |x+y+z| \leq 1$.

27. Find the volume of the object bounded by the surface $z = 1 + x^2 + y^2$, the cylinder $x^2 + 4y^2 = 4$ and the plane Oxy.

28. Find the volume of the object bounded by the surfaces $az = x^2 + y^2$, $z = \sqrt{x^2 + y^2}$ $(a > 0)$.

29. Find the area of the part of the sphere $x^2 + y^2 + z^2 = 4a^2$ that lies inside the cylinder $x^2 + y^2 - 2ay = 0$ $(a > 0)$.