# Template for Rolled Spherical Candles

Roman Firt

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We construct a parametric function for the candle middle surface as a function $m$ of angular coordinate $\phi$ defined as:

$$m(\phi) = (r(\phi), z(\phi)),$$

where $r$ and $z$ is the evolving radius and height of the wax sheet. The top and side view are illustrated in Figures 1 and 2. The individual components of $m$ are defined as:

$$\begin{aligned} z(\phi) &= \sqrt{R^2 - r^2(\phi)} && (1) \\ r(\phi) &= R_0 + \frac{T}{2} + \frac{\phi}{2\pi}T, && (2) \end{aligned}$$

where $R$, $R_0$ and $T$ are the desired candle radius, the wick radius and $T$ the wax sheet layer thickness respectively.

The main objective is to get the length of the mid surface $l$ as a function of $\phi$, so that it plotted as a cut template as illustrated in Figure 3. Using the arc length formula in polar coordinates we get:

$$\begin{aligned} l(\phi) &= \int_0^\phi \sqrt{\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 + r^2(\theta)}\mathrm{d}\theta && (3) \\ &= \int_0^\phi \sqrt{\frac{T^2}{4\pi^2} + \left(R_0 + \frac{T}{2} + \frac{\theta}{2\pi}\right)^2 T^2}\mathrm{d}\theta && (4) \\ &= \frac{T}{2\pi}\int_{\frac{2\pi}{T}R_0+\pi}^{\frac{2\pi}{T}R_0+\pi+\phi} \sqrt{\psi^2+1}\mathrm{d}\psi && (5) \\ &= \left[\frac{1}{2}\psi\sqrt{1+\psi^2} + \frac{1}{2}\log\left(\psi + \sqrt{1+\psi^2}\right)\right]_{\frac{2\pi}{T}R_0+\pi}^{\frac{2\pi}{T}R_0+\pi+\phi} && (6) \end{aligned}$$

Now the curves $(l(\phi), z(\phi))$ and $(l(\phi), -z(\phi))$ define the shape to be cut. For a particulat set of input parameters $(R, R_0, T)$ we calculate the discrete approximation of the curves using a fine enough discretization of $\phi$.

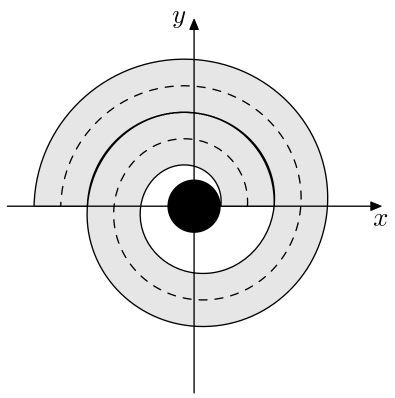


Figure 1: Candle top view. The wick is represented by the black circle. Grey area is the winding wax sheet. Dashed line is the middle surface we track.

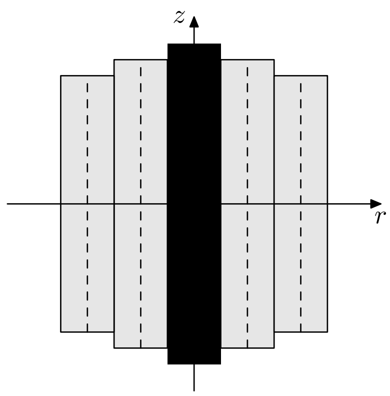


Figure 2: Candle side view. Dashed line again represents the tracked middle surface.

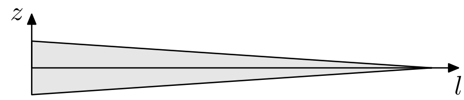


Figure 3: Example of a cut template.