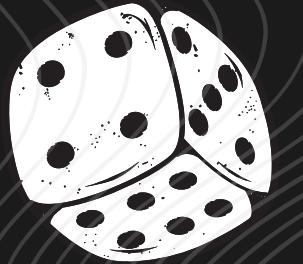




GAME THEORY



GAME THEORY

Game theory entails the examination of mathematical models governing strategic interactions among rational agents, finding applications across an array of disciplines including logic, systems science, and computer science. Originally confined to two-person zero-sum games where gains and losses nullify each other, contemporary game theory now encompasses a broader spectrum of behavioral dynamics. In contemporary contexts, it serves as a comprehensive framework for comprehending the rational decision-making processes employed by humans, animals, and computational systems.



AUCTIONS



Auction theory, a practical facet of economics, examines bidder behavior within auction markets and studies how specific market traits encourage foreseeable results. This theory serves as a guide for shaping actual auctions, aiding sellers in boosting revenues and enabling buyers to secure goods at minimized costs. The negotiation of price between buyers and sellers finds its economic balance. Auction theorists create auction rules to tackle potential market issues, fostering effective bidding strategies across diverse informational scenarios.

TYPES OF AUCTIONS

First-Price Auction



1. SEALED-BID FIRST-PRICE AUCTION

Second-Price Auction



2. SEALED-BID SECOND-PRICE AUCTION (VICKREY AUCTION)



FIRST PRICE AUCTION

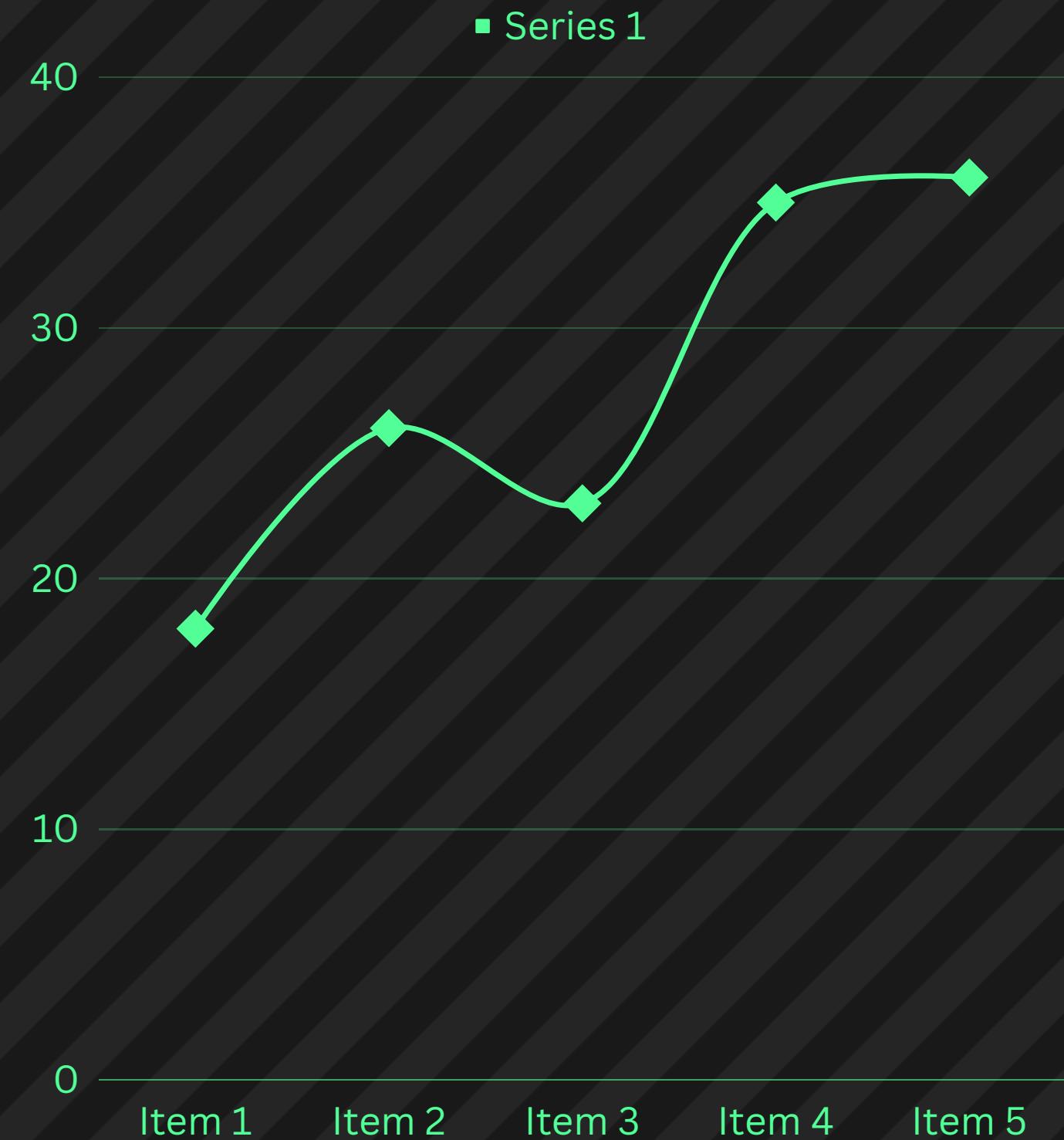
In a Sealed-Bid First-Price Auction, participants engage in a discreet yet strategic maneuvering of bids, unaware of their competitors' offers. The highest bidder emerges victorious, claiming the auctioned item and paying precisely the amount they had bid. This format creates a dynamic where bidders must tread the fine line between their aspiration to secure the item and their budgetary constraints.



The key lies in accurately assessing the item's value and placing a bid that outshines rivals, while avoiding an overestimation that could result in unnecessary expenditure. It's a game of wits, where participants balance ambition with precision, crafting bids that optimally position them for success.

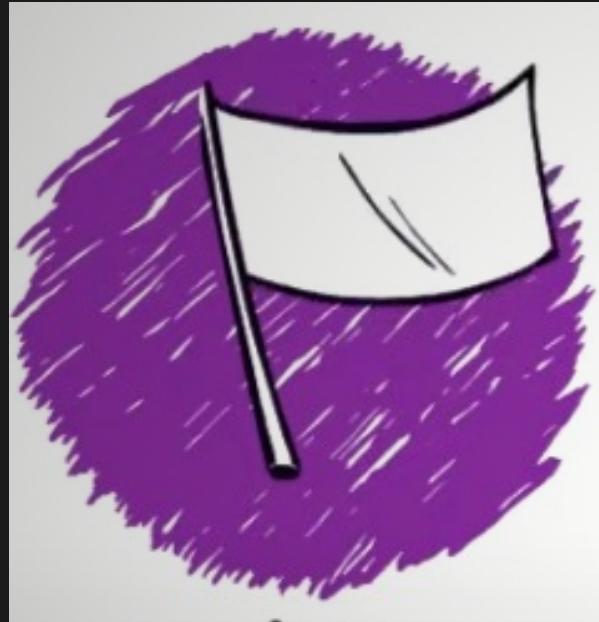
SECOND PRICE AUCTION

In a Sealed-Bid Second-Price Auction, also known as a Vickrey Auction, participants submit concealed bids without knowledge of others' offers. However, the twist lies in the winner's payment: it's not based on their bid, but on the second-highest bid. The participant with the highest bid secures the item but pays an amount equal to the second-highest bid. This format encourages bidders to bid truthfully, as they have no incentive to manipulate their offers to secure a better deal. Strategic planning involves valuing the item accurately and bidding sincerely, as the outcome is governed by the decisions of others. It's a subtle yet fascinating dance of estimation and cautious selection, where transparency and strategic restraint play key roles in achieving favorable outcomes.



RULES

- **Players** can be individuals ,companies ,governments etc., each pursuing their own interests.
- **Strategies** -> Player's choices and possible strategies they can take determine the possible outcomes.
- **Payoffs/Utilities**->Assign payoffs or utilities to each player for each possible outcome of the game. Payoffs represent the preferences of players for different outcomes.
- **Nash Equilibrium**-> A key concept in game theory, a Nash equilibrium is a set of strategies where no player has an incentive to unilaterally deviate from their chosen strategy, given the strategies of the other players.



#2

Payoffs



#3
Rules

ASSUMPTIONS

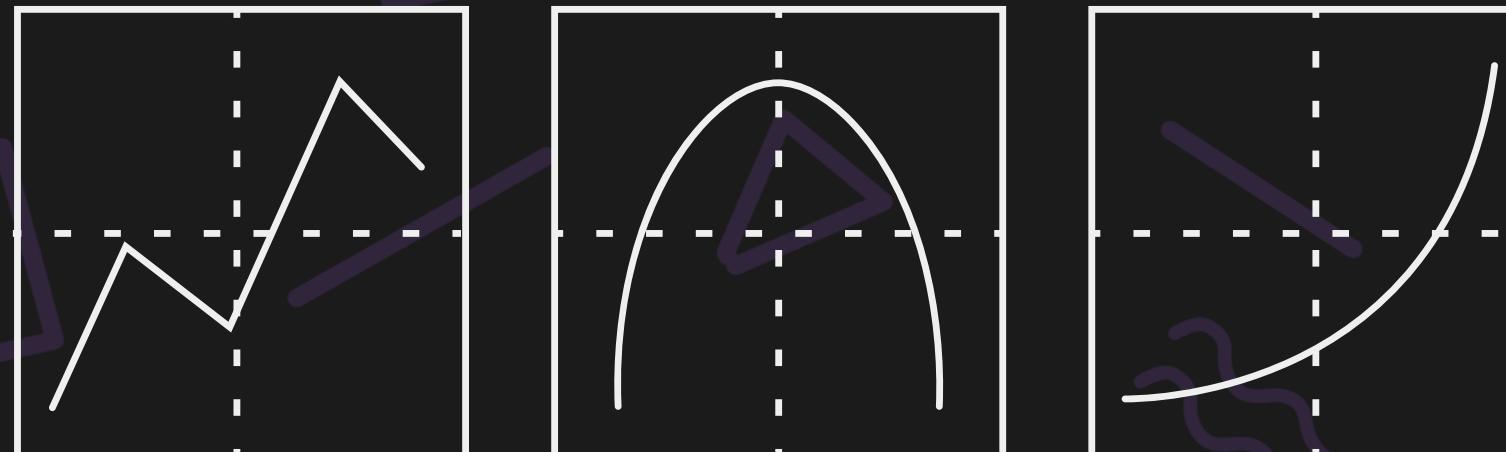


Assumptions help simplify the complexities of real-world situations and make them more tractable for analysis.

- Players are assumed to be **rational decision-makers** who act in their own best interests, attempting to maximise their own payoffs or utilities
- It is assumed that all players have complete and accurate information about the game, the strategies available, and the payoffs associated with each outcome.
- **Zero sum assumption:** Total payoff is always zero, one player's gain is balanced by another player's loss.
- Perfect information is assumed i.e. all players know the exact history of the game, including all previous moves and outcomes.

GAME 1: "WEIGHT HAS IT ALL"

Based on the First Price Auction



EXPLANATION OF THE GAME

Assumptions: Atleast 2 players must play the game,

- Consider the best of X where X is the final bid amount calculated according to a formula which will be mentioned in the coming points.
- The bidder with the highest total amount calculated after the final round will be considered as the winner.
- he/she will have to pay the total amount bid in all the rounds.
- if there is a tie after the final round, the scores calculated on the previous round will be used as the tie braker.
- In each round the bidder(s) will make simultaneously make a bid.
- We have defined a hyperparameter beta which decides how much emphasis we need to put on the past bids, recommended value of beta is 0.1 to 0.9.



the auctioneer has to input this value before the start of the game.

- The formula used will be as follows:

$$\text{curr_i} = ((1-\text{beta}) * \text{curr_i} + \text{prev_i} * (\text{beta})) / (1 - \text{beta}^{\text{round_number}})$$

where curr_i : current value bid

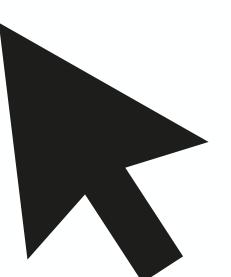
prev_i = previously calculated weighted average

beta: hyperparameter.

- The winner will be the one with highest value calculated using the hyperparameter.

LINK TO THE SPREADSHEET FOR GAME 1

[https://docs.google.com/
spreadsheets/d/12s92lmj
tpyUzlOMVUd3zFLqtMly
HsRL6rjbaw817hs4/edit?
usp=sharing](https://docs.google.com/spreadsheets/d/12s92lmjtpyUzlOMVUd3zFLqtMlyHsRL6rjbaw817hs4/edit?usp=sharing)



OUTCOMES

"Weight has it all" is a complex auction game where players strategically bid in each round, influenced by their previous bids and opponents. The winner accumulates the highest total value, factoring in bids, exponential calculations, and previous rounds.

Tiebreakers are determined by scores from earlier rounds, promoting consistent performance throughout the game.

In summary, "Weight has it all" is a dynamic and strategic auction game that requires players to adapt their bids and strategies while understanding complex formulas to outmaneuver opponents.





GAME 2 : "ROYAL BIDDING RIVALRY"

Based on the Second Price Auction
(Vickrey Auction)

EXPLANATION OF THE GAME

1. In the realm of "Royal Bidding Rivalry," a group of daring princes embarks on a treasure hunt game.
2. The competition spans several thrilling rounds, with one prince being the winner in each round and finally only one remains victorious.
3. At the start of their quest, all princes are bestowed with a purse of 100 gold coins, ready to dazzle in the auctions.
4. In each round, the princes bid an amount of coins.
 - The heart-pounding twist: You bid with your gold coins to win even more gold coins!
5. The prince who places the highest bid wins that round and they must pay the amount bid by the second-highest bidder for that choice
6. When two or more princes submit identical highest bids, both pay the amount bid by the second-highest bidder. The second highest bid is then divided by the number of highest bidders and that amount is to be paid by all the highest bidders.
7. Finally, the prince who won the maximum number of rounds, wins the game.
If there is more than one prince who wins a maximum number of rounds, then the prince with a higher number tag wins the game.



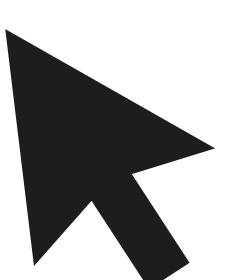
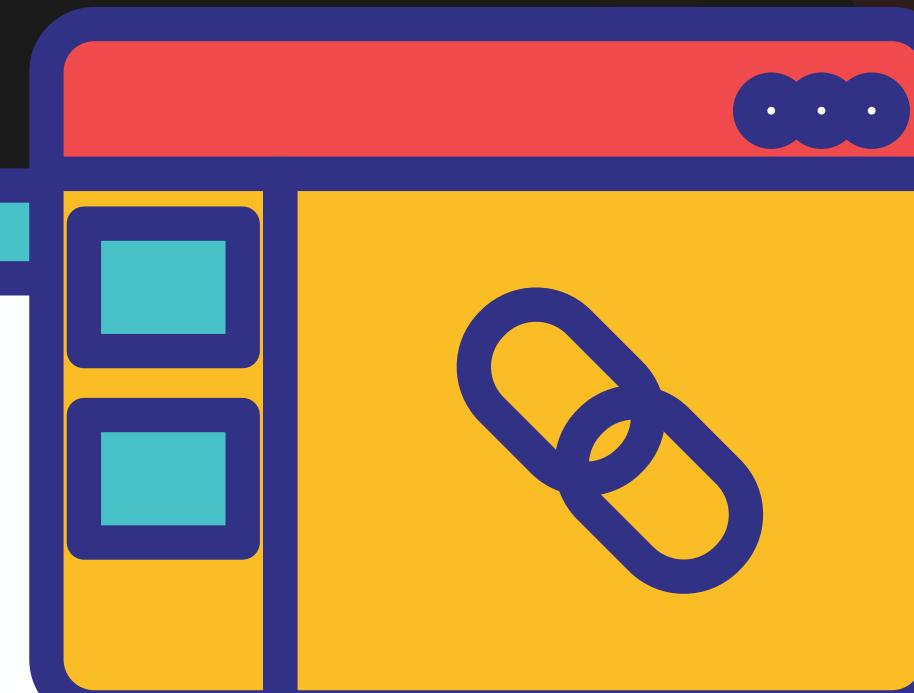
ASSUMPTIONS

1. Each prince knows the number of coins they have at the moment and do not bid more than that.

2. Each prince is given number tags randomly.
(Prince 1, Prince 2 and so on...)

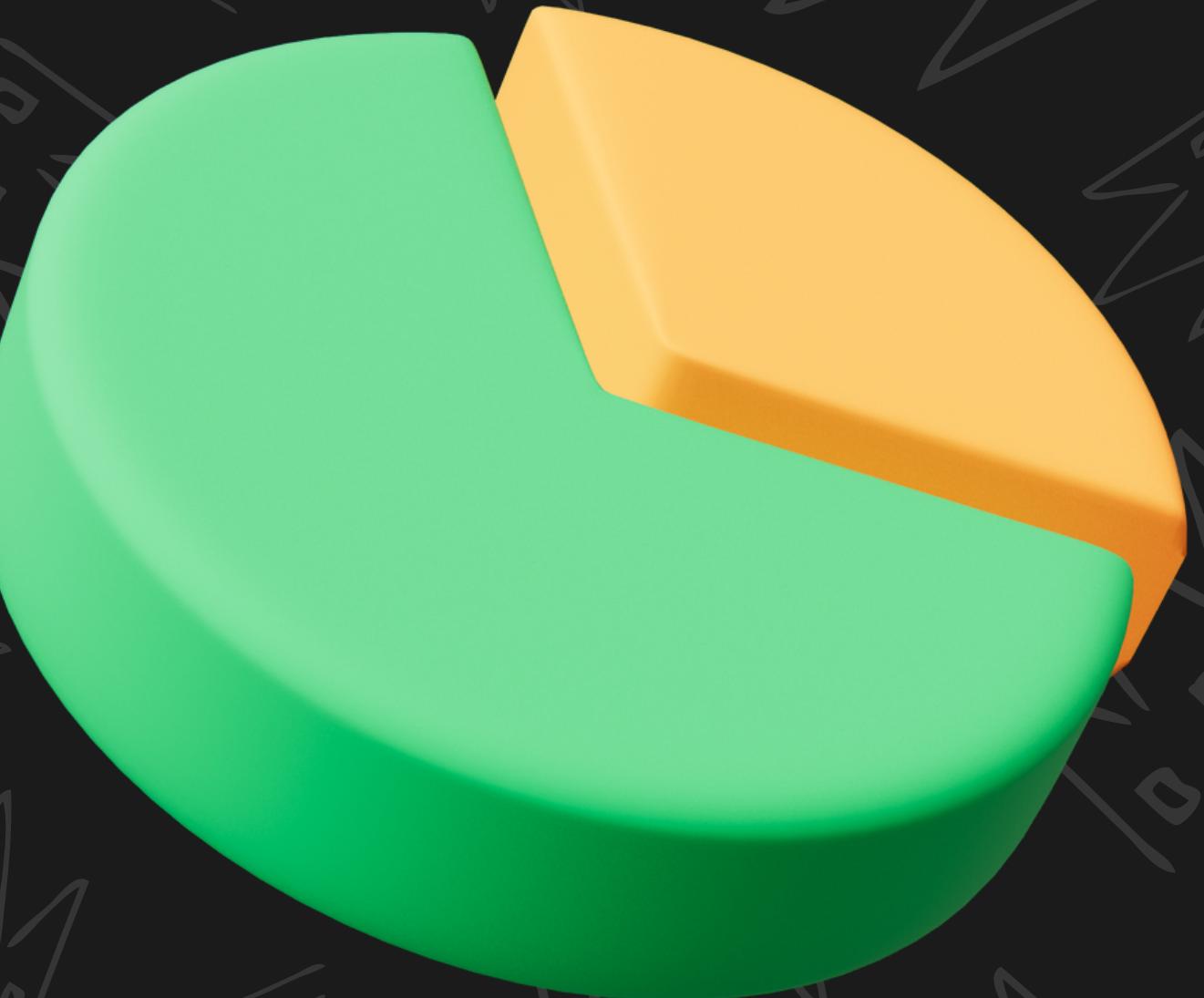
LINK TO THE SPREADSHEET FOR GAME 2

<https://docs.google.com/spreadsheets/d/1f18qXTD9ZBI4a899gVj1cH3uOEYIkvi-qmubEXkCZ5g/edit?usp=sharing>



OUTCOMES

1. The game is won by the prince who successfully follows a strategy to win maximum number of rounds.
2. Princes engage in fierce bidding wars, with the highest bidder for each choice claiming the coins but also paying the second-highest bidder's amount, leading to intense rivalries.
3. The biased nature of Royal bidding is depicted as in case of equal wins, the Prince with higher number tag is the final winner.



BEHAVIORAL ANALYSIS

We have assumed up to this point that the bidders' estimates of the object up for auction are independent. Nobody cares what the thing is worth to anyone else; each bidder knows his or her value for it. The seller might receive more money for the item if he had a first-price sealed-bid auction, according to a cursory comparison of the first-price and second-price sealed-bid auctions. In the end, he will be compensated with the highest offer rather than the second-highest offer. The key here is that bidders in a first-price auction will typically make lower bids than they would in a second-price auction, which will tend to counteract what might otherwise appear to be a difference in the size of the winning bid.

Another difference between the two games was that in the modified first price auction, the bidder had to pay the same amount at the end of each round regardless of how much he valued the item, whereas in the modified second price auction game, even if someone knew before the final round that he was not going to win, he would still purposefully raise his bid so that the winner had to pay a higher sum.

