Problem 6.1: Reading And/Or Weeping

Submit Assignment

Due Thursday by 11:59pm **Points** 20 **Submitting** a text entry box or a file upload
File Types pdf **Available** after Oct 29 at 11:59pm

You've decided to spend your free time this month reading Kleinberg and Tardos cover to cover. You've got D days this month, and on day i, you have m_i free minutes to read. There are n sections in the book, and section j takes t_j minutes to read. You want to read all the sections in order, and want to have read all the sections by the end of the month. Your task is to schedule your reading for the month.

If you read sections $j, \ldots, j+k$ on day i, there are two problems you might face. First, you might leave yourself with free time to do other things. Let $T_i = \sum_{\ell=j}^{j+k} t_\ell$ be the time spent reading on day i. If you have free time, then $T_i < m_i$. Let $F_i = \max(m_i - T_i, 0)$. That is, F_i is your free time on day i, or 0 if you have no free time. Having free time is a disaster, so you decide that part of your goal should be to minimize $\sum_{i=1}^{D} F_i^4$. The second, less bad thing that could happen is that you could decide to read too much on day i and have to skip a few hours of sleep. That is, $T_i > m_i$. Let S_i be the total sleep you lose from reading. Then $S_i = \max(T_i - m_i, 0)$. You don't want to lose too much sleep, so you decide that the other part of your objective should be to minimize $\sum_{i=1}^{D} S_i$. Your full objective, then, is to schedule your reading to minimize your total unhappiness, $\sum_{i=1}^{D} F_i^4 + \sum_{i=1}^{D} S_i$.

As an example, consider three days with $m_1 = 5$, $m_2 = 4$, and $m_3 = 6$ and four sections, with $t_1 = 3$, $t_2 = 1$, $t_3 = 5$, and $t_4 = 4$. An optimal solution will read sections 1 and 2 on day 1, section 3 on day 2, and section 4 on day 3. For this solution, $T_1 = 4$, $T_2 = 5$, and $T_3 = 4$, $F_1 = 1$, $F_2 = 0$, and $F_3 = 2$, and $S_1 = 0$, $S_2 = 1$, and $S_3 = 0$. It follows that your total unhappiness is $1^4 + 2^4 + 1 = 18$.

Give a dynamic programming algorithm which computes the total unhappiness of the optimal reading schedule. Include all four parts of a good writeup. You do not need to prove correctness, except in justifying your recurrence. Additionally, program your iterative algorithm, and run several test cases. Include a printout of your inputs and outputs with your writeup.