

4. Mean-Variance Model

Harry Markowitz [1952] developed the idea of portfolio selection based on maximizing returns while minimizing the variances and covariances of the assets. The formulation of the model is:

$$\text{Max } Z_1 = \sum_{i=1}^N X_i \mu_i \quad (4.1)$$

$$\text{Min } Z_2 = \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} X_i X_j \quad (4.2)$$

Subject to:

$$\sum_{i=1}^N X_i = 1 \quad (4.3)$$

$$X_i \geq 0 \quad (4.4)$$

The objective Z_1 represents the expected return of the one to 'N' assets, where 'X' is the percentage of the total investment in asset i and μ is the return of the asset i . The objective Z_2 is the risk of the portfolio represented by the variance-covariance of the assets, where σ_{ij} represents the variance of the asset for $i = j$ and the covariance between assets for $i \neq j$, and 'X' is the percentage of the total investment in the asset. Typically, this bi-criteria model is solved by constraining the return to a certain level and minimizing the risk. The efficient frontier is traced out of those combinations of assets that give a maximum return at a minimum variance. Potential portfolios may fall to the right of the frontier but not to the left. Those on the right of the frontier are "inefficient" because they obtain an inferior return level at the price of higher risk than those on the frontier. The idea of using the covariances, in addition to the variance, of assets yields diversification. If two assets have a negative covariance then this implies that they have a

negative relationship, meaning that as one increases, the other decreases. If two assets have a positive covariance, then they share a positive relationship, meaning that as one rises, the other one rises and vice versa. If two assets have a covariance of zero, then they are independent of each other.

The variance-covariance matrix in this paper is based on the twenty-year annual return data for the assets classes and the generated matrix is displayed in Table 4.1.

	Merrill Lynch 3-5 Yr. Govt	Merrill Lynch 15 yr and Up	MSCI 30-Day Treasury Bills	S&P 500 / Russell 2000	S&P 500 / NAREIT	Goldman Sachs Commodity Index				
Govt	45.51	84.01	8.86	9.94	25.71	41.8	28.32	32.4	44.92	-22.07
Up	84.01	182.80	6.44	47.78	58.03	97.0	69.05	85.6	85.50	-44.74
Bills										
Index										
Growth										
All										
Merrill Lynch 3-5 Yr. Govt	45.51	84.01	8.86	9.94	25.71	41.8	28.32	32.4	44.92	-22.07
Merrill Lynch 15 yr and Up	84.01	182.80	6.44	47.78	58.03	97.0	69.05	85.6	85.50	-44.74
U.S. 30-Day Treasury Bills	8.86	6.44	8.14	-0.63	-10.51	1.8	-9.80	-3.4	5.66	-6.01
MSCI EAFE Equity Index	9.94	47.78	-8.63	502.02	198.58	38.5	130.55	117.7	-40.07	9.76
Russell 2000 Growth	25.71	58.03	-10.51	166.58	403.31	192.2	257.62	164.9	65.94	84.10
Russell 2000 Value	41.88	97.09	1.85	38.53	192.26	302.2	69.63	140.8	191.07	-9.19
S&P 500 / BARRA Growth	28.32	69.05	-9.80	130.55	267.02	69.6	326.80	152.1	-15.40	-22.09
S&P 500 / BARRA Value	32.49	89.64	-3.41	117.74	164.95	140.8	152.18	138.9	77.50	20.83
NAREIT - REIT ALL	44.92	85.50	5.66	-40.07	65.94	191.0	-15.4043	77.5	249.03	51.75
Goldman Sachs Commodity	22.07	-44.74	-8.01	9.76	84.10	-9.1	-22.099	20.8	51.75	412.03

Table 4.1 Variance-Covariance Matrix

Table 4.2 shows the correlation coefficient matrix of the asset classes.

	Merrill Lynch 3-5 Yr. Govt	Merrill Lynch 15 yr and Up	U.S. 30-Day Treasury Bills	MSCI EAFE Equity Index	Russell 2000 Growth	S&P 500 / BARRA Value	S&P 500 / BARRA Growth	NAREIT - REIT ALL	Goldman Sachs Commodity	Correlation Coefficient Matrix
Correlation Coefficient Matrix	1	0.92	0.48	0.07	0.19	0.36	0.23	0.41	0.42	-0.16
Merrill Lynch 3-5 Yr. Govt	0.92	1	0.02	0.16	0.21	0.41	0.28	0.54	0.40	-0.16
Merrill Lynch 15 yr and Up	0.46	0.02	1	-0.14	-0.18	0.04	-0.19	-0.10	0.13	-0.10
U.S. 30-Day Treasury Bills	0.07	0.16	-0.14	1	0.37	0.10	0.32	0.45	-0.11	0.02
MSCI EAFE Equity Index	0.19	0.21	-0.18	0.37	1	0.55	0.74	0.70	0.21	0.21
Russell 2000 Growth	0.36	0.41	0.04	0.10	0.55	1	0.22	0.69	0.70	-0.03
Russell 2000 Value	0.23	0.28	-0.19	0.32	0.74	0.22	1	0.71	-0.05	-0.06
S&P 500 / BARRA Growth	0.41	0.54	-0.10	0.45	0.70	0.69	0.71	1	0.42	0.09
S&P 500 / BARRA Value	0.42	0.40	0.13	-0.11	0.21	0.70	-0.05	0.42	1	0.16
NAREIT - REIT ALL	-0.16	-0.16	0.02	0.21	-0.03	-0.06	0.09	0.16	1	
Goldman Sachs Commodity										

Table 4.2 Correlation Coefficient Matrix