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observations and takes the actual percentiles of the observation period as the VaR measures. It is calculated as follows:

$$Rp,\tau = \sum_{i=1}^{N} w_{it} R_{i,\tau} \quad \text{for } \tau = 1,\ldots,t \qquad (2.5)$$

Once the historical returns for the portfolio are obtained, the return corresponding to the percentile chosen is simply multiplied by the market value of the portfolio. The Monte Carlo technique provides a statistical estimate in the form of a sample frequency distribution. The future returns distribution is estimated based on the set of forecasts for volatilities and correlations. The calculation of VaR for this method is similar to the historical simulation in that it is the percentile corresponding to the chosen confidence level. The findings in this paper suggested that the beta model in relation to these three techniques for calculating VaR is appropriate for highly diversified portfolios. Dowd [1999] looks at value at risk in a different light by studying incremental VaR, IVaR. In this paper options are examined and a change to the portfolio is made if the incremental VaR associated with the change is low enough, relative to the expected return. The incremental VaR is based on the Sharpe decision rule. The Sharpe decision rule says to choose the asset with the higher Sharpe ratio, which is the expected return of the asset divided by the standard deviation of its return. IVaR is a measure of elasticity, therefore, the higher the elasticity, the greater the risk associated with the new asset and the higher the required return. This decision rule can be applied to investment decisions, hedging decisions and portfolio management decisions. In the case of portfolio management decisions, IVaR can be used to assess the efficiency of the portfolio and any future changes to it. Dowd cites in this paper that if the portfolio is efficient, any position included in the portfolio should have an expected return at least as great as any position

excluded from the portfolio. If this is not true, then the excluded asset should be incorporated into the portfolio. This approach is particularly useful for dealing with net exposures and has a greater benefit when applied to hedging decisions.

The scenario-based approach is yet another guide to portfolio selection. This method is also known as returns based or expected utility optimization. One of the main attractions of the scenario-based approach is that options and other distribution-altering assets may be included in the investment mix. The key to this procedure is to separate the conditional and unconditional expected return forecast. As concluded by Grinold [1999], the scenario-based model is more difficult than the mean-variance model. Therefore it is not a good substitution unless there is the presence of options or other distribution altering assets.

A purer application of operations research, Young [1998] develops a minimax rule for portfolio selection with a linear programming solution. In this method, framing the portfolio selection process as a linear optimization problem also makes it feasible to constrain certain decision variables to be integer valued. This feature facilitates the use of more complex decision-making models. This model can accommodate fixed transaction charges, may also have advantages when returns are non-normally distributed, and when the investor has a strong form of risk aversion. The minimax portfolio maximizes the quantity of the minimum return of the portfolio subject to the restriction that the average return on portfolio exceeds some minimum level and that the sum of the portfolio allocations does not exceed some total allocation budget. The minimax portfolio is given by the solution to the following linear program:

$$\text{Max } M_p \qquad (2.6)$$