# CP1 for NPRE 501

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## 1 Problem Definition

Table 1 lists the constants used in the problem.

| Parameter | Value | [Unit] |
|---|---|---|
| Diamater | 6 | [cm] |
| **Radius** | **3** | [cm] |
| Geometry | Sphere | |
| k | 15 | $[\frac{W}{mK}]$ |
| Density | 8000 | $[\frac{kg}{m^3}]$ |
| Specific Heat | 500 | $[\frac{J}{kgK}]$ |
| $\alpha$ | **3.75e-6** | $[\frac{m^2}{s}]$ |

*Table 1: Problem Constants. Derived constants are in bold.*

Differential Equation:

$$\frac{1}{\alpha} \cdot \frac{dT}{dt} = \frac{1}{r^2} \frac{d}{dr} r^2 \frac{dT}{dr}$$

Boundary Conditions:

$$T(0,t) = finite \quad OR \quad \frac{dT}{dr}(r=0,t) = 0$$

$$\frac{dT}{dr}(r=R) = 0$$

Initial Condition:

$$T(r,0) = \frac{T_0}{2}(1 - \cos{(\frac{\pi \cdot r}{R})})$$

# 2 C. Numerical Solution

Expanding the differential equation:

$$\frac{1}{\alpha} \cdot \frac{dT}{dt} = \frac{2}{r}\frac{dT}{dr} + \frac{d^2T}{dr^2}$$

Applying the finite difference method, central for r and explicit for t:

$$\frac{1}{\alpha} \cdot \frac{T_k^{u+1} - T_k^u}{\Delta t} = \frac{2}{r_k}\,(\frac{T_{k+1}^u - T_{k-1}^u}{2\Delta r}) + \frac{T_{k-1}^u - 2T_k^u + T_{k+1}^u}{\Delta r^2}$$

where u is the temporal step and k is the spacial step.
Solving for $T_k^u$:

$$T_k^u = \frac{\frac{2}{r_k}(\frac{T_{k+1}^u - T_{k-1}^u}{2\Delta r}) + \frac{T_{k-1}^u + T_{k+1}^u}{\Delta r^2} - \frac{T_k^{u+1}}{\alpha \Delta t}}{\frac{2}{\Delta r^2} - \frac{1}{\alpha \Delta t}}$$

# 3 Appendix A

## 3.1 a. Final Temperature Distribution in the Sphere

The final temperature distribution will be a cosine curve with the highest point at r = 0, gradually going down to the minimum value at r = R.

# 4 Appendix B

## 4.1 Analytical solution for solving T(r,t) directly

$$\frac{1}{\alpha} \cdot \frac{dT}{dt} = \frac{1}{r^2}\frac{d}{dr}r^2\frac{dT}{dr}$$

Boundary Conditions:

$$T(0,t) = finite$$

$$\frac{dT}{dr}(r = R) = 0$$

Initial Condition:

$$T(r,0) = \frac{T_0}{2}(1 - \cos{(\frac{\pi \cdot r}{R})})$$

set:

$$T(r,t) = \frac{\overline{T}}{r}$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt}\frac{1}{r} = \frac{1}{r^2}\frac{d}{dr}r^2(\frac{d\overline{T}}{dr}\frac{1}{r} - \frac{1}{r^2}\overline{T})$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} = \frac{1}{r}\frac{d}{dr}(\frac{d\overline{T}}{dr}r - \overline{T})$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} = \frac{1}{r}(\frac{d^2\overline{T}}{dr^2}r + \frac{d\overline{T}}{dr} - \frac{d\overline{T}}{dr})$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} = \frac{d^2\overline{T}}{dr^2}$$

turns into a cartesian problem.

Applying Separation of Variables:

$$\overline{T}(r,t) = \Gamma(t)\Psi(r)$$

Boundary Conditions:

$$\Psi(r=0) = finite$$

$$\frac{d\Psi}{dr}(r=R) = 0$$

Applying the new variables, dividing both sides by $\Gamma(t)\Psi(r)$, and setting it to a new variable $-\beta^2$:

$$\frac{1}{\alpha\Gamma} \cdot \frac{d\Gamma}{dt} = \frac{d^2\Psi}{dr^2}\frac{1}{\Psi} = -\beta^2$$

Solving for $\Psi$ first:

$$\frac{d^2\Psi}{dr^2}\frac{1}{\Psi} = -\beta^2$$

$$\frac{d^2\Psi}{dr^2} + \beta^2\Psi = 0$$

$$\Psi(r) = C_1 sin(\beta r) + C_2 cos(\beta r)$$

Applying the first boundary condition, interpreting as $\frac{d\Psi}{dr}(r=0) = 0$, $C_1 = 0$

Applying the second boundary condition,

$$0 = -C_2 \beta sin(\beta R)$$

$$sin(\beta R) = 0$$

$$\beta R = \pi, 2\pi, 3\pi ...$$

$$\beta_n = \frac{n\pi}{R} \quad where \quad n = 1 \; to \; \infty$$

Solving for $\Gamma(t)$:

$$\frac{1}{\alpha\Gamma} \cdot \frac{d\Gamma}{dt} = -\beta_n^2$$

$$\frac{1}{\alpha\Gamma} \cdot \frac{d\Gamma}{dt} = -\beta_n^2 \alpha\Gamma$$

$$\Gamma(t) = A_1 e^{-\alpha\beta_n^2 t}$$

This makes $\overline{T}(r,t)$:

$$\overline{T}(r,t) = A_n e^{-\alpha\beta_n^2 t} sin(\beta_n r)$$

$$\beta_n = \frac{n\pi}{R} \quad where \quad n = 1 \; to \; \infty$$

Solving back for $T(r,t)$:

$$T(r,t) = \frac{A_n}{r} e^{-\alpha\beta_n^2 t} sin(\beta_n r)$$

Applying initial condition and orthogonality:

$$T(r,0) = \frac{A_n}{r} sin(\beta_n r) = \frac{T_0}{2}(1 - \cos(\frac{\pi \cdot r}{R}))$$

$$A_n = \frac{\int_0^R \frac{T_0 r^3 \sin(\beta_n r)}{2}(1 - \cos(\frac{\pi \cdot r}{R}))dr}{\int_0^R r^2 \sin^2(\beta_n r) dr}$$

The analytical solution is:

$$T(r,t) = \frac{\int_0^R \frac{T_0 r^3 \sin(\beta_n r)}{2}(1 - \cos(\frac{\pi \cdot r}{R}))dr}{r \int_0^R r^2 \sin^2(\beta_n r) dr} e^{-\alpha\beta_n^2 t} sin(\beta_n r)$$

$$\beta_n = \frac{n\pi}{R} \quad where \quad n = 1 \; to \; \infty$$

### 4.2 Code1

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