UNIVERSITY OF ILLINOIS
DEPARTMENT OF NUCLEAR, PLASMA & RADIOLOGICAL ENGINEERING
NPRE - 501 Fundamentals of Nuclear Engineering
Fall 2017
Computer Project # 1

Due Date: ____________________

This project is based on the following problem from Test # 1.

Temperature distribution at $t = 0$ in a solid sphere of radius R is given by

$$T(r,t) = \frac{T_0}{2}\left(1 - \cos\left(\frac{\pi r}{R}\right)\right)$$

For $t > 0$, the surface of the sphere $(r = R)$ is insulated.

a) By simple energy balance (no differential equations), argue what will be the **final** (as *time* goes to infinity) temperature distribution in the sphere (both, profile as well as quantitative value).
b) Sketch $T(r, t)$ for 5 different values of time between 0 and infinity ($t = 0 < t_1 < t_2 < t_3 < t_4$; over $0 < r < R$) to capture the temperature evolution. Be respectful of the magnitudes and slopes.
c) How can you use the result of part *a* to simplify the solution for $T(r, t)$ *for* $t > 0$?
d) Now solve for the temperature distribution in the sphere $T(r, t)$ for $t > 0$.

For this computer project, do the following:

A. Do parts *a* through *c* above, and include them in Appendix A.

B. Obtain the analytical solutions for two different formulations: **(5 points)**
   a. $T(r, t)$, directly (include in Appendix B-1)
   b. By defining a new temperature as $T'(r, t) = (T(r, t) - T_{ss})$, and solving for $T'(r, t)$ first, where T_{ss} is the steady-state temperature (include in Appendix B-2)

C. Solve the problem *numerically* using a finite difference scheme (central difference for space, and explicit in time). Show your work and include your computer code in Appendix C. **(5 points)**

We want to study the characteristics of the analytical series solution (eigenvalues, number of terms needed for convergence at different t_i, etc), as well as the characteristics of the numerical solution (grid independence, convergence, etc).

D. Evaluate the temperature $T(r, t)$, using the two analytical solutions obtained above for $t_0 = 0 < t_1 < t_2 < t_3 < t_4 < t_5 < t_6$ **(40 points)**

i. For each t ($t = t_0, t_1, \ldots t_6$), estimate how many terms in the series solution are necessary to assure *reasonable* convergence.
ii. Show the impact of the number of terms kept in the series solution for each value of t_i on a separate graph. (Plot $T(r, t_i)$ for increasingly larger number of terms kept in the series.).
iii. Report the summary for all cases in a table.
iv. Plot the converged $T(r,t_i)$ as function of r $(0 < r < R)$ for all seven cases on the same graph.

v. Tabulate your eigenvalues, and show all steps in appendix D.
vi. Comment on the convergence properties of the two analytical solutions

E. For the numerical solution obtained using an explicit finite difference method: **(40 points)**

i. Carry out a systematic study of the effects of the grid size and the time step on the accuracy of the solution. Devise a strategy to convince the reader that the number of grid points you are using is adequate but not an overkill. Implement the strategy and report your results. [Carry out a mesh refinement study to determine an adequate grid size.]
ii. Also, convince the reader that the time step you used for each solution is small enough but not unnecessarily small. [Carry out a time step refinement study to determine adequate time steps for each simulation.]
iii. Plot the converged analytical and numerical solutions for each t_i and compare them.

Discuss your results. What conclusions can you draw from this exercise?

10 points are for organization and presentation of your project.

The sphere is 6 cm in diameter, and is made of steel ($k = 15$ $W/m\text{-}K$, $rho = 8000$ kg/m^3, $Cp = 500$ $J/kg\text{-}K$)

Select your t_i $i = 1, 2, \ldots, 6$ such that the entire transition from the initial condition to the steady-state is adequately spanned.

Read the instructions carefully, and write a report in a word processor (not in MATLAB) with your work, explanations, tables and plots. Codes and other intermediate work should be in an appendix.

ps: Evaluate your (analytical) temperatures at enough points within the domain to have a smooth temperature distribution. No kinks, please.

ps 2: All graphs should be labeled properly and must have a detailed caption.

ps 3: All plots that are intended to be compared with each other should be on the same graph.