CP1 for NPRE 501

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1 Problem Definition

Table 1 lists the constants used in the problem.

Parameter	Value	[Unit]
Diamater	6	[cm]
Radius	**3**	[cm]
Geometry	Sphere	
k	15	$[\frac{W}{mK}]$
Density	8000	$[\frac{kg}{m^3}]$
Specific Heat	500	$[\frac{J}{kgK}]$
α	**3.75e-6**	$[\frac{m^2}{s}]$

Table 1: Problem Constants. Derived constants are bolded.

Solve:

$$T(r,t) = \frac{T_0}{2}(1 - \cos(\frac{\pi \cdot r}{R}))$$

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3 Appendix A

3.1 a. Final Temperature Distribution in the Sphere

The final temperature distribution will be a cosine curve with the highest point at r = 0, gradually going down to the minimum value at r = R.

4 Appendix B

4.1 Analytical solution for solving T(r,t) directly

$$\frac{1}{\alpha} \cdot \frac{dT}{dt} = \frac{1}{r^2} \frac{d}{dr} r^2 \frac{dT}{dr}$$

Boundary Conditions:

$$T(0, t) = finite$$

$$\frac{dT}{dr}(r = R) = 0$$

Initial Condition:

$$T(r, 0) = \frac{T_0}{2}(1 - \cos(\frac{\pi \cdot r}{R}))$$

set:

$$T(r, t) = \frac{\overline{T}}{r}$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} \frac{1}{r} = \frac{1}{r^2} \frac{d}{dr} r^2 (\frac{d\overline{T}}{dr} \frac{1}{r} - \frac{1}{r^2} \overline{T})$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} = \frac{1}{r} \frac{d}{dr} (\frac{d\overline{T}}{dr} r - \overline{T})$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} = \frac{1}{r} (\frac{d^2\overline{T}}{dr^2} r + \frac{d\overline{T}}{dr} - \frac{d\overline{T}}{dr})$$

$$\frac{1}{\alpha} \cdot \frac{d\overline{T}}{dt} = \frac{d^2\overline{T}}{dr^2}$$

turns into a cartesian problem.
Applying Separation of Variables:

$$\overline{T}(r, t) = \Gamma(t)\Psi(r)$$

Boundary Conditions:

$$\Psi(r = 0) = finite$$

$$\frac{d\Psi}{dr}(r = R) = 0$$

Applying the new variables, dividing both sides by $\Gamma(t)\Psi(r)$, and setting it to a new variable

$-\beta^2$:

$$\frac{1}{\alpha\Gamma} \cdot \frac{d\Gamma}{dt} = \frac{d^2\Psi}{dr^2}\frac{1}{\Psi} = -\beta^2$$

Solving for Ψ first:

$$\frac{d^2\Psi}{dr^2}\frac{1}{\Psi} = -\beta^2$$

$$\frac{d^2\Psi}{dr^2} + \beta^2\Psi = 0$$

$$\Psi(r) = C_1 sin(\beta r) + C_2 cos(\beta r)$$

Applying the first boundary condition, interpreting as $\frac{d\Psi}{dr}(r=0) = 0$, $C_1 = 0$

Applying the second boundary condition,

$$0 = -C_2\beta sin(\beta R)$$

$$sin(\beta R) = 0$$

$$\beta R = \pi, 2\pi, 3\pi...$$

$$\beta_n = \frac{n\pi}{R} where \quad n = 1\ to\ \infty$$

4.2 Code1

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