

Fundamental sequences for the Veblen functions  $\varphi_\beta(\gamma) = \varphi(\beta, \gamma)$ :

- $(\varphi_{\beta_1}(\gamma_1) + \varphi_{\beta_2}(\gamma_2) + \cdots + \varphi_{\beta_k}(\gamma_k))[n] = \varphi_{\beta_1}(\gamma_1) + \cdots + \varphi_{\beta_{k-1}}(\gamma_{k-1}) + (\varphi_{\beta_k}(\gamma_k)[n])$ , where  $\varphi_{\beta_1}(\gamma_1) \geq \varphi_{\beta_2}(\gamma_2) \geq \cdots \geq \varphi_{\beta_k}(\gamma_k)$  and  $\gamma_m < \varphi_{\beta_m}(\gamma_m)$  for  $m \in \{1, 2, \dots, k\}$ ,
- $\varphi_0(0) = 1$ ,
- $\varphi_0(\gamma + 1)[n] = \varphi_0(\gamma)n$
- $\varphi_{\beta+1}(0)[0] = 0$  and  $\varphi_{\beta+1}(0)[n+1] = \varphi_\beta(\varphi_{\beta+1}(0)[n])$ ,
- $\varphi_{\beta+1}(\gamma+1)[0] = \varphi_{\beta+1}(\gamma) + 1$  and  $\varphi_{\beta+1}(\gamma+1)[n+1] = \varphi_\beta(\varphi_{\beta+1}(\gamma+1)[n])$ ,
- $\varphi_\beta(\gamma)[n] = \varphi_\beta(\gamma[n])$  for a limit ordinal  $\gamma < \varphi_\beta(\gamma)$ ,
- $\varphi_\beta(0)[n] = \varphi_{\beta[n]}(0)$  for a limit ordinal  $\beta < \varphi_\beta(0)$ ,
- $\varphi_\beta(\gamma + 1)[n] = \varphi_{\beta[n]}(\varphi_\beta(\gamma) + 1)$  for a limit ordinal  $\beta$ .