This function can be defined with fundamental sequences.

The fundamental sequences for the Veblen functions $\varphi_\beta(\gamma) = \varphi(\beta, \gamma)$ are :

1. $(\varphi_{\beta_1}(\gamma_1) + \varphi_{\beta_2}(\gamma_2) + \cdots + \varphi_{\beta_k}(\gamma_k))[n] = \varphi_{\beta_1}(\gamma_1) + \cdots + \varphi_{\beta_{k-1}}(\gamma_{k-1}) + (\varphi_{\beta_k}(\gamma_k)[n])$, where $\varphi_{\beta_1}(\gamma_1) \geq \varphi_{\beta_2}(\gamma_2) \geq \cdots \geq \varphi_{\beta_k}(\gamma_k)$ and $\gamma_m < \varphi_{\beta_m}(\gamma_m)$ for $m \in \{1, 2, ..., k\}$,
2. $\varphi_0(0) = 1$,
3. $\varphi_0(\gamma + 1)[n] = \varphi_0(\gamma)n$
4. $\varphi_{\beta+1}(0)[0] = 0$ and $\varphi_{\beta+1}(0)[n + 1] = \varphi_\beta(\varphi_{\beta+1}(0)[n])$,
5. $\varphi_{\beta+1}(\gamma+1)[0] = \varphi_{\beta+1}(\gamma)+1$ and $\varphi_{\beta+1}(\gamma+1)[n+1] = \varphi_\beta(\varphi_{\beta+1}(\gamma+1)[n])$,
6. $\varphi_\beta(\gamma)[n] = \varphi_\beta(\gamma[n])$ for a limit ordinal $\gamma < \varphi_\beta(\gamma)$,
7. $\varphi_\beta(0)[n] = \varphi_{\beta[n]}(0)$ for a limit ordinal $\beta < \varphi_\beta(0)$,
8. $\varphi_\beta(\gamma + 1)[n] = \varphi_{\beta[n]}(\varphi_\beta(\gamma) + 1)$ for a limit ordinal β.

These fundamental sequences can be reformulated to get a definition of the function φ.

1. This does not concern the definition of the φ function but the definition of addition
2. and
3. are equivalent to $\varphi_0(\gamma) = \omega^\gamma$.
4. $\varphi_{\beta+1}(0) = lim(n \mapsto {\varphi_\beta}^n(0)) = {\varphi_\beta}^\omega(0)$ which is the least fixed point of φ_β.
5. $\varphi_{\beta+1}(\gamma + 1) = lim(n \mapsto {\varphi_\beta}^n(\varphi_{\beta+1}(\gamma) + 1))$, which is the least fixed point of φ_β strictly greater than $\varphi_{\beta+1}(\gamma)$, so $\varphi_{\beta+1}(\gamma)$ is the $1 + \gamma$-th fixed point of φ_β.