

for

least part of

$$\alpha \mapsto 1 + \alpha \quad \omega$$

$$\alpha \mapsto \omega + \alpha \quad \omega^2$$

$$\alpha \mapsto \omega \times \alpha \quad \omega^\omega$$

$$\alpha \mapsto \omega^\alpha \quad \varepsilon_0$$

$$\alpha \mapsto \xi_\alpha = \varphi(1, \alpha) \quad \zeta_\omega = \varphi(2, \omega) \quad \xi_{\zeta_0} = \zeta_0$$

$$\alpha \mapsto \zeta_\alpha = \varphi(1, \alpha) \quad \eta_0 = \varphi(3, 0) \quad \zeta_0 \eta_0 = \eta_0$$

$$\alpha \mapsto \varphi(\alpha, 0) \quad \varphi(1, 0, 0)$$

$$\omega + \omega^2 = \omega(1 + \omega) = \omega^2$$

$$\omega \times \omega^\omega = \omega^{1+\omega} = \omega^\omega$$

$$\omega^{\varepsilon_0} = \varepsilon_0$$

$$\omega^2 + \omega^3 = \omega^2(1 + \omega) = \omega^3$$

$$= \omega^{2+2}$$

$$\omega^2 + \omega^2 \cdot 2$$

$$= \omega^2(1+2) = \omega^2 \cdot 3$$

$$= \omega^2 \cdot \omega = \omega^3$$

$\Phi$  is an ordinal

At least one

$$\alpha \mapsto 1 + \alpha \quad \text{less } \omega + 1, 1 + (\omega + 1), \dots = \omega + 1$$

$$\Phi[1 + \cdot] = [\omega + \cdot]$$

$$\alpha \mapsto \omega + \alpha \quad \text{less } \omega^2 + 1, \omega + \omega^2 + 1, \dots = \omega^2 + 1$$

$$\Phi[\omega + \cdot] = [\omega^2 + \cdot] = \Phi(\Phi[1 + \cdot])$$

$$\alpha \mapsto \omega \times \alpha \quad \text{less } \omega^\omega + 1, \omega(\omega^\omega + 1) = \omega^\omega + \omega$$

$$\omega(\omega^\omega + \omega) = \omega^{\omega+1} + \omega^{\omega+2}$$

$$> \omega^{\omega+2}$$

$$\Phi[\omega \times \cdot] = [\omega^\omega \times (1 + \cdot)]$$

$$\alpha \mapsto \omega^\alpha \quad \text{less } \varepsilon_0 + 1, \omega^{\varepsilon_0+1}, \omega^{\varepsilon_0+2}, \dots = \varepsilon_0 + 1$$

$$\Phi[\omega^\cdot] = [\varepsilon_\cdot]$$

$$\Phi[\varphi(\alpha, \cdot)] = [\varphi(\alpha+1, \cdot)]$$

$$\Phi[\varphi(\cdot, 0)] = [\varphi(1, 0, \cdot)]$$

$$\Phi^3[1 + \cdot] = [\omega^3 + \cdot]$$

$$\Phi^\omega[1 + \cdot] = [\omega^\omega + \cdot]$$

$$= \varepsilon_1$$

$$\omega^\omega + \alpha = \alpha$$