

# A Tutorial Overview of Ordinal Notations

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ocfrecdef.txt

10.1

$$\psi_\nu(\text{Lim}_{\kappa+1}h) = \lim[\psi_\nu(h((\psi_\kappa \circ h)^\bullet(\zeta)))]$$

Concerning the last formula, with ... we also get the previous one for  $\psi_0$  but it is not the same formula as for Buchholz function which we will see later.

buchholz psi functions 10.3.3 7.

$$\psi_\nu(\text{Lim}_{\mu+1}h) = \lim(\xi \mapsto \psi_\nu(h((\psi_\mu \circ h)^\xi(\Omega_\mu))))$$

Maksudov

first system

$$\psi_{\chi(0,\beta+1)}(0) = \chi(0,\beta) \cdot \omega$$

$$\psi_{\chi(0,\beta)}(\gamma+1) = \psi_{\chi(0,\beta)}(\gamma) \cdot \omega$$

$$\psi_{\chi(\beta+1,0)}(0) = \lim[(\chi \circ f)^\bullet(0)]$$

$$\psi_{\chi(\beta+1,\gamma+1)}(0) = \lim[(\chi \circ f)^\bullet(\chi(\beta+1,\gamma)+1)]$$

$$\psi_{\chi(\beta+1,\gamma)}(\delta+1) = \lim[(\chi \circ f)^\bullet(\psi_{\chi(\beta+1,\gamma)}(\delta)+1)]$$

$$\psi_{\chi(\text{Lim}_\mu f,0)}(0) = \text{Lim}_\mu[\chi(f(\bullet),0)] \text{ if } \omega_\mu \geq \omega$$

$$\psi_{\chi(\text{Lim}_\mu f,\gamma+1)}(0) = \text{Lim}_\mu[\chi(f(\bullet),\text{chi}(\text{Lim}_\mu f,\gamma)+1))] \text{ if } M > \omega_\mu \geq \omega$$

$$\psi_{\chi(\text{Lim}_\mu f,\gamma)}(\delta+1) = \text{Lim}_\mu[\chi(f(\bullet),\psi_{\chi(\beta,\gamma)}(\delta)+1)]$$

$$\psi_{\chi(\text{lim}_M f,0)}(0) = \lim[[\chi(f(\bullet),0)]^\bullet(1)]$$

$$\psi_{\chi(\text{lim}_M f,\gamma+1)}(0) = \lim[[\chi(f(\bullet),0)]^\bullet(\chi(\text{Lim}_M f,\gamma)+1)]$$

$$\psi_{\chi(\text{lim}_M f,\gamma)}(\delta+1) = \lim[[\chi(f(\bullet),0)]^\bullet(\psi_{\chi(\text{Lim}_M f,\gamma)}(\delta)+1)]$$

$$\chi(\beta, \text{Lim}_\mu) = \text{Lim}_\mu[\chi(\beta, f(\bullet))] \text{ if } \omega_\mu \geq \omega$$

$$\psi_\pi(\text{Lim}_\mu f) = \text{Lim}_\mu(\psi_\pi \circ f) \text{ if } \pi > \omega_\mu \geq \omega$$

$$\psi_\pi(\text{Lim}_\mu f) = \lim[\psi_\pi(f((\psi_\mu)^\bullet(1)))]$$

second system

$$\psi_{\chi(0)}(0) = 1$$

$$\psi_{\chi(\beta+1)}(0) = \chi(\beta) \cdot \omega$$

$$\psi_{\chi(\text{Lim}_\mu f)}(0) = \text{Lim}_\mu(\chi \circ f) \text{ if } \omega_\kappa < M$$

$$\psi_{\chi(\text{lim}_M f)}(0) = \lim[(\chi \circ f)^\bullet 1]$$

$$\psi_{\chi(\text{lim}_M f)}(\gamma+1) = \lim[(\chi \circ f)^\bullet(\psi_{\chi(\beta)}(\gamma)+1)]$$

$$\psi_{\chi(\beta)}(\gamma+1) = \psi_{\chi(\beta)}(\gamma) \cdot \omega$$

$$\psi_\pi(\text{Lim}_\mu f) = \text{Lim}_\mu(\psi_\pi \circ f) \text{ if } \pi > \omega_\mu \geq \omega$$

$$\psi_\pi(\text{Lim}_\mu f) = \lim[\psi_\pi(f((\psi_\mu \circ f)^\bullet(1)))] \text{ if } \omega_\mu \geq \pi$$