A Tutorial Overview of Ordinal Notations

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June 11, 2018

ocfrecdef.txt
10.1
$\psi_\nu(Lim_{\kappa+1} h) = lim[\psi_\nu(h((\psi_\kappa \circ h)^\bullet(\zeta)))]$
Concerning the last formula, with ... we also get the previous one for ψ_0 but it is not the same formula as for Buchholz function which we will see later.
buchholz psi functions 10.3.3 7.
$\psi_\nu(Lim_{\mu+1} h) = lim(\xi \mapsto \psi_\nu(h((\psi_\mu \circ h)^\xi(\Omega_\mu))))$

Maksudov
first system

3) $\psi_{\chi(0,0)}(0) = 1$
4) $\psi_{\chi(0,\beta+1)}(0) = \chi(0,\beta) \cdot \omega$
5) $\psi_{\chi(0,\beta)}(\gamma+1) = \psi_{\chi(0,\beta)}(\gamma) \cdot \omega$

6) $\psi_{\chi(\beta+1,0)}(0) = [\chi(\beta,\bullet)]^\omega(0)$
7) $\psi_{\chi(\beta+1,\gamma+1)}(0) = [\chi(\beta,\bullet)]^\omega(\chi(\beta+1,\gamma)+1)$
8) $\psi_{\chi(\beta+1,\gamma)}(\delta+1) = [\chi(\beta,\bullet)]^\omega(\psi_{\chi(\beta+1,\gamma)}(\delta)+1)$

9) $\psi_{\chi(Lim_\mu f,0)}(0) = Lim_\mu[\chi(f(\bullet),0)]$ if $\omega_\mu \geq \omega$
10) $\psi_{\chi(Lim_\mu f,\gamma+1)}(0) = Lim_\mu[\chi(f(\bullet), chi(Lim_\mu f,\gamma)+1))]$ if $M > \omega_\mu \geq \omega$
11) $\psi_{\chi(Lim_\mu f,\gamma)}(\delta+1) = Lim_\mu[\chi(f(\bullet), \psi_{\chi(Lim_\mu f,\gamma)}(\delta)+1)]$

12) $\psi_{\chi(lim_M f,0)}(0) = [\chi(f(\bullet),0]^\omega(1)$
13) $\psi_{\chi(lim_M f,\gamma+1)}(0) = [\chi(f(\bullet),0]^\omega(\chi(Lim_M f,\gamma)+1)$
14) $\psi_{\chi(lim_M f,\gamma)}(\delta+1) = [\chi(f(\bullet),0]^\omega(\psi_{\chi(Lim_M f,\gamma)}(\delta)+1)$

18) $\chi(\beta, Lim_\mu f) = Lim_\mu[\chi(\beta, f(\bullet))]$ if $\omega_\mu \geq \omega$
19) $\psi_\pi(Lim_\mu f) = Lim_\mu(\psi_\pi \circ f)$ if $\pi > \omega_\mu \geq \omega$
20) $\psi_\pi(Lim_\mu f) = lim[\psi_\pi(f((\psi_\mu \circ f)^\bullet(1)))]$ if $\omega_\mu \geq \pi$

second system

7) $\psi_{\chi(0)}(0) = 1$
2) $\psi_{\chi(\beta+1)}(0) = \chi(\beta) \cdot \omega$
3) $\psi_{\chi(Lim_\mu f)}(0) = Lim_\mu(\chi \circ f)$ if $\omega_\kappa < M$
4) $\psi_{\chi(lim_M f)}(0) = (\chi \circ f)^\omega(1)$
6) $\psi_{\chi(\mathrm{lim}_M f)}(\gamma+1) = (\chi \circ f)^\omega(\psi_{\chi(\beta)}(\gamma)+1)$
5) $\psi_{\chi(\beta)}(\gamma+1) = \psi_{\chi(\beta)}(\gamma) \cdot \omega$
11) $\psi_\pi(Lim_\mu f) = Lim_\mu(\psi_\pi \circ f)$ if $\pi > \omega_\mu \geq \omega$
12) $\psi_\pi(Lim_\mu f) = lim[\psi_\pi(f((\psi_\mu \circ f)^\bullet(1)))]$ if $\omega_\mu \geq \pi$