A Tutorial Overview of Ordinal Notations

Jacques Bailhache (jacques.bailhache@gmail.com)

January-April 2018

Functions collapsing large cardinals

Hypcos
$\psi_\pi(0) = 1$
$\psi_{\Omega_1}(\alpha) = \omega^\alpha$ if $\alpha < \varepsilon_0$
$\psi_{\Omega_{\nu+1}}(\alpha) = \omega^{\Omega_\nu+\alpha}$ if $1 \le \alpha \le \varepsilon_{\Omega_\nu+1}$ and $\nu > 0$
$\psi_{\Omega_{\nu+1}}(1) = \Omega_\nu \cdot \omega$
$\psi_{\Omega_{\nu+1}}(\beta+1) = \psi_{\Omega_{\nu+1}}(\beta) \cdot \omega$
$\psi_{I_{\nu+1}}(1) = [\Omega_\bullet]^\omega(I_\nu+1) = H[\Omega_\bullet](I_\nu+1)$
$\psi_{I_{\nu+1}} = [\Omega_\bullet]^\omega(\psi_{I_{\nu+1}}(\beta))$

Functions collapsing α-weakly inaccessible cardinals
$I(0,\alpha) = \Omega_{1+\alpha}$
$I(1,\alpha) = I_{1+\alpha}$
$\psi_{I(0,0)}(\alpha) = \omega^\alpha$ if $\alpha < \varepsilon_0$
$\psi_{I(0,\alpha+1)}(\beta) = \omega^{I(0,\alpha)+1+\beta}$
$\psi_{I(\beta+1,0)}(0) = [I(\beta,\bullet)]^\omega 0$
$\psi_{I(\beta+1,\gamma+1)}(0) = [I(\beta,\bullet)]^\omega I(\beta+1,\gamma)$
$\psi_{I(\beta+1,\gamma)}(\delta+1) = [I(\beta,\bullet)]^\omega(\psi_{I(\beta+1,\gamma)}(\delta)+1)$

Jäger
$\varphi(0,\beta) = \omega^\beta$
$\varphi(\beta+1,0) = [\varphi(\beta,\bullet)]^\omega 0$
$\varphi(\beta+1,\gamma+1) = [\varphi(\beta,\bullet)]^\omega(\varphi(\beta+1,\gamma)+1)$
$\psi_{I(0,0)}(0) = [\varphi(\bullet,0)]^\omega 0 = \Gamma_0$
$\psi_{I(0,\beta+1)}(0) = [\varphi(\bullet,0)]^\omega(I(0,\beta)+1)$
$\psi_{I(0,\beta)}(\gamma+1) = [\varphi(\bullet,0)]^\omega(\psi_{I(0,\beta)}(\gamma)+1)$
$\psi_{I(\beta+1,0)}(0) = [I(\beta,\bullet)]^\omega 0$
$\psi_{I(\beta+1,\gamma+1)}(0) = [I(\beta,\bullet)]^\omega(I(\beta+1,\gamma)+1)$
$\psi_{I(\beta+1,\gamma)}(\delta+1) = [I(\beta,\bullet)]^\omega(\psi_{I(\beta+1,\gamma)}(\delta)+1)$