Equations and Factorising

Jordan Baillie

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Problems

No calculators, unless stated otherwise. Motto - we *really really really* don't like fractions!

Warm up - Working with polynomials

1. Expand and simplify the following expressions:
   (a) $(x+1)(x+2)$
   (b) $\left(x + \frac{3}{2}\right)(x+2)$
   (c) $(x+5)(2x^2 - 7x - 3)$
   (d) $(3x+1)(x-1) + 2(x^2 - 5)$
   (e) $(x+1)^3$

Factorising quadratic expressions

1. Factorise the following expressions into products of linear polynomials:
   (a) $x^2 + 2x + 1$
   (b) $x^2 + 4x + 3$
   (c) $x^2 + 6x + 8$
   (d) $x^2 + 6x + 5$
   (e) $x^2 + 7x + 6$
   (f) $\frac{1}{2}x^2 + \frac{9}{2}x + 4$
   (g) $2x^2 + 14x + 12$
   (h) $4x^2 - 20x - 56$
   (i) $2x^2 - 5x - 3$
   (j) $3x^2 + x - 2$

2. Complete the square in $(1)(a-e)$.

3. This question concerns expressions which can be factorised using the 'difference of two squares' technique. Recall that an expression of the form
$$a^2 - b^2$$
can be written in the form
$$(a-b)(a+b).$$
Factorise the following expressions fully:
   (a) $x^2 - 1$

(b) $y^2 - 9$

(c) $z^2 - 25$

(d) $2p^2 - 32$

(e) $4x^2 - y^2$

(f) $16x^2 - 25y^2$

Solving quadratic equations

1. Convince yourself that you would have been able to solve any part of (2) or (3) had I instead set the expression equal to zero. How would you amend your written solution accordingly?

2. What do the solutions of a quadratic expression 'look like' on a graph? Where are they always found?

3. Suppose $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$. Recall the quadratic formula, which states that the solutions are given by:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We use the quadratic formula whenever an expression does not admit a 'nice' factorisation.

Use the quadratic formula to solve the following equations, giving your answer correct to 1 decimal place:

(a) $x^2 + 6x + 2 = 0$

(b) $4x^2 - 7x + 1 = 0$

(c) $4 - 4x - x^2 = 0$

(d) $2x^2 = 3x + 3$

4. The graph of the curve $f(x) = 2x^2 - 4x - 3$ is shown below.
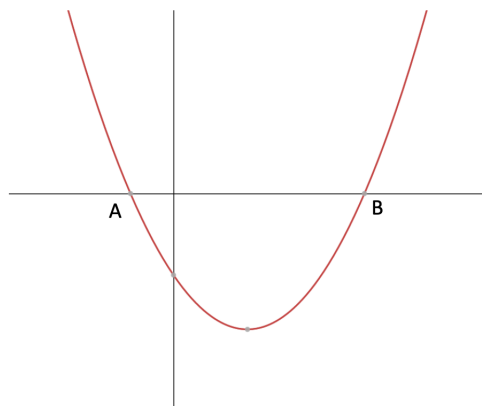
What are the coordinates of A and B? Give the **coordinates** correct to 4 significant figures.

Some harder questions

1. Simplify fully the expression:
$$\frac{x^2 - 3x - 28}{x^2 - 16}.$$

2. Simplify fully the expression:
$$\frac{x^2 - 4x}{x^2 + x - 20}.$$

3. (a) Find the gradient of the line between the points $(4m^2, 2m)$ and (n^2, n).

   (b) Using your answer to part (a), or otherwise, find the gradient of the line between the points $(4, 2)$ and $(1, 1)$.