Equations and Factorising

Jordan Baillie

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Session Review

When factorising, your general plan of attack should be:

1. Can you pull anything out from the get-go? *Remember to keep this at the front in your final answer.*
2. Is it a difference of two squares? *Remember that, for example,* $4x^2 = (2x)^2$.
3. Can you use the 'first times last' technique? *Remember after we split, both brackets should be the same*

When completing the square, your general plan of attack should be:

1. Half the coefficient of x and put it inside the bracket.
2. Square this number. What do we need to add/subtract to get to the number we want? Stick this outside the bracket.

When solving quadratic equations, your general plan of attack should be:

1. Remember that the solutions of quadratic equations concur with roots of the corresponding parabola.
2. Make sure you have it in the form $ax^2 + bx + c = 0$.
3. Can you factorise it? Use the steps above to try. Set both brackets equal to 0 to solve.
4. Struggling to factorise? This is likely to be in a calculator paper - use the quadratic formula.

'Harder questions' look tedious but you can definitely do them! They usually rely on you applying the techniques above multiple times. Just take these questions slowly and look out for hints. For example in the 'harder question 2' below, you know there will be a factor of $(x - 4)$ on the bottom which will cancel with the top.

Warm up - Working with polynomials

1. Expand and simplify the following expressions:
   (a) $(x + 1)(x + 2)$
   (b) $\left(x + \frac{3}{2}\right)(x + 2)$
   (c) $(x + 5)(2x^2 - 7x - 3)$
   (d) $(3x + 1)(x - 1) + 2(x^2 - 5)$
   (e) $(x + 1)^3$

Factorising quadratic expressions

1. Factorise the following expressions into products of linear polynomials:
   (a) $x^2 + 2x + 1$
   (b) $x^2 + 4x + 3$
   (c) $x^2 + 6x + 8$
   (d) $x^2 + 6x + 5$
   (e) $x^2 + 7x + 6$
   (f) $\frac{1}{2}x^2 + \frac{9}{2}x + 4$
   (g) $2x^2 + 14x + 12$
   (h) $4x^2 - 20x - 56$
   (i) $2x^2 - 5x - 3$
   (j) $3x^2 + x - 2$

2. Complete the square in $(1)(a - e)$.

3. This question concerns expressions which can be factorised using the 'difference of two squares' technique. Recall that an expression of the form
$$a^2 - b^2$$
can be written in the form
$$(a - b)(a + b).$$
Factorise the following expressions fully:
   (a) $x^2 - 1$
   (b) $y^2 - 9$
   (c) $z^2 - 25$
   (d) $2p^2 - 32$
   (e) $4x^2 - y^2$
   (f) $16x^2 - 25y^2$

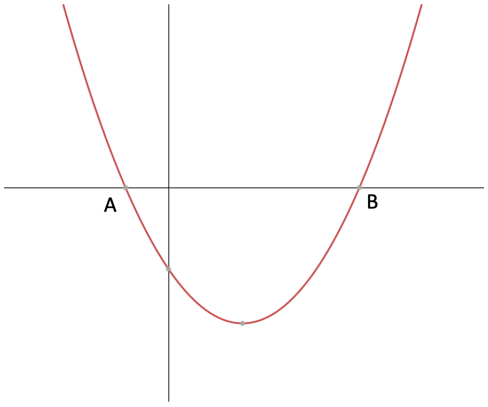
Solving quadratic equations

1. Convince yourself that you would have been able to solve any part of (2) or (3) had I instead set the expression equal to zero. How would you amend your written solution accordingly?

2. What do the solutions of a quadratic expression 'look like' on a graph? Where are they always found?

3. Suppose $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$. Recall the quadratic formula, which states that the solutions are given by:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
We use the quadratic formula whenever an expression does not admit a 'nice' factorisation.
Use the quadratic formula to solve the following equations, giving your answer correct to 1 decimal place:
   (a) $x^2 + 6x + 2 = 0$
   (b) $4x^2 - 7x + 1 = 0$
   (c) $4 - 4x - x^2 = 0$
   (d) $2x^2 = 3x + 3$

4. The graph of the curve $f(x) = 2x^2 - 4x - 3$ is shown below.



What are the coordinates of A and B? Give the **coordinates** correct to 4 significant figures.

Some harder questions

1. Simplify fully the expression:
$$\frac{x^2 - 3x - 28}{x^2 - 16}.$$

2. Simplify fully the expression:
$$\frac{x^2 - 4x}{x^2 + x - 20}.$$

3. (a) Find the gradient of the line between the points $(4m^2, 2m)$ and (n^2, n).
   (b) Using your answer to part (a), or otherwise, find the gradient of the line between the points $(4, 2)$ and $(1, 1)$.

4. (a) Factorise $3j^2 - 3k^2$.
   (b) Using your result, evaluate this expression whenever $j = 2.3$ and $k = 0.7$.

5. Solve the equation
$$2x^2 + 7x - 15 = 0.$$

6. Solve the equation
$$4x(x + 1) = 15.$$

7. A rock is thrown off of a cliff at time $t = 0$ and follows a parabolic shape. Its height h at a given time $t \geq 0$ is given by
$$h = -t^2 + 4t + 12.$$
   (a) At what time will the rock hit the ground?
   (b) What was the maximum height the rock reached?