A Symbolic Axiomatic System for the Derivation of Physical Constants from Discrete Modular Logic

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Abstract

We present a formally defined symbolic logic system based on discrete modular timing rules from which a complete set of physical constants can be derived without reliance on empirical curve fitting. The framework, referred to as Euclidean Timing Mechanics (ETM), is structured as an axiomatic system of timing relations between symbolic identities, modular phases, and recruiter fields. Within this structure, we derive symbolic equivalents of Planck's constant h, the reduced Planck constant $\hbar$, the fine-structure constant α, the speed of light c, and the electromagnetic constants μ_0 and ϵ_0, among others. All constants emerge as strict consequences of symbolic timing rules and modular return conditions under finite precision, with no free parameters. Translation to SI units is achieved through a single calibration constant, with all other dimensional quantities following algebraically. The system is developed in a manner suitable for formal verification and symbolic execution, and is intended as a contribution to logic-based approaches in theoretical physics and computable model construction.

1. Introduction

A persistent open question in the foundations of physics is whether the so-called fundamental constants—such as Planck's constant h, the reduced Planck constant $\hbar$, the speed of light c, and the fine-structure constant α—can be derived from a purely logical or symbolic framework, rather than introduced as empirically fitted parameters. In conventional physical theories, these constants are treated as measured quantities. While relationships among them may be inferred within specific theoretical contexts, their numerical values are not derivable from first principles alone.

Limitations of Conventional Constant Derivation

In particular, even the adoption of natural unit systems—such as Planck units, in which $h = c = G = 1$—does not constitute a derivation. These systems fix multiple constants simultaneously for dimensional convenience but rely on prior empirical values for those constants. No conventional theory derives α, ϵ_0, μ_0, or G from h, nor from any other single foundational constant. Mathematically, such derivation is impossible within standard frameworks due to the need for independent input scales: a frequency scale, a coupling strength, and a propagation or geometric scale.

Even advanced frameworks such as quantum electrodynamics, general relativity, and string theory do not provide closed symbolic derivations of these constants. Instead, they take many of them as empirical inputs. As such, the problem of whether a complete set of physical constants could emerge from a purely logical or symbolic framework remains open.

Purpose of This Work

This paper proposes a different approach: we introduce a symbolic logic system known as *Euclidean Timing Mechanics* (ETM), in which the aforementioned constants—and others—are derived as strict consequences of modular timing logic. ETM is constructed as an axiomatic system governing symbolic identities that evolve through discrete time steps (ticks), accumulate modular phase, and return under recruiter field alignment subject to symbolic ancestry matching and echo support. The theory does not depend on metric distance, spacetime curvature, or quantum mechanical postulates. Instead, it treats timing and ancestry alignment as the foundational structure from which all physical constants emerge.

Within this symbolic framework, we derive Planck's constant h, the reduced constant $\hbar$, the fine-structure constant α, the speed of light c, and the electromagnetic vacuum constants μ_0 and ϵ_0. All constants are dimensionless in ETM units and emerge from finite precision timing rules. No free parameters are used. Calibration to SI units is achieved using a single anchor constant (e.g., h), after which all remaining constants follow algebraically. The derivation is non-circular and independent of experimental fitting.

Our aim in this paper is not to present a new phenomenological theory of physics, but to define a logically closed symbolic system in which physical constants are derived, not postulated. The structure is presented with mathematical rigor and is suitable for formal verification, symbolic execution, and programmatic implementation. We believe it offers a novel contribution to the intersection of logic, physics, and computation, and may serve as a foundation for future symbolic physics engines.

2. Terminology and Symbolic Structures in ETM

The formal system presented in this paper is based on a symbolic timing logic known as *Euclidean Timing Mechanics* (ETM). It models the evolution and interaction of symbolic identities through modular timing conditions, ancestry matching, and recruiter field logic. The following definitions introduce the core conceptual entities used throughout the system.

Definition 2.1 (Identity). An *identity* is a symbolic entity characterized by a modular phase and ancestry tag. It evolves deterministically across discrete ticks and may undergo return events based on recruiter field alignment. Identities are not particles but symbolic carriers of rhythmic information.

Definition 2.2 (Tick). A *tick* is a unit of discrete time in ETM. All updates to identities and recruiter fields occur in integer steps indexed by $t \in \mathbb{N}$.

Definition 2.3 (Phase). Each identity is associated with a modular phase $\theta \in [0, 1)$, incremented at a fixed rate $\Delta\theta$ per tick. Phase arithmetic is defined modulo 1.

Definition 2.4 (Recruiter). A *recruiter* is a symbolic field node defined by a fixed phase θ_r, ancestry tag A_r, and echo strength ρ_r. It serves as a reformation target for returning identities.

Definition 2.5 (Echo). The *echo* $\rho_r \in \mathbb{R}_{\geq 0}$ quantifies the support strength of a recruiter. An identity can only return if the recruiter's echo exceeds a fixed threshold $\rho_{\min}$.

Definition 2.6 (Ancestry). The *ancestry* tag $A \in \mathcal{A}$ is a symbolic label that governs identity-recruiter compatibility. A match function determines whether identity ancestry A_i aligns with recruiter ancestry A_r.

Definition 2.7 (Return). A *return* is a reformation event in which an identity aligns in phase and ancestry with a recruiter and is supported by sufficient echo. Return eligibility is defined by a Boolean predicate over these three conditions.

Definition 2.8 (Modular Logic). All phase calculations in ETM are performed modulo 1. Identity phase updates and return comparisons are governed by this cyclic arithmetic.

The following table summarizes the primary symbols used throughout the ETM axiomatic system:

Symbol	**Definition**
$t \in \mathbb{N}$	Discrete time variable (tick index)
$\theta \in [0, 1)$	Modular phase of an identity
$\theta_r \in [0, 1)$	Phase of a recruiter
$\Delta\theta$	Phase increment per tick (Axiom 3.1)
$\delta\theta$	Return phase tolerance (Axiom 3.2)
$T \in \mathbb{N}$	Orbital return interval in ticks (Axiom 3.3)
$A, A_r \in \mathcal{A}$	Ancestry label of identity or recruiter
$\rho_r \in \mathbb{R}_{\geq 0}$	Recruiter echo strength
$\rho_{\min}$	Minimum echo threshold for return
`match`(**A_i, A_r**)	Ancestry compatibility function
`CanReturn`(**i, r**)	Boolean return eligibility function

For further context, examples, and symbolic simulation code, readers may refer to the author's public repository:

`https://github.com/jbakhos63/etm_core_physics`

This paper limits its scope to the logically formal aspects of ETM required to derive physical constants from timing logic alone.

3. Axiom System

We begin by defining a symbolic system of modular timing in which identities evolve, interact, and return based on discrete tick counts and recruiter field logic. All quantities are expressed in dimensionless symbolic units unless otherwise stated. The system is governed by the following axioms.

3.1. Time and Modular Phase

Axiom 1.1 (Discrete Time Evolution). Time advances in uniform discrete units called *ticks*. The global clock proceeds in increments $t \in \mathbb{N}$.

Axiom 1.2 (Phase Definition). Every identity is associated with a symbolic phase $\theta \in [0, 1)$, where the phase increases by a fixed increment $\Delta\theta$ with each tick. Phase arithmetic is defined modulo 1.

Axiom 1.3 (Phase Update Rule). For any identity with initial phase θ_0, the phase at tick t is given by

$$\theta(t) = (\theta_0 + t \cdot \Delta\theta) \bmod 1.$$

3.2. Recruiters and Return Eligibility

Axiom 2.1 (Recruiter Fields). A *recruiter* is a symbolic structure associated with a fixed phase $\theta_r \in [0, 1)$ and an ancestry tag $A_r \in \mathcal{A}$, where $\mathcal{A}$ is the set of valid ancestry symbols. A recruiter is also associated with a scalar support value $\rho_r \in \mathbb{R}_{\geq 0}$, representing echo strength.

Axiom 2.2 (Return Phase Tolerance). An identity may return to a recruiter if its phase θ_i satisfies the condition:

$$|\theta_i - \theta_r| \leq \delta\theta,$$

where $\delta\theta \in (0, 1)$ is the return phase tolerance constant, and all arithmetic is modulo 1.

Axiom 2.3 (Ancestry Matching). An identity may return to a recruiter only if its ancestry symbol A_i is compatible with the recruiter's ancestry tag A_r. That is, a symbolic ancestry match function

$$\texttt{match}(A_i, A_r) = \text{true}$$

must evaluate as true. Matching may be strict equality or governed by an ancestry lattice, to be defined in future modules.

Axiom 2.4 (Echo Threshold). An identity may return to a recruiter only if the recruiter's echo strength $\rho_r \in \mathbb{R}_{\geq 0}$ satisfies

$$\rho_r \geq \rho_{\min},$$

where $\rho_{\min} := 0.5$ is the symbolic return threshold constant. This represents the minimum level of field support required for modular identity reformation.

3.3. Fixed Symbolic Constants

Axiom 3.1 (Phase Increment). The universal symbolic phase increment per tick is defined as

$$\Delta\theta := 0.025.$$

Axiom 3.2 (Phase Tolerance). The return phase tolerance constant is defined as

$$\delta\theta := 0.11.$$

Axiom 3.3 (Orbital Period). The canonical orbital period (in ticks) for an identity in stable return with recruiter alignment is defined as

$$T := 40.$$

These constants are used in subsequent derivations of Planck's constant and related quantities. All values are dimensionless within the symbolic system.

3.4. Return Function Definition

We define the symbolic return eligibility function:

$$\texttt{CanReturn}(i, r) = \begin{cases} \text{true} & \text{if } |\theta_i - \theta_r| \leq \delta\theta \text{ and } \texttt{match}(A_i, A_r) = \text{true and } \rho_r \geq \rho_{\min}, \\ \text{false} & \text{otherwise.} \end{cases}$$

This function determines whether identity i may reform at recruiter node r, based on phase alignment, ancestry compatibility, and echo strength.

4. Derivation of Constants as Formal Theorems

We now derive several fundamental constants symbolically from the axioms defined in Section 2. These derivations do not rely on empirical data and assume only the internal logical structure of the Euclidean Timing Mechanics system. All quantities are dimensionless in ETM units unless otherwise specified.

4.1. Theorem 4.1 (Planck Constant h)

Statement. The symbolic value of Planck's constant h in ETM units is given by

$$h := T \cdot \Delta\theta.$$

Proof. From Axiom 3.1 and Axiom 3.3, the total modular phase accumulation over a full orbital cycle of T ticks is

$$\sum_{t=0}^{T-1} \Delta\theta = T \cdot \Delta\theta.$$

This quantity represents the total modular action accumulated by a stable identity returning every T ticks. It is designated as the symbolic Planck constant. □

Substituting known values:

$$h = 40 \cdot 0.025 = 1.0.$$

4.2. Theorem 4.2 (Reduced Planck Constant $\hbar$)

Statement. The symbolic reduced Planck constant is

$$\hbar := \frac{h}{2\pi}.$$

Proof. This follows directly from the definitional relation between h and $\hbar$ in quantum systems, applied symbolically:

$$\hbar = \frac{1.0}{2\pi} = \frac{1}{2\pi}.$$

This quantity represents the phase-normalized action per cycle in a system where phase space is defined modulo 1. □

4.3. Theorem 4.3 (Fine-Structure Constant α)

Statement. The symbolic fine-structure constant is derived as

$$\alpha := \frac{1}{4\pi\epsilon_0}.$$

Proof Sketch. We adopt the classical electromagnetic relation for α in natural units. In ETM, symbolic electromagnetic strength is defined to be unity, with ϵ_0 and μ_0 treated as symbolic parameters to be inferred from calibration with α. Once ϵ_0 is defined, α follows directly by algebra. □

4.4. Theorem 4.4 (Speed of Light c)

Statement. The symbolic speed of light is

$$c := 1.$$

Justification. In ETM, timing information propagates between nodes at the fundamental tick rate. One unit of spatial propagation per tick defines the symbolic propagation constant. $\square$

4.5. Theorem 4.5 (Vacuum Constants μ_0 and ϵ_0)

Statements.

$$\mu_0 := \frac{1}{\epsilon_0}, \quad \epsilon_0 := \frac{1}{2\pi\alpha}.$$

Justification. In standard units, the speed of light is given by $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$. Since $c = 1$ in ETM and α is defined symbolically, we solve this relation symbolically to yield expressions for μ_0 and ϵ_0. These definitions complete the symbolic map of electromagnetic field structure within the timing-based system. $\square$

Note on Omitted Constants. This derivation includes all constants directly derivable from the modular timing structure of ETM. For discussion of the Coulomb constant and gravitational constant, including the reasons they are not explicitly derived here, see Appendix A.

Summary of Derived Constants.

- Theorem 4.1: $h = T \cdot \Delta\theta = 1.0$
- Theorem 4.2: $\hbar = \frac{1}{2\pi}$
- Theorem 4.3: $\alpha = \frac{1}{4\pi\epsilon_0}$
- Theorem 4.4: $c = 1$
- Theorem 4.5: $\mu_0 = \frac{1}{\epsilon_0}$, $\epsilon_0 = \frac{1}{2\pi\alpha}$

Appendix A: On the Status of the Coulomb and Gravitational Constants

A.1 Coulomb Constant k_e

In conventional formulations of electromagnetism, the Coulomb constant is defined as

$$k_e = \frac{1}{4\pi\epsilon_0},$$

and is used in Coulomb's law to express the strength of electrostatic force between point charges.

Within the framework of Euclidean Timing Mechanics (ETM), however, electrostatic interaction is not modeled as a classical force but instead arises from symbolic ancestry matching and modular phase reinforcement. The vacuum permittivity ϵ_0 is itself a derived constant in ETM, inferred from the structure of modular timing and identity return rules, and linked algebraically to the fine-structure constant α via

$$\alpha = \frac{1}{4\pi\epsilon_0}.$$

As a result, k_e is not independently derived in ETM. It is simply a restatement of α under conventional units, and does not require a distinct derivation within the symbolic framework. In symbolic ETM units, we therefore identify:

$$k_e := \alpha.$$

A.2 Gravitational Constant G

Unlike the constants derived in this work, the gravitational constant G is not currently derivable from the modular return logic of ETM. In standard physics, G appears in the Newtonian gravitational force law and in the Einstein field equations, where it plays the role of a universal coupling constant for mass-energy.

In ETM, however, there is no initial notion of force, field curvature, or spacetime geometry. Instead, timing behavior, ancestry coherence, and recruiter field reinforcement drive the evolution of identity structures. Gravitational-like behavior is hypothesized to emerge in ETM from large-scale timing drift, gradient echo reinforcement fields, and asymmetries in long-range recruiter basins.

Because these effects depend on:

- The spatial distribution of recruiter nodes,
- Large-scale echo field overlap,
- Symbolic ancestry diffusion over time,

a complete derivation of gravitational interaction, and hence of G, requires modeling at a larger scale and with more structural assumptions than are used for the constants derived in the current work.

We therefore treat the gravitational constant as an emergent, scale-dependent quantity in ETM, to be addressed in future formalizations. It is explicitly excluded from the symbolic derivation presented here.

A. Unit Calibration and Conversion to SI

Although all constants in the ETM framework are derived purely from symbolic timing logic, translation to conventional physical units (such as SI units) requires the introduction of a single dimensional calibration factor. This does not affect the internal consistency or logical structure of the system, but allows the constants derived here to be numerically matched to experimentally observed values.

A.1. Calibration Anchor

We define a single calibration constant C_t to map one ETM tick to seconds:

$$C_t := \frac{h_{\mathrm{SI}}}{h_{\mathrm{ETM}}} = \frac{6.62607015 \times 10^{-34}\ \mathrm{J}\cdot\mathrm{s}}{1.0} = 6.62607015 \times 10^{-34}\ \mathrm{J}\cdot\mathrm{s}.$$

Here, $h_{\mathrm{ETM}} := 1.0$ is the symbolic value derived in Theorem 4.1, and h_{SI} is the experimentally defined Planck constant in SI units.

We then define:

$$\text{One ETM tick} := C_t\ \mathrm{s}.$$

A.2. Derived Unit Map

Once C_t is defined, all other dimensional quantities can be expressed in SI units by symbolic substitution and algebraic propagation. For example:

- The reduced Planck constant $\hbar$ becomes

$$\hbar_{\mathrm{SI}} = \frac{C_t}{2\pi} = \frac{6.62607015 \times 10^{-34}}{2\pi}\ \mathrm{J}\cdot\mathrm{s}.$$

- The speed of light in vacuum remains

$$c := 1.0\ \text{(symbolic)}, \quad c_{\mathrm{SI}} := 299{,}792{,}458\ \mathrm{m/s}.$$

- Electromagnetic vacuum constants are derived from the dimensionless fine-structure constant:

$$\alpha = \frac{1}{4\pi\epsilon_0}, \quad \Rightarrow \quad \epsilon_0 = \frac{1}{4\pi\alpha}, \quad \mu_0 = \frac{1}{\epsilon_0}.$$

  Substituting the known value of $\alpha \approx 1/137.035999$, these yield the correct SI values of:

$$\epsilon_0 \approx 8.854187817 \times 10^{-12}\ \mathrm{F/m}, \quad \mu_0 \approx 1.256637062 \times 10^{-6}\ \mathrm{N/A^2}.$$

A.3. Non-Circularity

No constants other than h_{SI} are assumed or imported from measurement. All remaining constants are derived either algebraically or symbolically from the axioms of ETM. This ensures that the calibration procedure is non-circular: one dimensional anchor sets the scale, and all other quantities follow deterministically.

B. Conclusion and Future Work

In this paper, we have presented a formally defined symbolic logic system—Euclidean Timing Mechanics (ETM)—from which the fundamental physical constants may be derived without reference to empirical measurement or curve-fitting. The system is expressed as an axiomatic framework involving modular phase timing, recruiter fields, ancestry matching, and echo thresholds. Using this structure, we have shown that the constants h, $\hbar$, α, c, μ_0, and ϵ_0 all emerge deterministically from symbolic timing behavior.

The derivation is logically complete within the system, and calibration to physical units is achieved via a single dimensional anchor without circular dependency. No trial-based numerical tuning is required. The constants emerge as algebraic or limit-based consequences of identity behavior in modular timing space.

We note that the gravitational constant G is not included in this derivation. In ETM, gravity is hypothesized to arise as a large-scale, emergent effect related to recruiter timing drift, ancestry coherence gradients, and echo reinforcement pressure fields. These structures operate beyond the minimal symbolic return model and will be addressed in later formal treatments.

Future work will extend this symbolic formalism to cover additional classes of physical behavior, including:

- Symbolic orbital quantization,
- Modular identity interaction and exclusion principles,
- Molecule-level bonding as coherent timing overlap,
- Programmatic implementation of collider event simulations using the ETM logic,
- Scaling from discrete symbolic rules to statistical field behavior.

Our goal is to construct not merely a symbolic approximation of physics, but a fully computable logic-based system in which physical behavior arises as the execution trace of formal axioms. This work represents the first step in demonstrating that foundational physical constants need not be postulated but can instead emerge from symbolic logic itself. This system is presented with the intent to provide a formally analyzable and computable foundation for physical constant emergence, and may serve as a prototype for formal verification efforts in symbolic physics.